



**DCGI**

**KATEDRA POČÍTAČOVÉ GRAFIKY A INTERAKCE**

# Ray Tracing

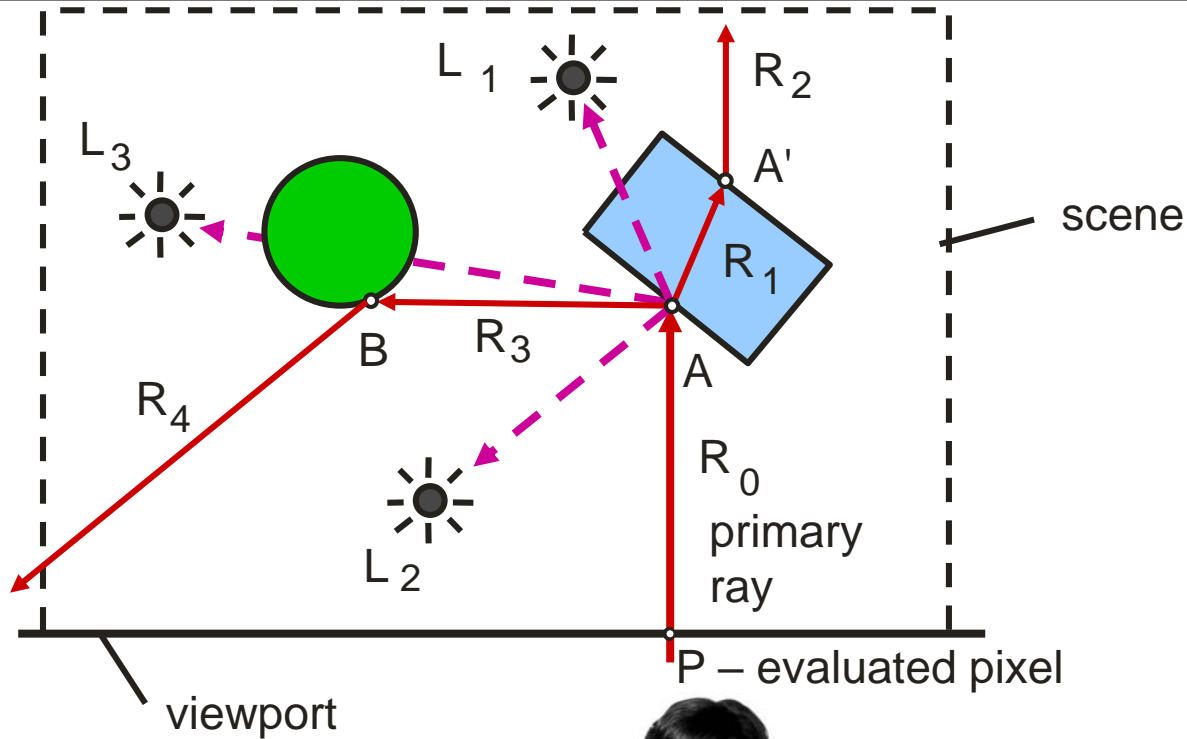
Jiří Bittner

# Outline

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- Whitted Ray Tracing MPG 15.9
- Path Tracing
- Ray Tracing Acceleration MPG 15.9.3

# Ray Tracing Principle



[Whitted 1980]

# (Backward) Ray Tracing - Algorithm

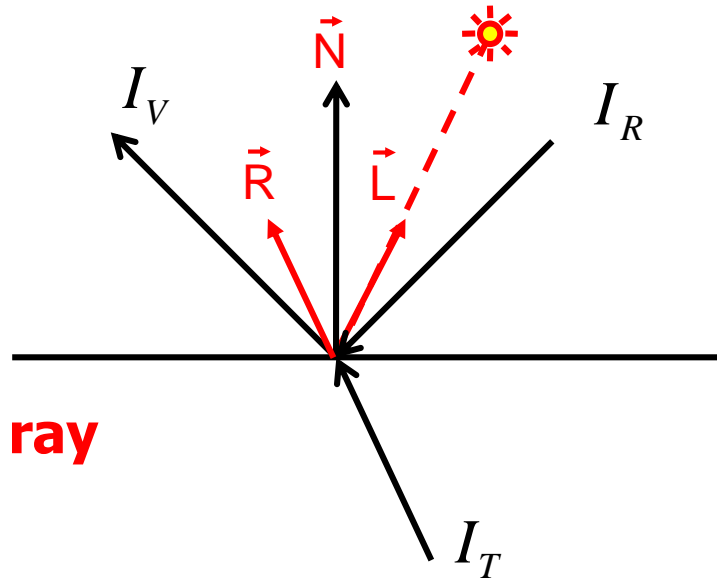
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**TraceRay** (Ray  $R$ , recursion depth  $H$ )

1. Find intersection  $P$  of  $R$  with the nearest object
2. If no intersection // ray leaves the scene  
assign  $R$  background color and terminate
3. For all light sources:  
cast a **shadow ray** from  $P$ .  
if the shadow ray reaches the light source mark it visible
4. Evaluate light contribution at  $P$  from all **visible light sources**
5. If  $H < \text{max depth}$  :
  - (a) **TraceRay** (Reflected ray  $RR$ ,  $H + 1$ )
  - (b) **TraceRay** (Refracted ray  $RT$ ,  $H + 1$ )
6. Assign to  $R$  the resulting color using sum of illumination from light sources and  $RR$  and  $RT$

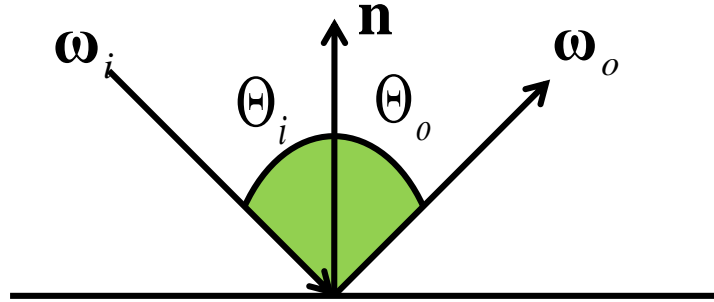
# Extending Illumination Model

- $I_V = I_a + I_r + I_t + \sum I_d + I_s$
- $I_a$  ambient component
- $I_d$  diffuse component
- $I_s$  specular component
- $I_r = k_s I_R$
- $I_t = k_t I_T$
- $k_s$  ... specular coef.
- $k_t$  ... transmittance coef.



# Direction of Reflected Ray

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$$\Theta_o = \Theta_i$$

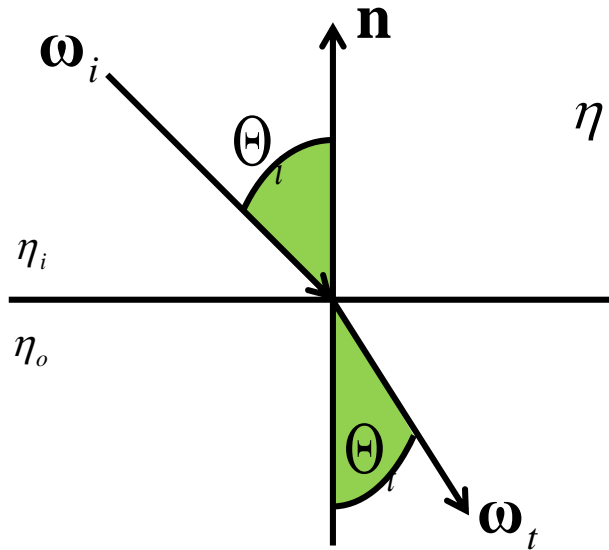
$$\boldsymbol{\omega}_o = \boldsymbol{\omega}_i + 2 \cos \Theta_i \mathbf{n}$$

$$\boldsymbol{\omega}_o = \boldsymbol{\omega}_i - 2(\boldsymbol{\omega}_i \circ \mathbf{n})\mathbf{n}$$

# Refracted Ray

$$\eta_i \sin \theta_i = \eta_o \sin \theta_t \quad \text{Snell's law}$$

$$\omega_t = \eta_{io} \omega_i - \left[ \sqrt{1 - \eta_{io}^2 (1 - \cos^2 \theta_i)} + \eta_{io} \cos \theta_i \right] \mathbf{n} \quad \eta_{io} = \frac{\eta_i}{\eta_o}$$



$\eta$  index of refraction (air 1.000293, water 1.33, glass 1.6, diamond 2.4)

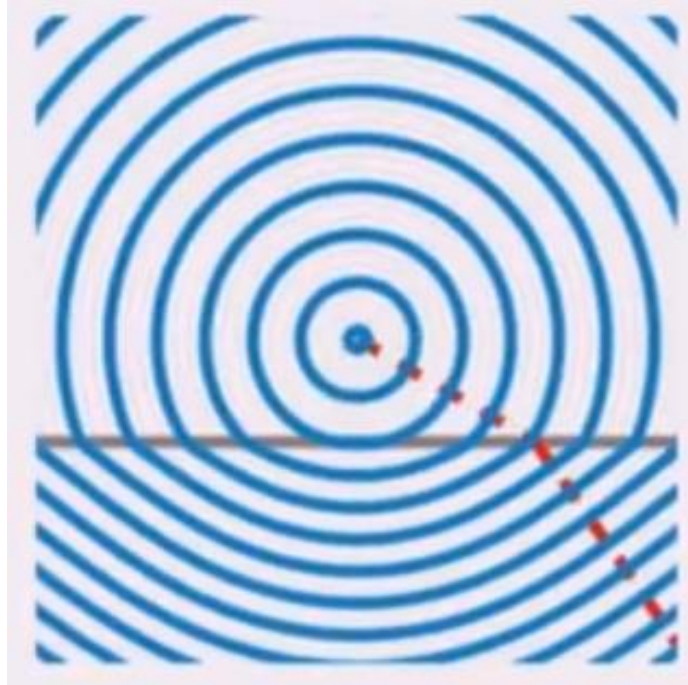
$$\sin^2 \theta_t = \eta_{io}^2 (1 - \cos^2 \theta_i) > 1$$

Total internal reflection – no refraction



# Refraction - Example

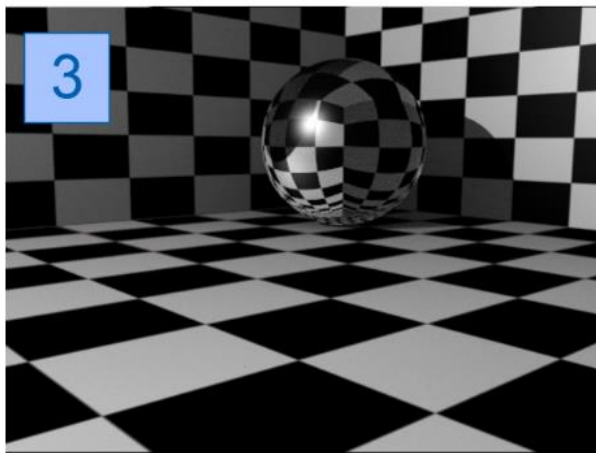
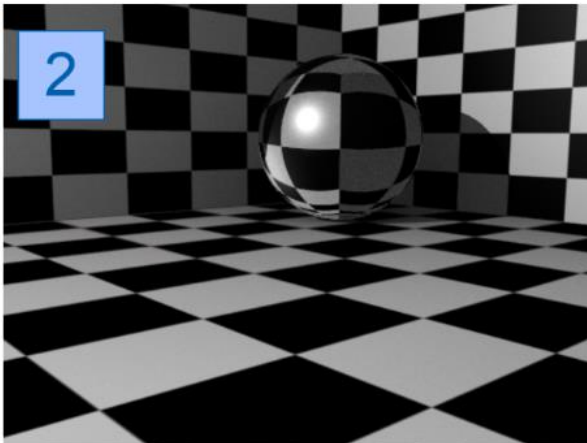
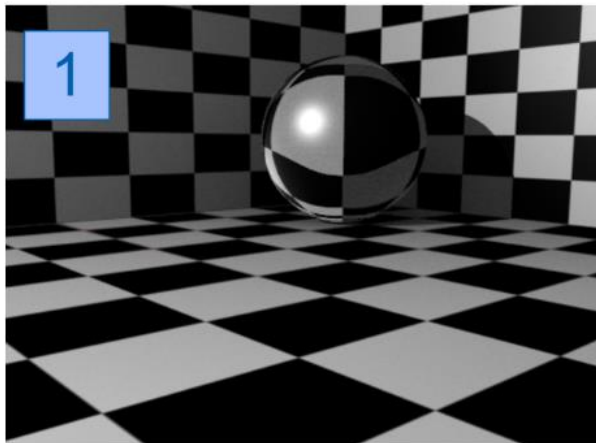
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# Index of Refraction - examples

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1. IOR = 1.333 (water)
2. IOR = 1.517 (glass)
3. IOR = 2.417 (diamond)

Examples courtesy of L. Cmolik

# Fresnel Equations

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- Reflectivity (odrazivost) / transmissivity (propustnost) varies with incident angle! (also polarization matters)

reflected  $F(\theta_i) = \left| \frac{\eta_{io} \cos \theta_i - \sqrt{1 - (\eta_{io} \sin \theta_i)^2}}{\eta_{io} \cos \theta_i + \sqrt{1 - (\eta_{io} \sin \theta_i)^2}} \right|^2$

transmitted  $T(\theta_i) = 1 - F(\theta_i)$

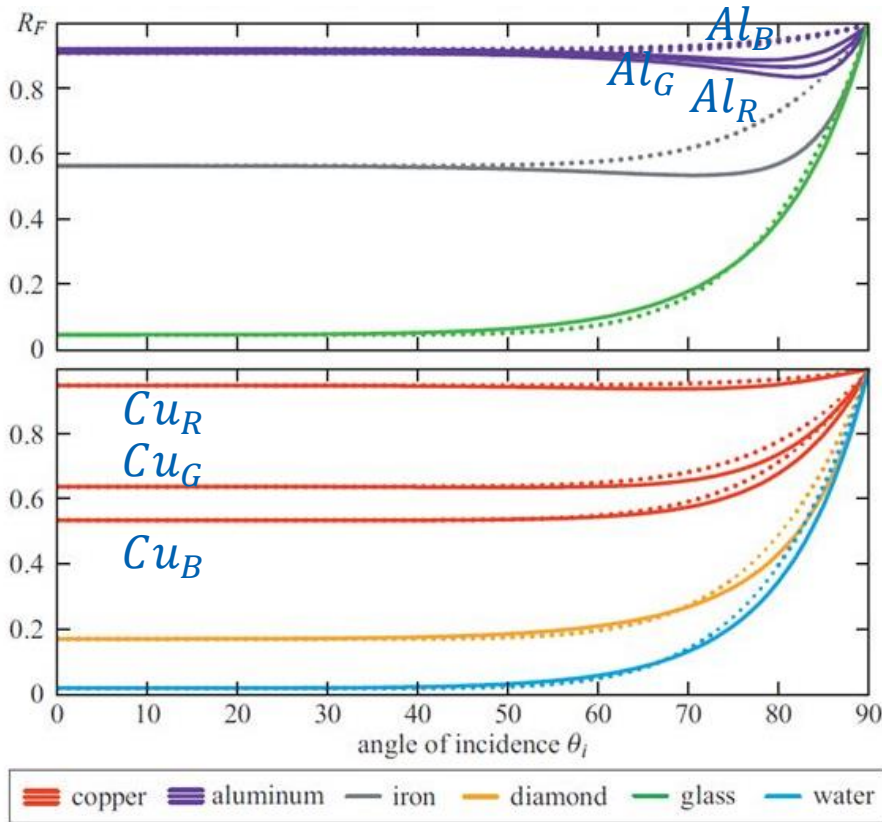
- Schlick approximation [1994]

$$F(\theta_i) = F_0 + (1 - F_0)(1 - \cos \theta_i)^5 \quad F_0 = \left( \frac{\eta_{io} - 1}{\eta_{io} + 1} \right)^2$$

# Examples

solid = Fresnel  
dotted = Schlick

$$R(\theta_i)$$



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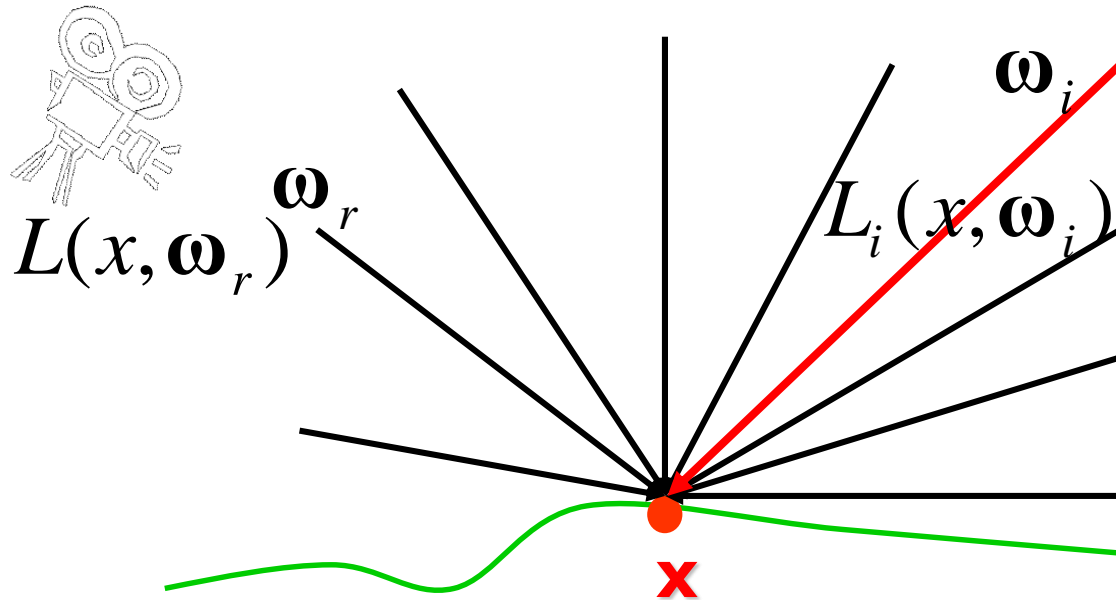
# Path Tracing

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- Correct evaluation of indirect illumination
- Monte-Carlo integration → Unbiased image estimate
- Generating random reflection/refraction directions
- Appropriate sample distribution (importance sampling)
- One secondary ray per hit → single light path
- Many paths per pixel (samples per pixel)

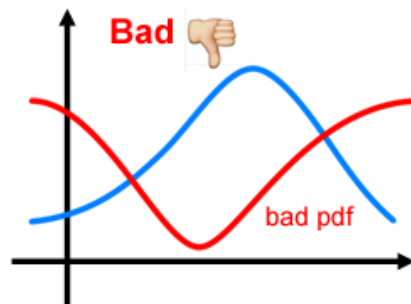
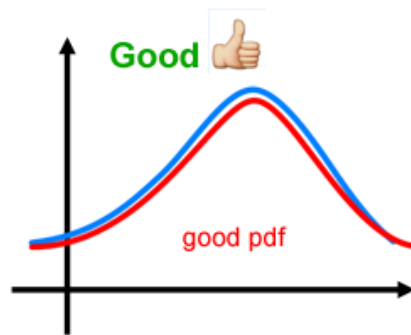
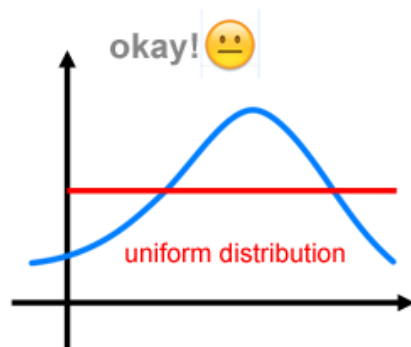
# Path Tracing – Rendering Equation

$$L_r(x, \omega_r) = L_e(x, \omega_r) + \int_{\Omega} f_r(x, \omega_i, \omega_r) L_i(x, \omega_i) \cos \theta_i d\omega_i$$



# Monte Carlo Integration

$$\int h(x) dx \approx \frac{1}{N} \sum_{k=1}^N \frac{h(x_i)}{pdf(x_i)}$$



# Monte Carlo Integration

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$$L_r(x, \omega_r) = L_e(x, \omega_r) + \int_{\Omega} f_r(x, \omega_i, \omega_r) L_i(x, \omega_i) \cos \theta_i d\omega_i \approx$$

$$L_e(x, \omega_r) + \frac{1}{N} \sum_{k=1}^N \frac{f_r(x, \omega_{i_k}, \omega_r) L_i(x, \omega_{i_k}) \cos \theta_{i_k}}{\text{pdf}(\omega_{i_k})}$$



# MC Integration – Diffuse Surface, Uniform Distribution

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$$L_r(\mathbf{x}, \boldsymbol{\omega}_r) \approx \frac{1}{N} \sum_{k=1}^N \frac{f_r(\mathbf{x}, \boldsymbol{\omega}_{i_k}, \boldsymbol{\omega}_r) L_i(\mathbf{x}, \boldsymbol{\omega}_{i_k}) \cos \theta_i}{pdf(\boldsymbol{\omega}_{i_k})}$$

$$L_r(\mathbf{x}, \boldsymbol{\omega}_r) \approx \frac{1}{N} \sum_{k=1}^N \frac{\frac{k_d}{\pi} L_i(\mathbf{x}, \boldsymbol{\omega}_{i_k}) \cos \theta_i}{\frac{1}{2\pi}}$$

# MC Integration – Difuse Surface, Cosine Distribution

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$$L_r(\mathbf{x}, \boldsymbol{\omega}_r) \approx \frac{1}{N} \sum_{k=1}^N \frac{f_r(\mathbf{x}, \boldsymbol{\omega}_{i_k}, \boldsymbol{\omega}_r) L_i(\mathbf{x}, \boldsymbol{\omega}_{i_k}) \cos \theta_i}{pdf(\boldsymbol{\omega}_{i_k})}$$

$$L_r(\mathbf{x}, \boldsymbol{\omega}_r) \approx \frac{1}{N} \sum_{k=1}^N \frac{\frac{k_d}{\pi} L_i(\mathbf{x}, \boldsymbol{\omega}_{i_k}) \cos \theta_i}{\frac{\cos \theta_i}{\pi}}$$

$$L_r(\mathbf{x}, \boldsymbol{\omega}_r) \approx \frac{k_d}{N} \sum_{k=1}^N L_i(\mathbf{x}, \boldsymbol{\omega}_{i_k})$$

# Path Tracing

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- Monte Carlo integration
- Many primary samples
- 1 secondary sample per hit point
- Stochastic decisions
  - Reflect / Refract
  - Diffuse / Specular
  - Importance sampling (pdf according to BSDF component)

# Path Tracing

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- Terminated by ray leaving the scene
  - HDR map lookup of  $L_i(x, \omega_{i_k})$
- Russian roulette
  - Terminate the path with probability  $p_t$
  - If survives multiply the incoming radiance by  $\frac{1}{p_t}$
- Next event estimation
  - Explicit sampling of lights with shadow rays (one to few lights per hit point)
  - Add to path contribution

smallpt: path tracing in 99 lines of C++ code

<https://www.kevinbeason.com/smallpt/>

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# Acceleration Methods

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Ray tracing is costly – must accelerate!

## 1. Accelerating intersection computation

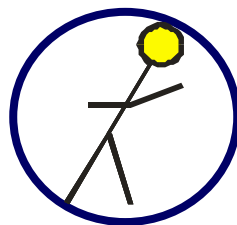
- a) Faster ray / object intersection (low level optimizations)
- b) Less ray / object intersections (spatial data structures)
- c) Tracing more rays together

## 2. Cast less rays

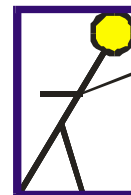
- Importance sampling
- Adaptive antialiasing, adaptive depth of recursion, ...

# Less Ray / Object Intersections

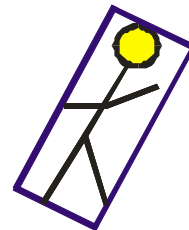
- Kd-tree, octree
- Bounding Volume Hierarchy
- Hot research topic at DCGI!



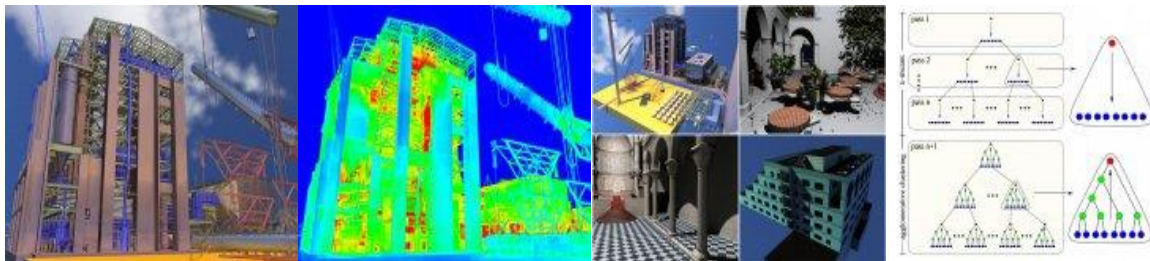
Sphere



AABB



OBB

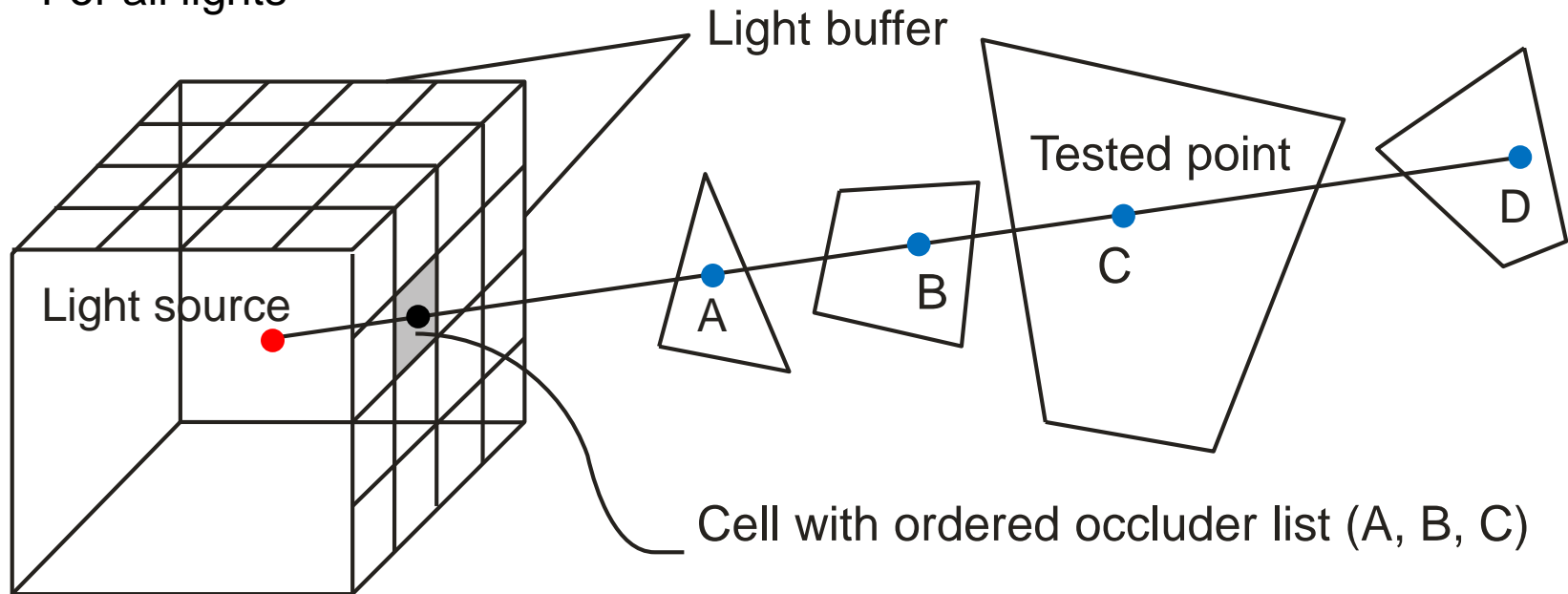


<https://github.com/meistdan/ploc>

<https://github.com/meistdan/prbvh>

# Less Ray / Object Intersections

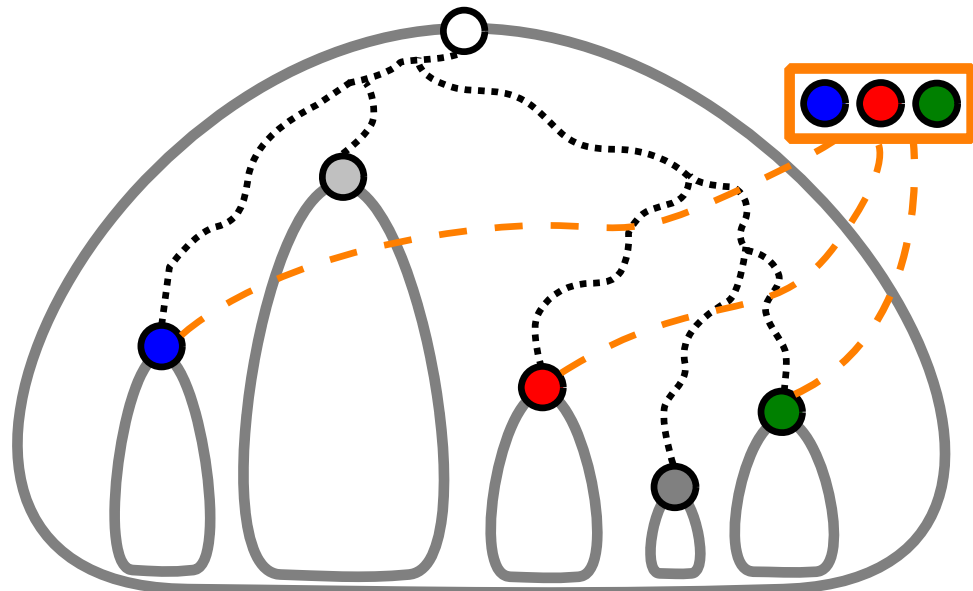
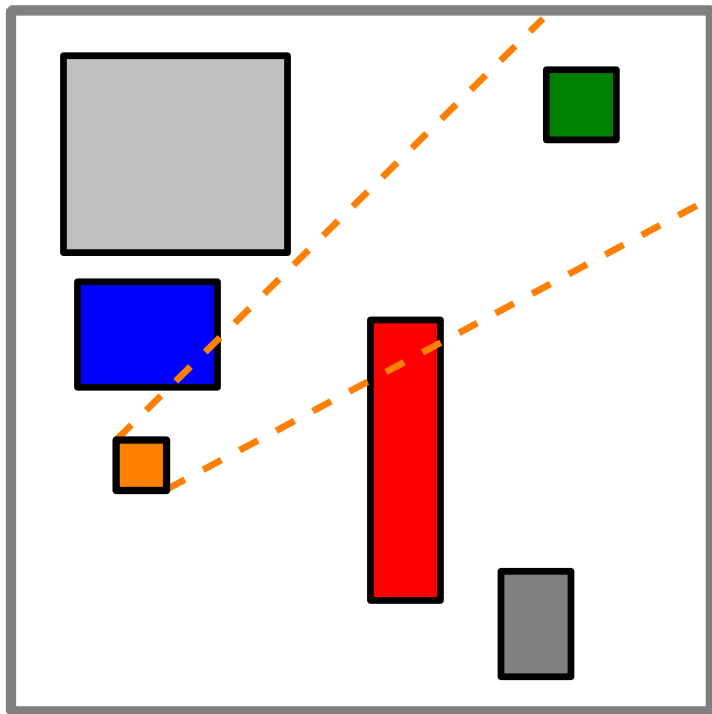
- Light buffer [Haines & Greenberg 1986]
  - Faster shadow rays
  - For all lights



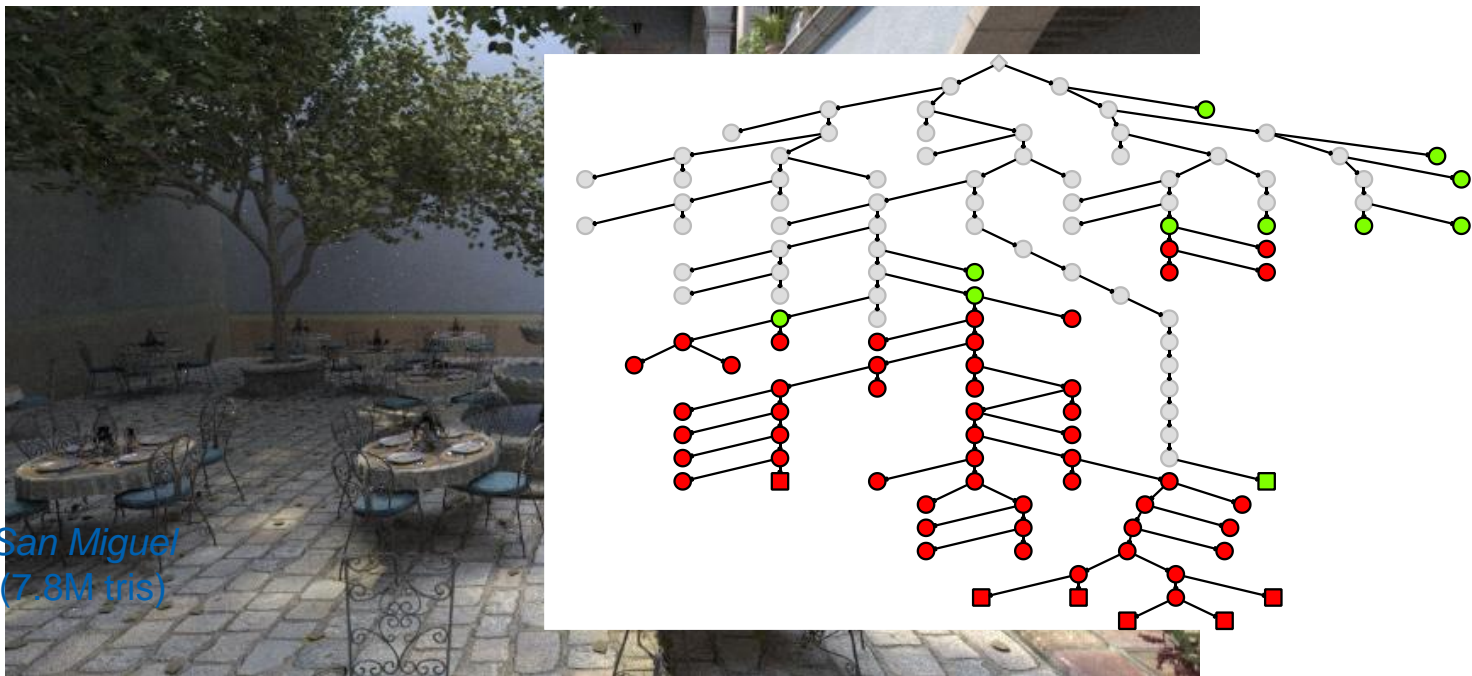


# Less Ray / Object Intersections

- Ray Classification for BVH [Hendrich et al. 2019]



# Ray Classification for BVH



# Ray Sorting

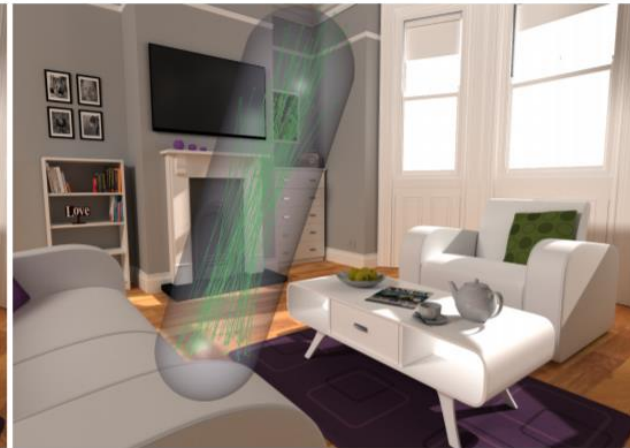
- Meister et al. On Ray Reordering Techniques for Faster GPU Ray Tracing (I3D 2020)



2355 MRays/s



3593 MRays/s



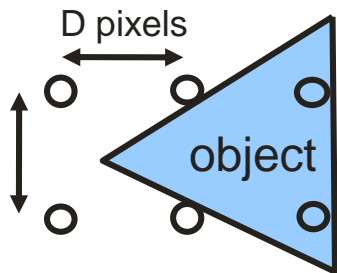
3914 MRays/s

# Cast Less Rays

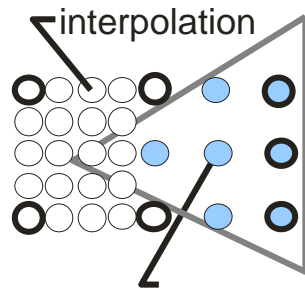
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- Importance sampling
  - Material BSDF
  - Potential light contribution (scenes with many lights)
- Controlling recursion depth
  - Static using a constant (e.g. 5)
    - too deep for non reflecting surfaces
    - Systematic error (bias)
  - Adaptive using importance of contribution
    - Initial contribution 100%, reflection/refraction multiply with  $r_s$  ( $<1$ )
    - Russian roulette – unbiased (but increases noise)!
- Adaptive sampling of primary rays

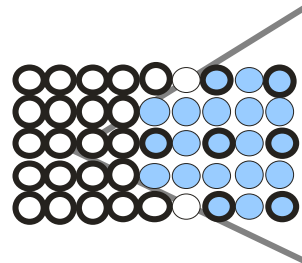
# Adaptive Sampling



coarse sampling



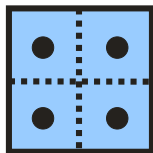
fine sampling



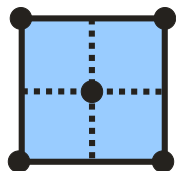
result

## Supersampling

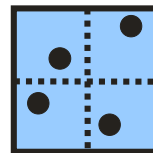
1 pixel:



subpixel centers



center and corners

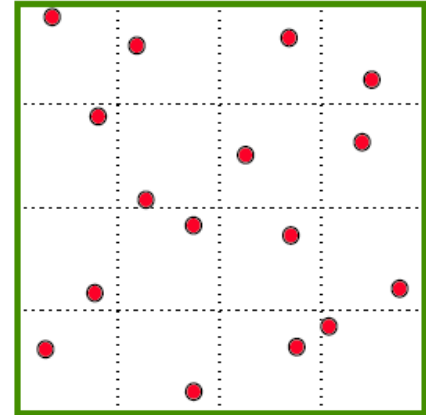


jittering

# Jittering

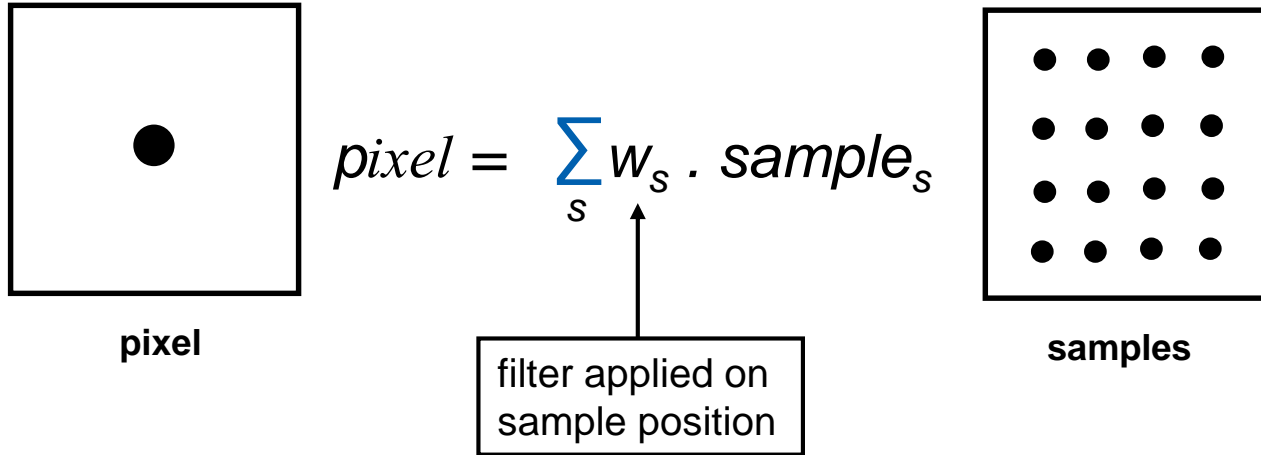
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- Jittering = organized sampling in a grid
  - Stratified sampling
  - Avoids creating larger sample clusters
  - Better distribution than random sampling
  - *Disadvantage* – up to four samples can get clustered
- Sample relaxation



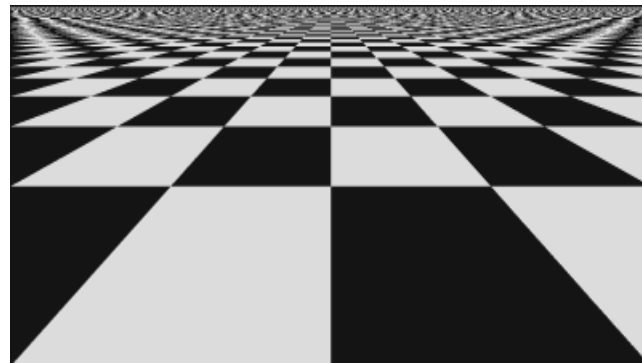
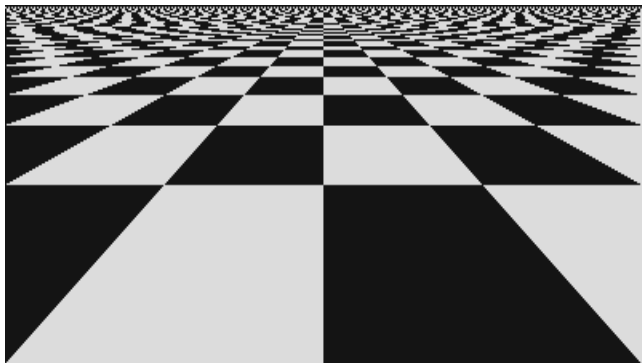
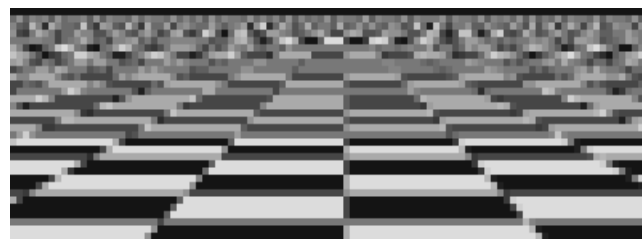
# Antialiasing using Supersampling

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# Point Sampling vs. Supersampling

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**Point**

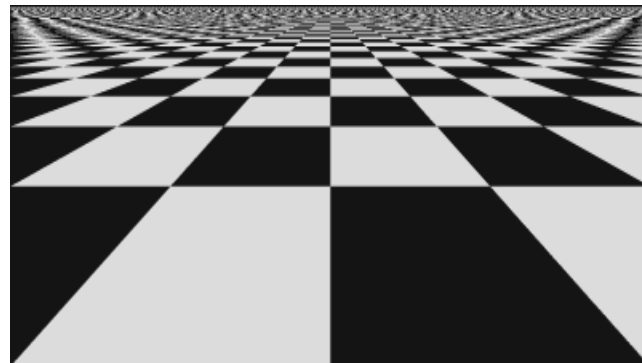
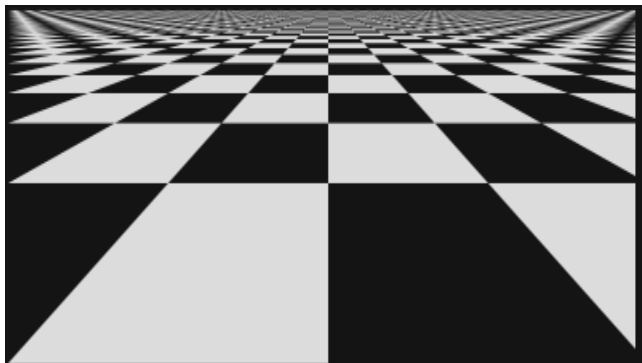
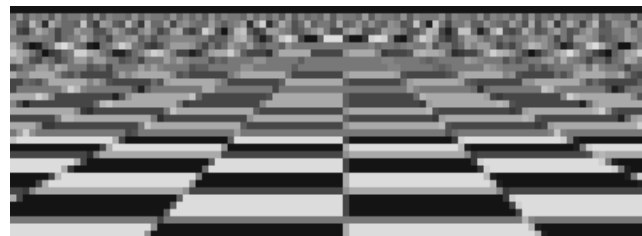
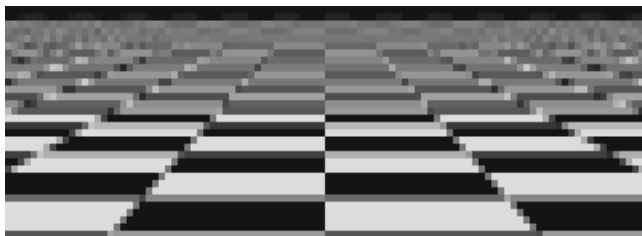
**Supersampling 4x4**

Checkerboard sequence by Tom Duff



# Exact Solution vs. Supersampling

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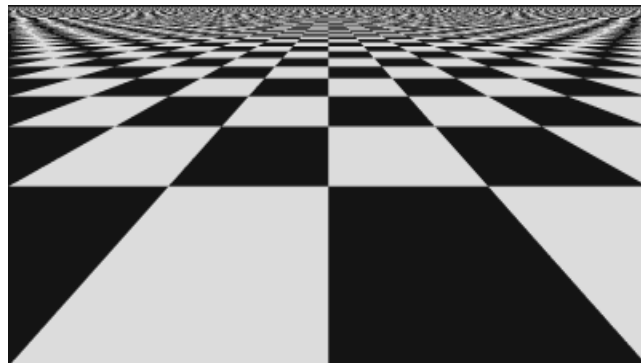
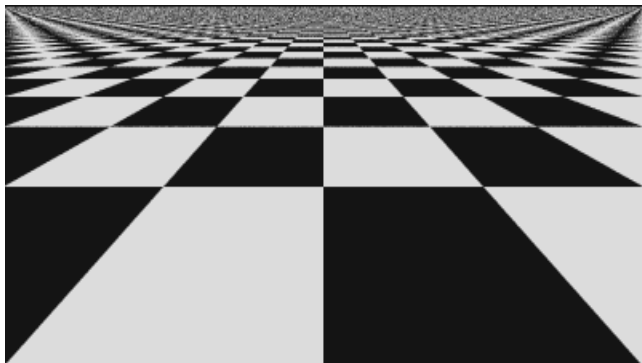
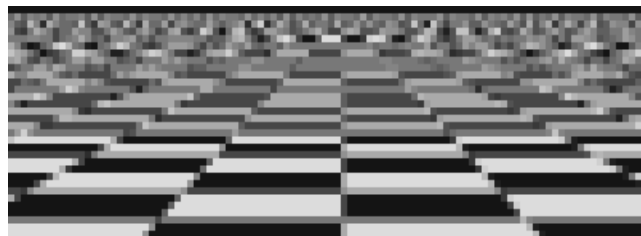
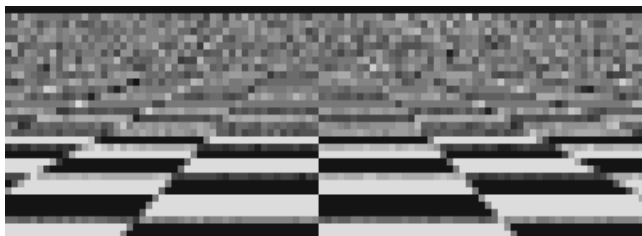


**Exact visible area calculation**

**Supersampling 4x4**

# Jittering vs. Regular Supersampling

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**Jittering 4x4**

**Supersampling 4x4**

# Interactive Path Tracing - Example

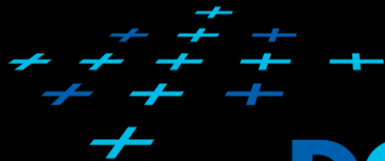
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Questions?