Languages, grammars, automata

English sources:

[1] B. Melichar, J. Holub, T. Polcar: **Text Search Algorithms**
http://cw.felk.cvut.cz/lib/exe/fetch.php/courses/a4m33pal/melichar-tsa-lectures-1.pdf
Chapters 1.4 and 1.5, it is probably reasonably short, there is nothing to skip.

[2] J. E. Hopcroft, R. Motwani, J. D. Ullman: **Introduction to Automata Theory**
folow the link at http://cw.felk.cvut.cz/doku.php/courses/a4m33pal/literatura_odkazy
Chapters 1., 2., 3., there is a lot to skip, consult the teacher preferably.

Czech instant sources:

[3] M. Demlová: **A4B01JAG**
http://math.feld.cvut.cz/demlova/teaching/jag/
Pages 1-27, in PAL, you may wish to skip: Proofs, chapters 2.4, 2.6, 2.8.

[4] I. Černá, M. Křetínský, A. Kučera: **Automaty a formální jazyky I**
http://is.muni.cz/do/1499/el/estud/fi/js06/ib005/Formalni_jazyky_a_automaty_I.pdf
Chapters 1 and 2, skip same parts as in [1].

For more references see PAL links pages
https://cw.fel.cvut.cz/wiki/courses/be4m33pal/references (EN)
http://cw.felk.cvut.cz/doku.php/courses/b4m33pal/odkazy-zdroje (CZ)

Alphabet

Alphabet … finite (unempty) set of symbols

|A| … size of alphabet A

Examples: A = { ‘A’, ‘D’, ‘G’, ‘O’, ‘U’}, |A| = 5

A = {0,1}, |A| = 2

A = {○, □, △}, |A| = 3

word

Word (over alphabet A) … finite (maybe empty) sequence
also string of symbols of alphabet (A)

|w| … length of word w

Examples: w = OUAGADOUGOU, |w| = 11

w = 1001, |w| = 4

w = □△○△□, |w| = 5

Language

Language … set of words (=strings)
(not necessarily finite, can be empty too)
over a given alphabet

|L| … **cardinality** of language L

---

1 Language specification -- List of all words of the language (only for finite languages!)

Examples:

A_1 = {‘A’, ‘D’, ‘G’, ‘O’, ‘U’}

L_1 = {ADA, DOG, GOUDA, D, GAG}, $|L_1| = 5$

$A_2 = \{0,1\}$

L_2 = {0, 1, 00, 01, 10, 11}, $|L_2| = 6$

A_3 = { ○, □, △}

L_3 = { △△, ○□○, □□△○ }, $|L_2| = 3$

2 Language specification -- Informal (but unambiguous) description in natural human language (usually for infinite language)

Examples: A_1 = {'A', 'D', 'G', 'O', 'U'}

L_1: Set of all words over A_1, which begin with DA, end with G and do not contain substring AA.

L_1 = {DAG, DADG, DAGG, DAOG, DAUG, DADAG, DADDG… }

$|L_1| = \infty$

$A_2 = \{0,1\}$

L_2: Set of all words over A_2, where each 0 is followed by at least two 1s.

L_2 = { 1, 11, 011, 0111, 1011, 1111, … , 011011, 011111, … }

$|L_2| = \infty$

③ Language specification -- By finite automaton

Definition

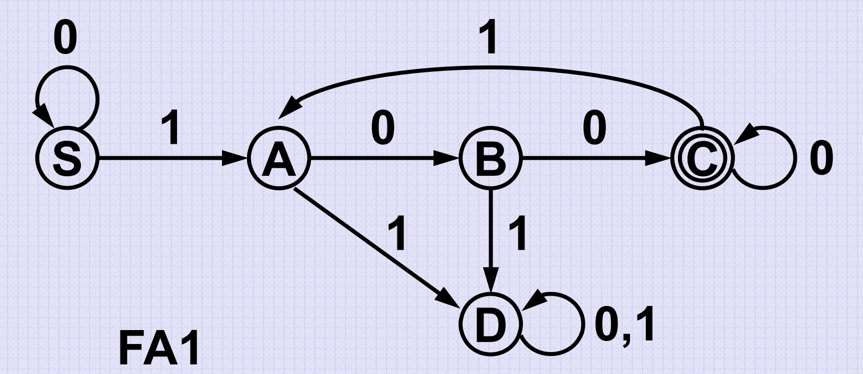
Finite automaton **is a five-tuple (A, Q, σ, S_0, Q_F), where:**

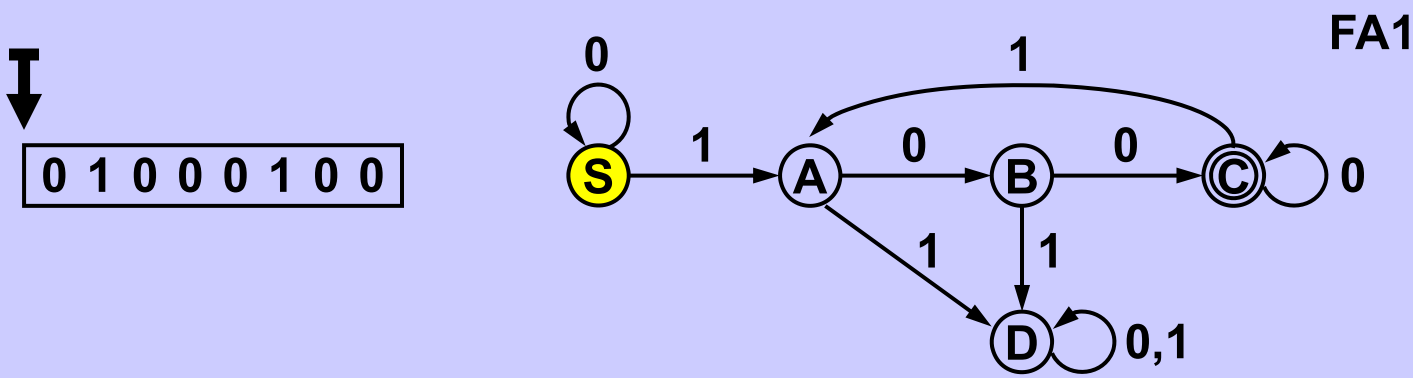
A … alphabet … finite set of symbols
$|A|$ ... size of alphabet
Q ... set of states (often numbered)
σ ... transition function ... $\sigma: Q \times A \rightarrow Q$
S_0 ... start state $S_0 \in Q$
Q_F ... unempty set of final states $\emptyset \neq Q_F \subseteq Q$

Automaton FA1:

A … <u>alphabet</u> … {0,1}, $|A| = 2$
Q ... set of <u>states</u> {S, A, B, C, D }
σ ... <u>transition function</u> ... $\sigma: Q \times A \to Q$: {
$\sigma(S,0) = S$, $\sigma(A,0) = B$, $\sigma(B,0) = C$, $\sigma(C,0) = C$, $\sigma(D,0) = D$,
$\sigma(S,1) = A$, $\sigma(A,1) = D$, $\sigma(B,1) = D$, $\sigma(C,1) = A$, $\sigma(D,1) = D$ }
S_0 ... <u>start state</u> $S \in Q$
Q_F … unempty set of <u>final states</u> $\varnothing \neq \{ C \} \subseteq Q$

Transition diagram
of the automaton FA1




FA1
0 1 0 0 0 1 0 0
S
A
B
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D
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1
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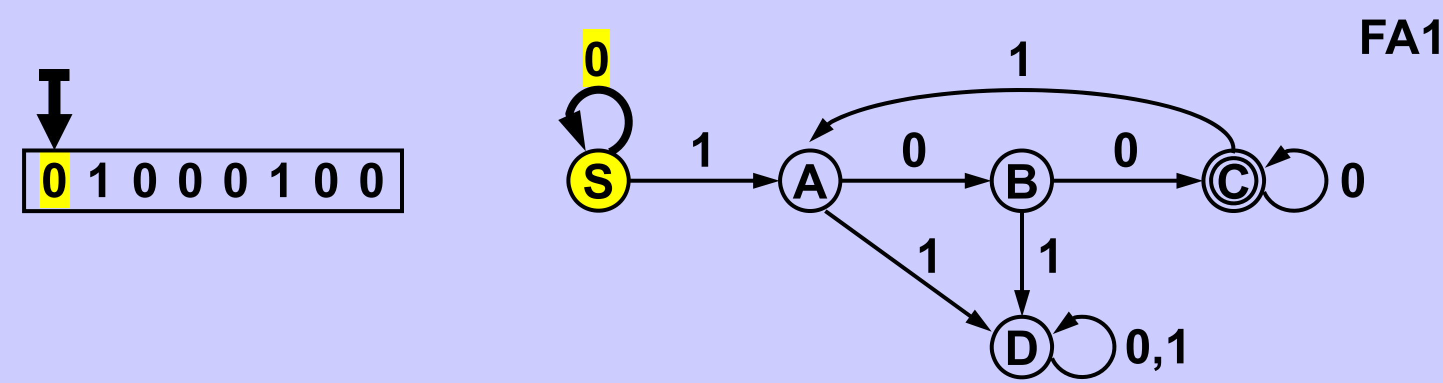
FA1
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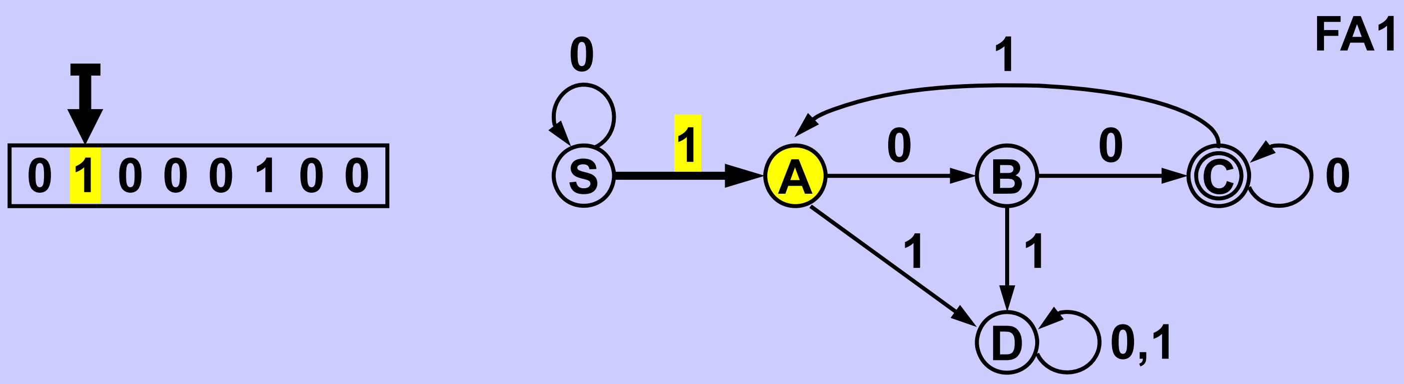
0 1 0 0 0 1 0 0
FA1
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S
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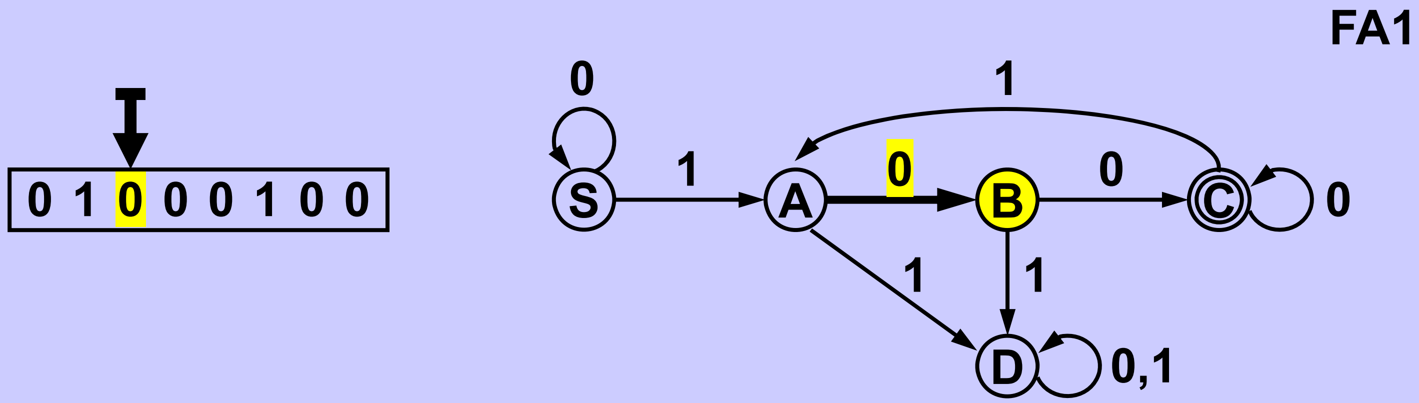
0 1 0 0 0 1 0 0
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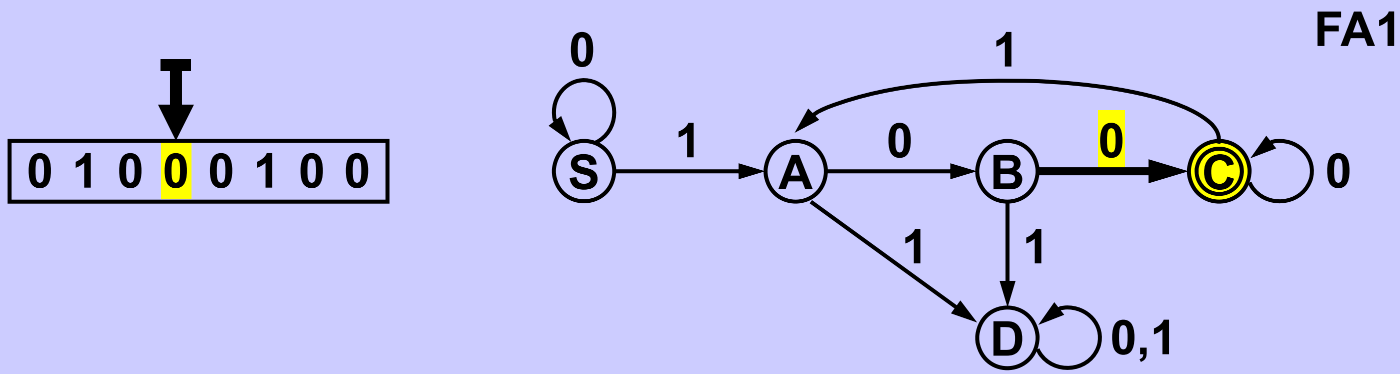
FA1
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S
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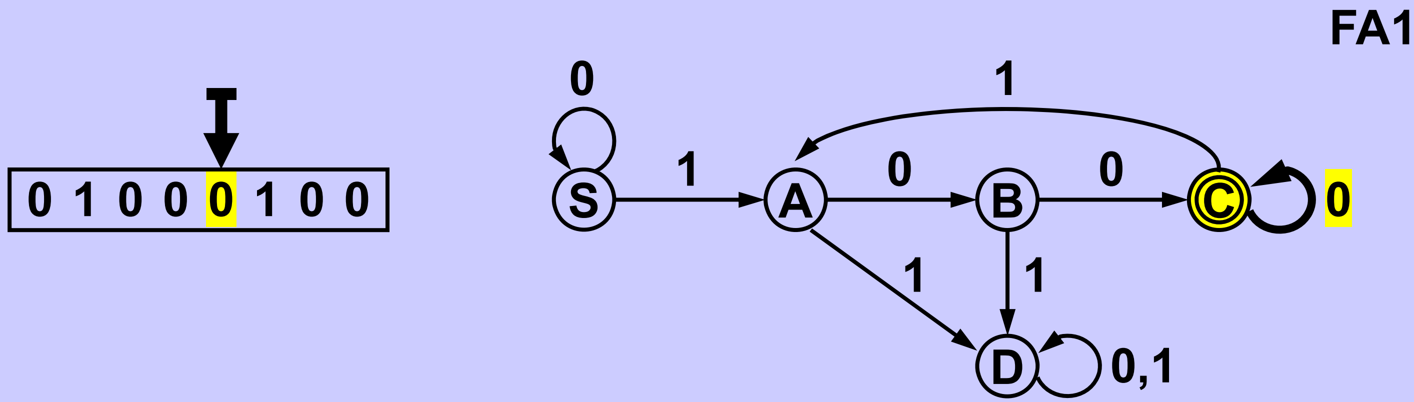
FA1
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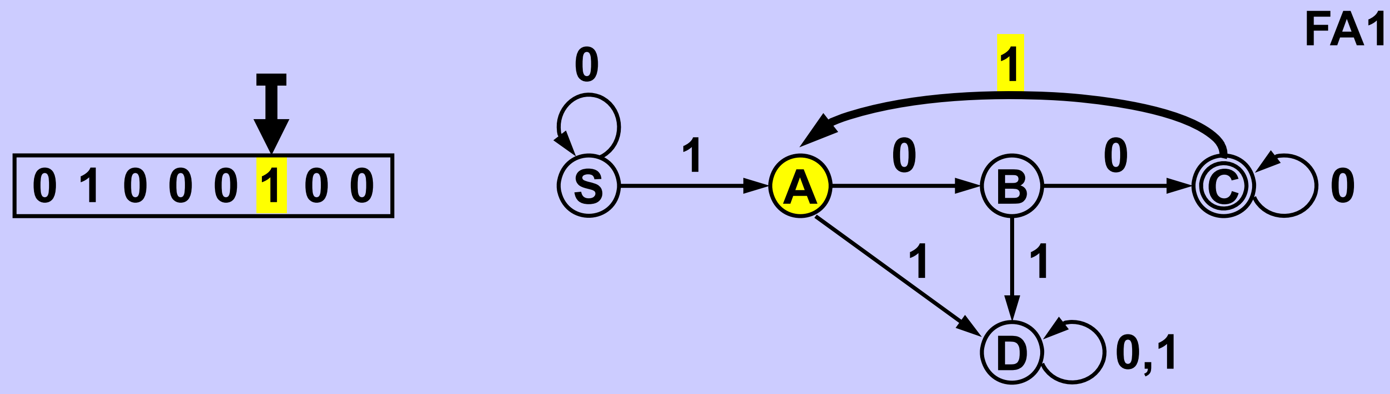
FA1
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S
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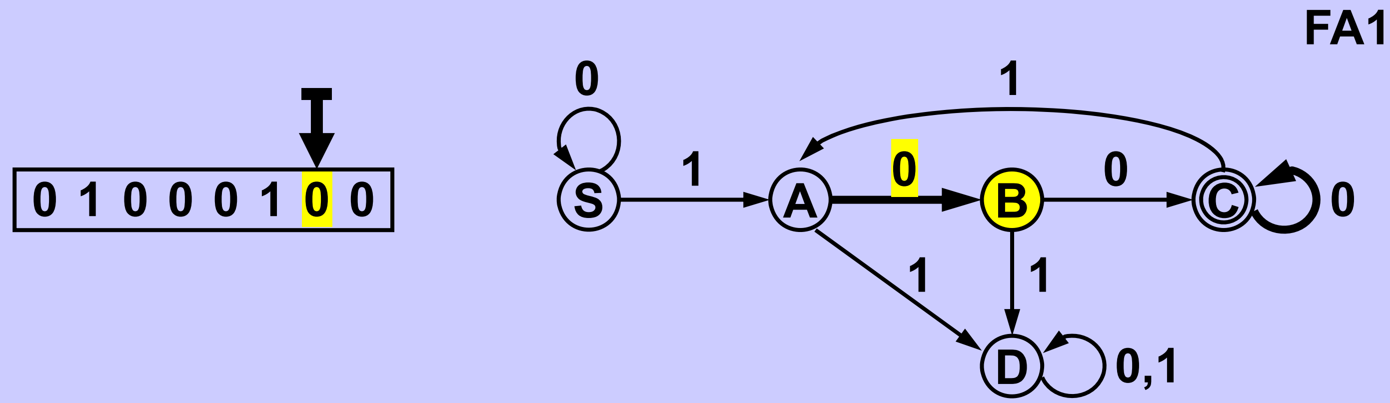
FA1
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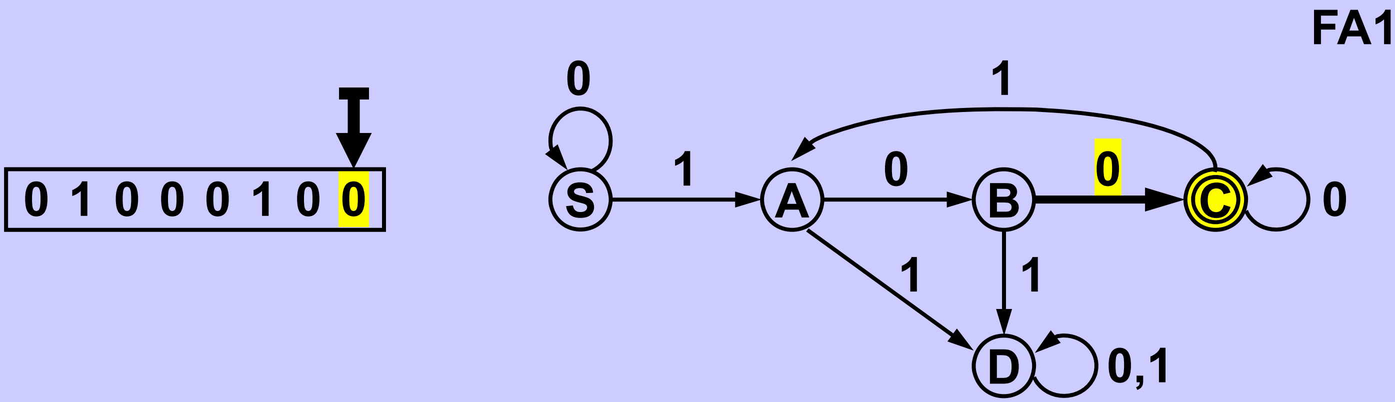
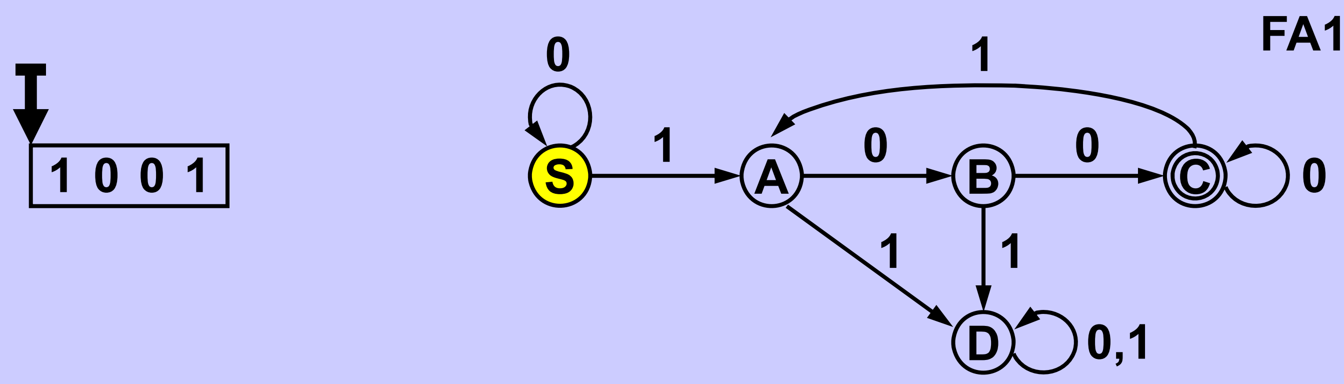


When the last word symbol is read automaton FA1 is in final state ◎

$\Rightarrow$

Word **0 1 0 0 0 1 0 0** **is <u>accepted</u> by automaton FA1**


FA1
1 0 0 1
S
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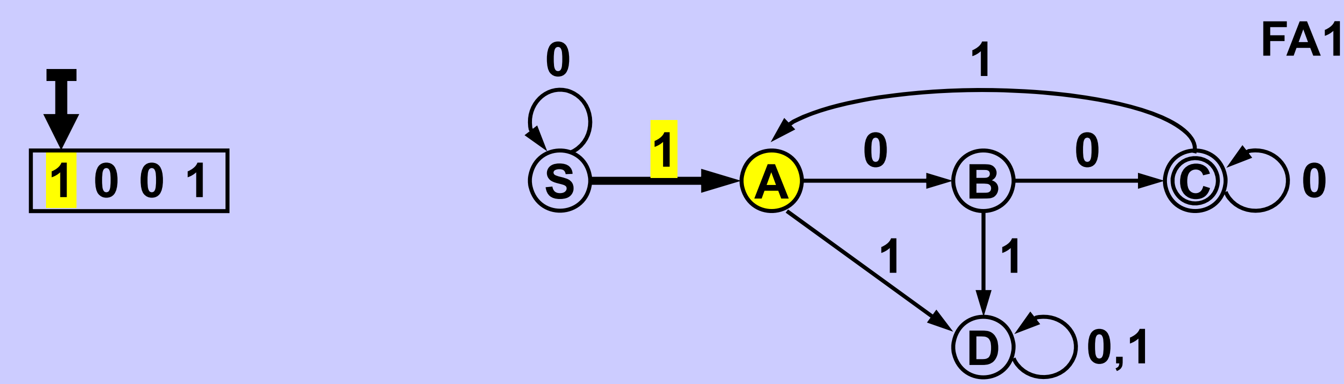
0,1


FA1
1 0 0 1
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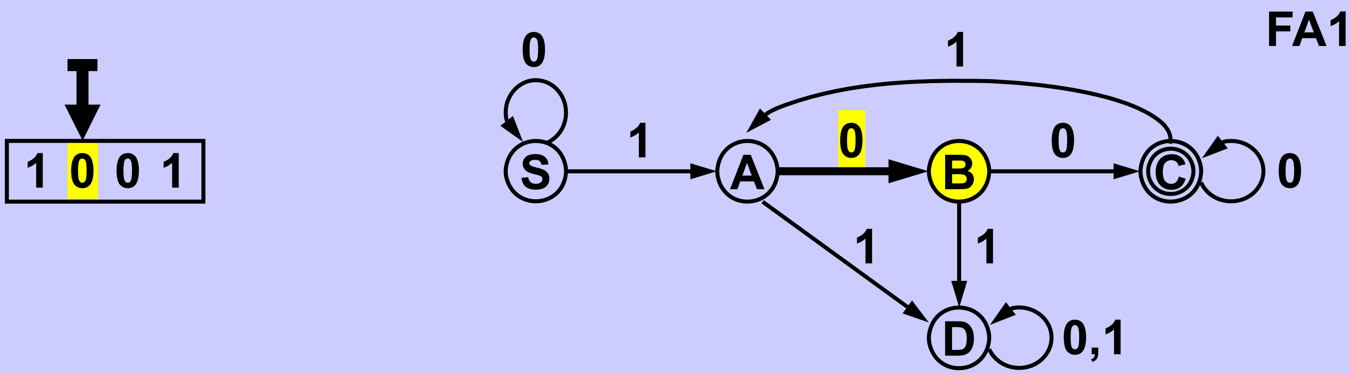
1 0 0 1
FA1
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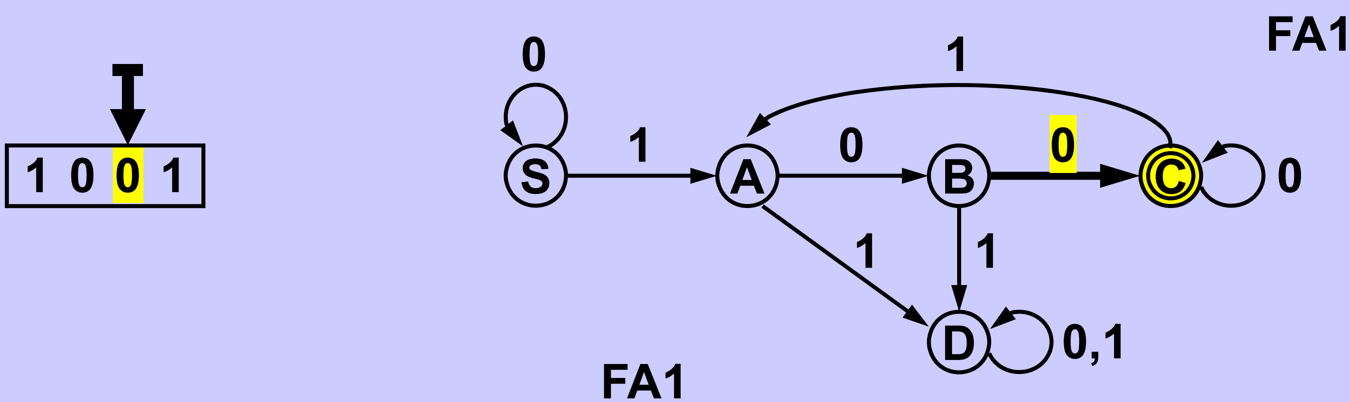
1 0 0 1
FA1
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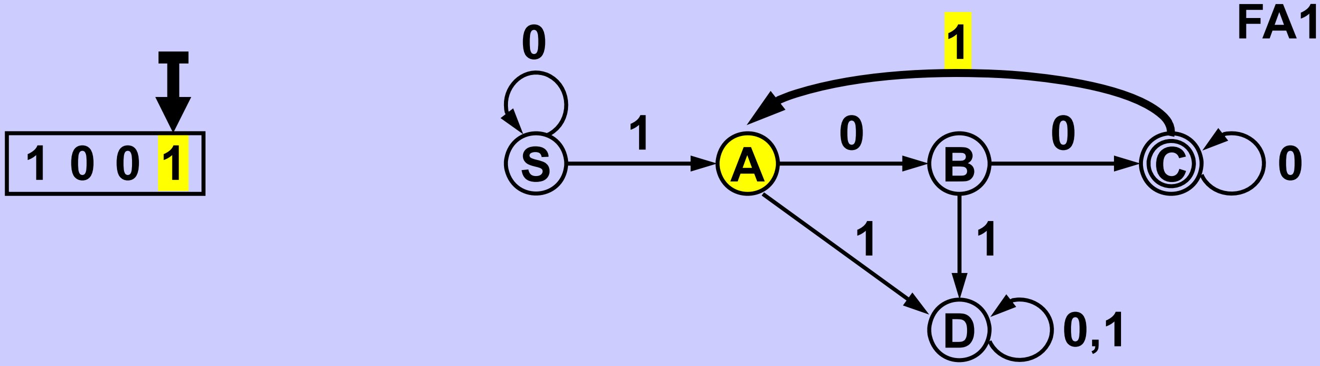
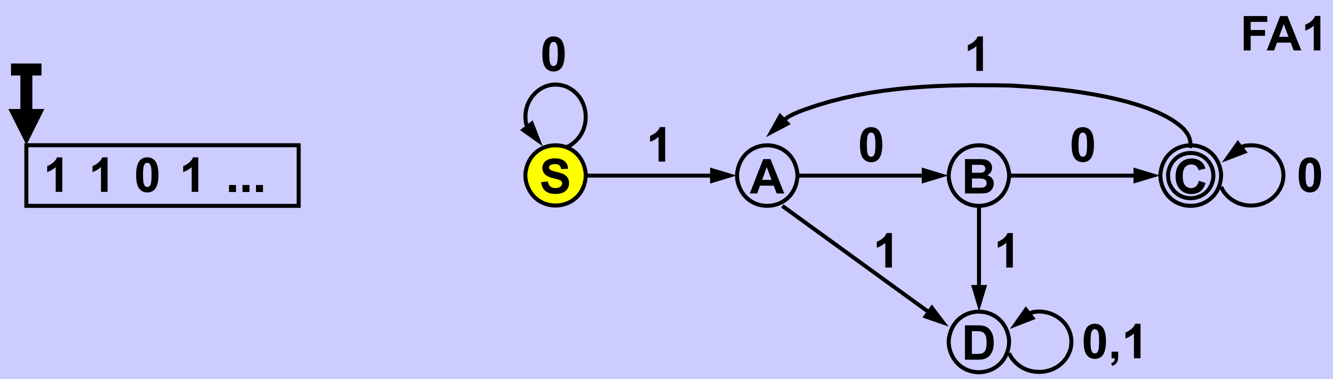
FA1



When the last word symbol is read automaton FA1 is in a state which is not final

$\Rightarrow$

Word $\boxed{1\ 0\ 0\ 1}$ **<u>is not accepted</u> by automaton FA1**


1 1 0 1 ...
FA1
0
1
1
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1
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0,1
S
A
B
C
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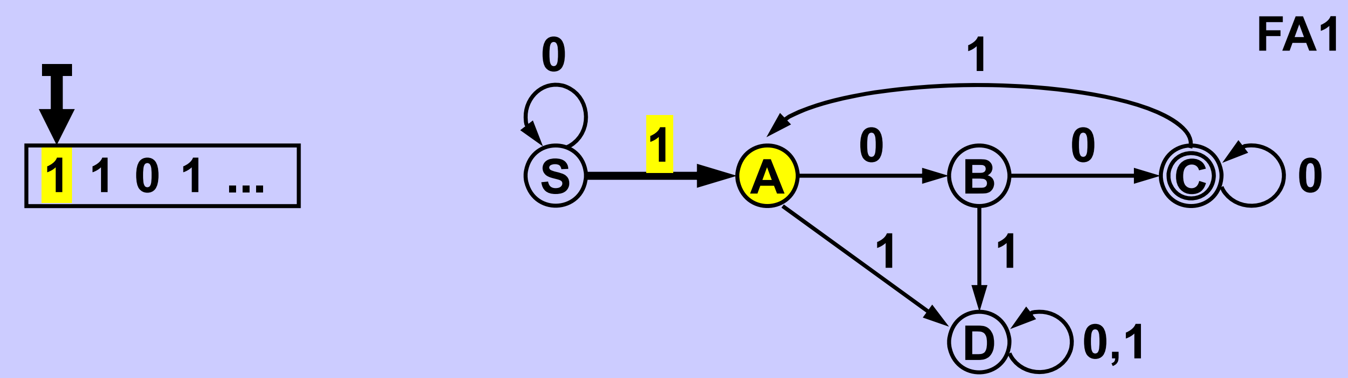
1 1 0 1 ...
FA1
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S
A
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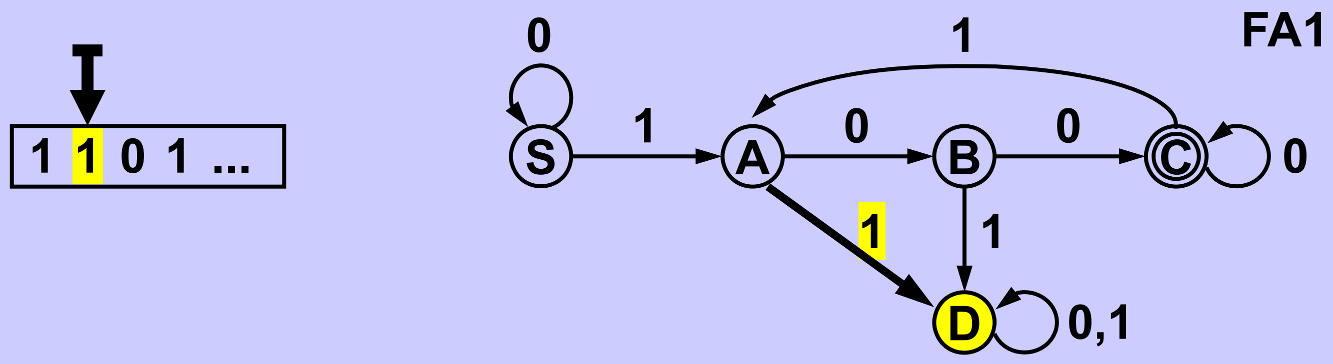
1 1 0 1 ...
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0
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D
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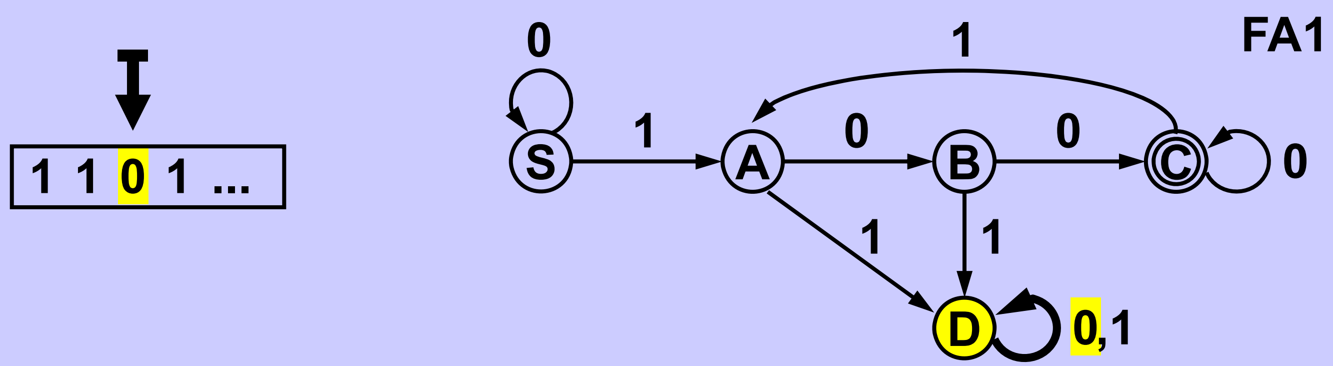
FA1


1 1 0 1 ...
0
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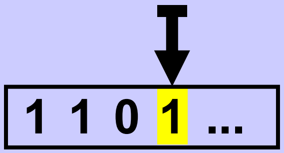
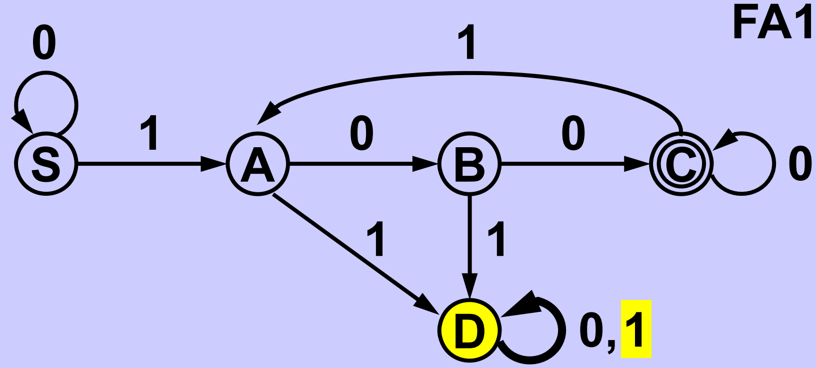
0,1
FA1





No word starting with **1 1 ...** **is accepted** by automaton FA1

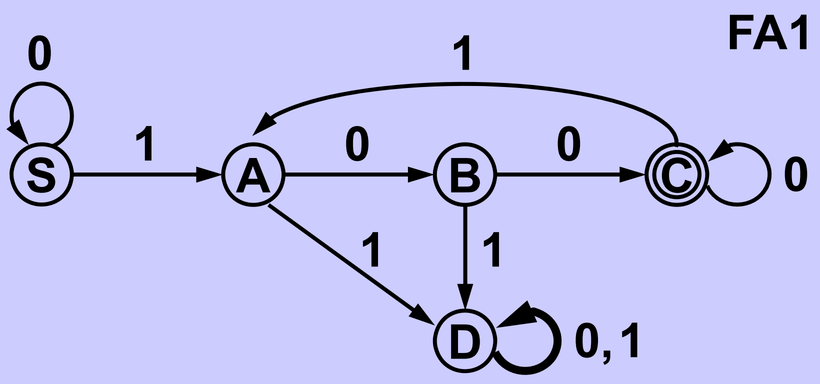
No word containing **... 1 1 ...** **is accepted** by automaton FA1

No word containing **... 1 0 1 ...** **is accepted** by automaton FA1

Automaton FA1 accepts only words
- containing at least one 1
- containing at least two 0s after each 1

Language accepted by automaton = set of all words accepted by automaton

Alphabet		0	1	Final
States	S	S	A	0
	A	B	D	0
	B	C	D	0
	C	C	A	1
	D	D	D	0



Transition table (including Final)
describes the automaton completely.

Usually (if not specified otherwise),
the first row corresponds to the start state.

Automaton A activity:

At the beginning, A is in the start state.

Next, A reads the input word symbol by symbol and transits

to other states according to the transition function.

When the word is completely read, A is again in some state.

If A is in a final state, we say that A <u>accepts</u> the word,

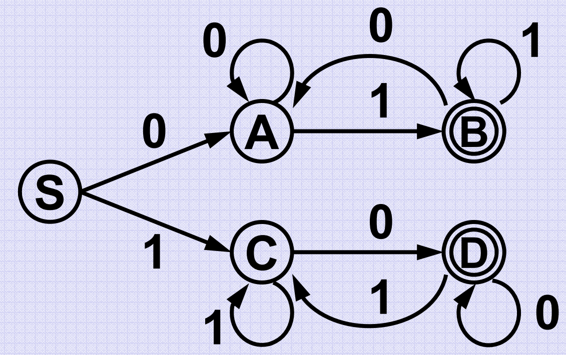
if A is not in a final state, we say that A <u>does not accept</u> the word.

All words accepted by A represent

<u>a language accepted (or recognized) by A.</u>

Language over alphabet {0,1} :

If the word starts with 0, it ends with 1,
If the word starts with 1, it ends with 0.



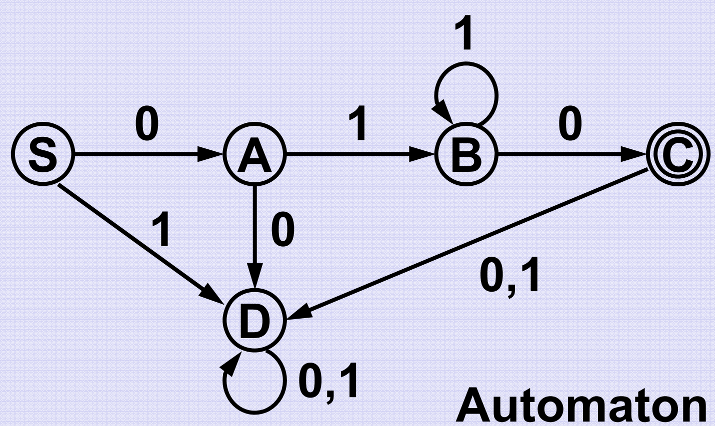
Automat FA2

Example of analysis of different words by FA2:

0 1 0 1 0 : (S),0 → (A),1 → (B),0 → (A),1 → (B),0 → (A)
(A) is not a final state, word 0 1 0 1 0 is rejected by FA2.

1 0 1 1 0 : (S),1 → (C),0 → (D),1 → (C),1 → (C),0 → (D)
(D) is a final state, word 1 0 1 1 0 is accepted by FA2.

Language:
{
0 1 0,
0 1 1 0,
0 1 1 1 0,
0 1 1 1 1 0,
0 1 1 1 1 1 0,
... }



Automaton FA3

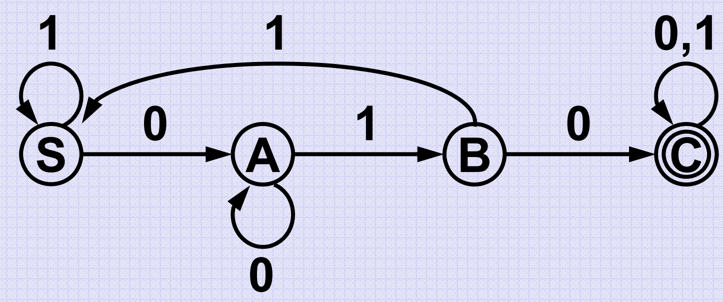
Example of analysis of different words by FA3:

0 1 0 1 0 : (S),0 → (A),1 → (B),0 → (C),1 → (D),0 → (D)

(D) is not a final state, word 0 1 0 1 0 is rejected by FA3.

0 1 1 1 0 : (S),0 → (A),1 → (B),1 → (B),1 → (B),0 → (C)

(C) is a final state, word 0 1 1 1 0 is accepted by FA3.



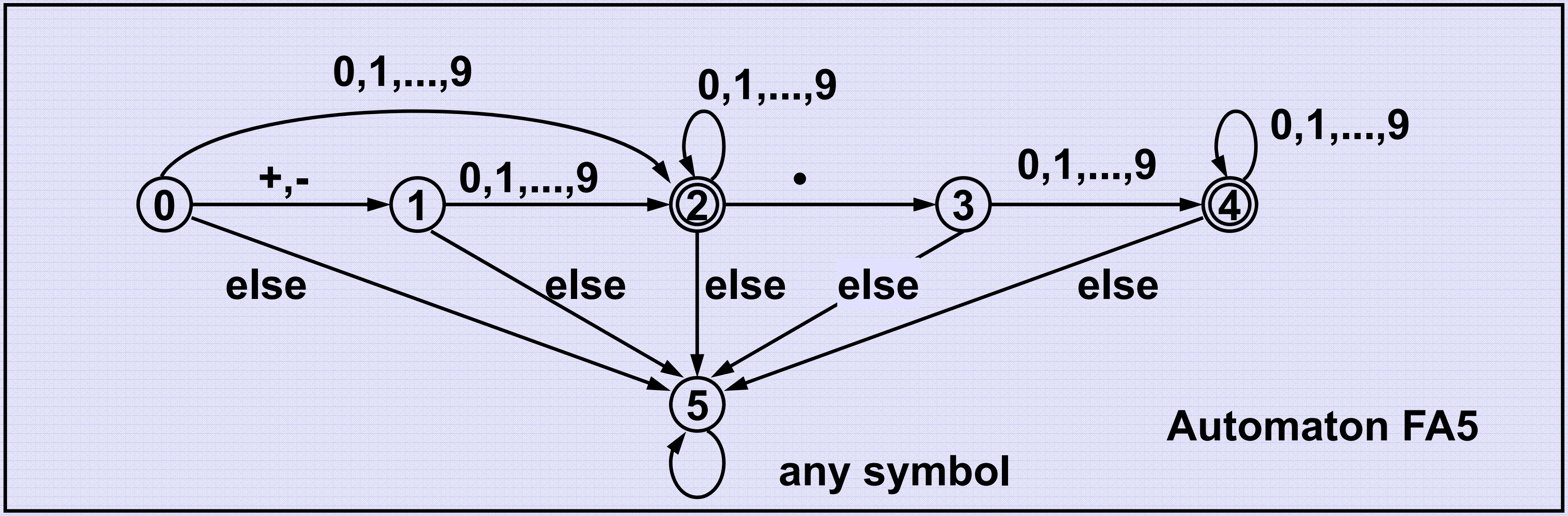
Automaton FA4 accepts each word over alphabet {0,1}
which contains substring $\boxed{\text{... 0 1 0 ...}}$

Example of analysis of different words by FA4:

0 0 1 0 1 : (S),0 → (A),0 → (A),1 → (B),0 → (C),1 → (C)
(C) is a final state, word 0 0 1 0 1 is accepted by FA4.

0 1 1 1 0 : (S),0 → (A),1 → (B),1 → (S),1 → (S),0 → (A)
(A) is not a final state, word 0 1 1 1 0 is rejected by FA4.

Language over alphabet { +, - , . , 0, 1, …, 8, 9, … } whose words represent decimal numbers



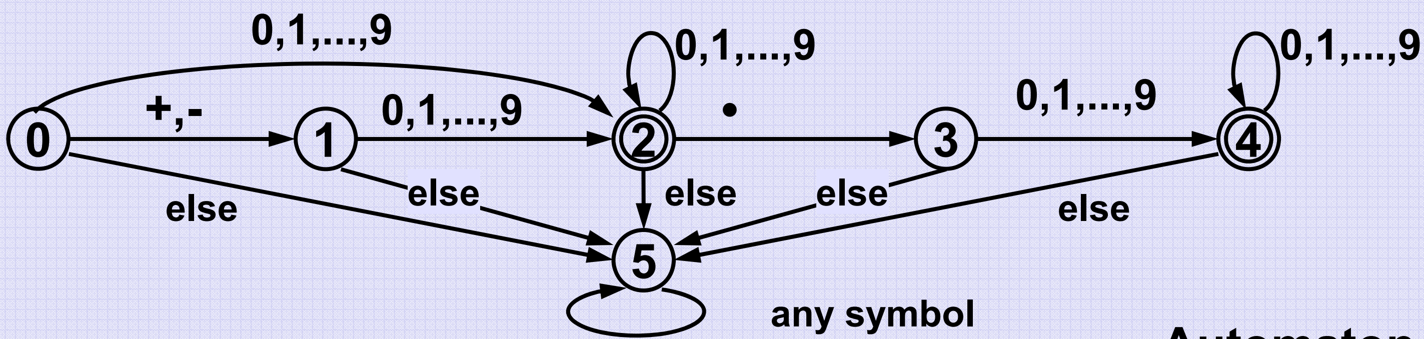
Example of word analysis

+87.09: (0),+ → (1),8 → (2),7 → (2), . → (3),0 → (4),9 → (4)

(4) is a final state, word +87.05 is accepted by FA5.

76+2: (0),7 → (2),6 → (2),+ → (5),2 → (5)

(5) is not a final state, word 76+2 is not accepted by FA5.



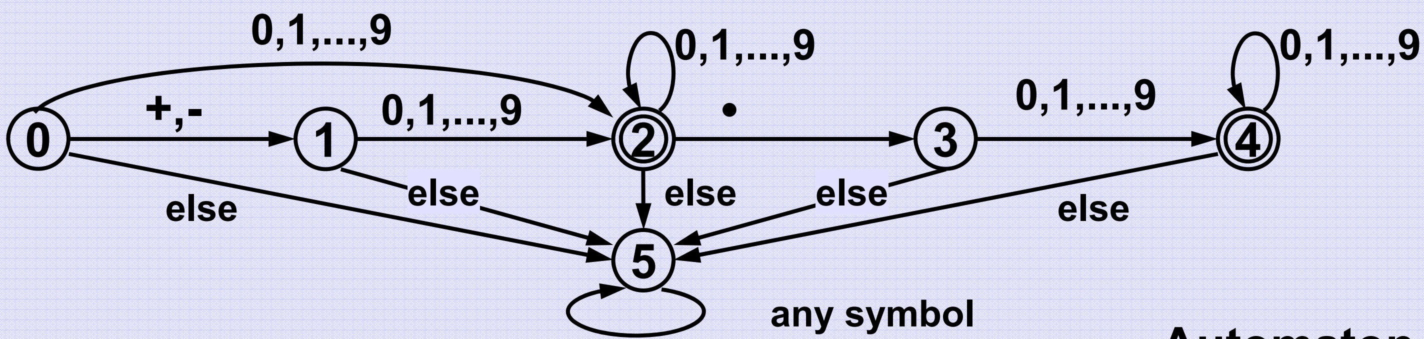
Automaton FA5

Code of the finite automaton

(The word which is being read is stored in the array text):

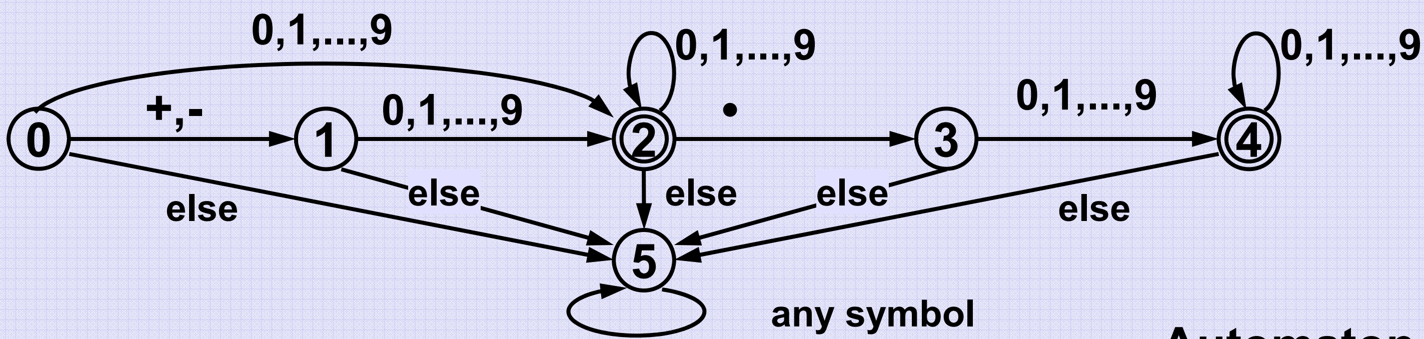
```
boolean isDecimal( char [] text ) {
int state = 0;

for(int i = 0; i < text.length; i++) {  // check each symbol
  switch (state) {
  ...
```



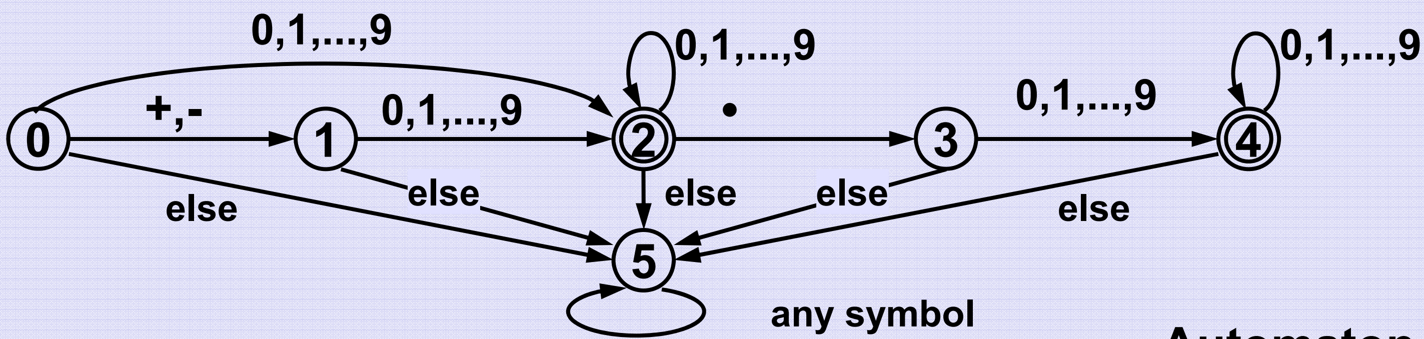
Automaton FA5

```
0  case 0:
     if ((text[i] == '+') || (text[i] == '-')) state = 1;
     else
     if ((text[i] >= '0') && (text[i] <= '9')) state = 2;
     else state = 5;
     break;
1  case 1:
     if ((text[i] >= '0') && (text[i] <= '9')) state = 2;
     else state = 5;
     break;
```



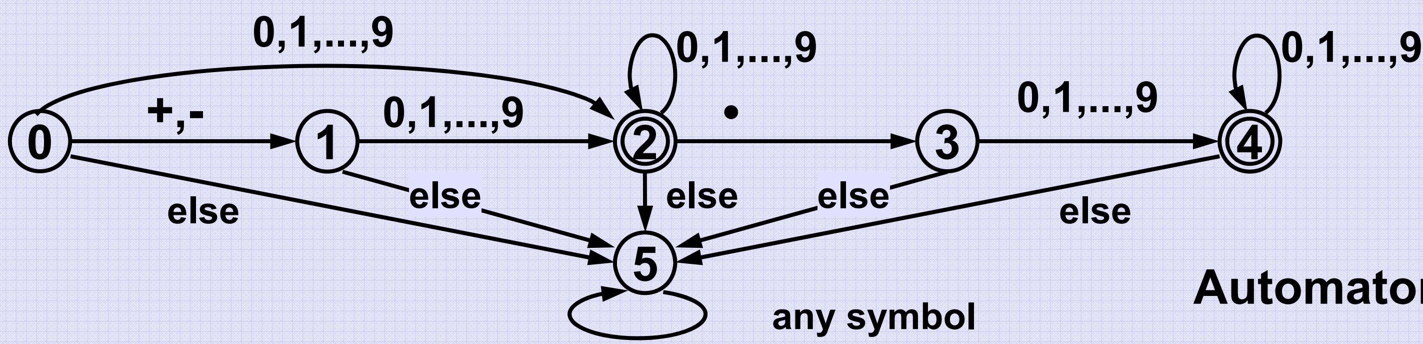
Automaton FA5

```
(2) case 2:
      if ((text[i] >= '0') && (text[i] <= '9')) state = 2;
      else
      if (text[i] == '.') state = 3;
      else state = 5;
      break;
(3) case 3:
      if ((text[i] >= '0') && (text[i] <= '9')) state = 4;
      else state = 5;
      break;
```



Automaton FA5

```
④  case 4:
     if ((text[i] >= '0') && (text[i] <= '9')) state = 4;
     else state = 5;
     break;
⑤  case 5: break;               // no need to react anyhow
    default : break;
    } // end  switch
  } // end for
return (state == 2) || (state == 4);   // final states
}
```



Automaton FA5

Transition table of automaton FA5

states \ alphabet	0	1	2	3	4	5	6	7	8	9	.	+	–	%	=	...	}	final
0	2	2	2	2	2	2	2	2	2	2	5	1	1	5	5	...	5	0
1	2	2	2	2	2	2	2	2	2	2	5	5	5	5	5	...	5	0
2	2	2	2	2	2	2	2	2	2	2	3	5	5	5	5	...	5	1
3	4	4	4	4	4	4	4	4	4	4	5	5	5	5	5	...	5	0
4	4	4	4	4	4	4	4	4	4	4	5	5	5	5	5	...	5	1
5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	...	5	0

```
boolean isAccepted( char [] text, int [][] TT, boolean [] F ){
  int state = 0;  // start state
  for( char c: text ){
    state = TT[state][Integer.valueOf(c)];  // c: char -> int
  }
  return F[state];
}
```

Tables TT and F specify the automaton completely (provided start state is typically 0) , their construction is problem/implementation dependent and should not influence the operation(s) of the automaton.

States (TT) \ alphabet	0	1	2	3	4	5	6	7	8	9	.	+	–	%	=	...	}	Final (F)
0	2	2	2	2	2	2	2	2	2	2	5	1	1	5	5	...	5	0
1	2	2	2	2	2	2	2	2	2	2	5	5	5	5	5	...	5	0
2	2	2	2	2	2	2	2	2	2	2	3	5	5	5	5	...	5	1
3	4	4	4	4	4	4	4	4	4	4	5	5	5	5	5	...	5	0
4	4	4	4	4	4	4	4	4	4	4	5	5	5	5	5	...	5	1
5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5	...	5	0

Pros:

Simplicity an completeness
An automaton defines all words in the language unambiguously.
There is no need of additional code/methods to check if a word is "correct" or "acceptable" or whatever.

Speed
Time spent on each symbol in an input word (text) is constant (and very short, typically).
Input of length **N**
is processed in **one** single pass in

O**(N)** time.

Cons:

Limited class of languages
A finite automaton can recognize (and process)
only so-called **regular** languages,
the smallest class of languages in Chomsky hierarchy.

Out of question are e.g.:
-- natural languages
-- programming languages
-- expressions with unlimited parenthesis depth
-- … **:-((**