1 Description Logics

1.1 Formal Ontologies

Formalizing Ontologies

- We heard about ontologies as "some shared knowledge structures often visualized through UML-like diagrams" ...
- How to express more complicated constructs like cardinalities, inverses, disjointness, etc.?
- How to check they are designed correctly? How to reason about the knowledge inside?
- We need a **formal language**.

Logics for Ontologies

- propositional logic

  Example

  "John is clever.″ $\Rightarrow$ ¬"John fails at exam.″

- first order predicate logic

  Example

  $(\forall x)(Clever(x) \Rightarrow \neg((\exists y)(Exam(y) \wedge Fails(x, y))))$.

- modal logic

  Example

  $\Box((\forall x)(Clever(x) \Rightarrow \Diamond\neg((\exists y)(Exam(y) \wedge Fails(x, y)))))$.

- ... what is the meaning of these formulas ?

Logics for Ontologies (2)

Logics are defined by their

- Syntax – to *represent* concepts (*defining symbols*)
- Semantics – to capture meaning of the syntactic constructs (*defining concepts*)
- Proof Theory – to enforce the semantics

Logics trade-off
A logical calculus is always a trade-off between *expressiveness* and *tractability of reasoning.*

Propositional Logic

Example
How to check satisfiability of the formula $A \vee (\neg(B \wedge A) \vee B \wedge C)$?

syntax – atomic formulas and $\neg$, $\wedge$, $\vee$, $\Rightarrow$

semantics ($\models$) – an interpretation assigns true/false to each formula.

proof theory ($\vdash$) – resolution, tableau

complexity – NP-Complete (Cook theorem)

First Order Predicate Logic

Example
What is the meaning of this sentence ?

$$(\forall x_1)((Student(x_1) \wedge (\exists x_2)(GraduateCourse(x_2) \wedge isEnrolledTo(x_1, x_2))) \\ \Rightarrow (\forall x_3)(isEnrolledTo(x_1, x_3) \Rightarrow GraduateCourse(x_3)))$$

$$Student \sqcap \exists isEnrolledTo.GraduateCourse \sqsubseteq \forall isEnrolledTo.GraduateCourse$$

First Order Predicate Logic – quick informal review

syntax – constructs involve

term (variable x, constant symbol $JOHN$, function symbol applied to terms $fatherOf(JOHN)$)

axiom/formula (predicate symbols applied to terms $hasFather(x, JOHN)$, possibly glued together with $\neg$, $\wedge$, $\vee$, $\Rightarrow$, $\forall$,$\exists$)

universally closed formula formula without free variable ($(\forall x)(\exists y)hasFather(x, y) \wedge Person(y)$)

semantics – an interpretation (with valuation) assigns:

domain element to each term

true/false to each closed formula

proof theory – resolution; *Deduction Theorem*, *Soundness Theorem*, *Completeness Theorem*

complexity – undecidable (Goedel)

Open World Assumption

OWA
FOPL accepts Open World Assumption, i.e. whatever is not known is not necessarily false.

As a result, FOPL is *monotonic*, i.e.

monotonicity
No conclusion can be invalidated by adding extra knowledge.

This is in contrary to relational databases, or Prolog that accept Closed World Assumption.

1.2 Towards Description Logics

Languages sketched so far aren't enough ?

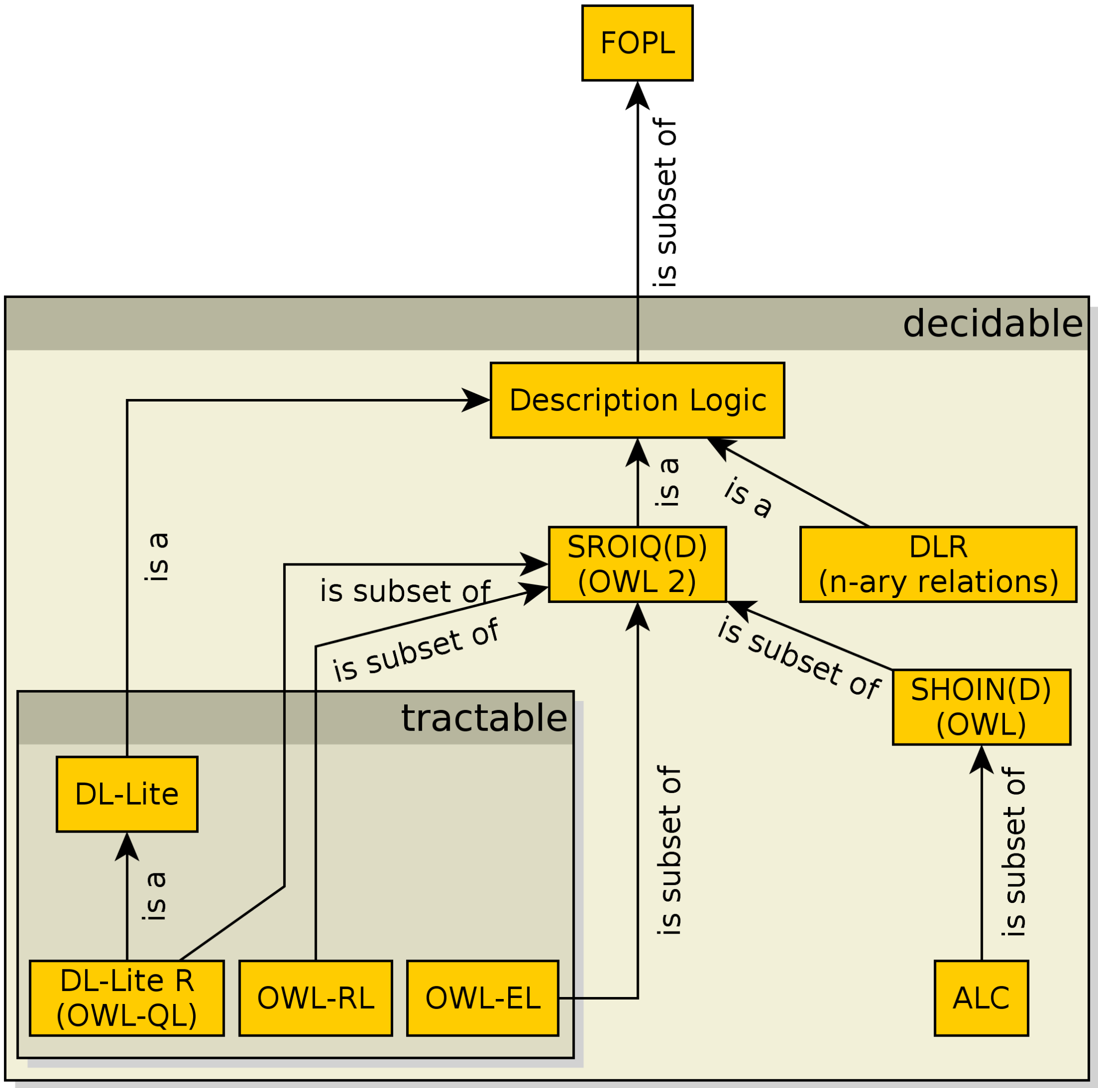
- Why not First Order Predicate Logic ?
  - ☹ FOPL is undecidable – many logical consequences cannot be verified in finite time.
  - – We often do not need full expressiveness of FOL.
- Well, we have Prolog – wide-spread and optimized implementation of FOPL, right ?
  - ☹ Prolog is not an implementation of FOPL – OWA vs. CWA, negation as failure, problems in expressing disjunctive knowledge, etc.

What are Description Logics ?

Description logics (DLs) are (almost exclusively) decidable subsets of FOPL aimed at modeling *terminological incomplete knowledge.*

- first languages emerged as an experiment of giving formal semantics to semantic networks and frames. First implementations in 80's – KL-ONE, KAON, Classic.

- 90's $\mathcal{ALC}$
- 2004 $\mathcal{SHOIN}(\mathcal{D})$ – OWL
- 2009 $\mathcal{SROIQ}(\mathcal{D})$ – OWL 2



1.3 $\mathcal{ALC}$ Language

Concepts and Roles

- Basic building blocks of DLs are :

(atomic) concepts - representing (named) *unary predicates* / classes, e.g. $Parent$, or $Person \sqcap \exists hasChild \cdot Person$.

(atomic) roles - represent (named) *binary predicates* / relations, e.g. $hasChild$

individuals - represent ground terms / individuals, e.g. $JOHN$

- Theory $\mathcal{K} = (\mathcal{T}, \mathcal{A})$ (in OWL refered as Ontology) consists of a

  TBOX $\mathcal{T}$ - representing axioms generally valid in the domain, e.g. $\mathcal{T} = \{Man \sqsubseteq Person\}$

  ABOX $\mathcal{A}$ - representing a particular relational structure (data), e.g. $\mathcal{A} = \{Man(JOHN), loves(JOHN, M$

- DLs differ in their expressive power (concept/role constructors, axiom types).

Semantics, Interpretation

- as $\mathcal{ALC}$ is a subset of FOPL, let's define semantics analogously (and restrict interpretation function where applicable):
- **Interpretation** is a pair $\mathcal{I} = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}})$, where $\Delta^{\mathcal{I}}$ is an interpretation domain and $\cdot^{\mathcal{I}}$ is an interpretation function.
- Having *atomic* concept A, *atomic* role R and individual a, then

$$
\begin{aligned}
A^{\mathcal{I}} &\subseteq \Delta^{\mathcal{I}} \\
R^{\mathcal{I}} &\subseteq \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}} \\
a^{\mathcal{I}} &\in \Delta^{\mathcal{I}}
\end{aligned}
$$

$\mathcal{ALC}$ (= attributive language with complements)

Having concepts C, D, atomic concept A and atomic role R, then for interpretation $\mathcal{I}$:

concept	*concept*$^{\mathcal{I}}$	*description*
$\top$	$\Delta^{\mathcal{I}}$	(universal concept)
$\bot$	$\emptyset$	(unsatisfiable concept)
$\neg C$	$\Delta^{\mathcal{I}} \setminus C^{\mathcal{I}}$	(negation)
$C_1 \sqcap C_2$	$C_1^{\mathcal{I}} \cap C_2^{\mathcal{I}}$	(intersection)
$C_1 \sqcup C_2$	$C_1^{\mathcal{I}} \cup C_2^{\mathcal{I}}$	(union)
$\forall R \cdot C$	$\{a \mid \forall b((a,b) \in R^{\mathcal{I}} \implies b \in C^{\mathcal{I}})\}$	(universal restriction)
$\exists R \cdot C$	$\{a \mid \exists b((a,b) \in R^{\mathcal{I}} \wedge b \in C^{\mathcal{I}})\}$	(existential restriction)

TBOX

axiom	$\mathcal{I} \models$ axiom iff	*description*
$C_1 \sqsubseteq C_2$	$C_1^{\mathcal{I}} \subseteq C_2^{\mathcal{I}}$	(inclusion)
$C_1 \equiv C_2$	$C_1^{\mathcal{I}} = C_2^{\mathcal{I}}$	(equivalence)

ABOX (UNA = unique name assumption[1])

axiom	$\mathcal{I} \models$ axiom iff	*description*
$C(a)$	$a^{\mathcal{I}} \in C^{\mathcal{I}}$	(concept assertion)
$R(a_1, a_2)$	$(a_1^{\mathcal{I}}, a_2^{\mathcal{I}}) \in R^{\mathcal{I}}$	(role assertion)

[1]two different individuals denote two different domain elements

$\mathcal{ALC}$ – Example

Example

Consider an information system for genealogical data integrating multiple geneological databases. Let's have atomic concepts $Person, Man, GrandParent$ and atomic role $hasChild$.

- Set of persons that have just men as their descendants (if any)
  - $Person \sqcap \forall hasChild \cdot Man$
- How to define concept $GrandParent$? (specify an *axiom*)
  - $GrandParent \equiv Person \sqcap \exists hasChild \cdot \exists hasChild \cdot \top$
- How does the previous axiom look like in FOPL ?

$$\begin{aligned}\forall x\,(GrandParent(x) \equiv (Person(x) \wedge \exists y\,(hasChild(x,y)\\ \wedge \exists z\,(hasChild(y,z)))))\end{aligned}$$

$\mathcal{ALC}$ Example – $\mathcal{T}$

Example

$$\begin{aligned}
Woman &\equiv Person \sqcap Female\\
Man &\equiv Person \sqcap \neg Woman\\
Mother &\equiv Woman \sqcap \exists hasChild \cdot Person\\
Father &\equiv Man \sqcap \exists hasChild \cdot Person\\
Parent &\equiv Father \sqcup Mother\\
Grandmother &\equiv Mother \sqcap \exists hasChild \cdot Parent\\
MotherWithoutDaughter &\equiv Mother \sqcap \forall hasChild \cdot \neg Woman\\
Wife &\equiv Woman \sqcap \exists hasHusband \cdot Man
\end{aligned}$$

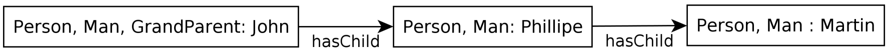
Interpretation – Example

Example

- Consider a theory $\mathcal{K}_1 = (\{GrandParent \equiv Person \sqcap \exists hasChild \cdot \exists hasChild \cdot \top\}, \{GrandParen$
  Find some model.
- a model of $\mathcal{K}_1$ can be interpretation $\mathcal{I}_1$:
  - $\Delta^{\mathcal{I}_1} = Man^{\mathcal{I}_1} = Person^{\mathcal{I}_1} = \{\text{John}, \text{Phillipe}, \text{Martin}\}$
  - $hasChild^{\mathcal{I}_1} = \{(\text{John}, \text{Phillipe}), (\text{Phillipe}, \text{Martin})\}$

- $GrandParent^{\mathcal{I}_1} = \{\text{John}\}$
- $JOHN^{\mathcal{I}_1} = \{\text{John}\}$

- this model is finite and has the form of a tree with the root in the node John :



Shape of DL Models

The last example revealed several important properties of DL models:

Tree model property (TMP)

Every consistent $\mathcal{K} = (\{\}, \{C(I)\})$ has a model in the shape of a *rooted tree*.

Finite model property (FMP)

Every consistent $\mathcal{K} = (\mathcal{T}, \mathcal{A})$ has a *finite model*.

Both properties represent important characteristics of $\mathcal{ALC}$ that significantly speed-up reasoning.

In particular (generalized) TMP is a characteristics that is shared by most DLs and significantly reduces their computational complexity.

Example – CWA $\times$ OWA

Example

ABOX
$hasChild(JOCASTA, OEDIPUS)$
$hasChild(OEDIPUS, POLYNEIKES)$
$Patricide(OEDIPUS)$
$hasChild(JOCASTA, POLYNEIKES)$
$hasChild(POLYNEIKES, THERSANDROS)$
$\neg Patricide(THERSANDROS)$

Edges represent role assertions of *hasChild*; red/green colors distinguish concepts instances – *Patricide* a $\neg Patricide$

$$\begin{tikzcd} JOCASTA \arrow[rr] \arrow[dr] & & POLYNEIKES \arrow[r] & THERSANDROS \\ & OEDIPUS \arrow[ur] & & \end{tikzcd}$$

Q1 $(\exists hasChild \cdot (Patricide \sqcap \exists hasChild \cdot \neg Patricide))(JOCASTA)$,

$$JOCASTA \longrightarrow \bullet \longrightarrow \bullet$$

Q2 Find individuals x such that $\mathcal{K} \models C(x)$, where C is

$$\neg Patricide \sqcap \exists hasChild^{-} \cdot (Patricide \sqcap \exists hasChild^{-} \cdot \{JOCASTA\})$$

What is the difference, when considering CWA ?

$$JOCASTA \longrightarrow \bullet \longrightarrow x$$

References

Bibliography

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