VIR3: Neural networks

Karel Zimmermann

http://cmp.felk.cvut.cz/~zimmerk/

Vision for Robotics and Autonomous Systems
https://cyber.felk.cvut.cz/vras/

Center for Machine Perception
https://cmp.felk.cvut.cz

Department for Cybernetics
Faculty of Electrical Engineering
Czech Technical University in Prague

Outline

- Neuron+ computational graph
- Fully connected neural network

Linear classifier and neuron

Labels

RGB images

$+1$

-1

def classify():

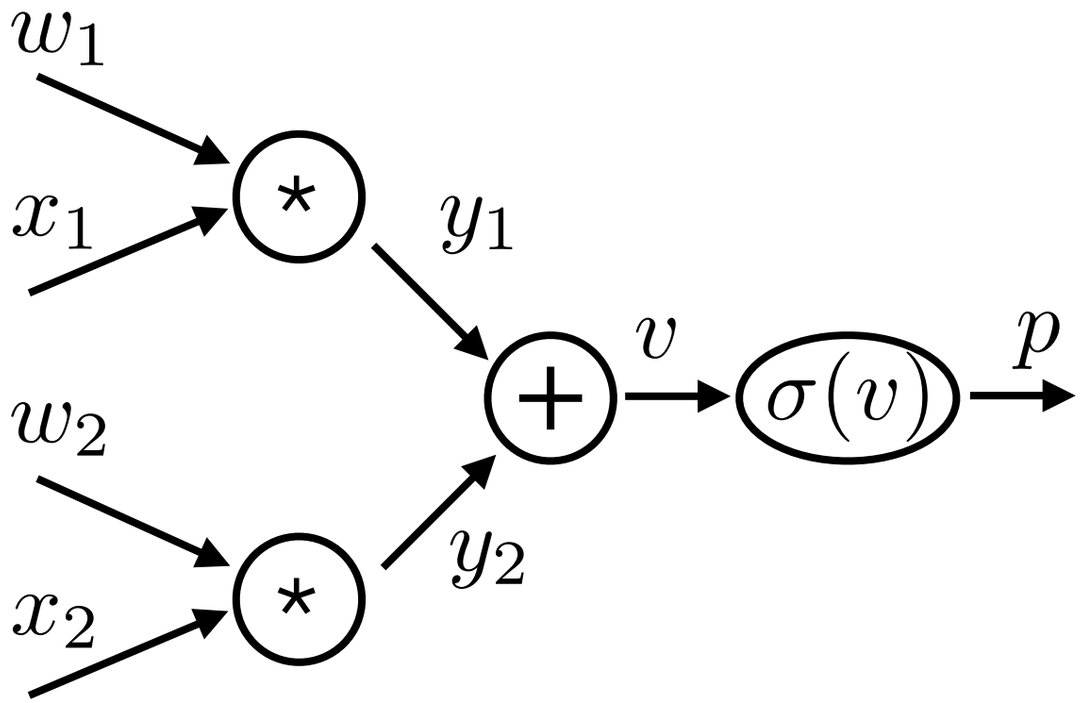
 # Linear classifier

$\mathbf{x} = \text{vec}($  $)$

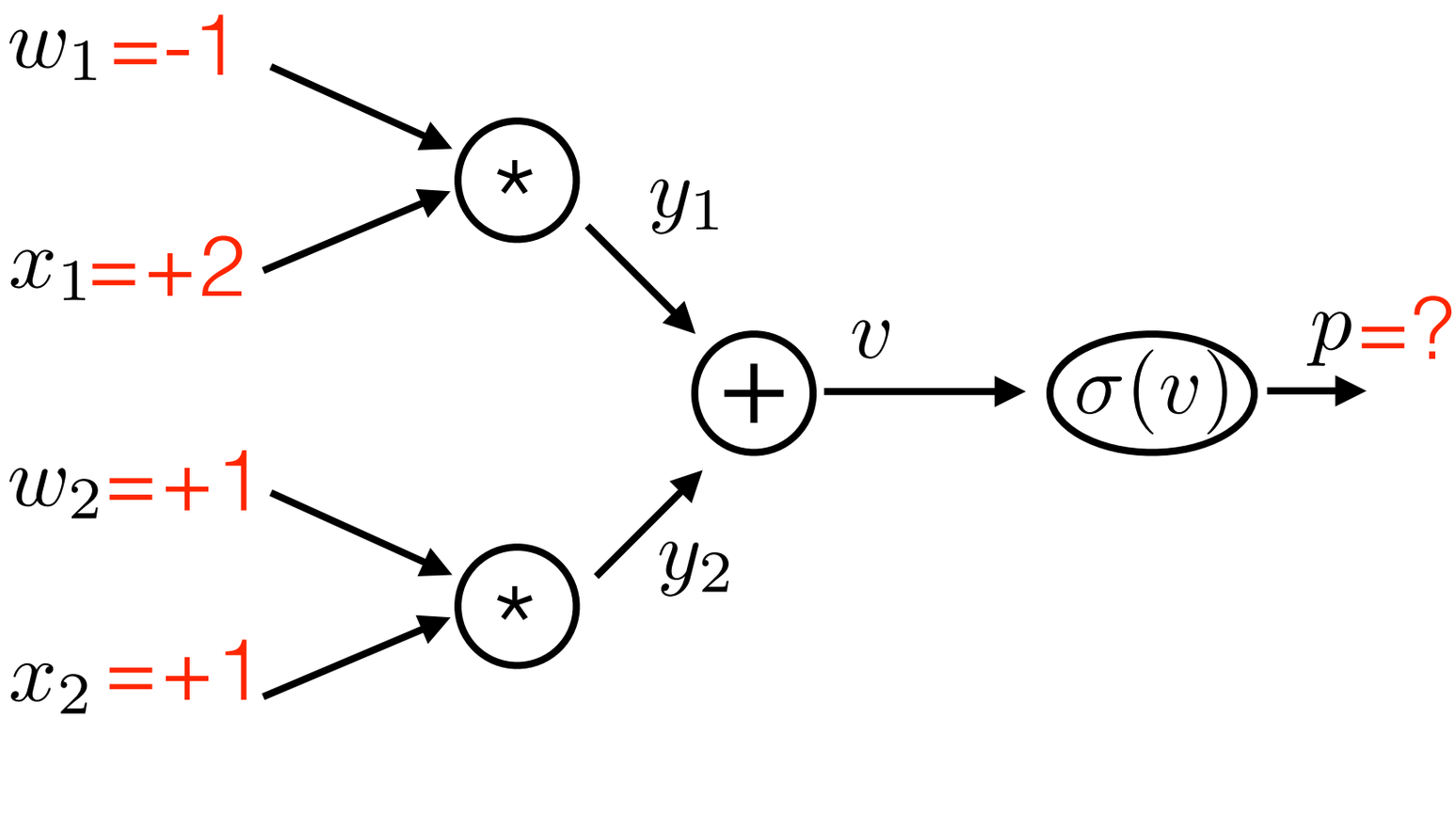
$p = \sigma\left(\mathbf{w}^\top \mathbf{x}\right)$

return p

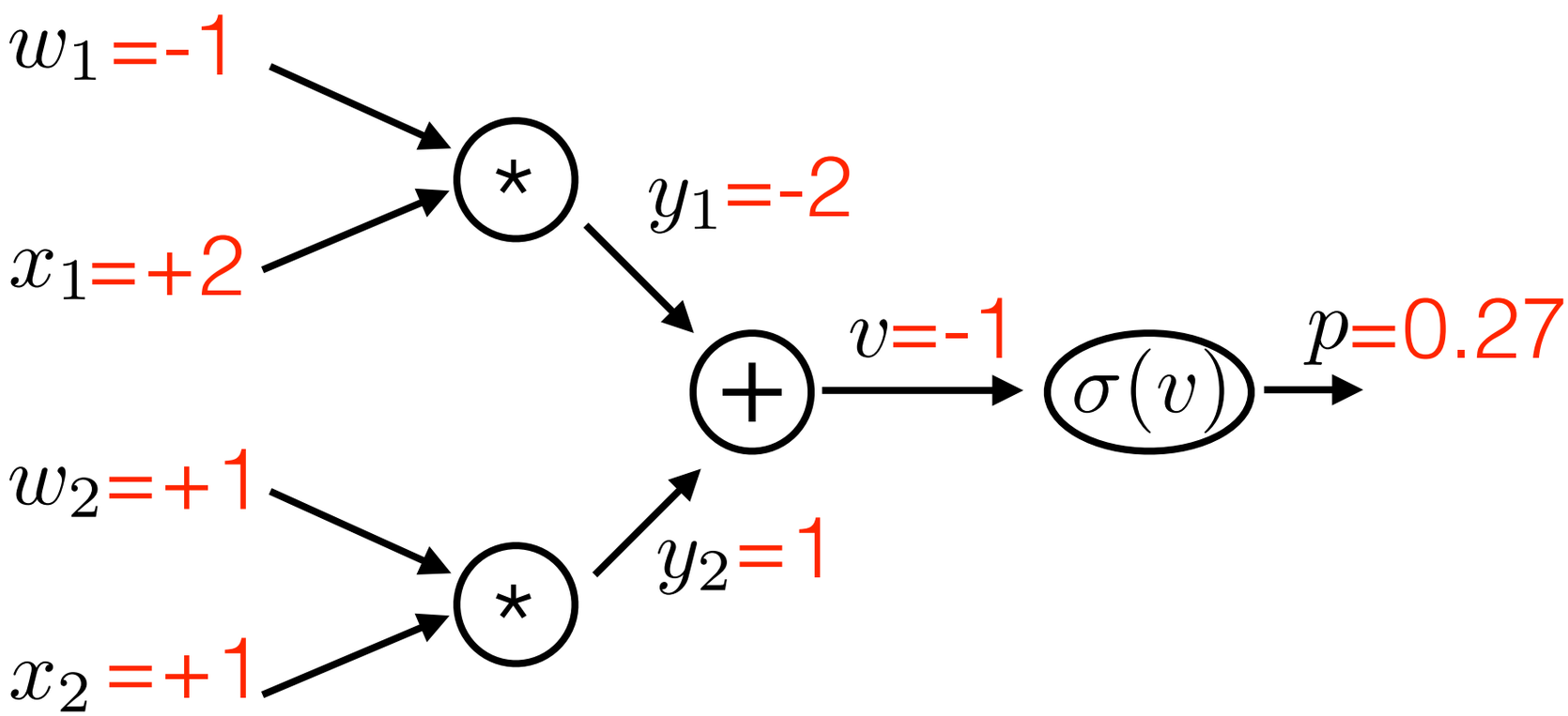
Computational graph of linear classifier



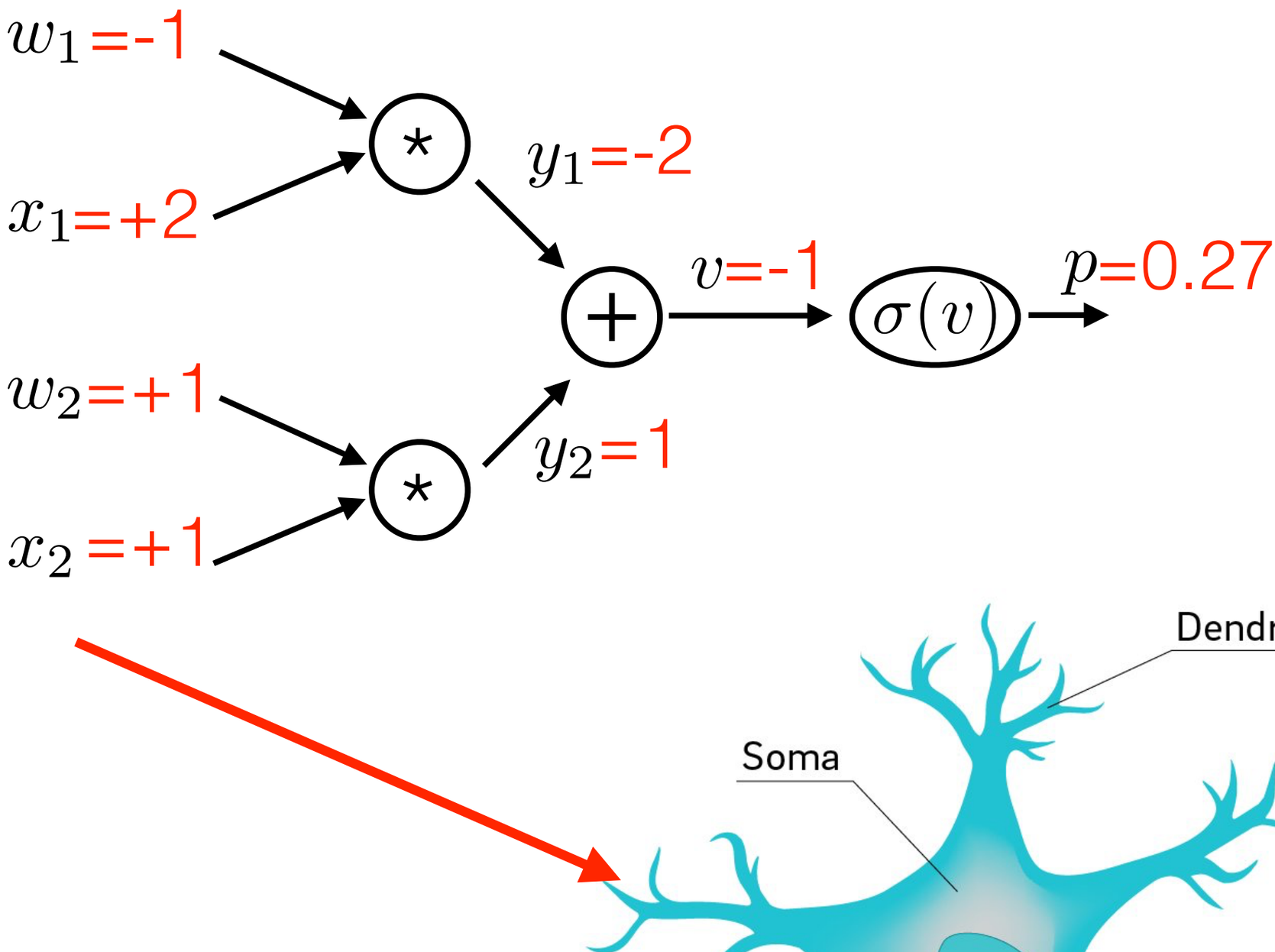
Example I: given trained neuron, and input, what is output?

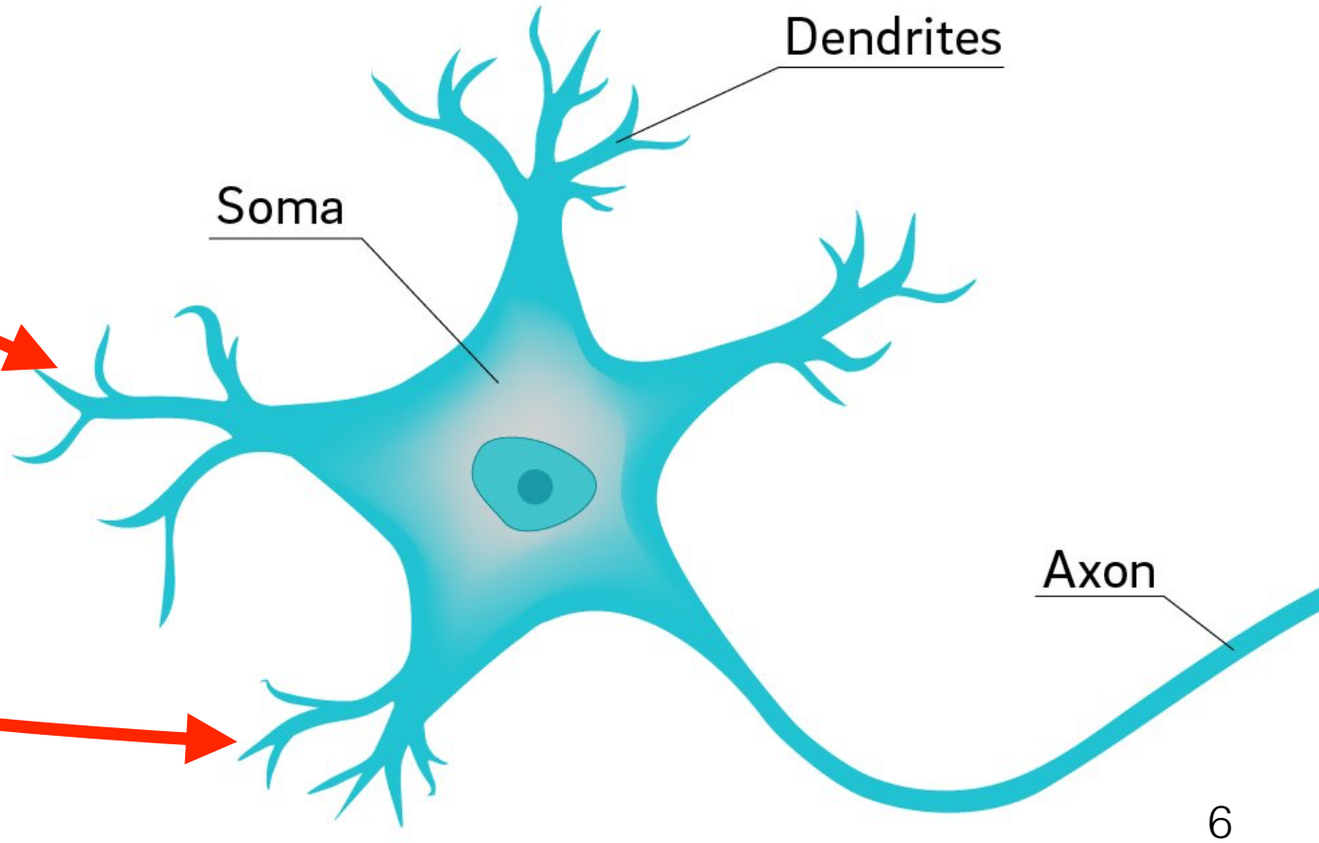


Example I: given trained classifier, and input, what is output?

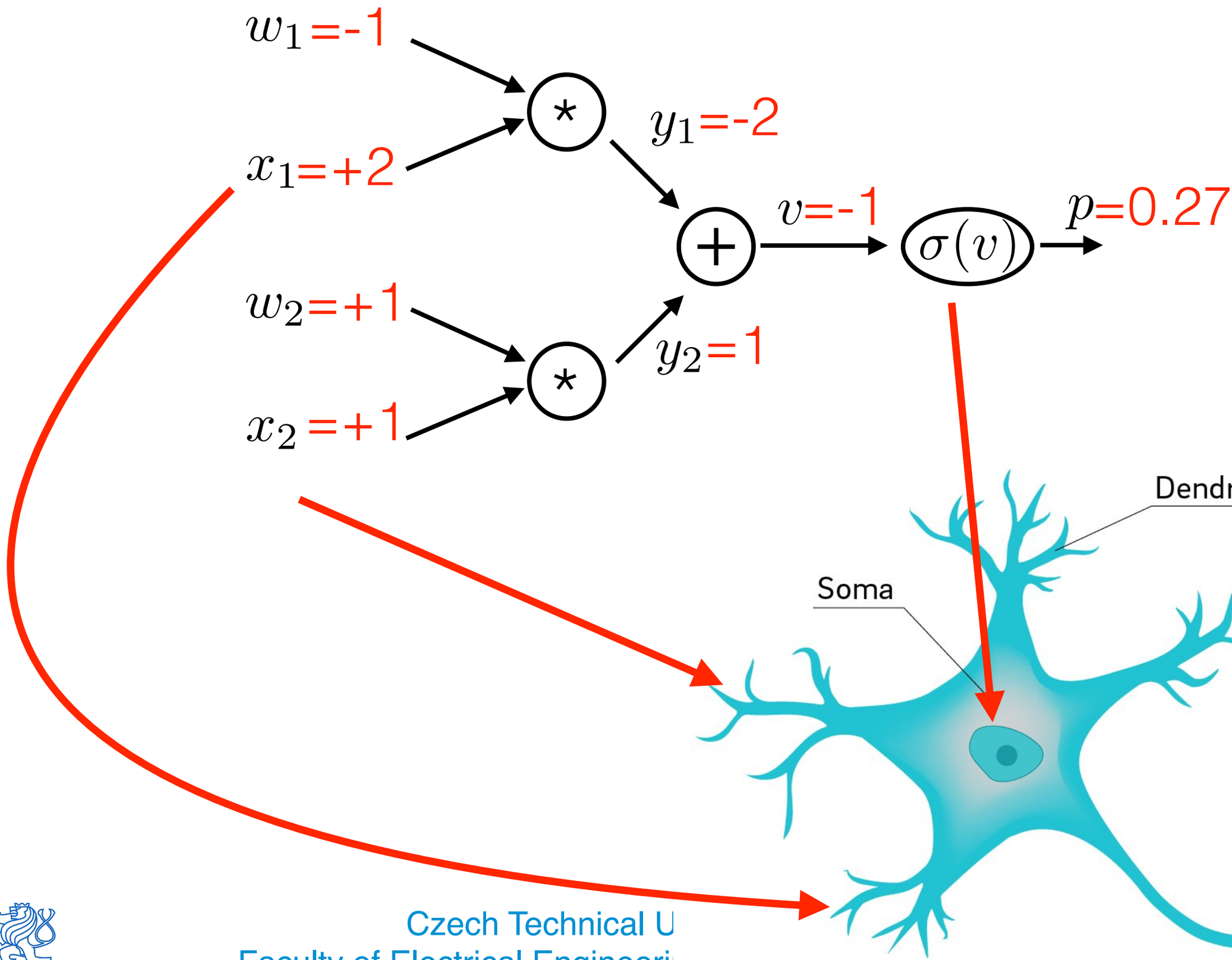


Relation to biological neuron

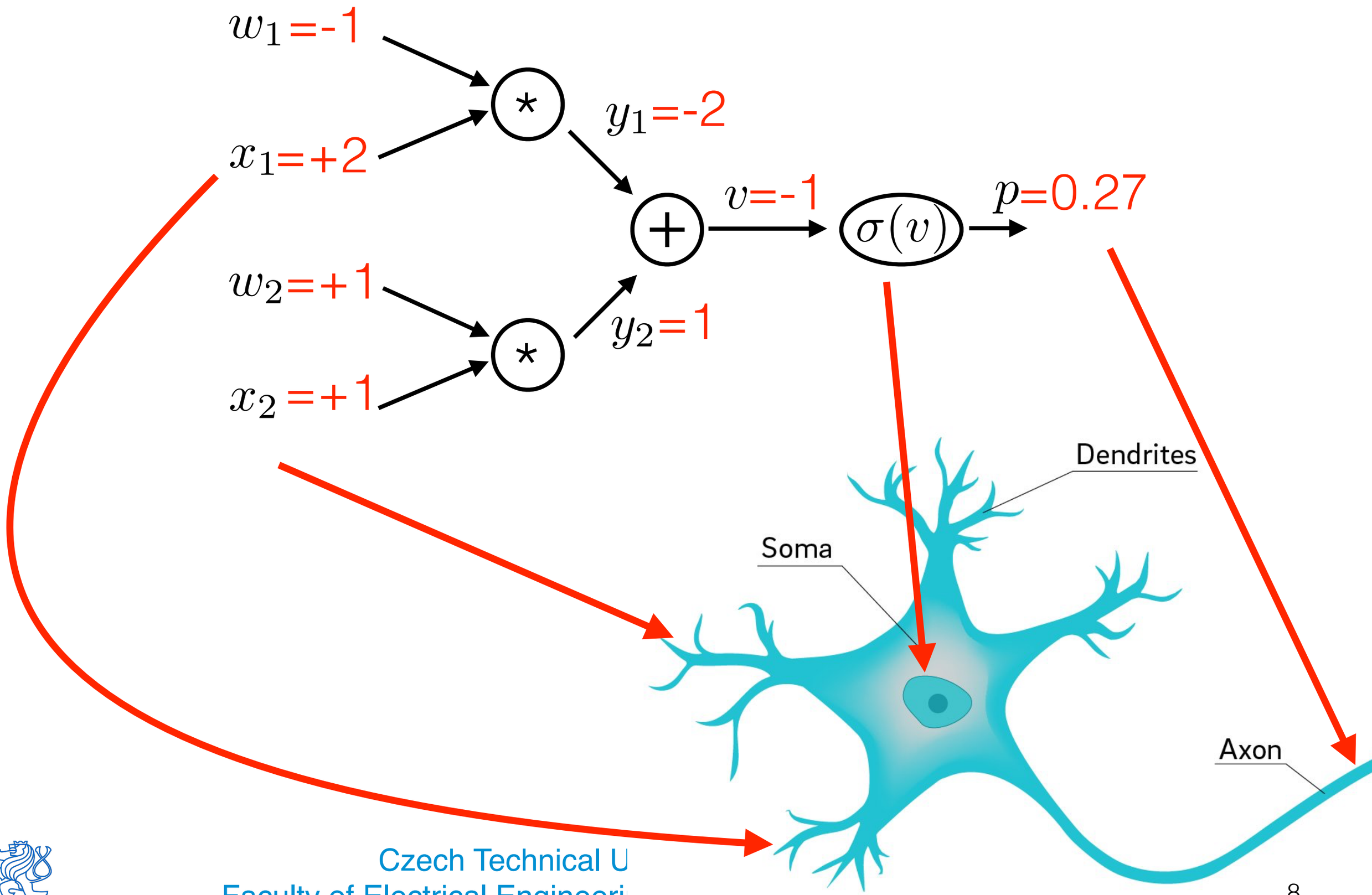




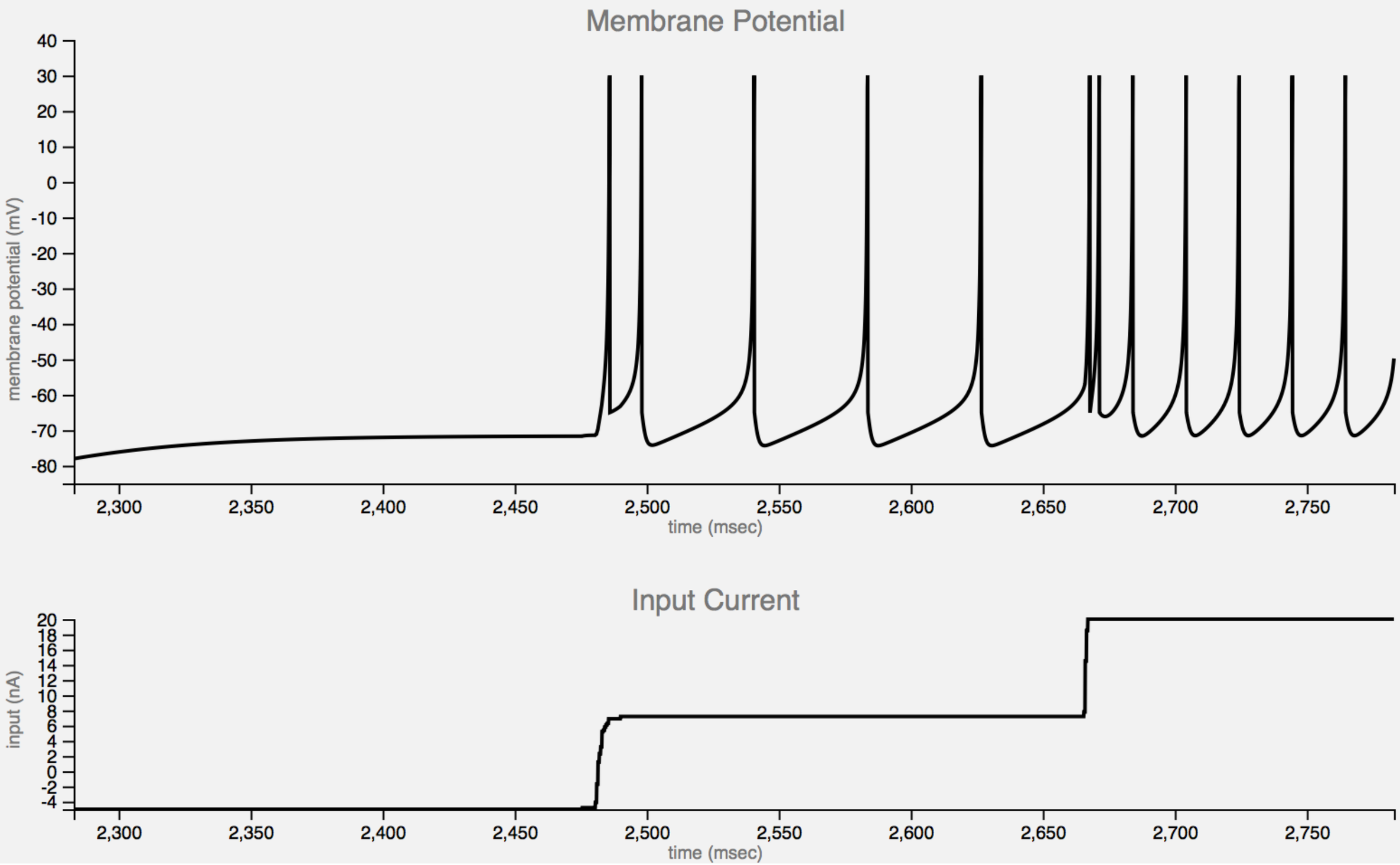
Relation to biological neuron



Relation to biological neuron



Modeling dynamic neuron behaviour



http://jackterwilliger.com/biological-neural-networks-part-i-spiking-neurons/

Linear classifier and neuron

Labels

RGB images

$+1$

-1

def classify():

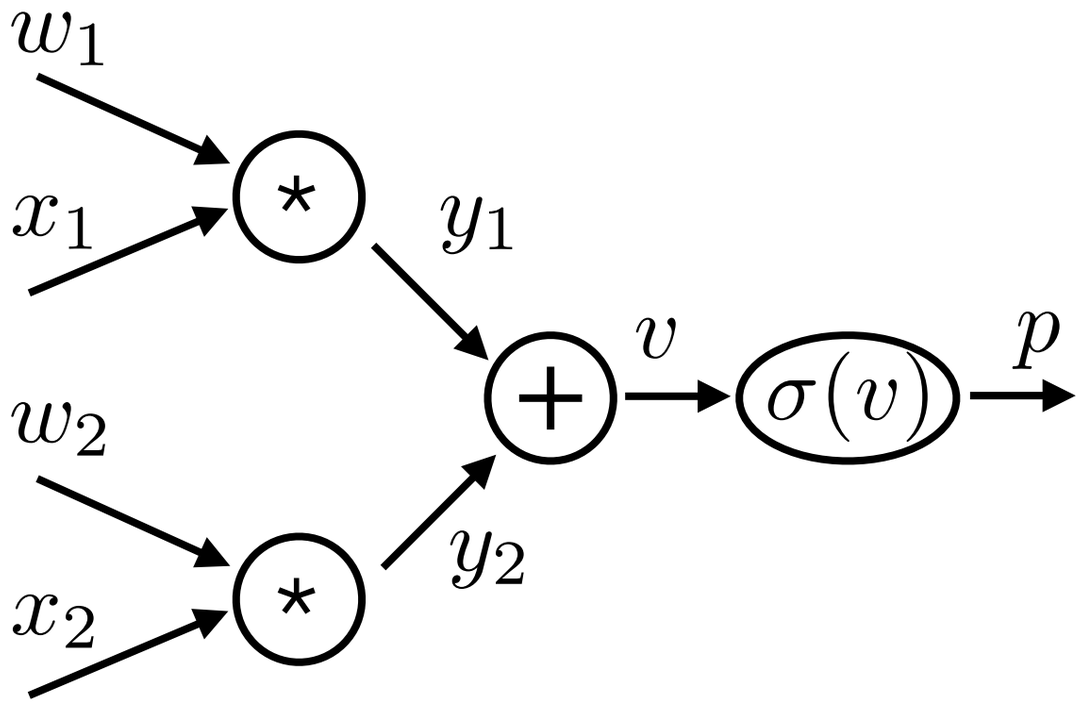
 # Linear classifier

$\mathbf{x} = \text{vec}($  $)$

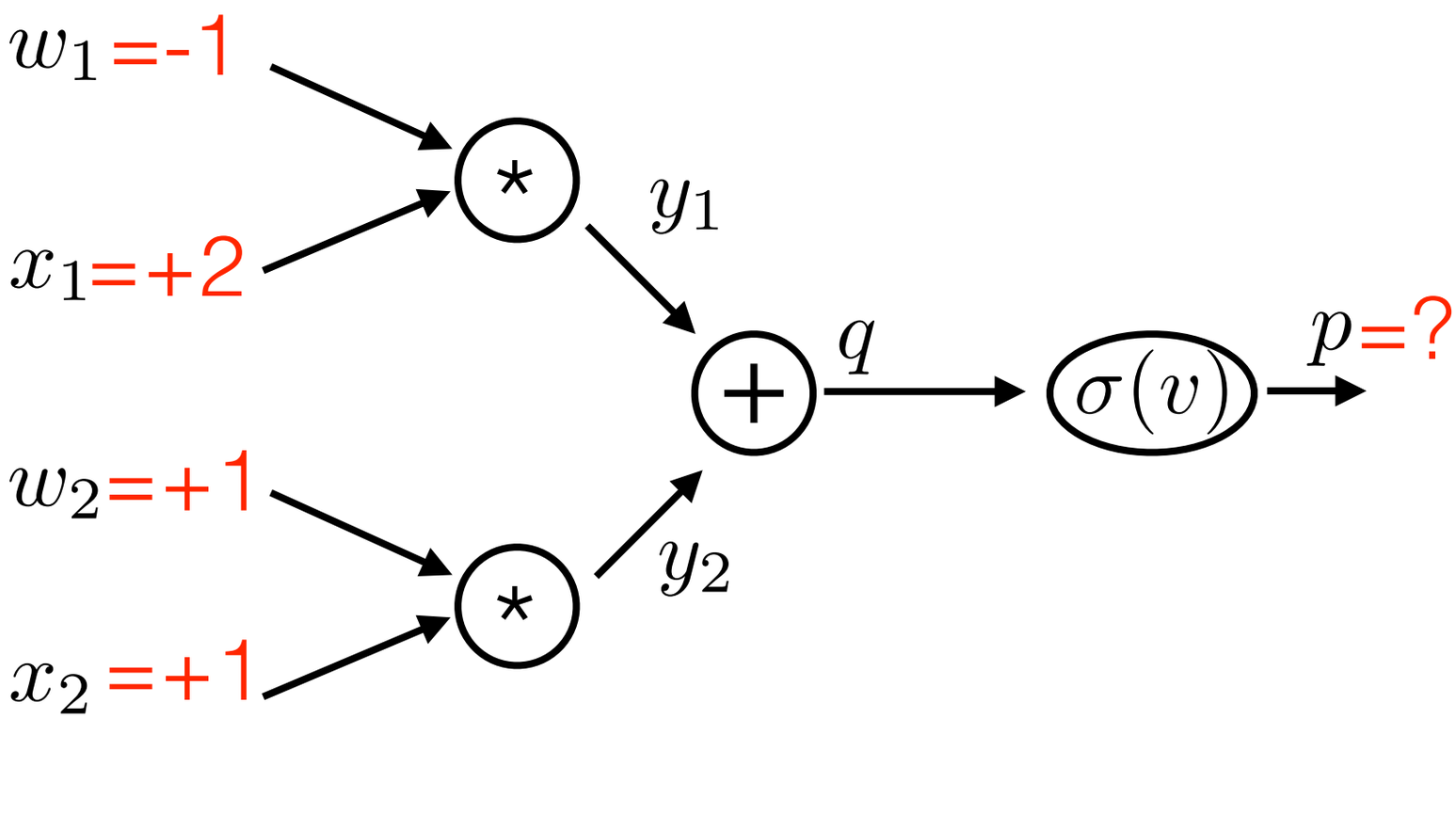
$p = \sigma\left(\mathbf{w}^\top \mathbf{x}\right)$

return p

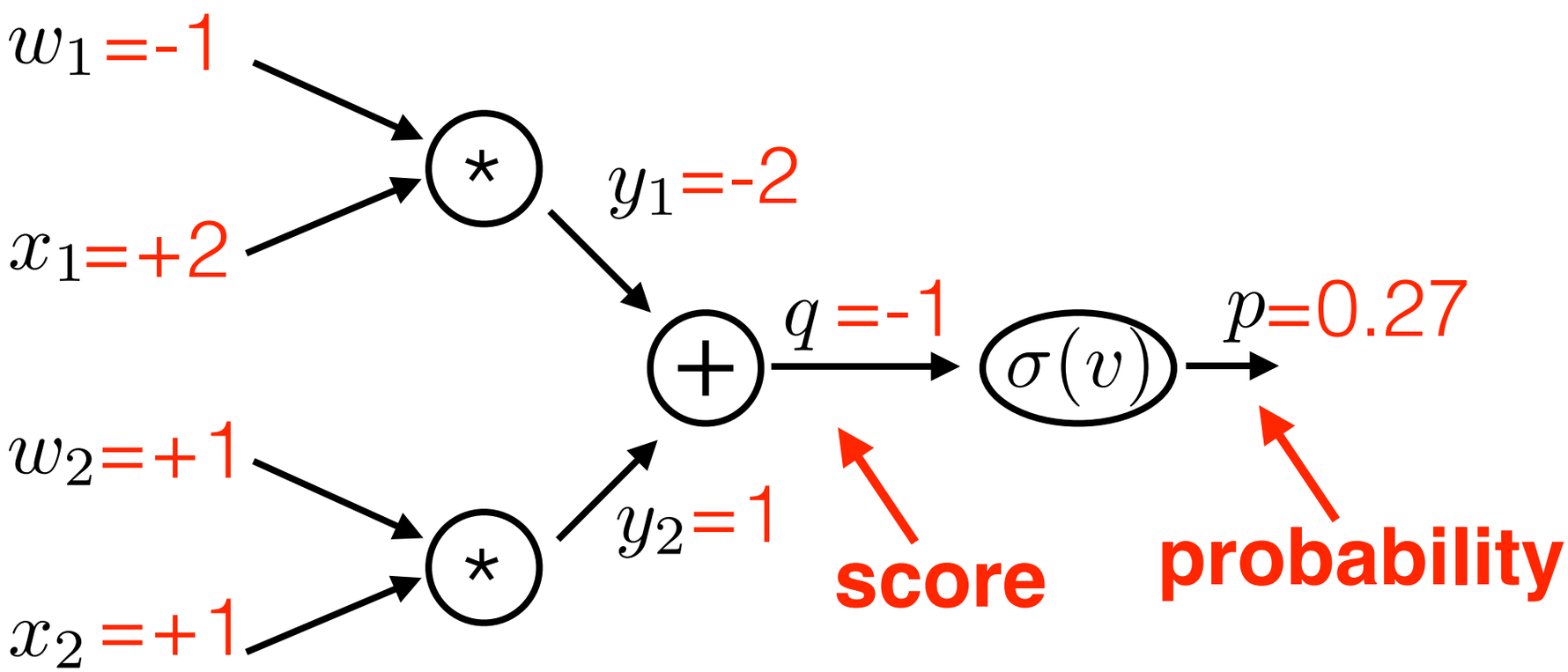
Computational graph of linear classifier



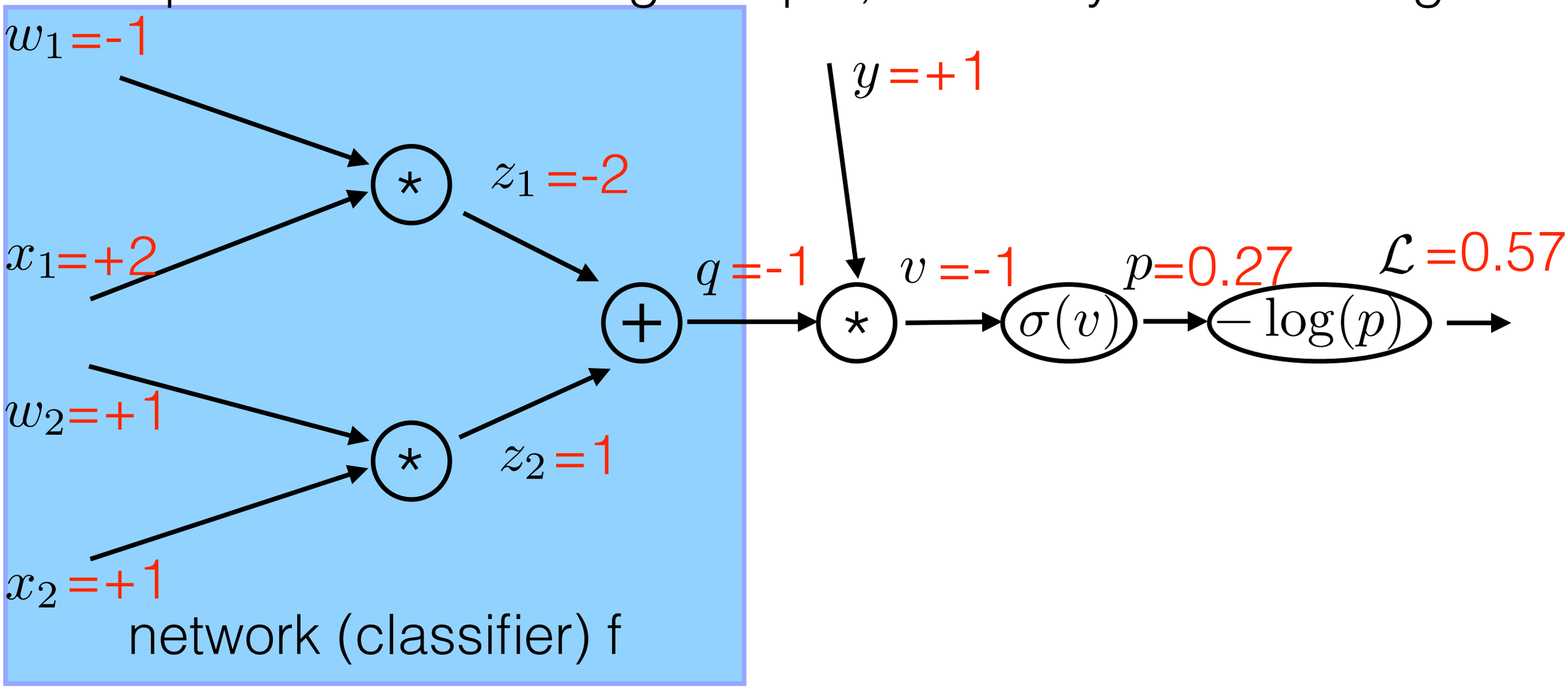
Example I: given trained neuron, and input, what is output?



Example I: given trained classifier, and input, what is output?

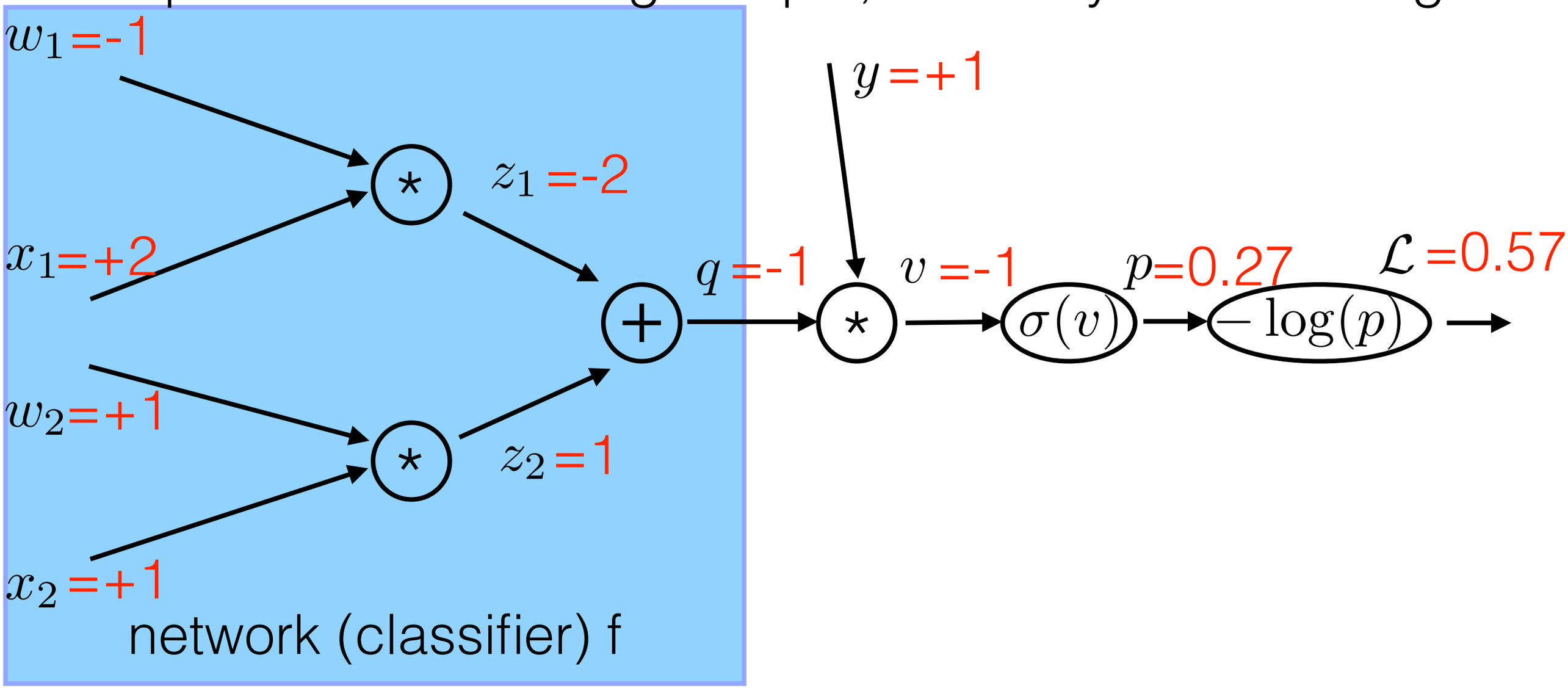


Example II: Given training sample, how do you learn weights?

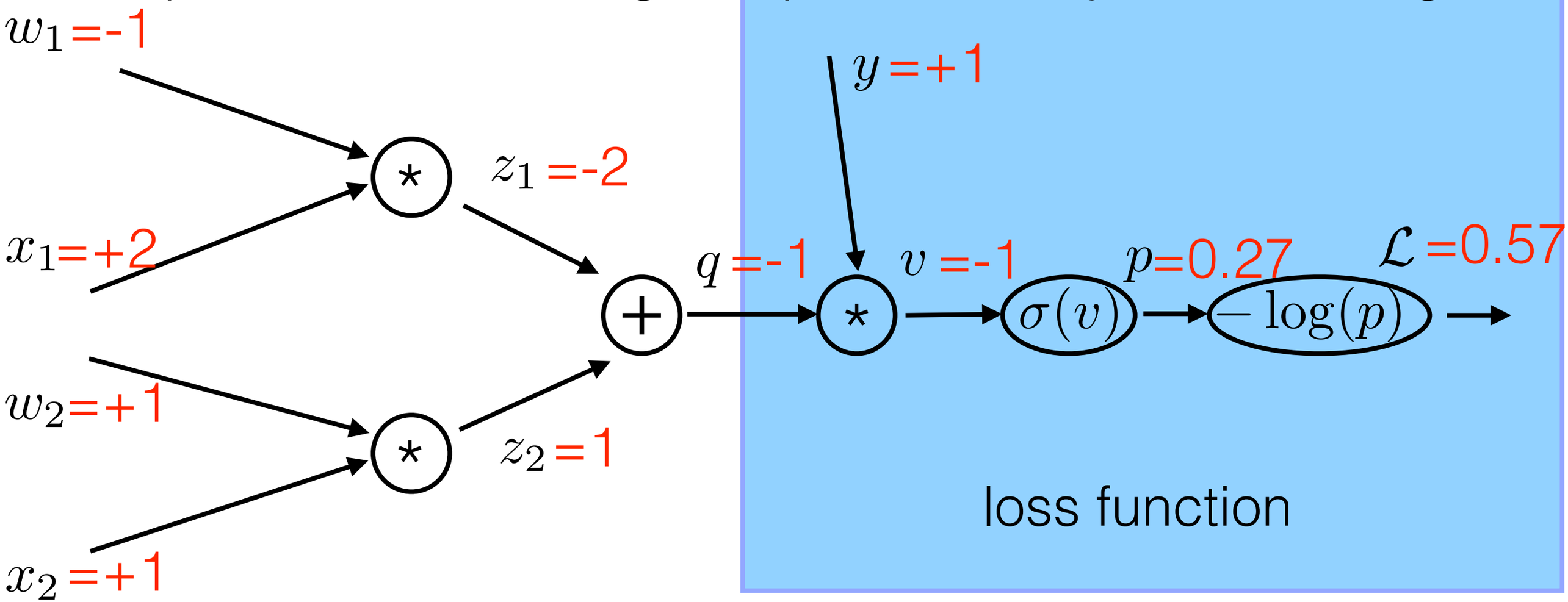


$$\arg\min_{\mathbf{w}} \Big(-\log\left[\sigma\left(y_i\, f(\mathbf{x}_i, \mathbf{w})\right)\right] \Big)$$

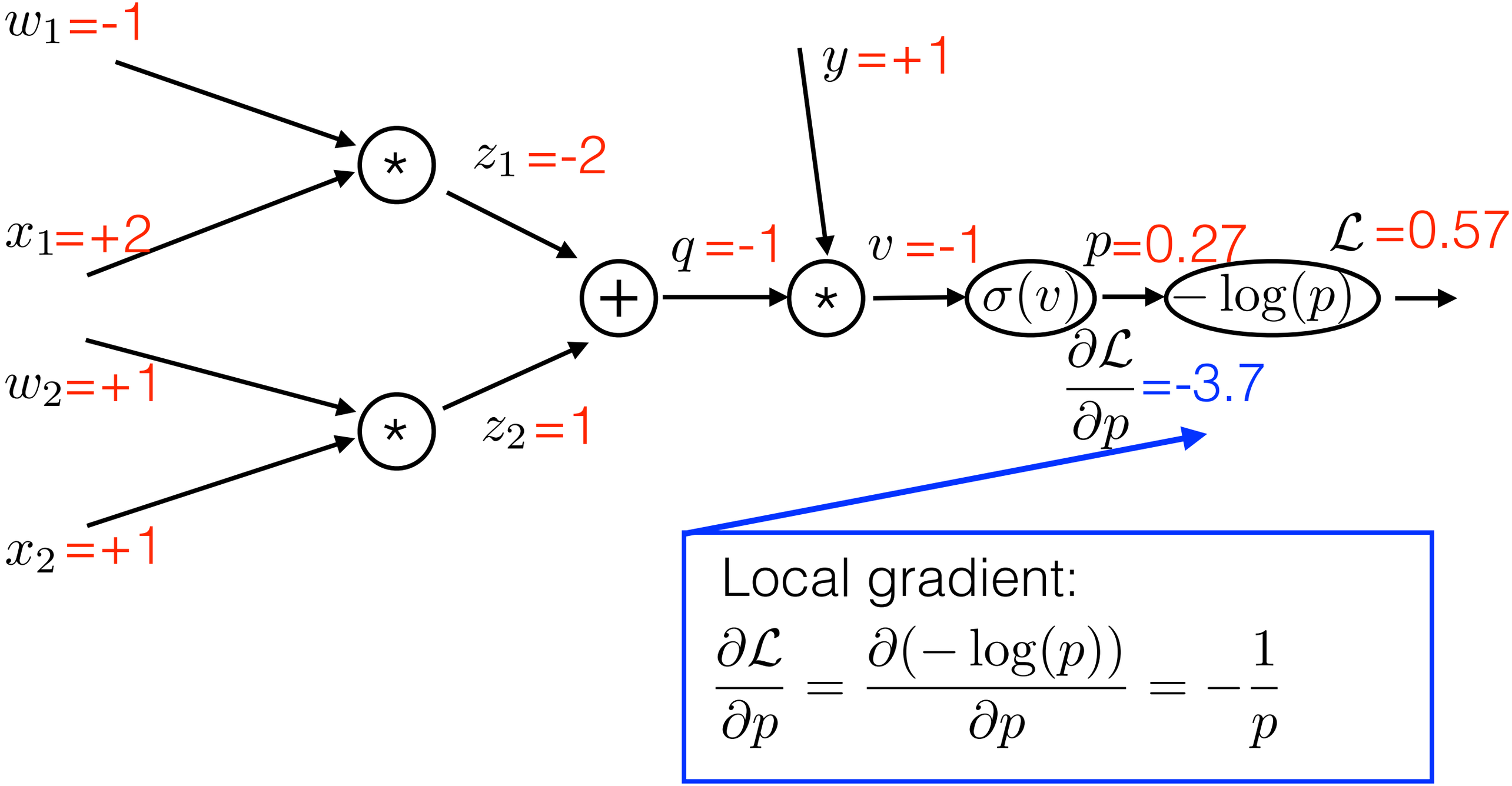
Example II: Given training sample, how do you learn weights?



Example II: Given training sample, how do you learn weights?



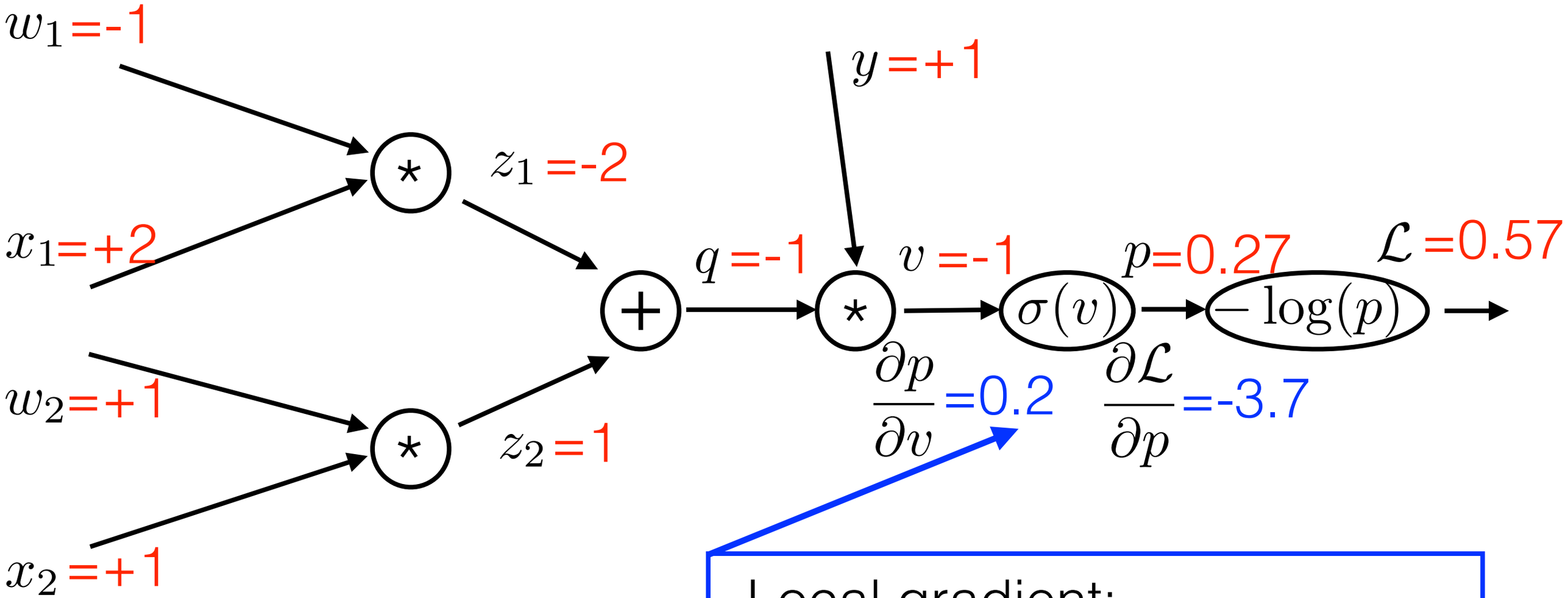
Example II: Given training sample, how do you learn weights?



Local gradient:

$$\frac{\partial \mathcal{L}}{\partial p} = \frac{\partial(-\log(p))}{\partial p} = -\frac{1}{p}$$

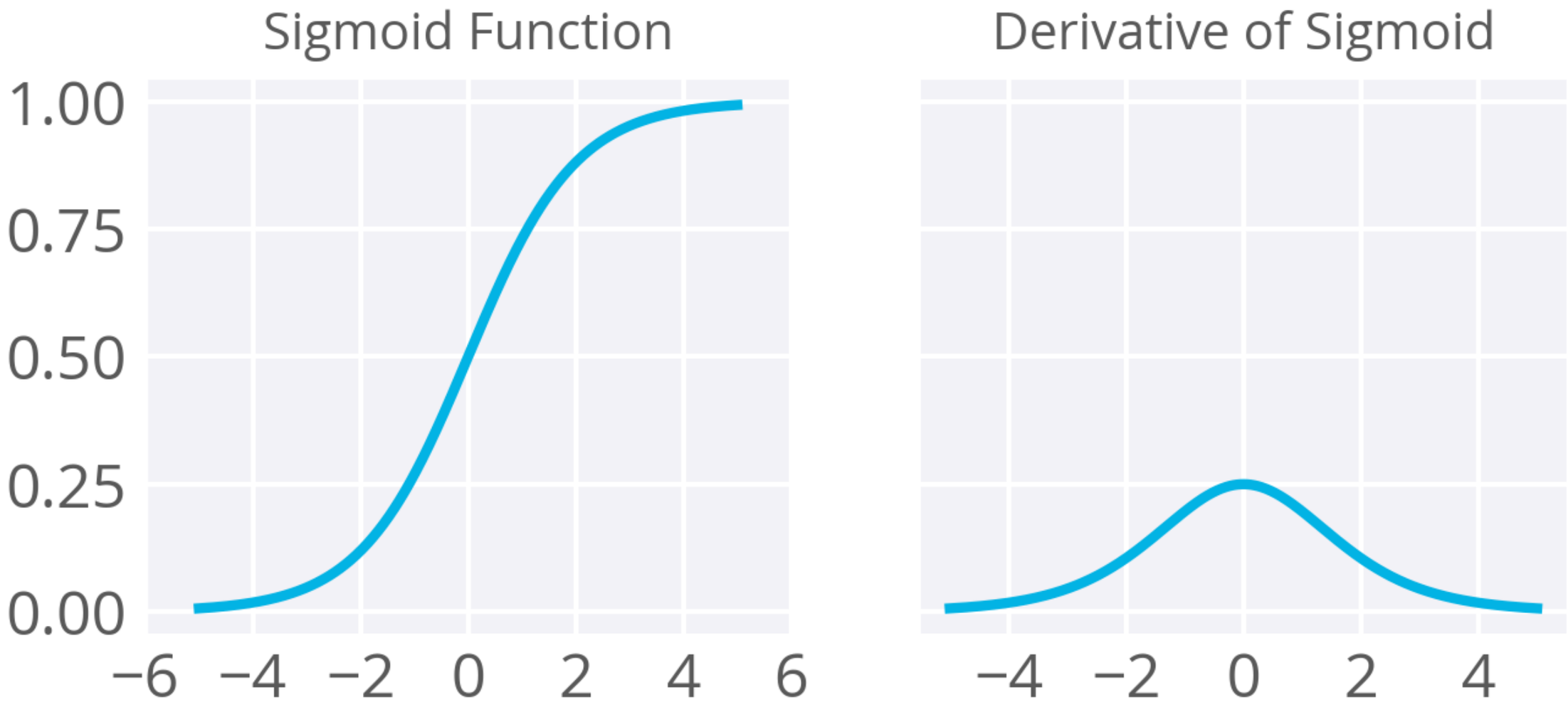
Example II: Given training sample, how do you learn weights?



Local gradient:

$$\frac{\partial p}{\partial v} = \frac{\partial \sigma(v)}{\partial v} = \sigma(v)(1 - \sigma(v))$$

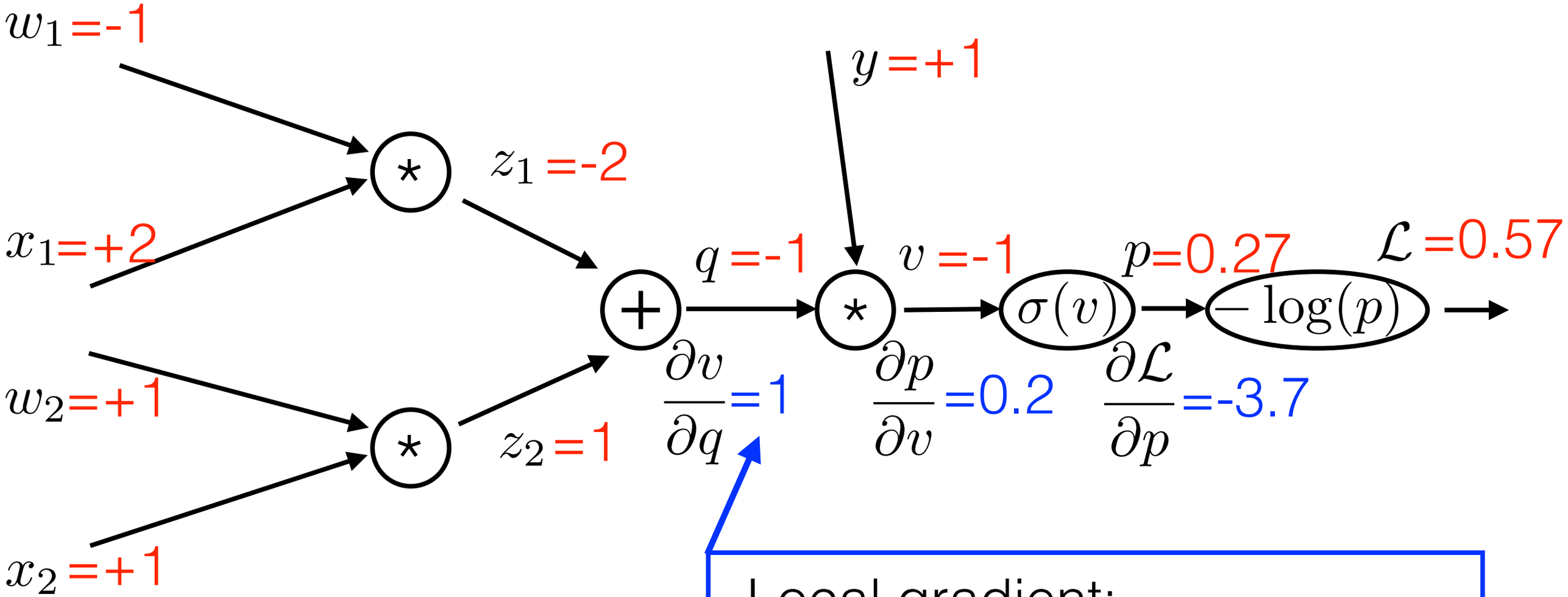
Example II: Given training sample, how do you learn weights?



Local gradient:

$$\frac{\partial p}{\partial v} = \frac{\partial \sigma(v)}{\partial v} = \sigma(v)(1 - \sigma(v))$$

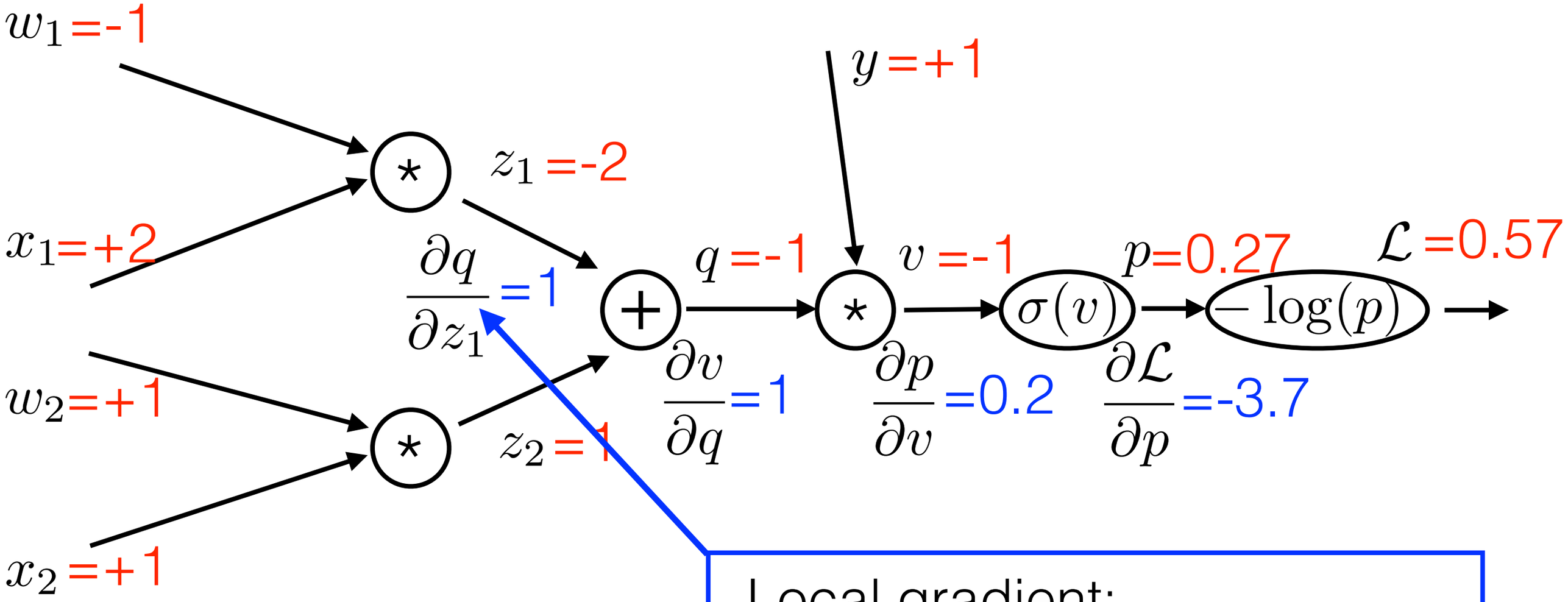
Example II: Given training sample, how do you learn weights?



Local gradient:

$$\frac{\partial v}{\partial q} = \frac{\partial (yq)}{\partial q} = y$$

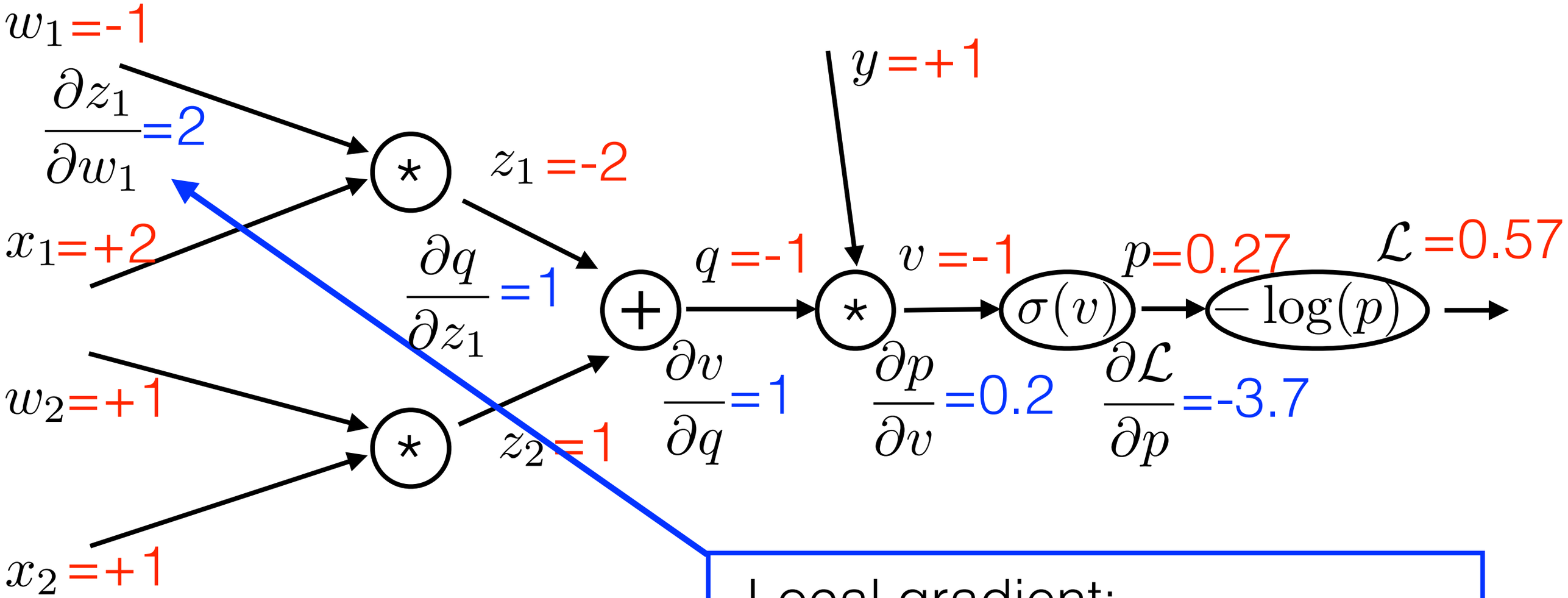
Example II: Given training sample, how do you learn weights?



Local gradient:

$$\frac{\partial q}{\partial z_1} = \frac{\partial (z_1 + z_2)}{\partial z_1} = 1$$

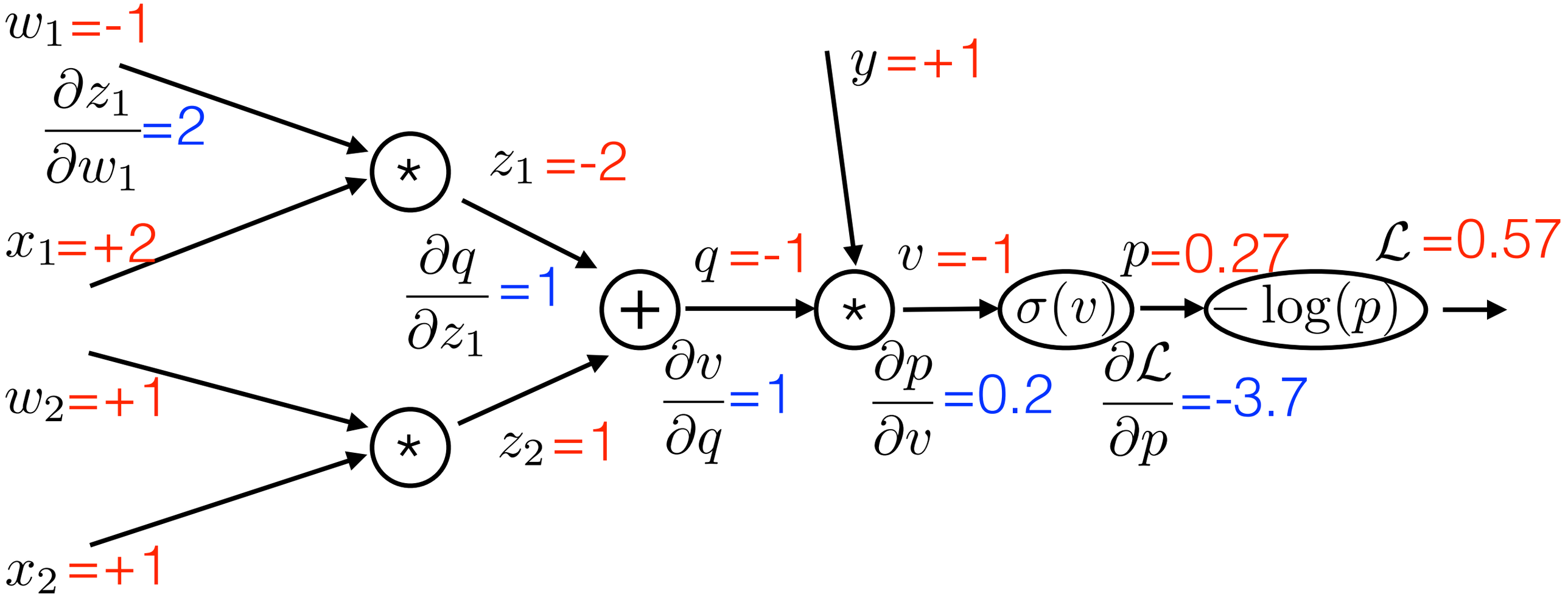
Example II: Given training sample, how do you learn weights?



Local gradient:

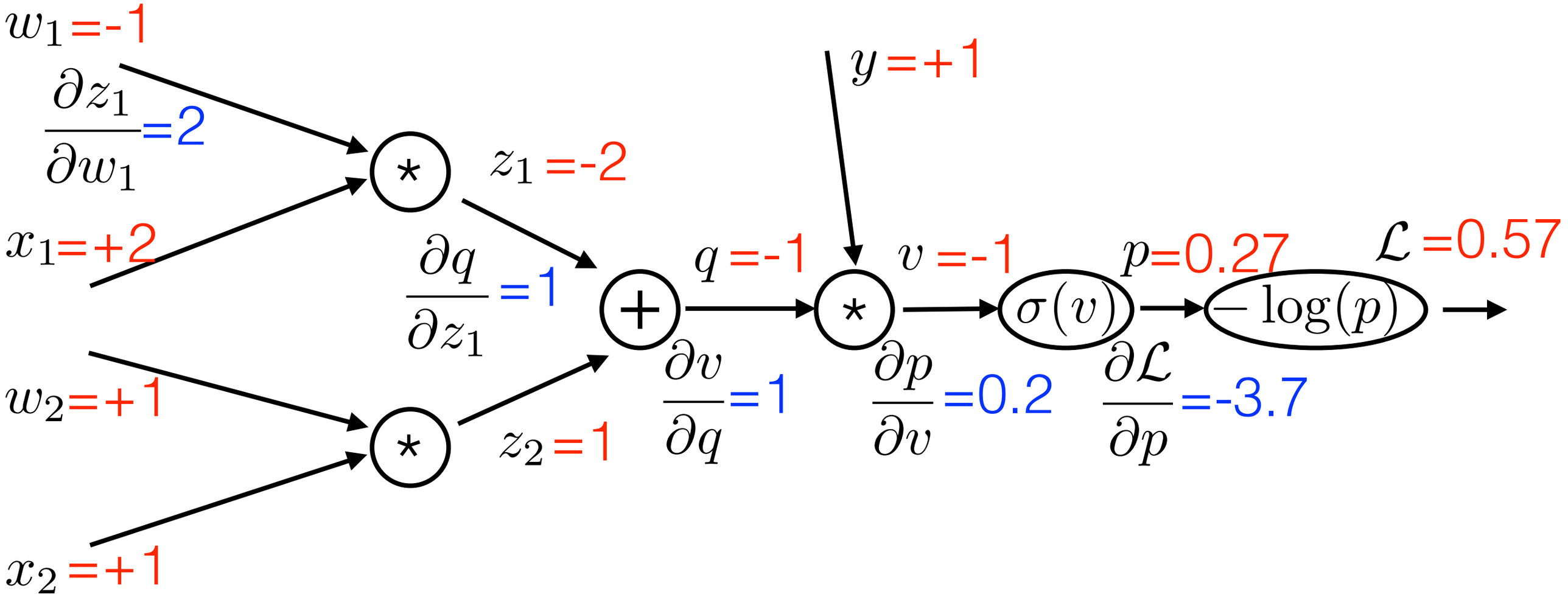
$$\frac{\partial z_1}{\partial w_1} = \frac{\partial (w_1 x_1)}{\partial w_1} = x_1$$

Example II: Given training sample, how do you learn weights?



$$\frac{\partial \mathcal{L}}{\partial w_1} = \frac{\partial \mathcal{L}}{\partial p}\frac{\partial p}{\partial v}\frac{\partial v}{\partial q}\frac{\partial q}{\partial z_1}\frac{\partial z_1}{\partial w_1} = -3.7*0.2*1*1*2 = -1.48$$

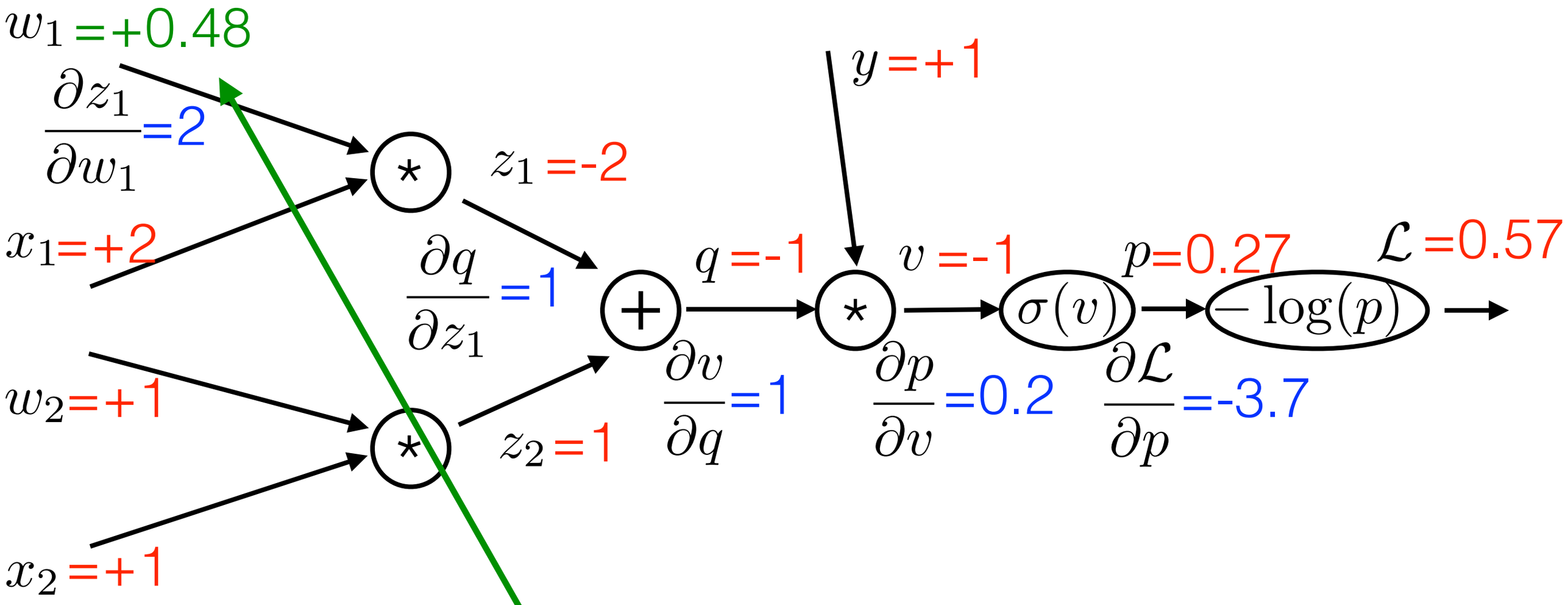
Example II: Given training sample, how do you learn weights?



$$\frac{\partial \mathcal{L}}{\partial w_1} = \frac{\partial \mathcal{L}}{\partial p}\frac{\partial p}{\partial v}\frac{\partial v}{\partial q}\frac{\partial q}{\partial z_1}\frac{\partial z_1}{\partial w_1} = -3.7*0.2*1*1*2 = -1.48$$

$$w_1 = w_1 - \alpha\frac{\partial \mathcal{L}}{\partial w1} = +0.48$$

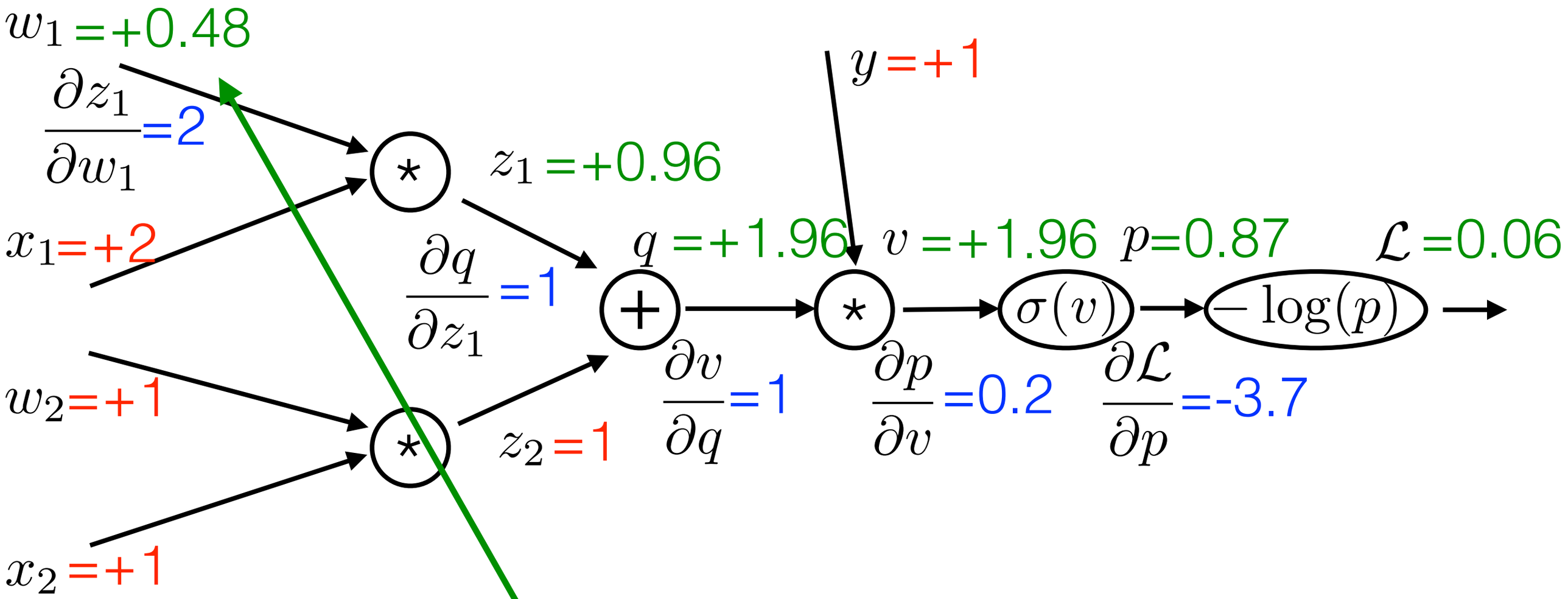
Example II: Given training sample, how do you learn weights?



$$\frac{\partial \mathcal{L}}{\partial w_1} = \frac{\partial \mathcal{L}}{\partial p}\frac{\partial p}{\partial v}\frac{\partial v}{\partial q}\frac{\partial q}{\partial z_1}\frac{\partial z_1}{\partial w_1} = -3.7*0.2*1*1*2 = -1.48$$

$$w_1 = w_1 - \alpha\frac{\partial \mathcal{L}}{\partial w1} = +0.48$$

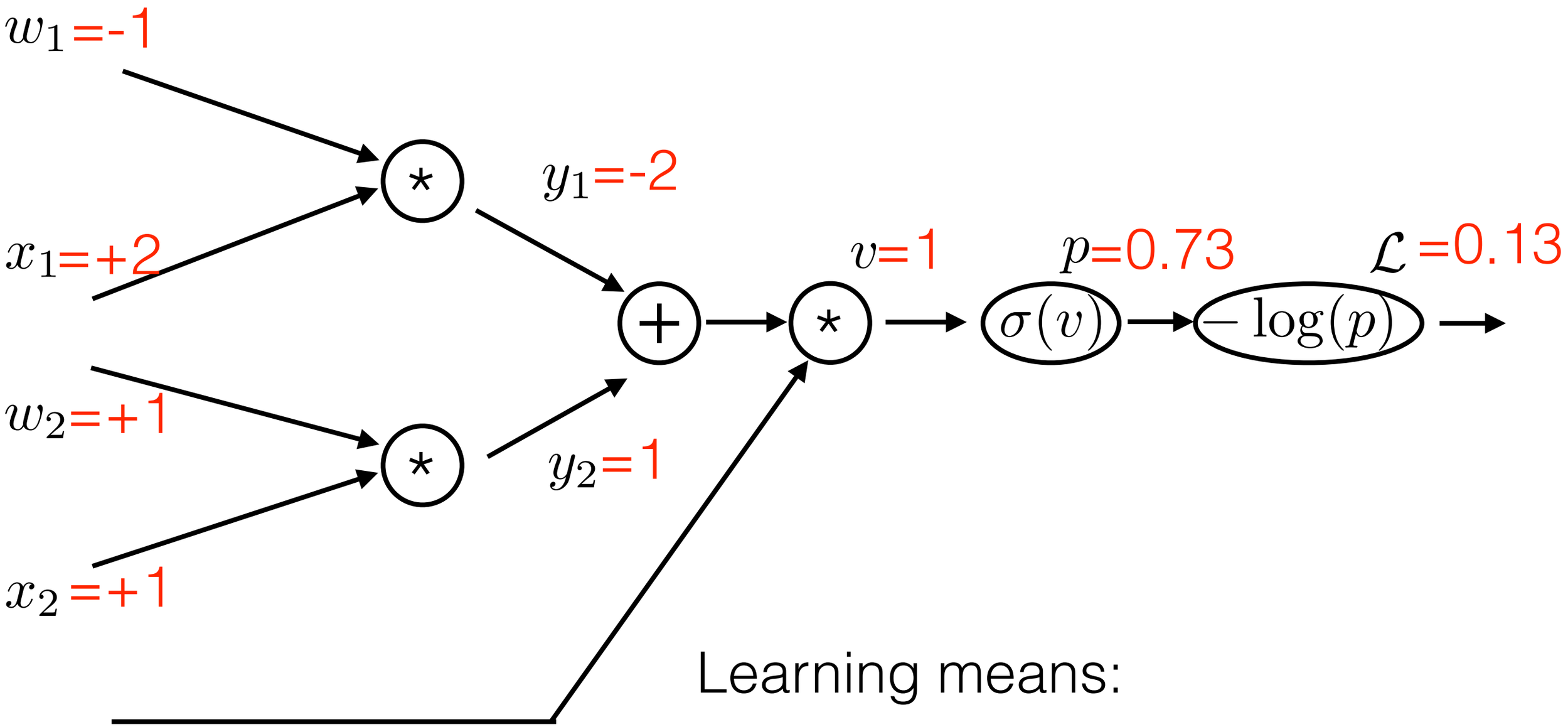
Example II: Given training sample, how do you learn weights?



$$\frac{\partial \mathcal{L}}{\partial w_1} = \frac{\partial \mathcal{L}}{\partial p}\frac{\partial p}{\partial v}\frac{\partial v}{\partial q}\frac{\partial q}{\partial z_1}\frac{\partial z_1}{\partial w_1} \text{=-3.7*0.2*1*1*2 =-1.48}$$

$$w_1 = w_1 - \alpha\frac{\partial \mathcal{L}}{\partial w1} \text{ = +0.48}$$

Example III: vector representation

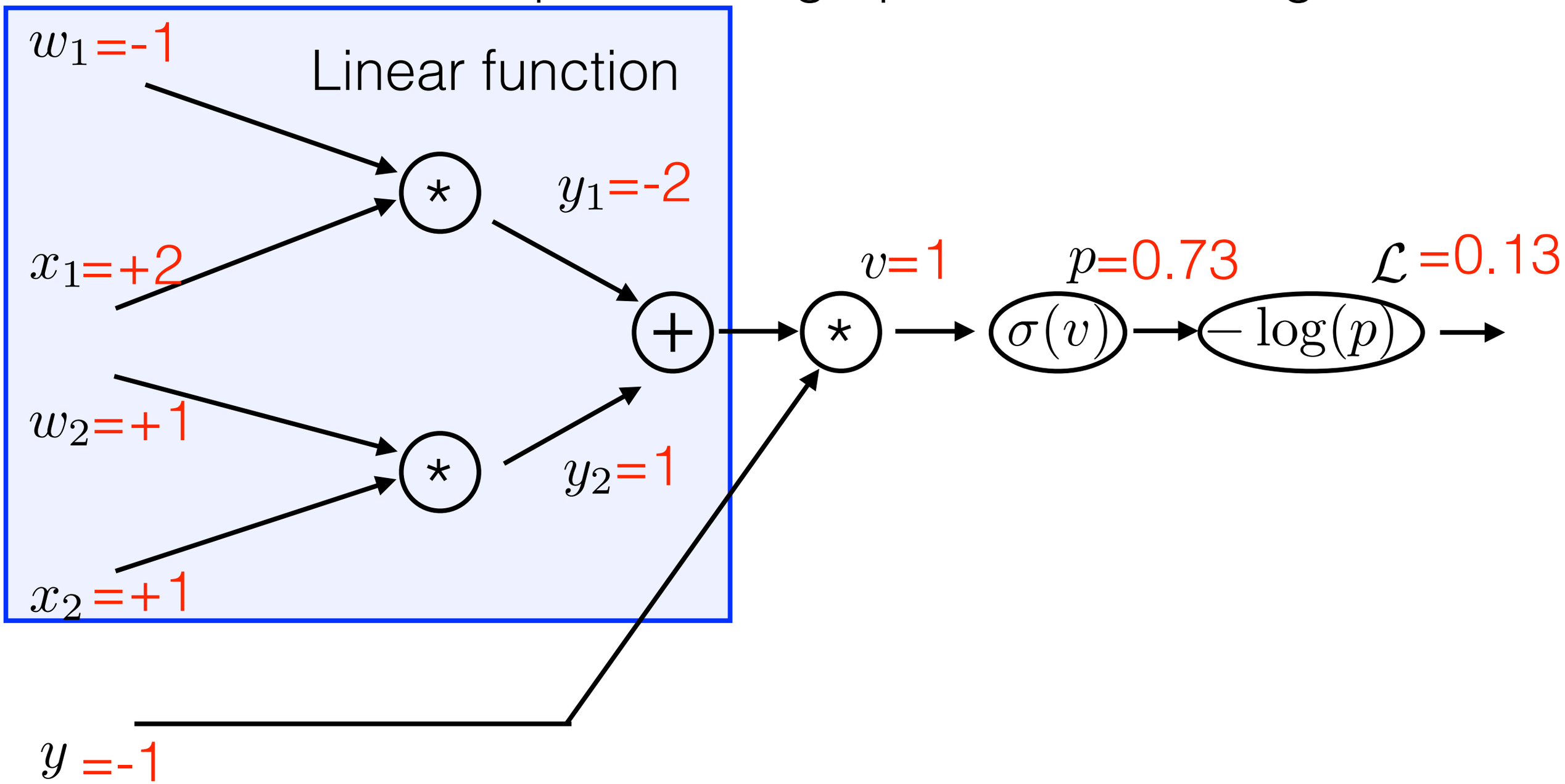


Learning means:

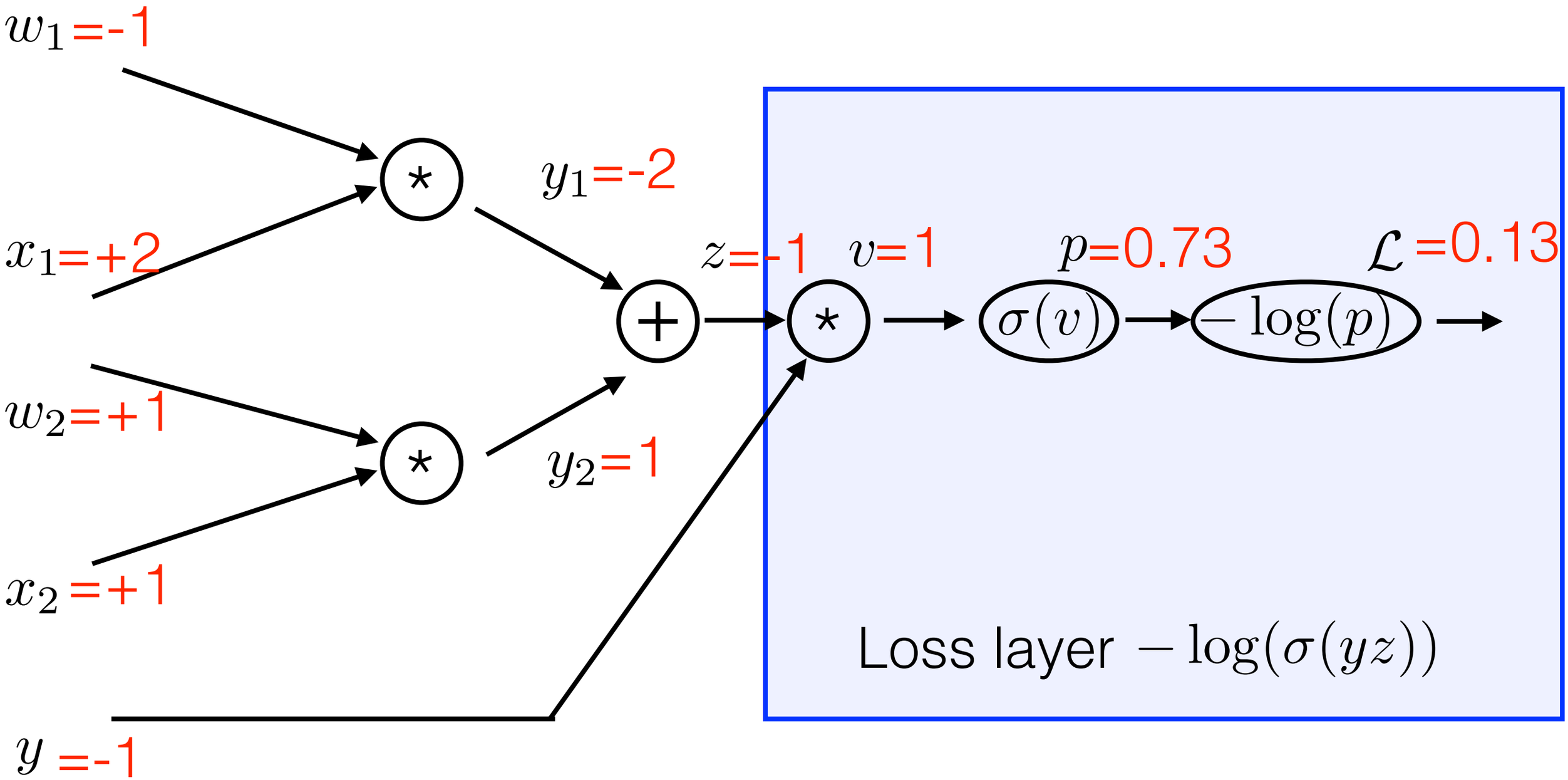
Iteratively change all weights $\mathbf{w}$ to minimize $\mathcal{L}$

$$\mathbf{w} = \mathbf{w} - \alpha \left[\frac{\partial \mathcal{L}(\mathbf{w})}{\partial \mathbf{w}}\right]^\top \quad \text{where} \quad \frac{\partial \mathcal{L}}{\partial \mathbf{w}} = \left[\frac{\partial \mathcal{L}}{\partial w_1}, \frac{\partial \mathcal{L}}{\partial w_2}, \dots\right]$$

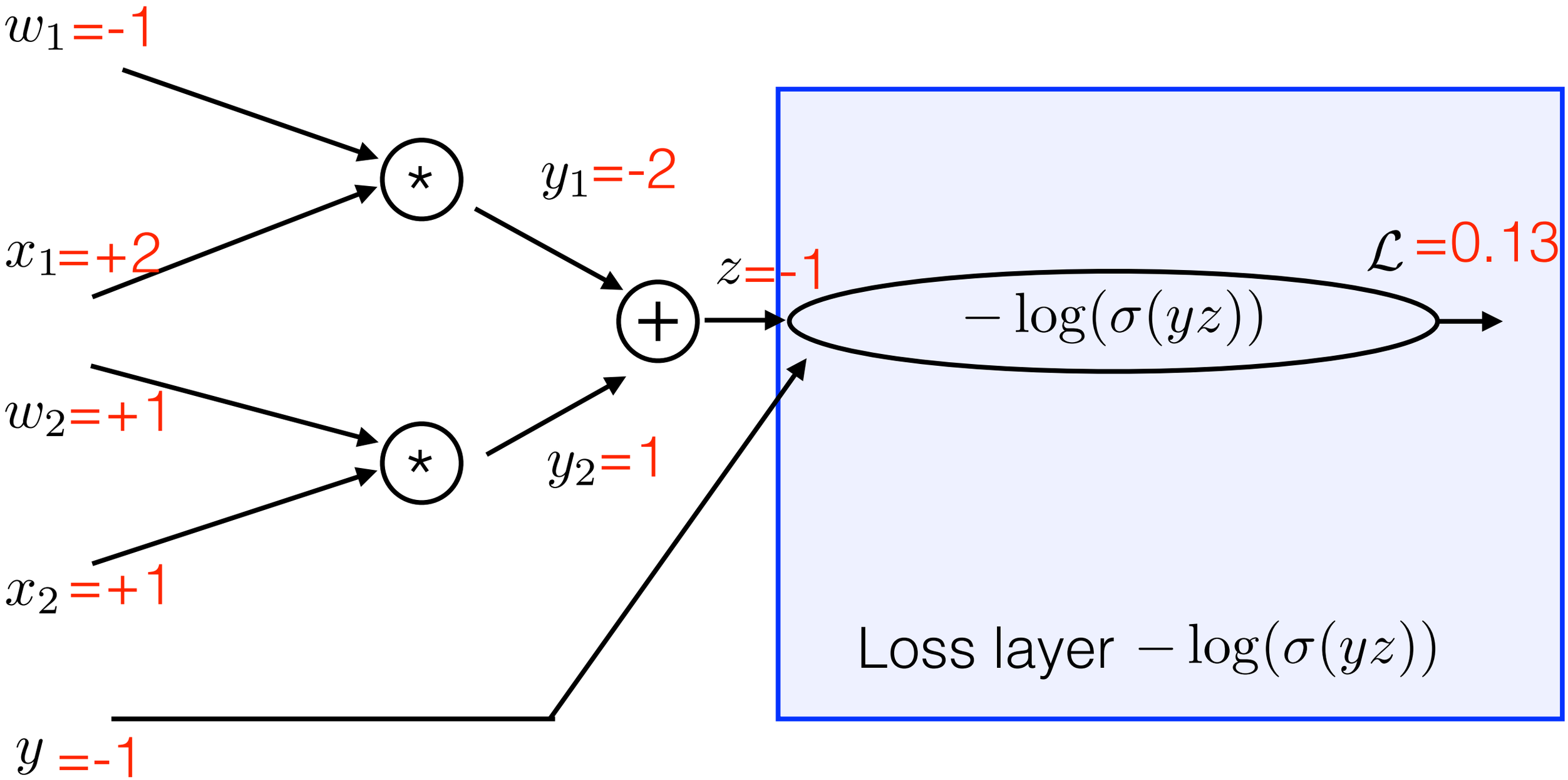
Computational graph of the learning



Computational graph of the learning



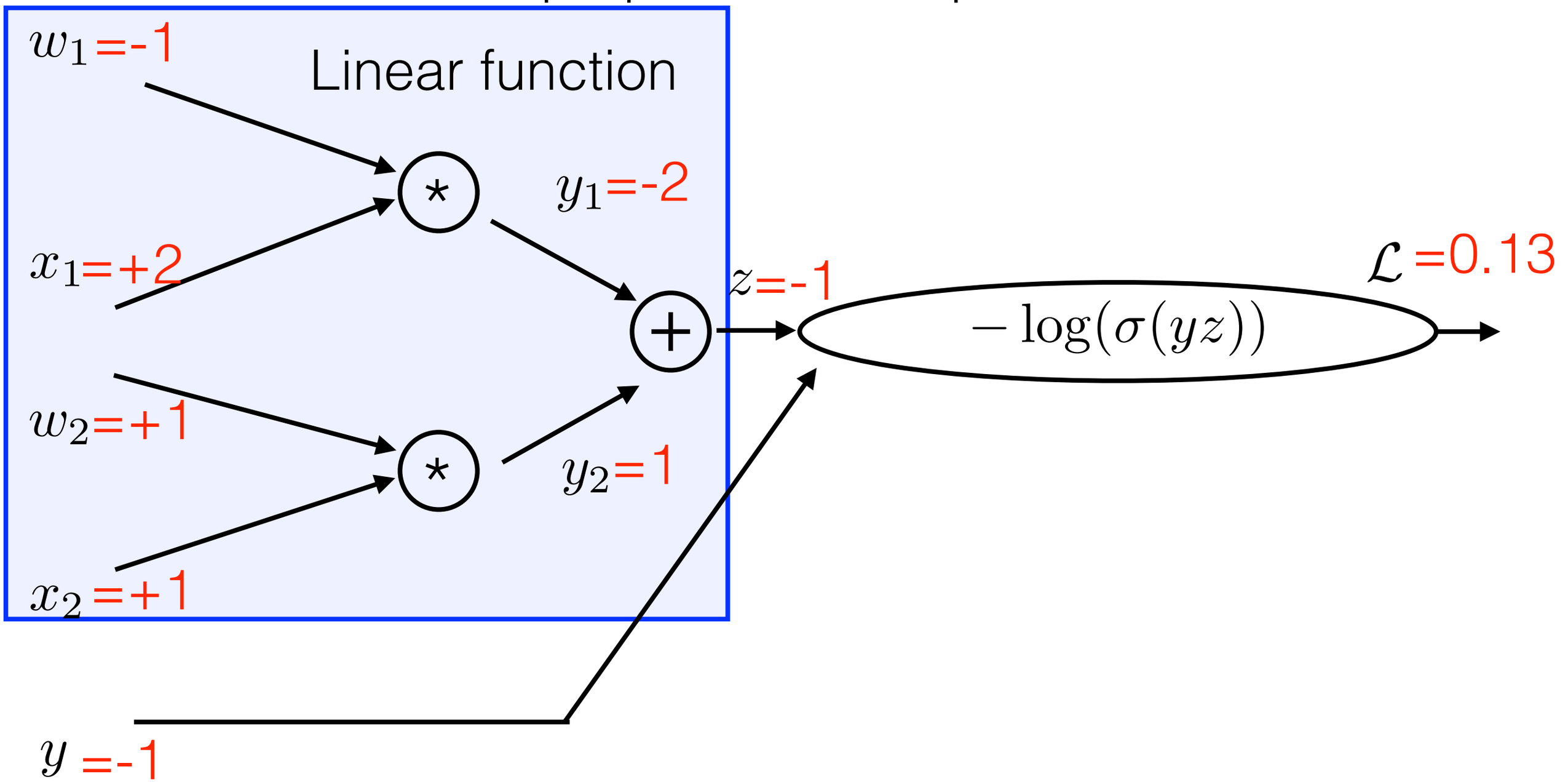
Backprop in vector representation



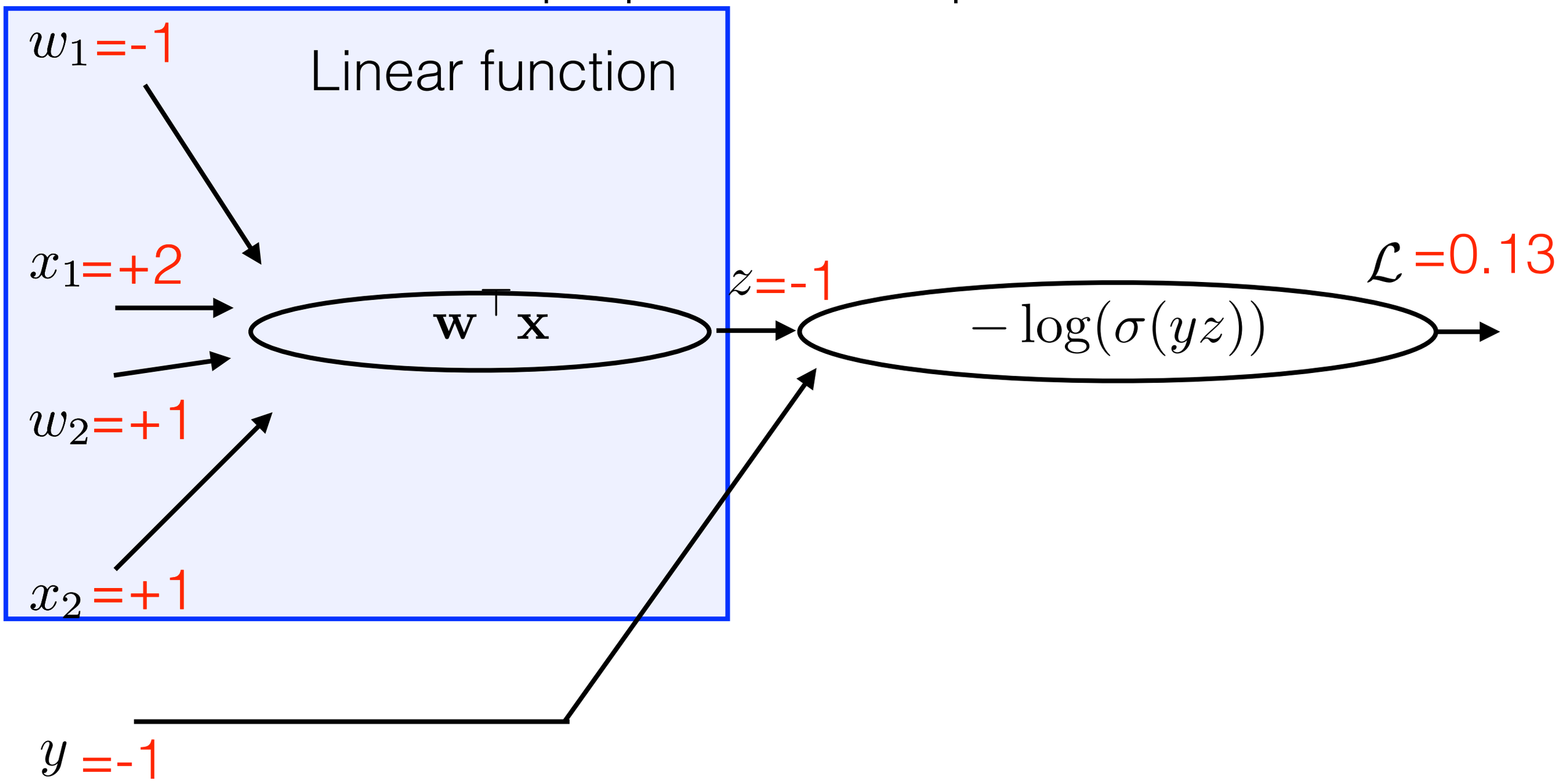
This is the logistic loss!

$$\mathcal{L}(y, z) = -\log(\sigma(yz)) = \log(1 + \exp(-yz))$$

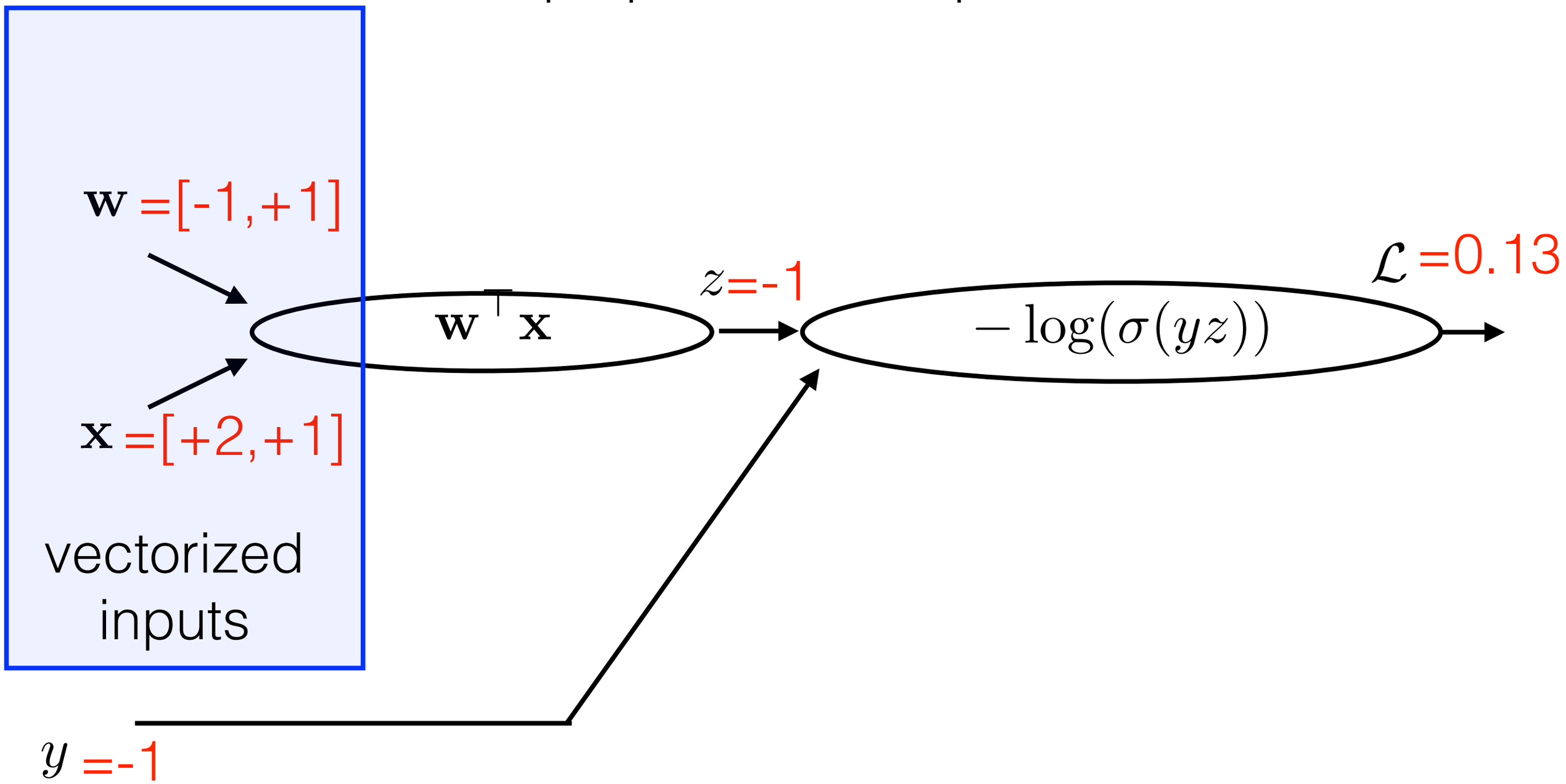
Backprop in vector representation



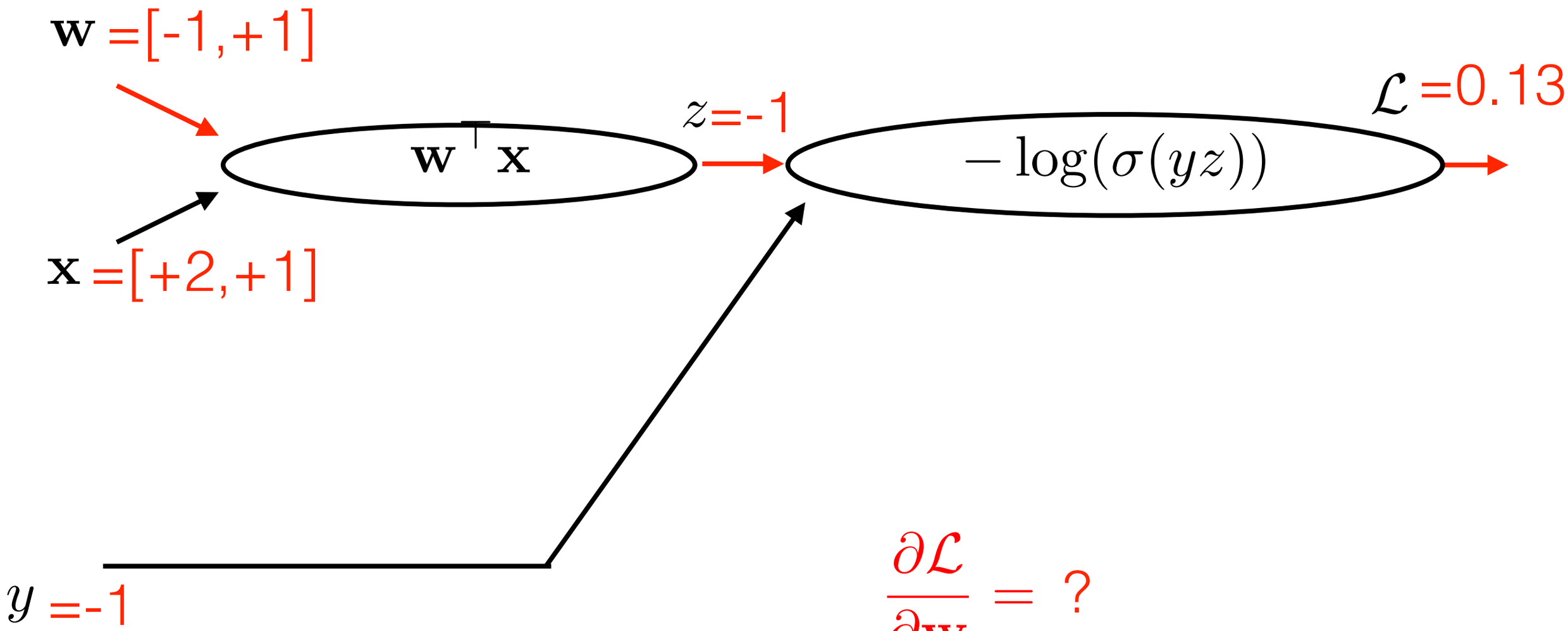
Backprop in vector representation



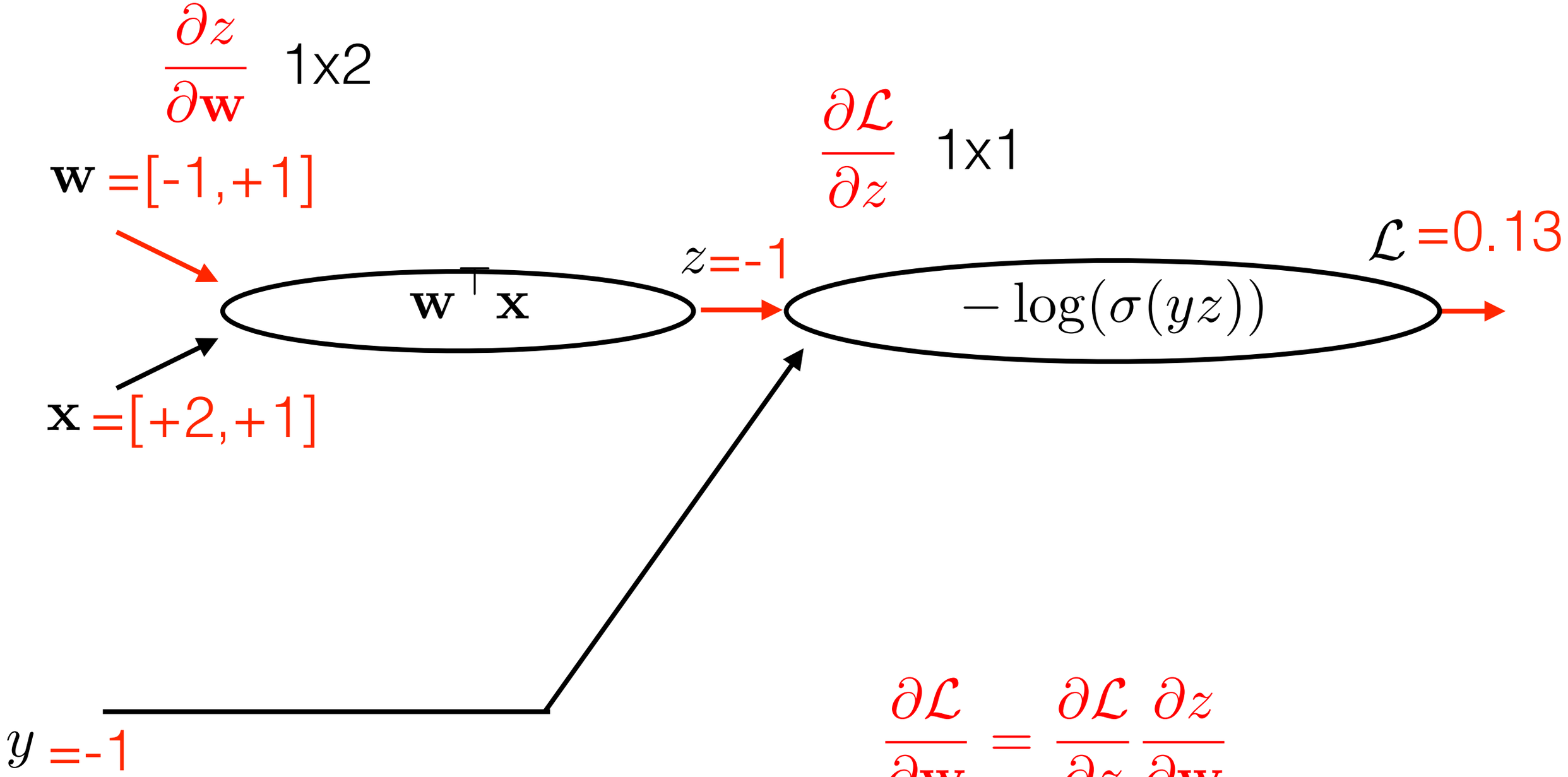
Backprop in vector representation



Backprop in vector representation

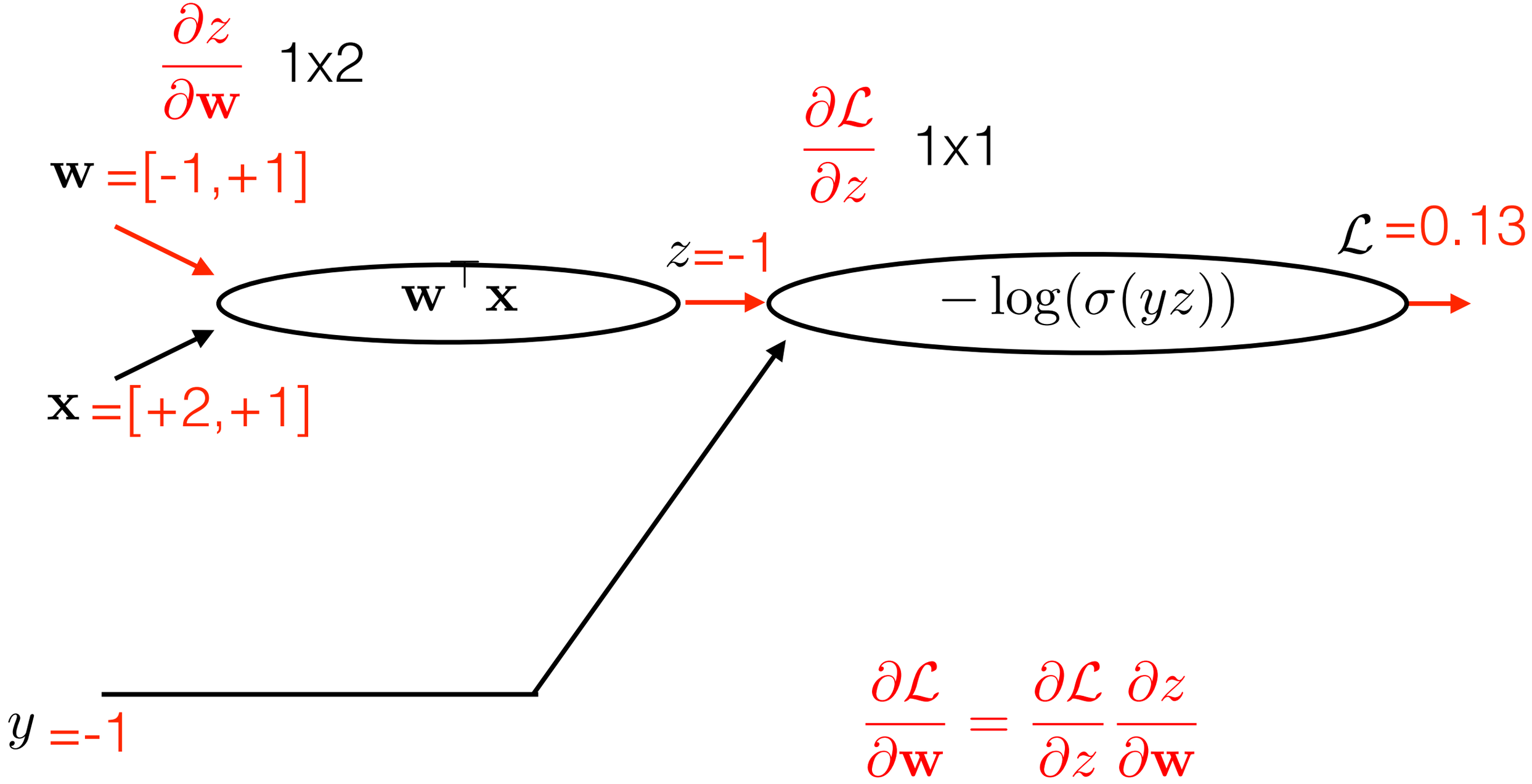


Backprop in vector representation



$$\frac{\partial \mathcal{L}}{\partial \mathbf{w}} = \frac{\partial \mathcal{L}}{\partial z} \frac{\partial z}{\partial \mathbf{w}}$$

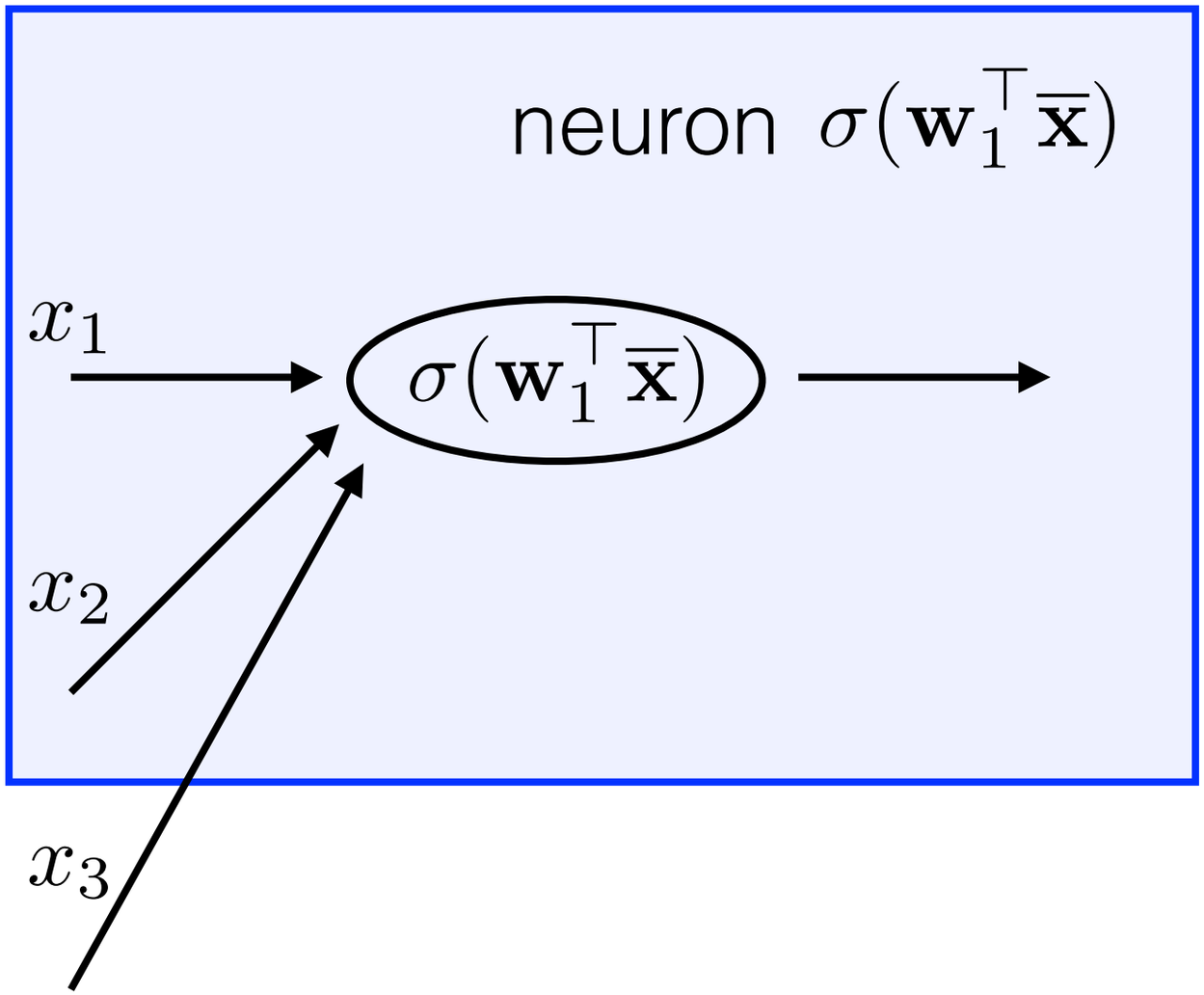
Backprop in vector representation



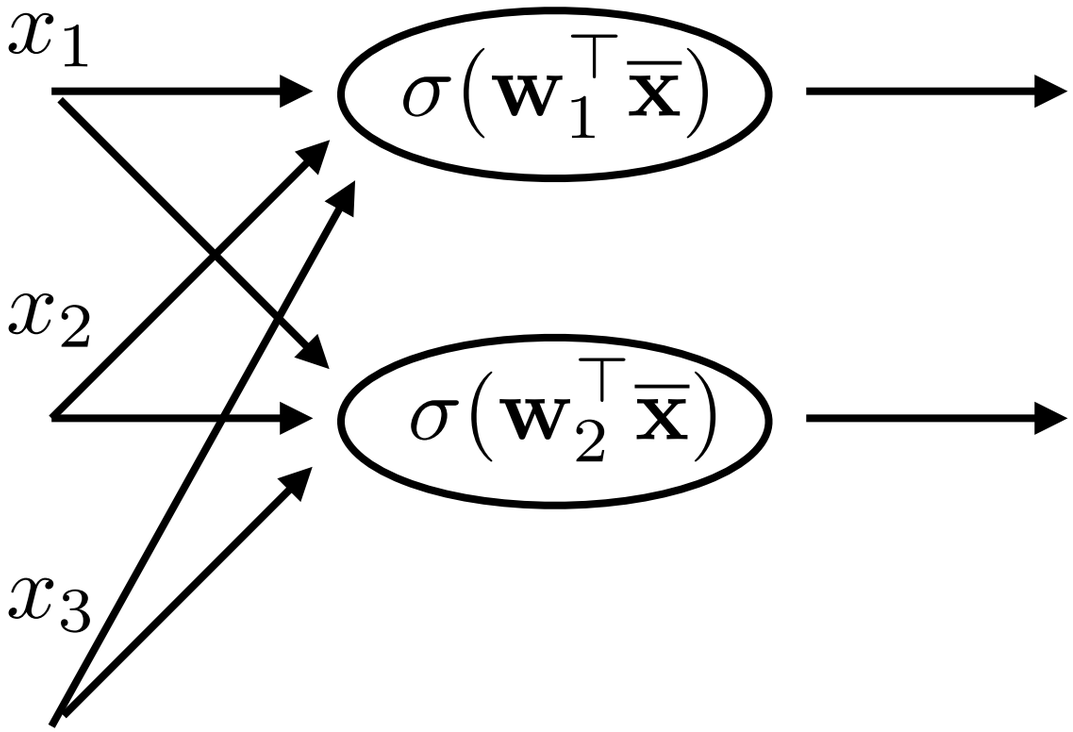
$$\frac{\partial \mathcal{L}}{\partial \mathbf{w}} = \frac{\partial \mathcal{L}}{\partial z} \frac{\partial z}{\partial \mathbf{w}}$$

Learning from multiple training samples means summing up the gradient over all samples

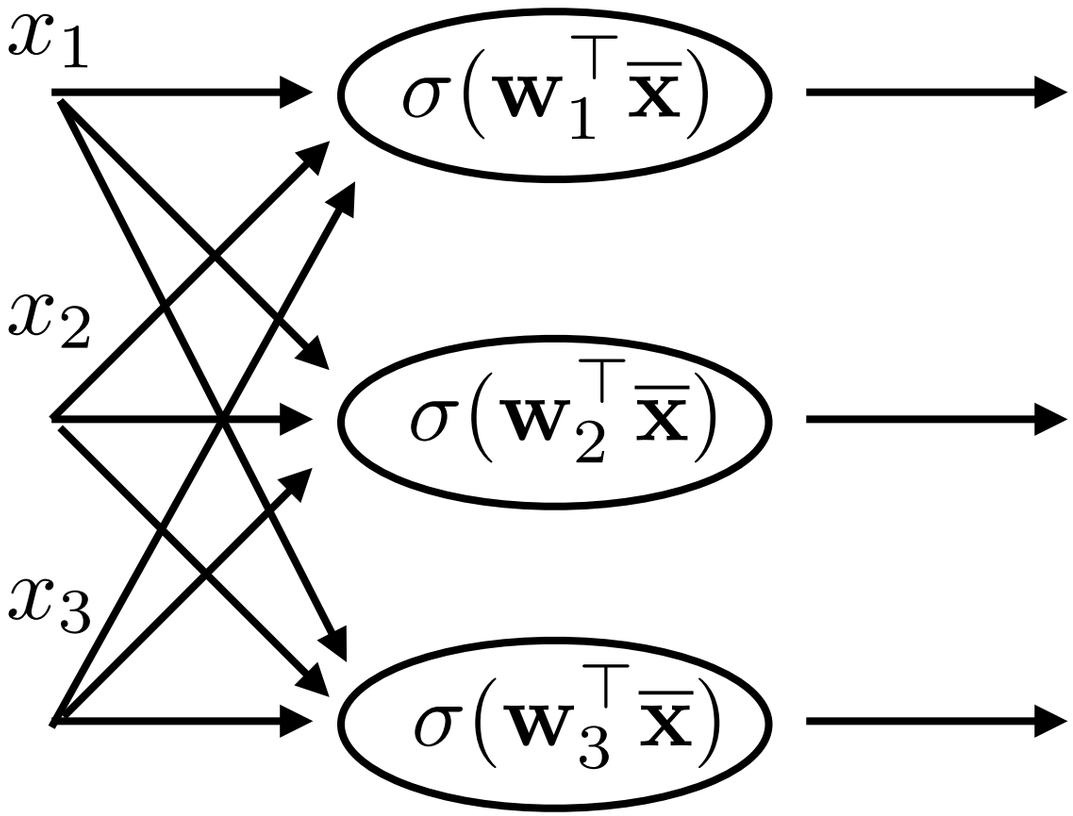
Fully connected neural network



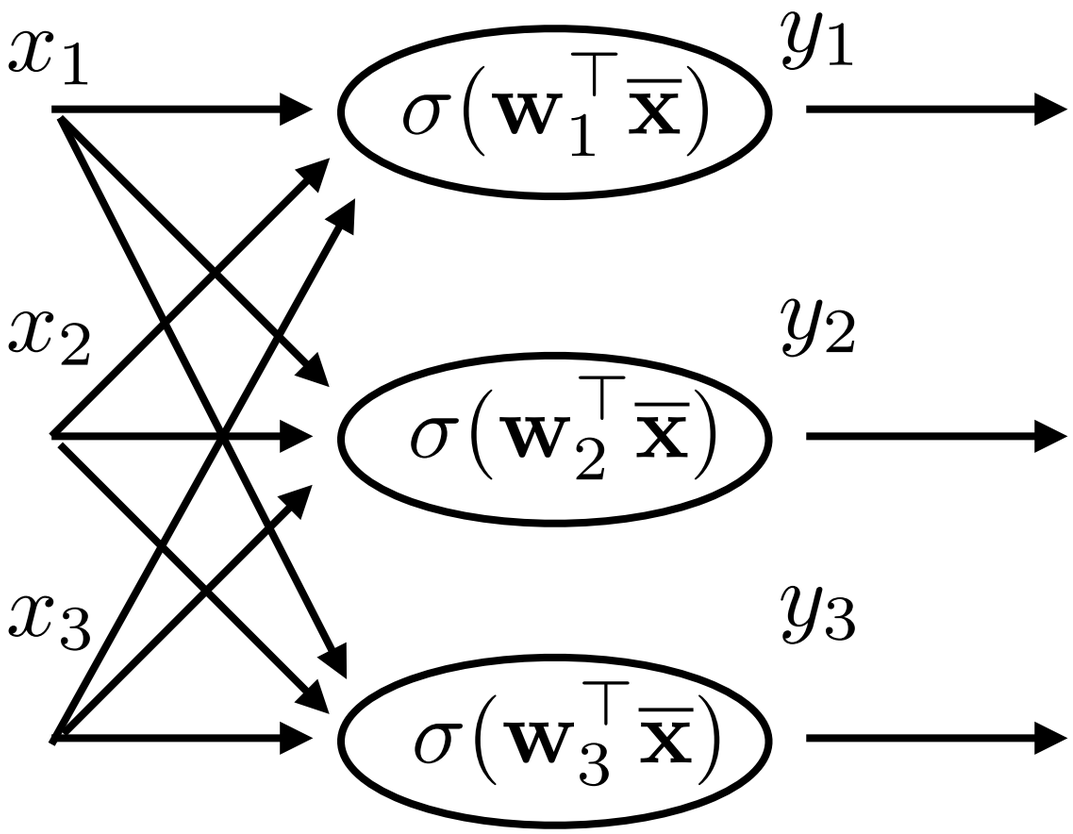
Fully connected neural network



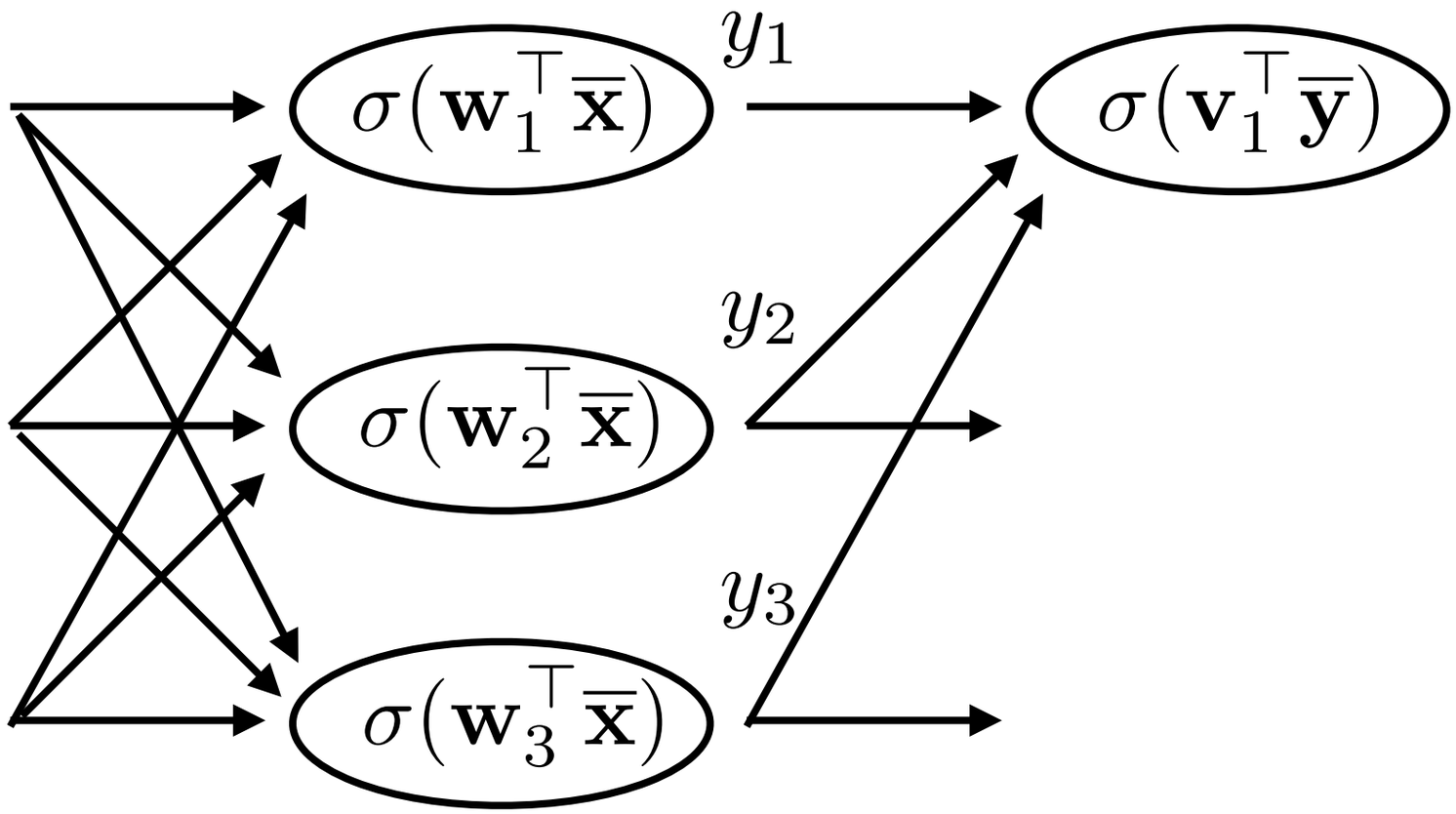
Fully connected neural network



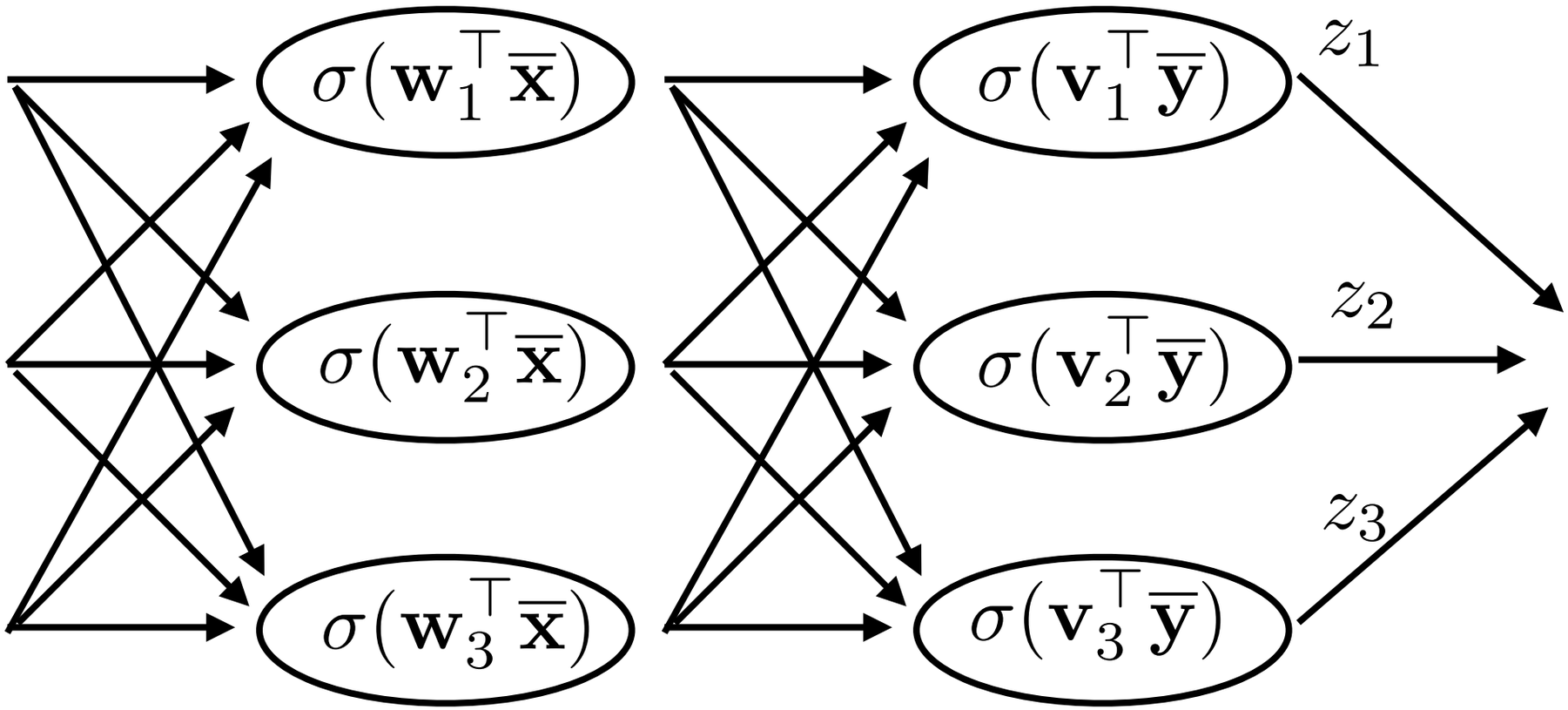
Fully connected neural network



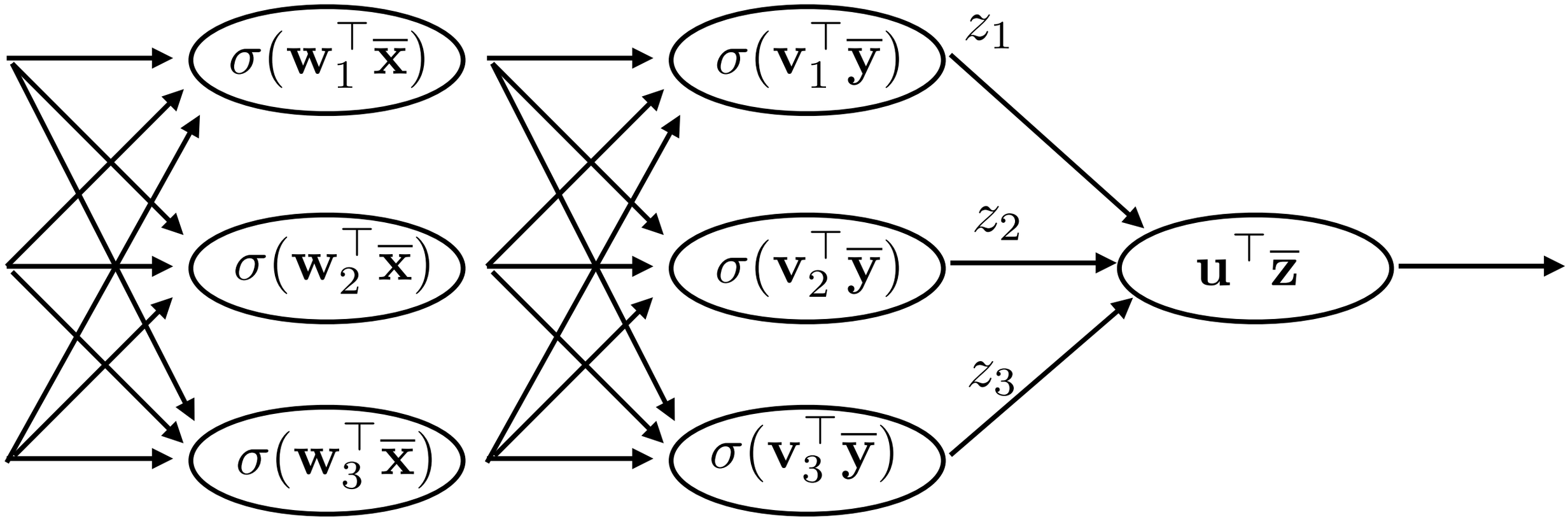
Fully connected neural network



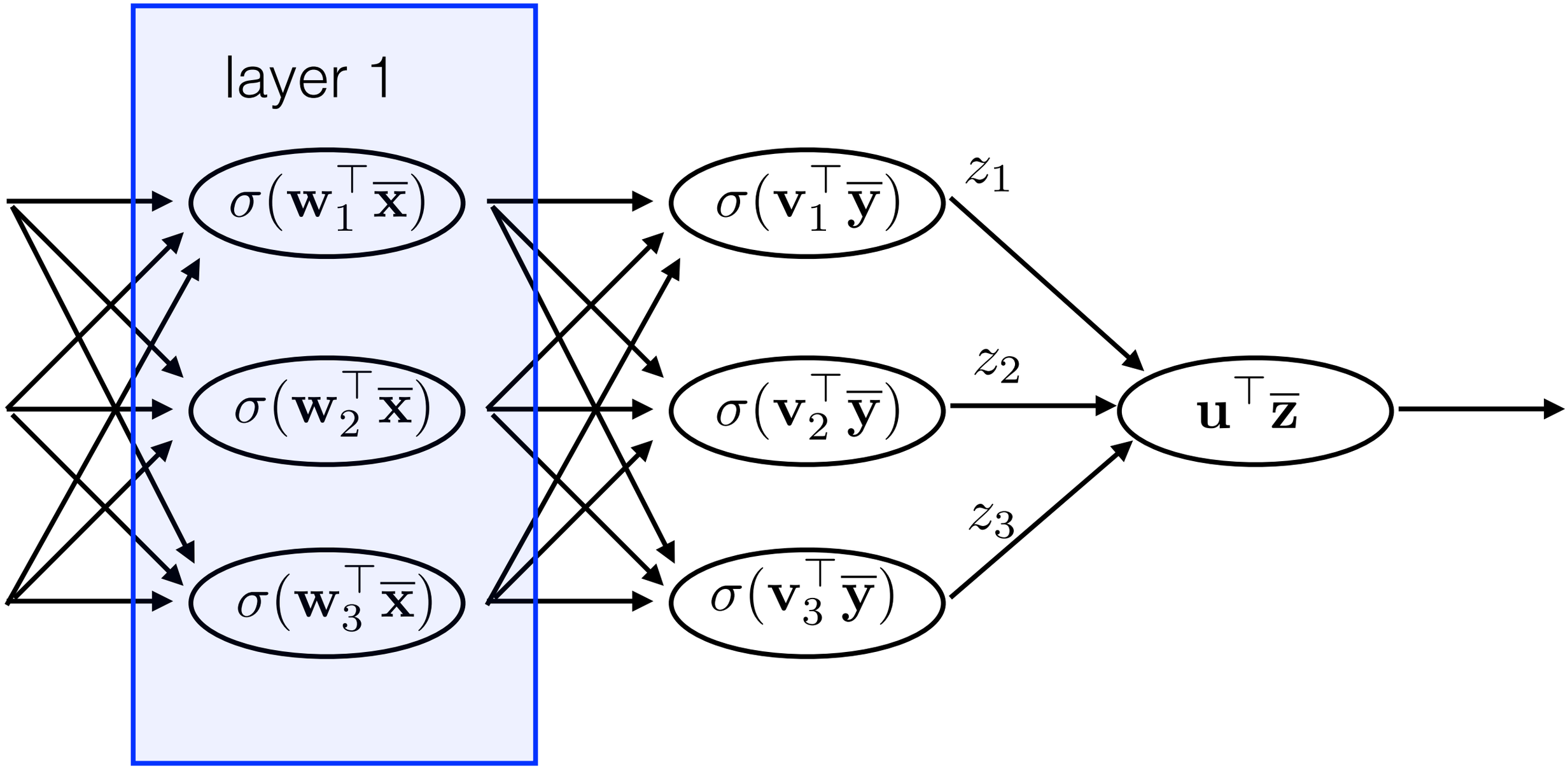
Fully connected neural network



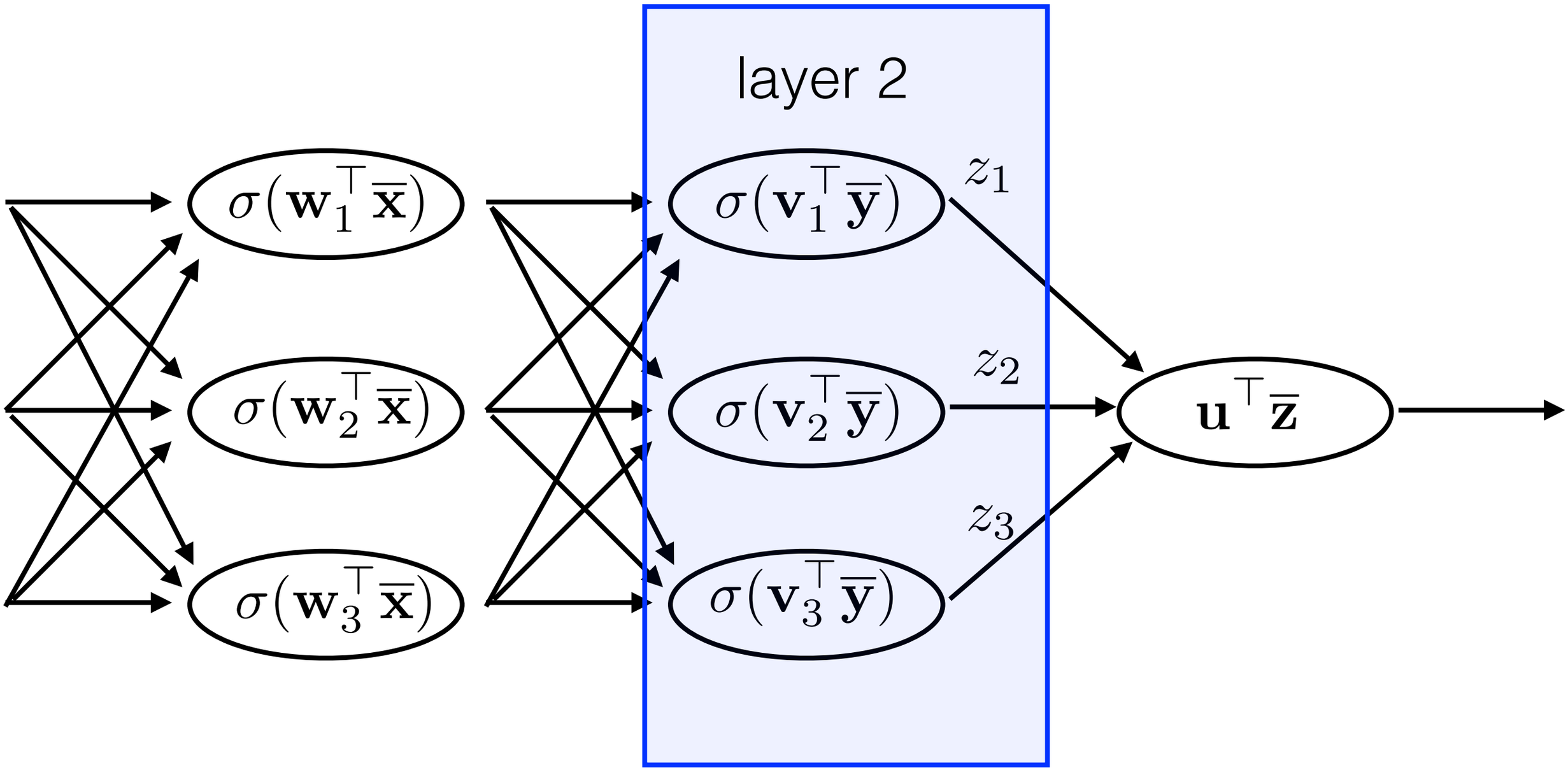
Fully connected neural network



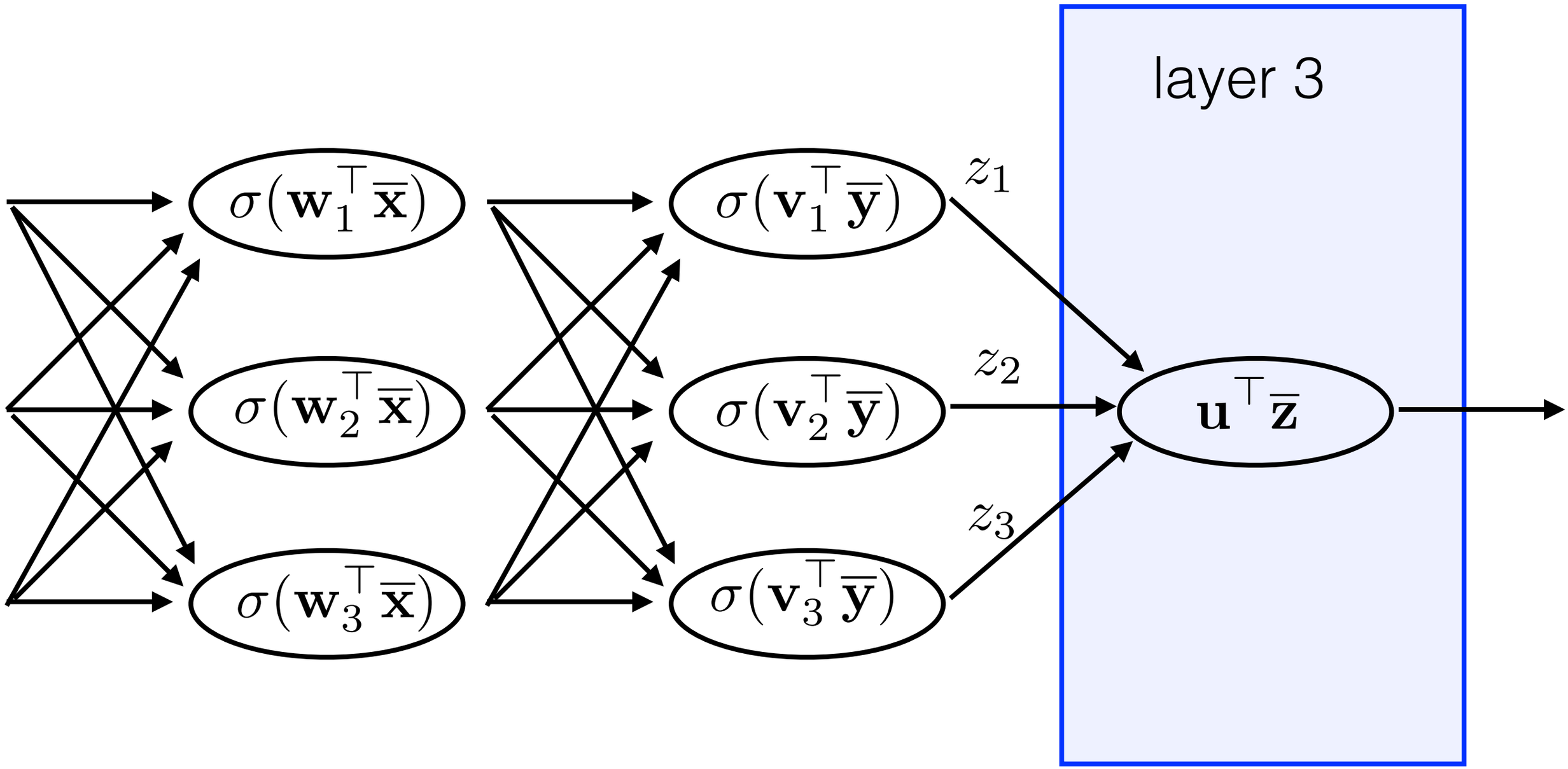
Fully connected neural network



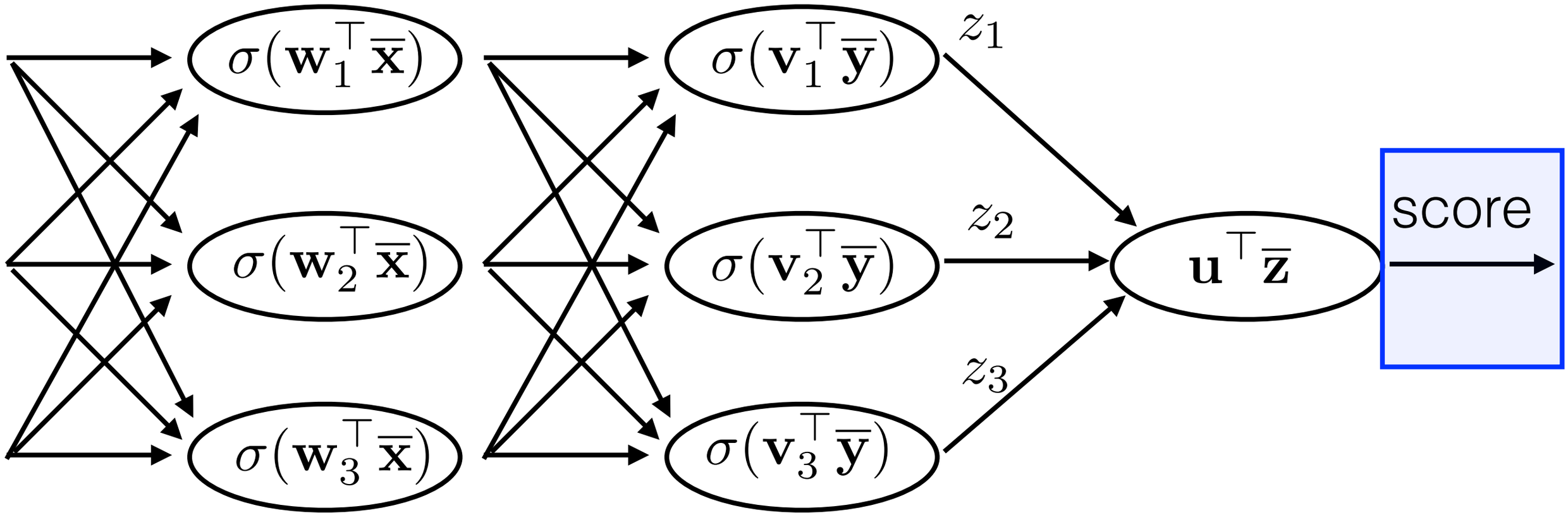
Fully connected neural network



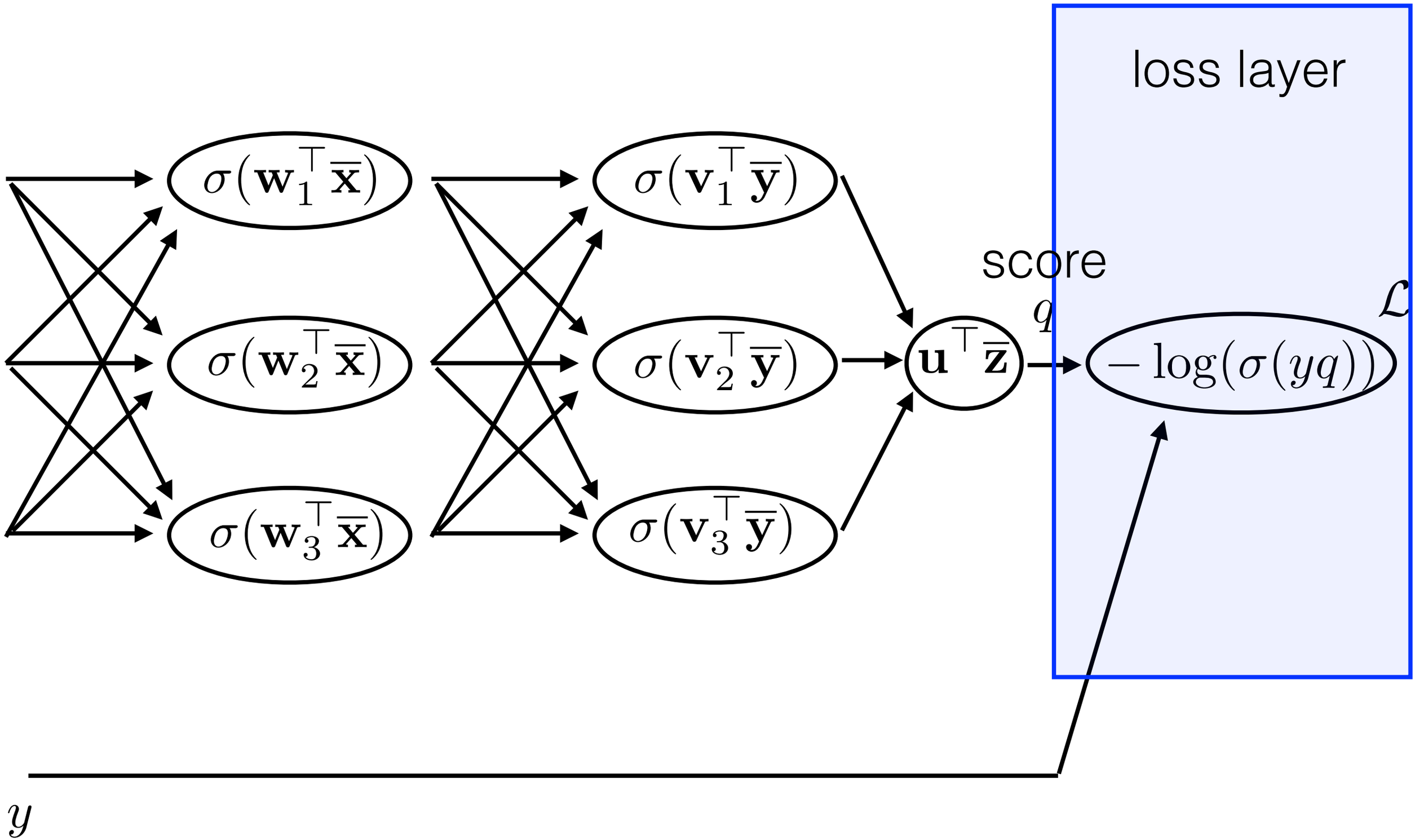
Fully connected neural network



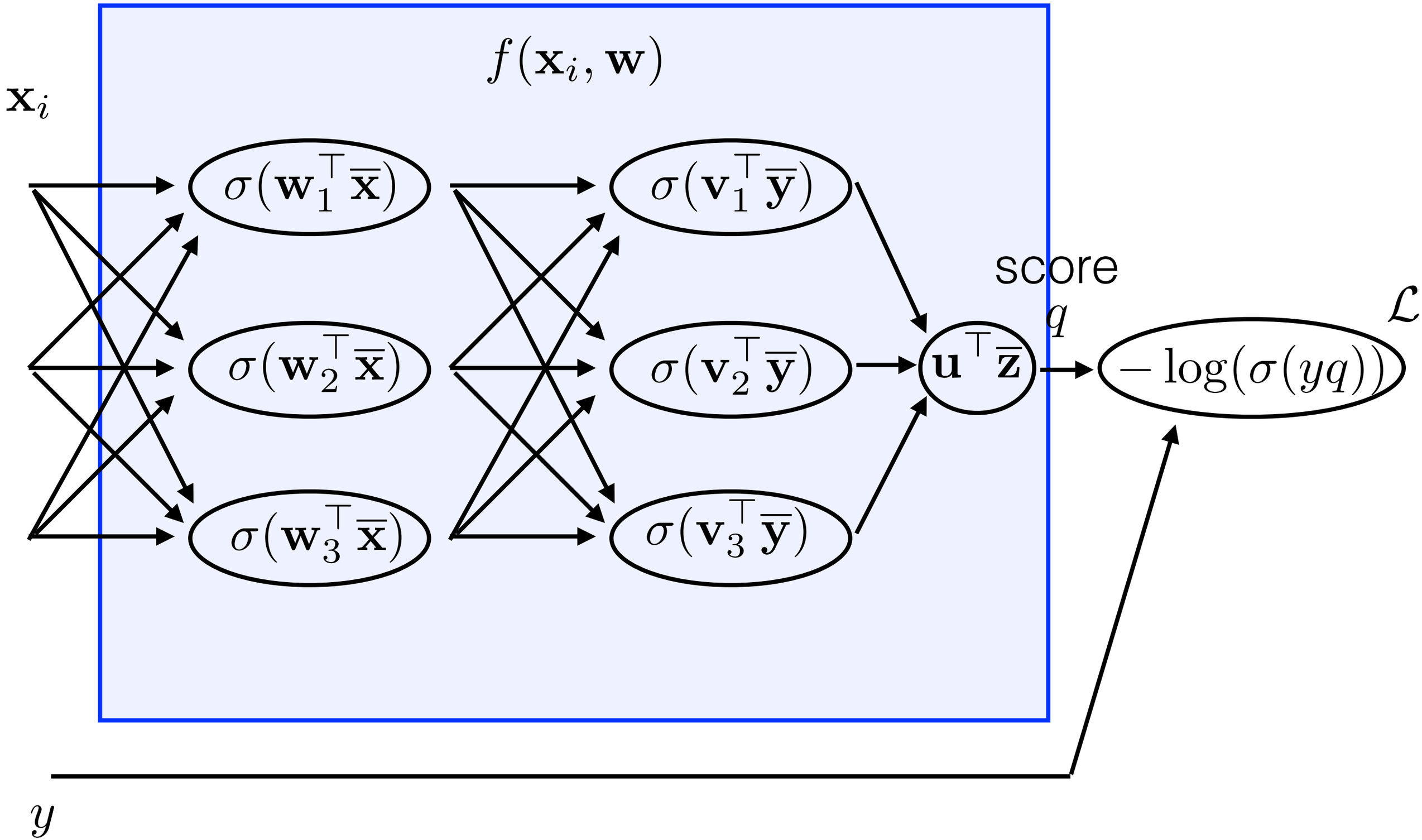
Fully connected neural network



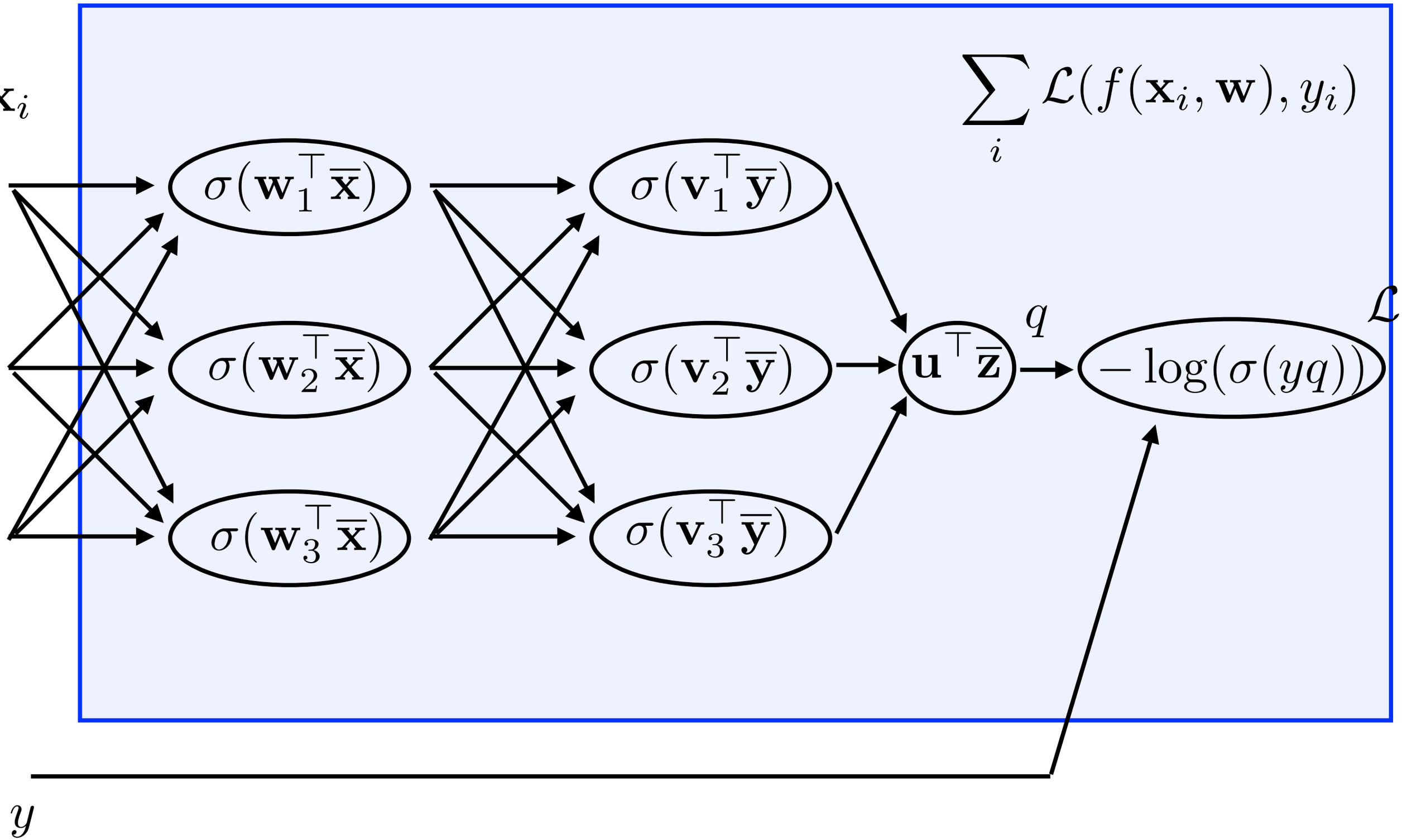
Fully connected neural network



Fully connected neural network



Fully connected neural network



Learning of fully connected neural network

1. Estimate gradient

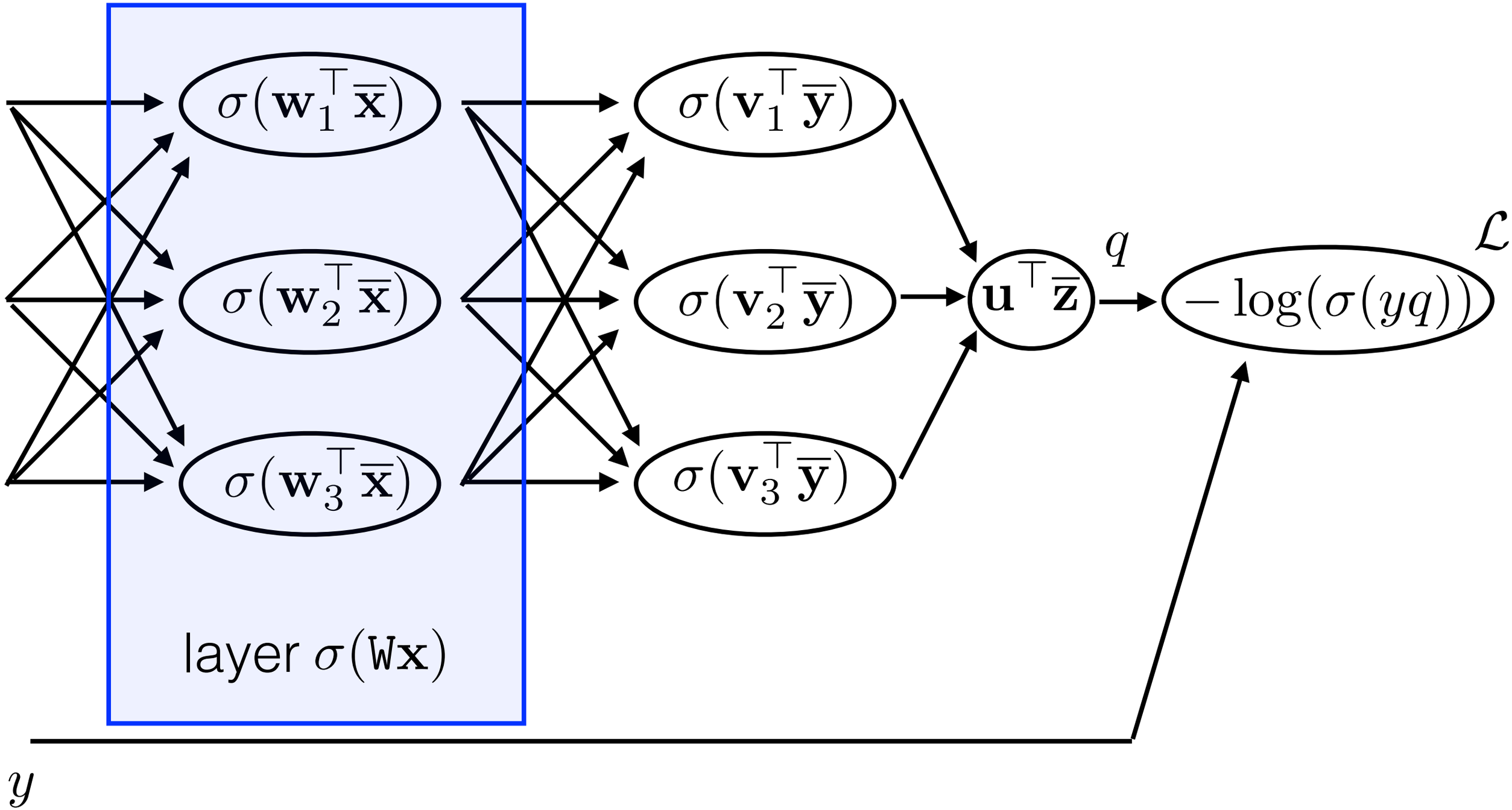
$$\sum_i \frac{\partial \mathcal{L}(f(\mathbf{x}_i, \mathbf{w}), y_i)}{\partial \mathbf{w}}$$

2. Update weights:

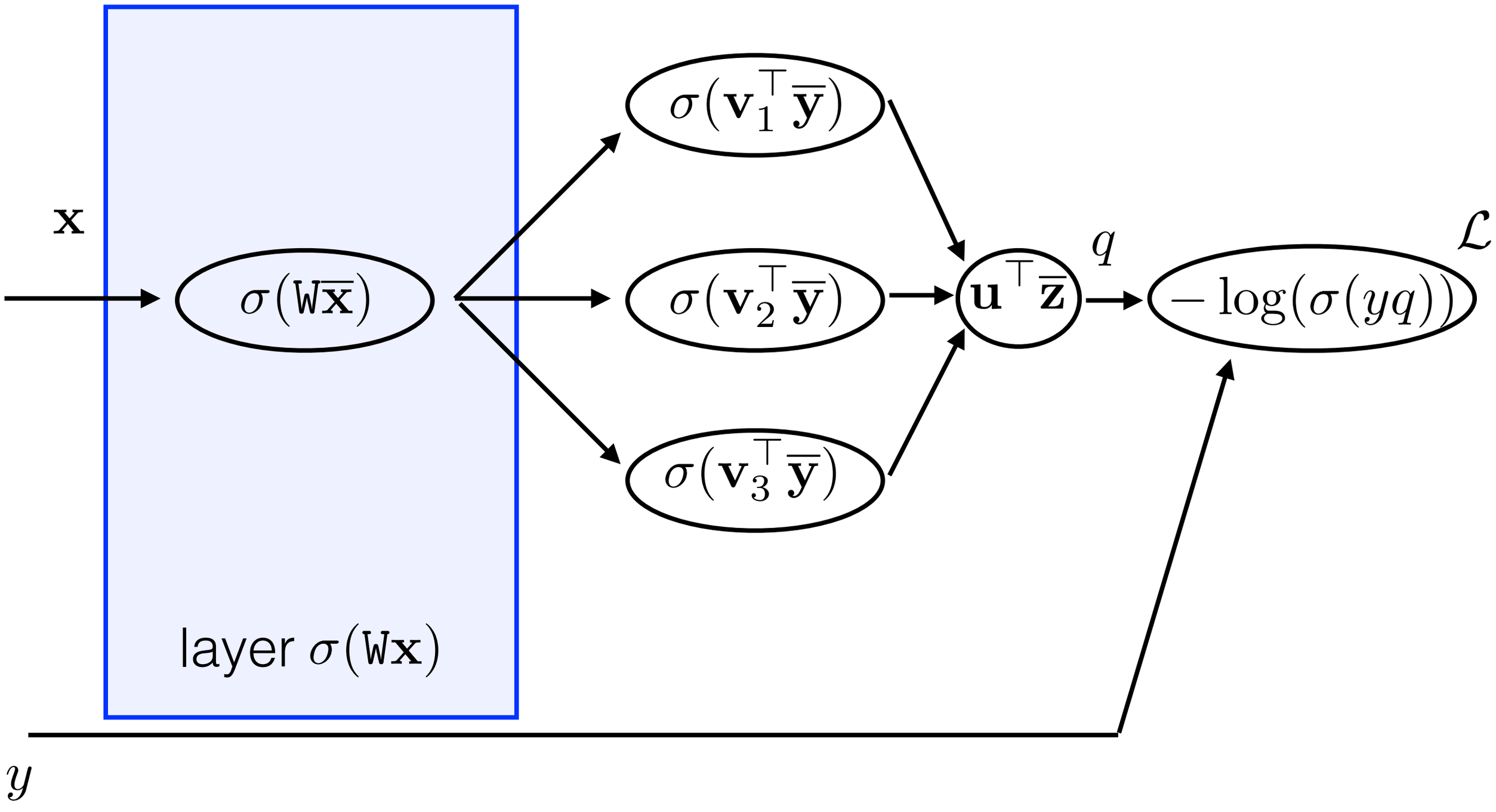
$$\mathbf{w} = \mathbf{w} - \alpha \sum_i \frac{\partial \mathcal{L}(f(\mathbf{x}_i, \mathbf{w}), y_i)}{\partial \mathbf{w}}$$

3. Optionally update learning rate α
4. Repeat until convergence

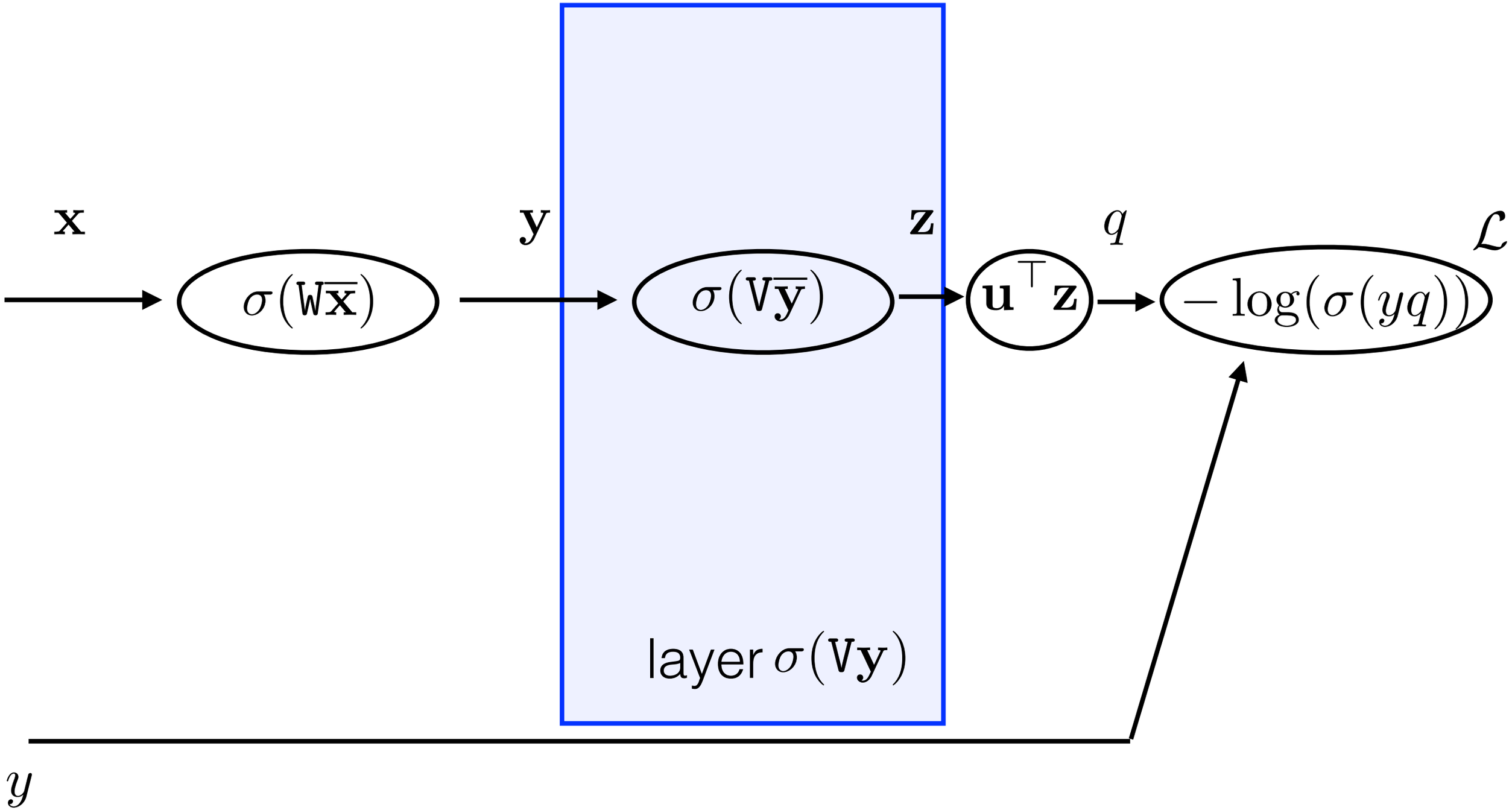
Learning of fully connected neural network



Learning of fully connected neural network

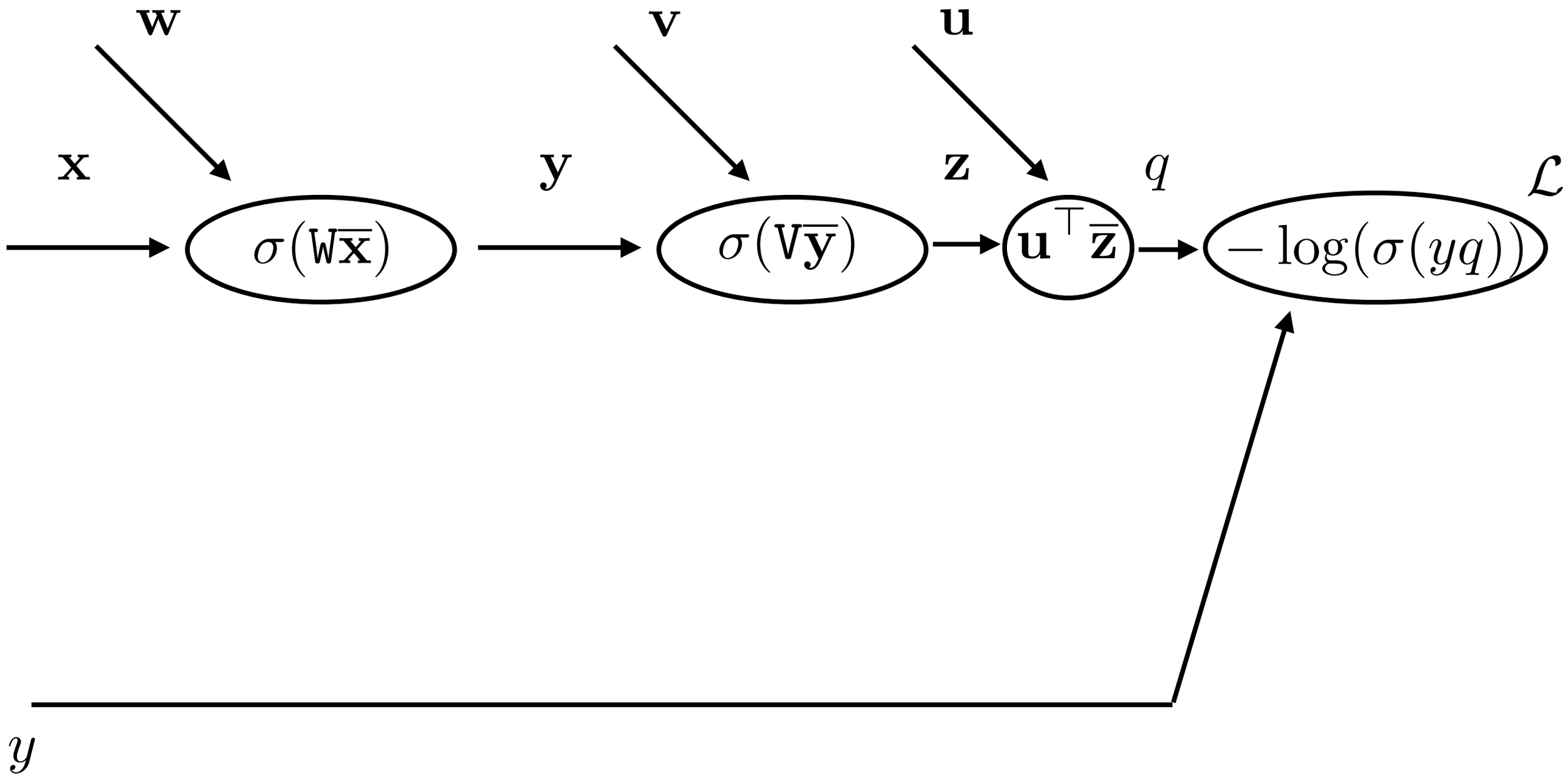


Learning of fully connected neural network



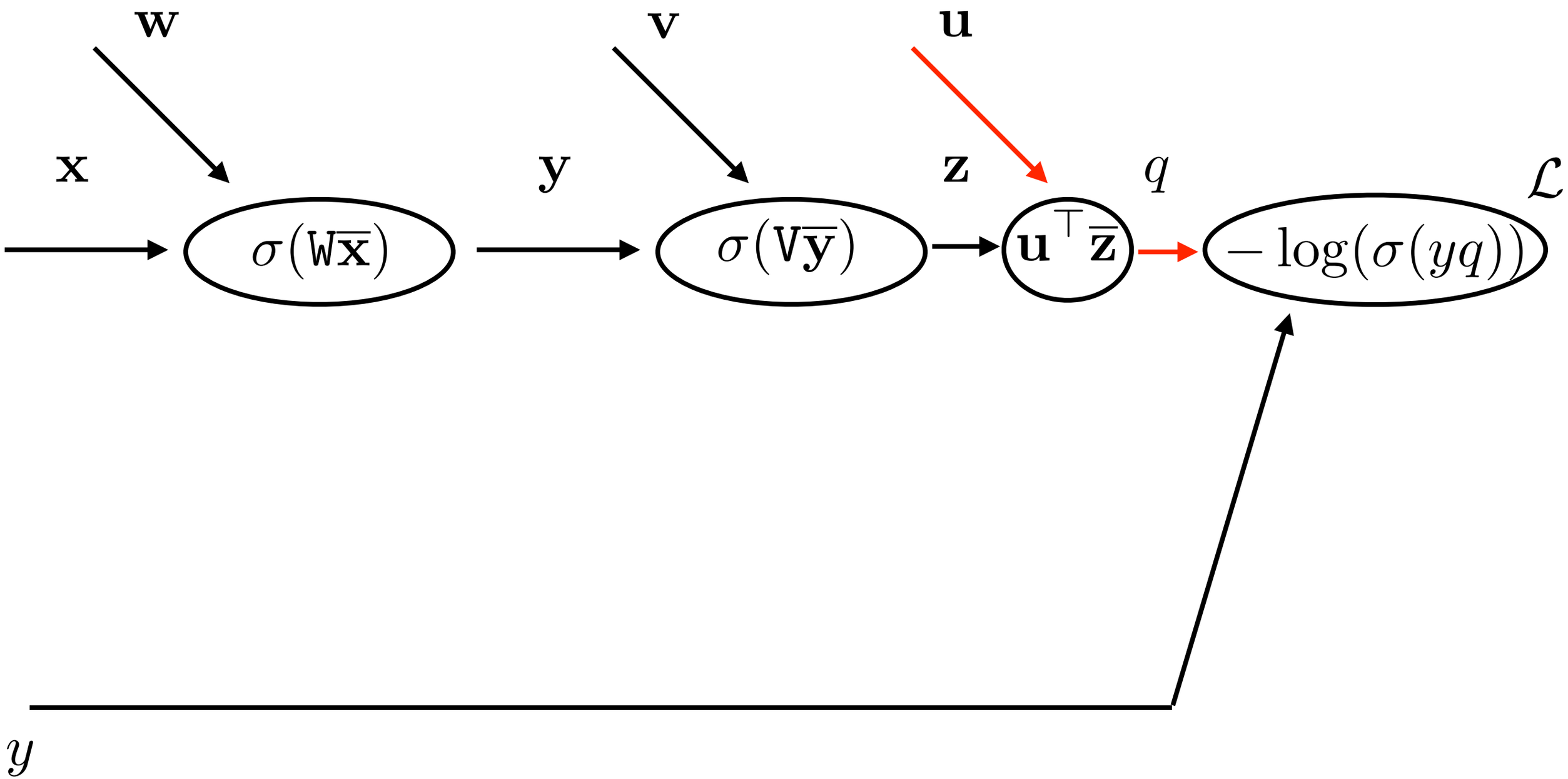
Learning of fully connected neural network

$$\mathbf{w} = \mathrm{vec}(\mathtt{W}) \qquad \mathbf{v} = \mathrm{vec}(\mathtt{V})$$



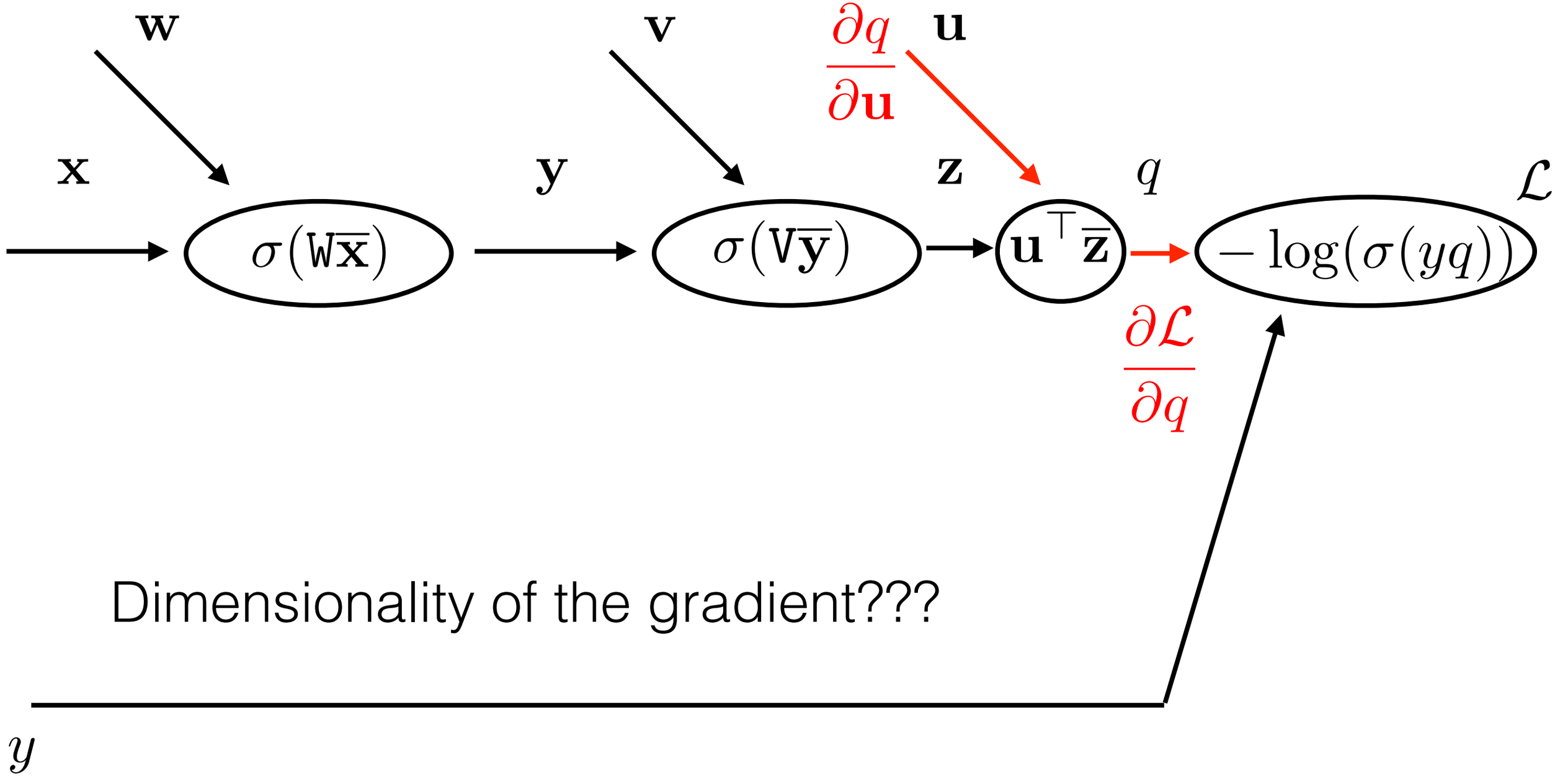
Learning of fully connected neural network

Derivative wrt $\mathbf{u}$: $\dfrac{\partial \mathcal{L}}{\partial \mathbf{u}} = \; ?$



Learning of fully connected neural network

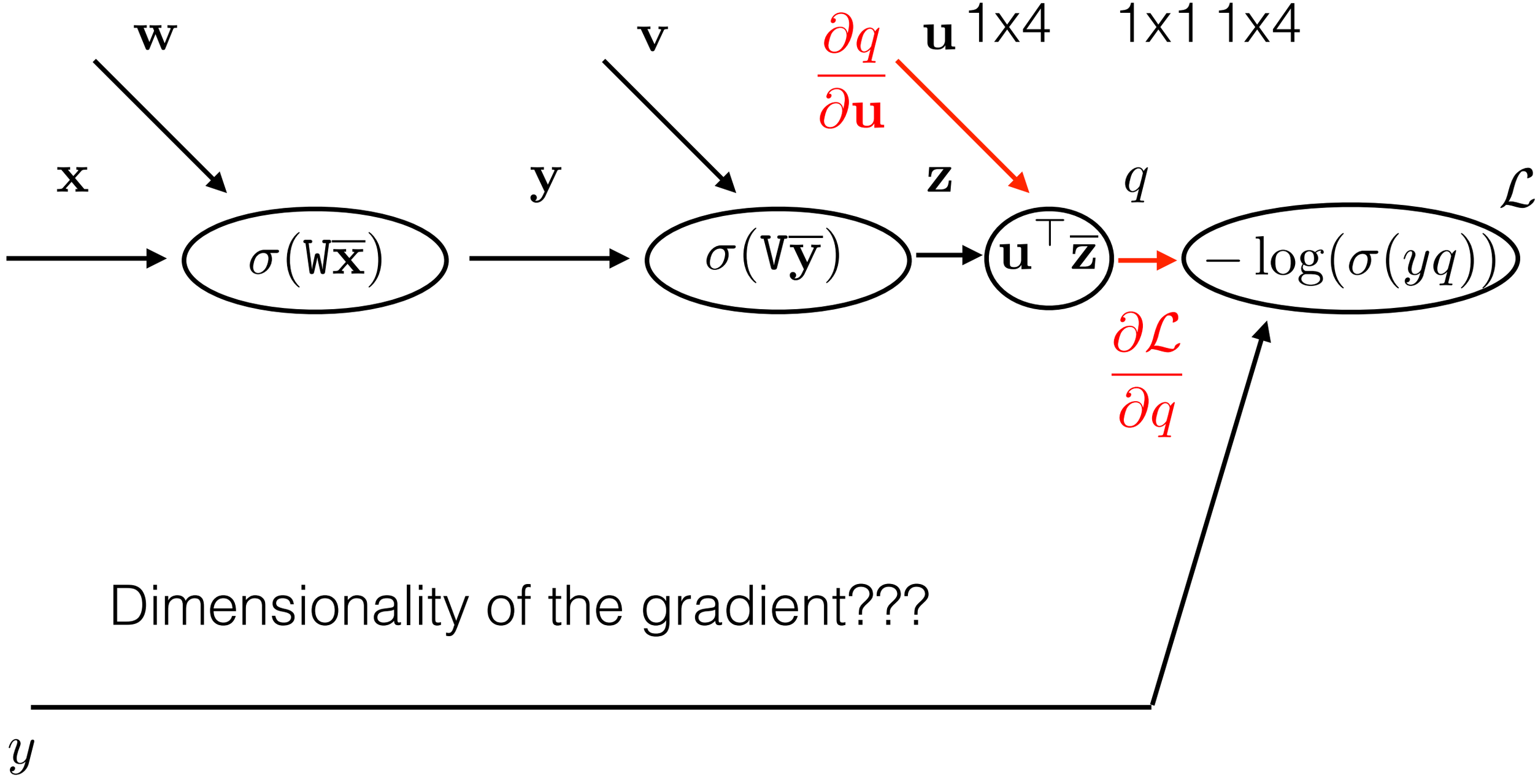
Derivative wrt $\mathbf{u}$: $\frac{\partial \mathcal{L}}{\partial \mathbf{u}} = \frac{\partial \mathcal{L}}{\partial q} \frac{\partial q}{\partial \mathbf{u}}$



Learning of fully connected neural network

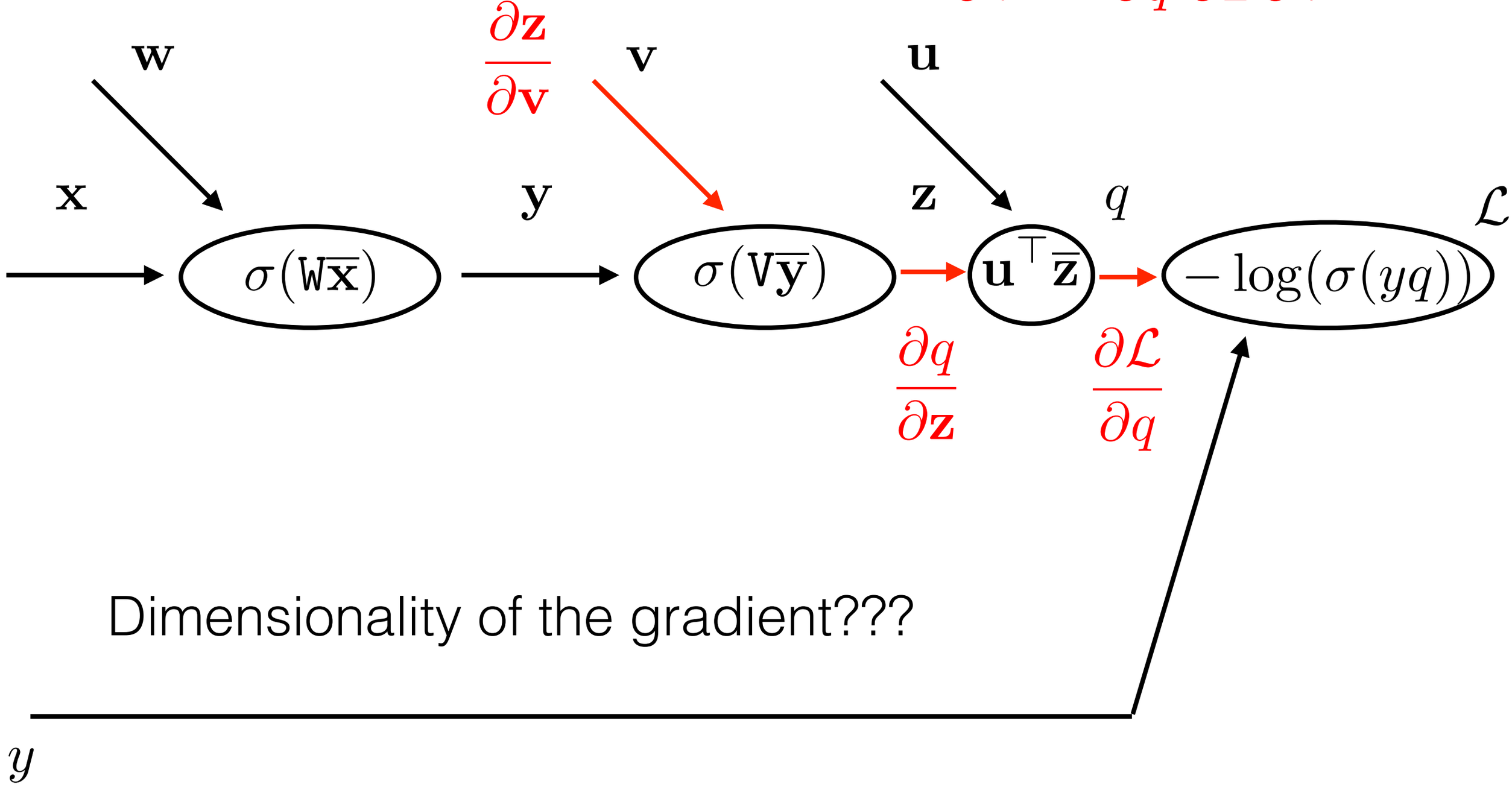
Derivative wrt $\mathbf{u}$: $\frac{\partial \mathcal{L}}{\partial \mathbf{u}} = \frac{\partial \mathcal{L}}{\partial q}\frac{\partial q}{\partial \mathbf{u}}$

1x4 1x1 1x4



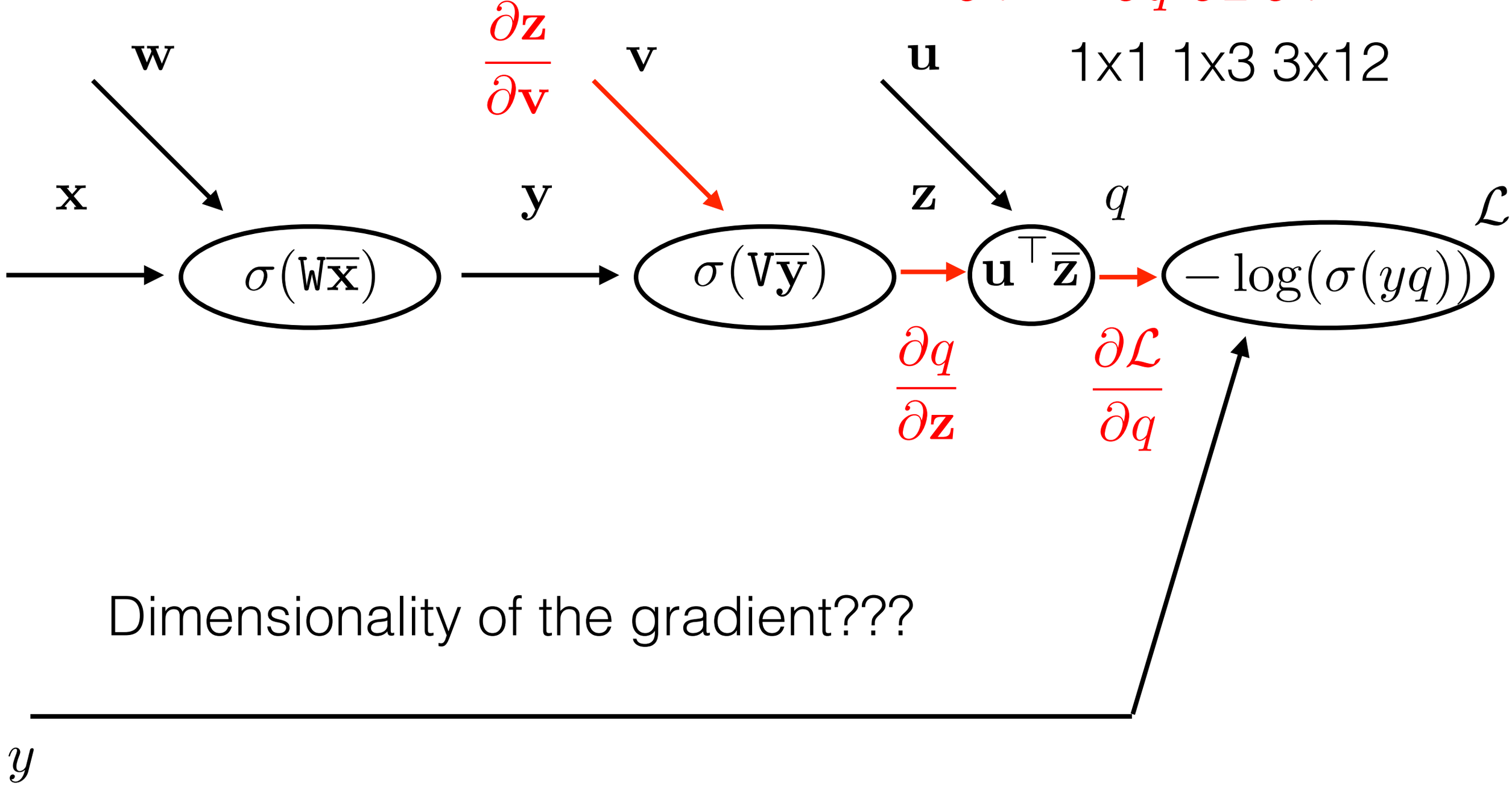
Learning of fully connected neural network

Derivative wrt $\mathbf{v}$: $\frac{\partial \mathcal{L}}{\partial \mathbf{v}} = \frac{\partial \mathcal{L}}{\partial q}\frac{\partial q}{\partial \mathbf{z}}\frac{\partial \mathbf{z}}{\partial \mathbf{v}}$



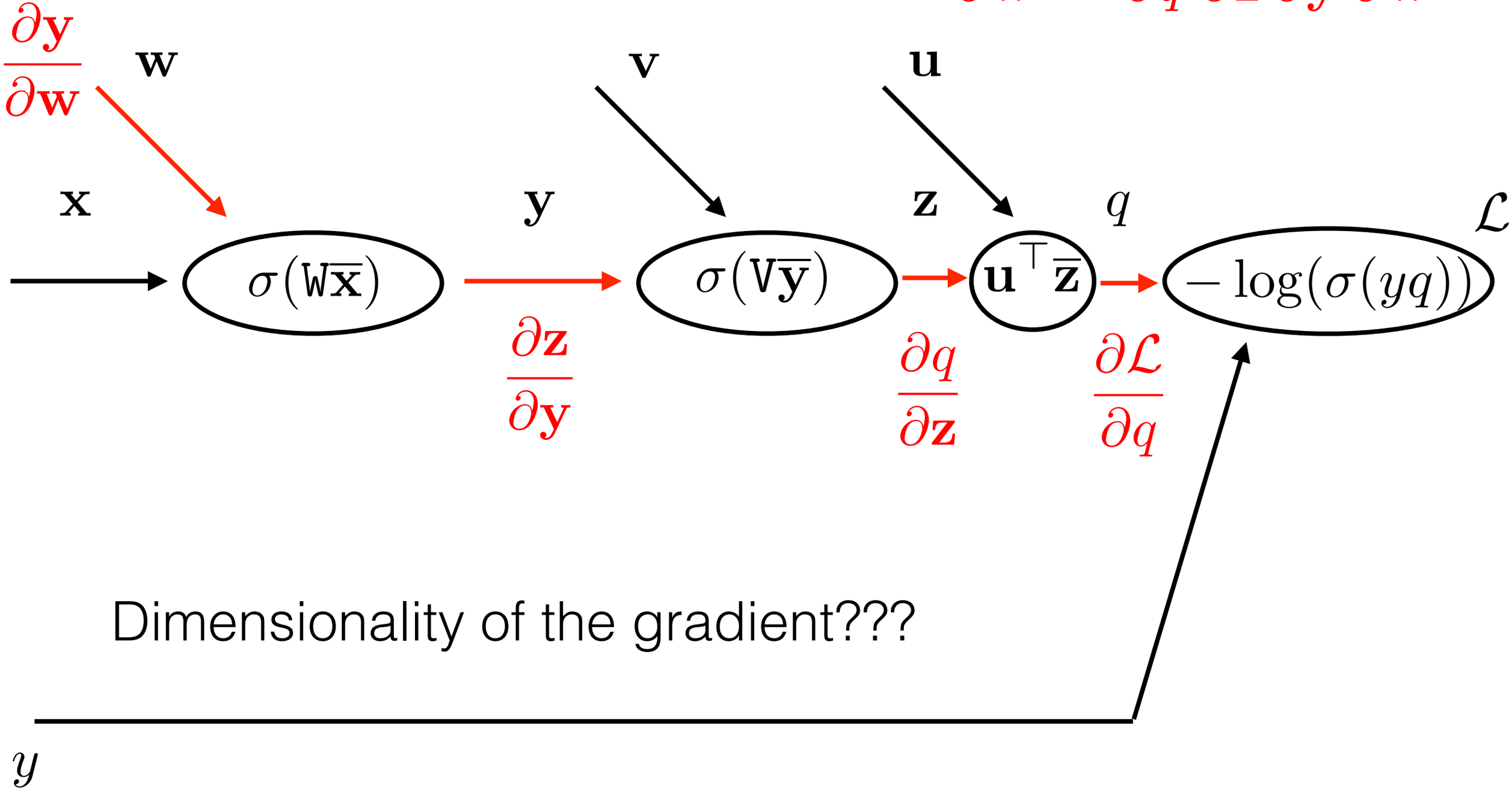
Learning of fully connected neural network

Derivative wrt $\mathbf{v}$: $\frac{\partial \mathcal{L}}{\partial \mathbf{v}} = \frac{\partial \mathcal{L}}{\partial q}\frac{\partial q}{\partial \mathbf{z}}\frac{\partial \mathbf{z}}{\partial \mathbf{v}}$



Learning of fully connected neural network

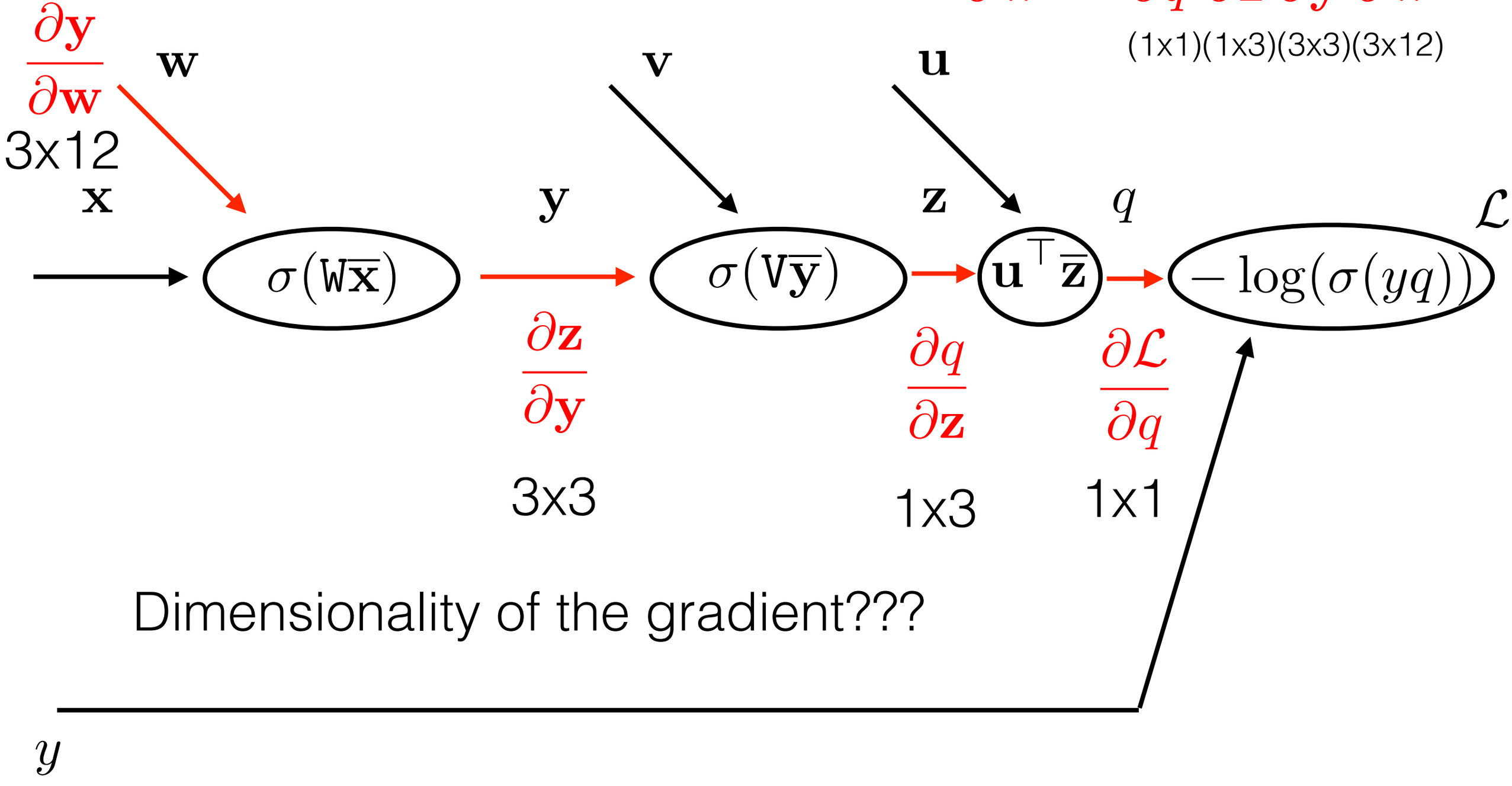
Derivative wrt $\mathbf{w}$: $\frac{\partial \mathcal{L}}{\partial \mathbf{w}} = \frac{\partial \mathcal{L}}{\partial q}\frac{\partial q}{\partial \mathbf{z}}\frac{\partial \mathbf{z}}{\partial \mathbf{y}}\frac{\partial \mathbf{y}}{\partial \mathbf{w}}$



Learning of fully connected neural network

Derivative wrt $\mathbf{w}$: $\frac{\partial \mathcal{L}}{\partial \mathbf{w}} = \frac{\partial \mathcal{L}}{\partial q}\frac{\partial q}{\partial \mathbf{z}}\frac{\partial \mathbf{z}}{\partial \mathbf{y}}\frac{\partial \mathbf{y}}{\partial \mathbf{w}}$

(1x1)(1x3)(3x3)(3x12)



Learning of fully connected neural network

1. Estimate all required local gradients
2. Update weights:

$$\frac{\partial \mathcal{L}}{\partial \mathbf{u}} = \frac{\partial \mathcal{L}}{\partial q}\frac{\partial q}{\partial \mathbf{u}} \qquad \mathbf{u} = \mathbf{u} - \alpha \left[\frac{\partial \mathcal{L}}{\partial \mathbf{u}}\right]^\top$$

$$\frac{\partial \mathcal{L}}{\partial \mathbf{v}} = \frac{\partial \mathcal{L}}{\partial q}\frac{\partial q}{\partial \mathbf{z}}\frac{\partial \mathbf{z}}{\partial \mathbf{v}} \qquad \mathbf{v} = \mathbf{v} - \alpha \left[\frac{\partial \mathcal{L}}{\partial \mathbf{v}}\right]^\top$$

$$\frac{\partial \mathcal{L}}{\partial \mathbf{w}} = \frac{\partial \mathcal{L}}{\partial q}\frac{\partial q}{\partial \mathbf{z}}\frac{\partial \mathbf{z}}{\partial \mathbf{y}}\frac{\partial \mathbf{y}}{\partial \mathbf{w}} \qquad \mathbf{w} = \mathbf{w} - \alpha \left[\frac{\partial \mathcal{L}}{\partial \mathbf{w}}\right]^\top$$

3. Optionally update learning rate α
4. Repeat until convergence

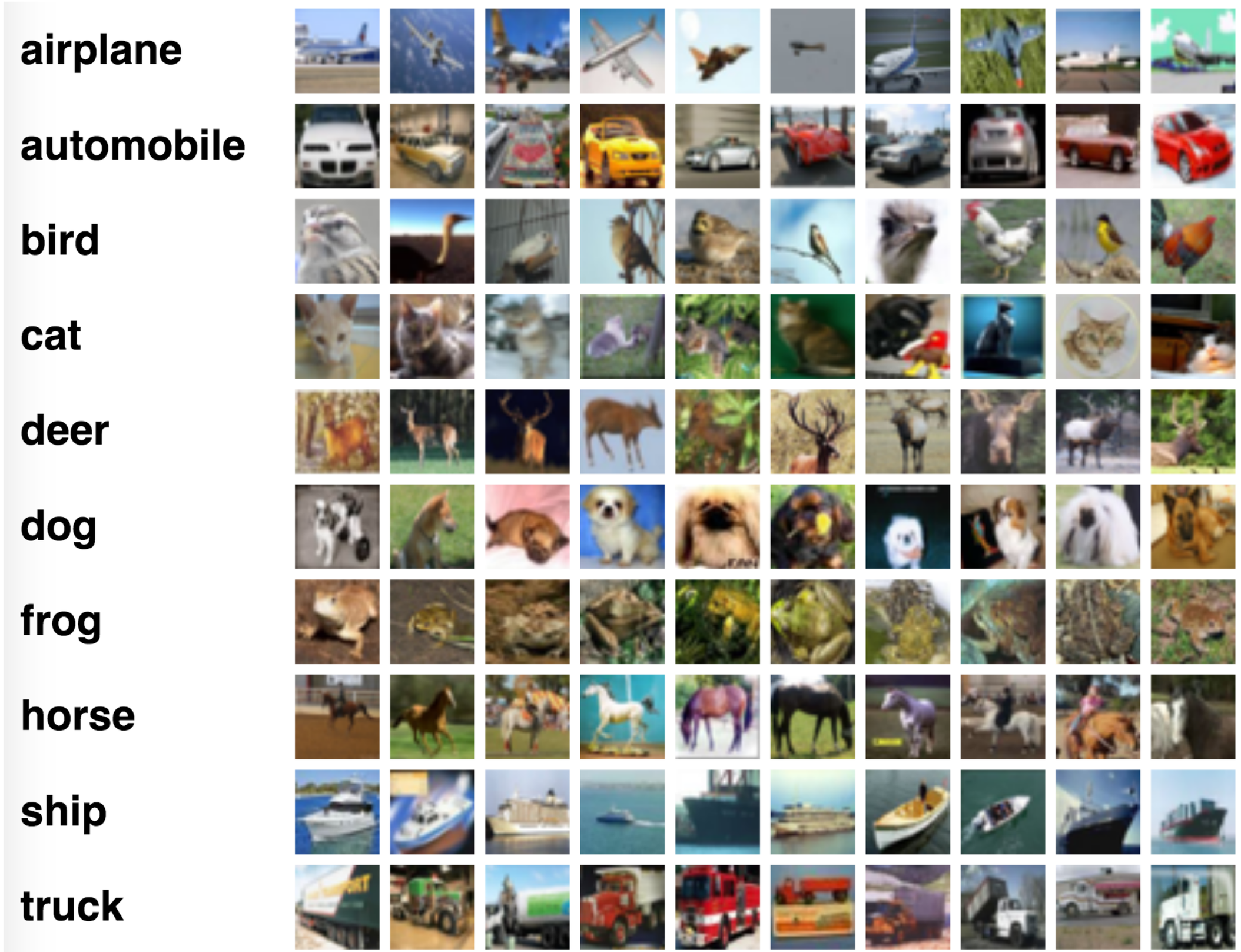
Neural nets summary

- Neural net is a function created as concatenation of simplier functions (e.g. neurons or layers of neurons)
- Gradient optimization of the neural net is called backpropagation
- Neural net frameworks has many predefined layers
- **Spoiler alert:** It does not work (on images) at all - why?

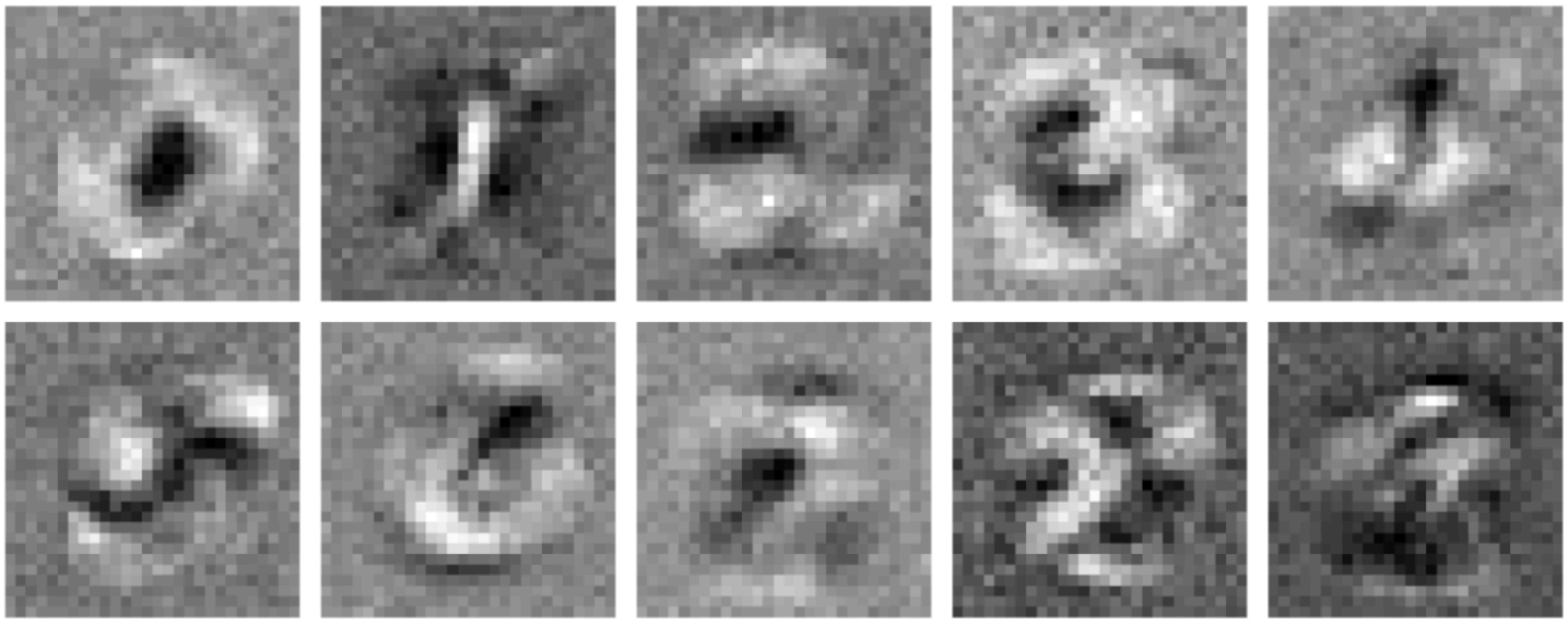
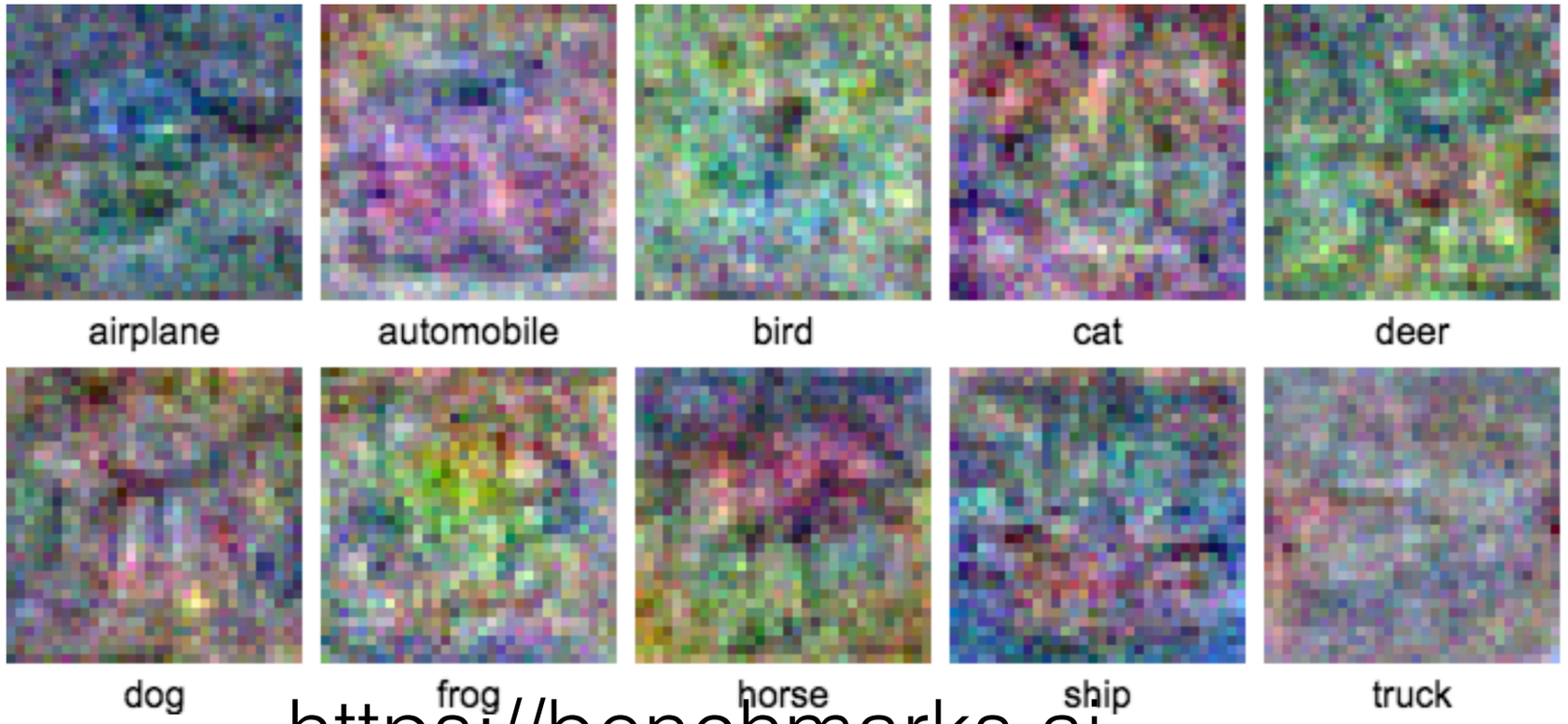
Neural nets summary

- Neural net is a function created as concatenation of simplier functions (e.g. neurons or layers of neurons)
- Gradient optimization of the neural net is called backpropagation
- Neural net frameworks has many predefined layers
- **Spoiler alert:** It does not work (on images) at all - why?

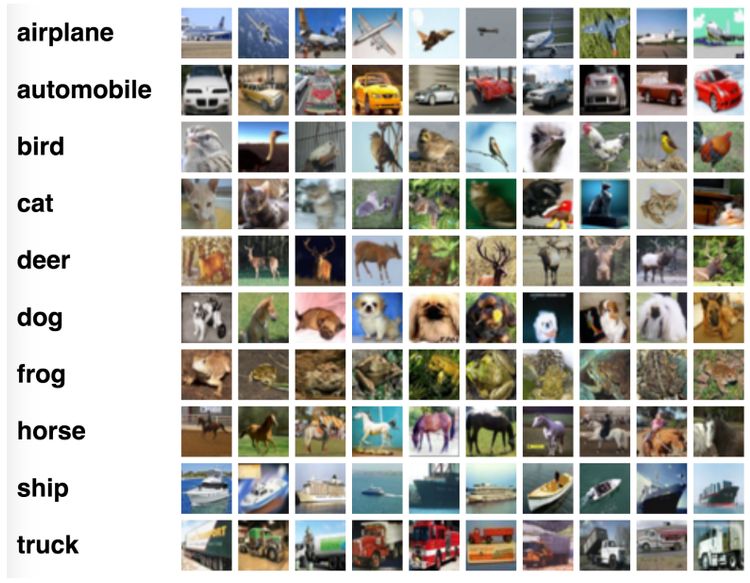
https://benchmarks.ai



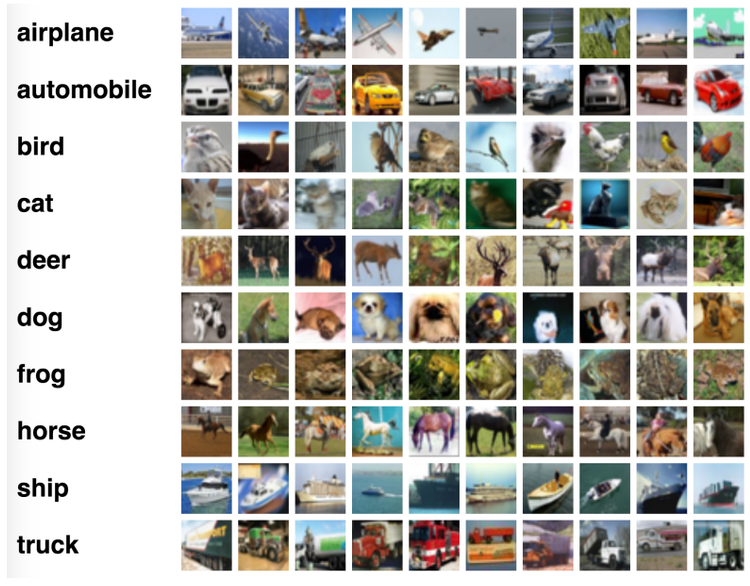
CIFAR-10: classify 32x32 RGB images into 10 categories
https://www.cs.toronto.edu/~kriz/cifar.html

Dataset	Learned weights of linear classifier	Error
MNIST		8%
CIFAR-10		63%

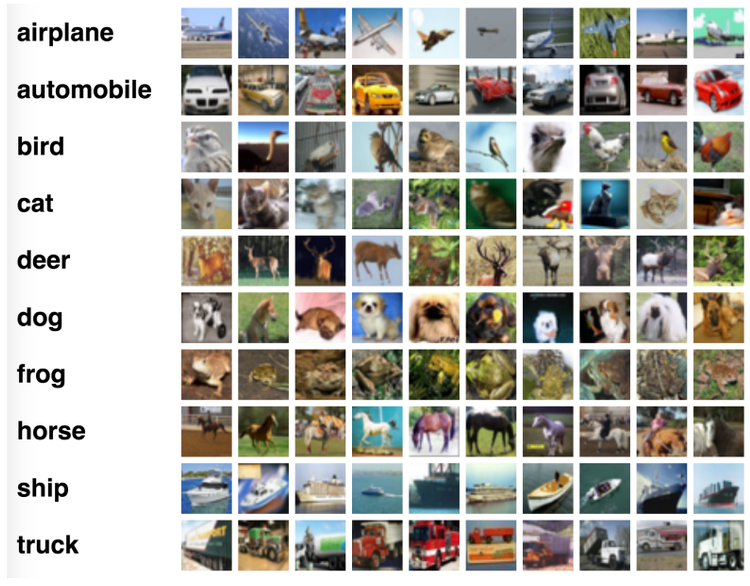
https://benchmarks.ai

Dataset		Linear Error
MNIST		8%
CIFAR-10		63%

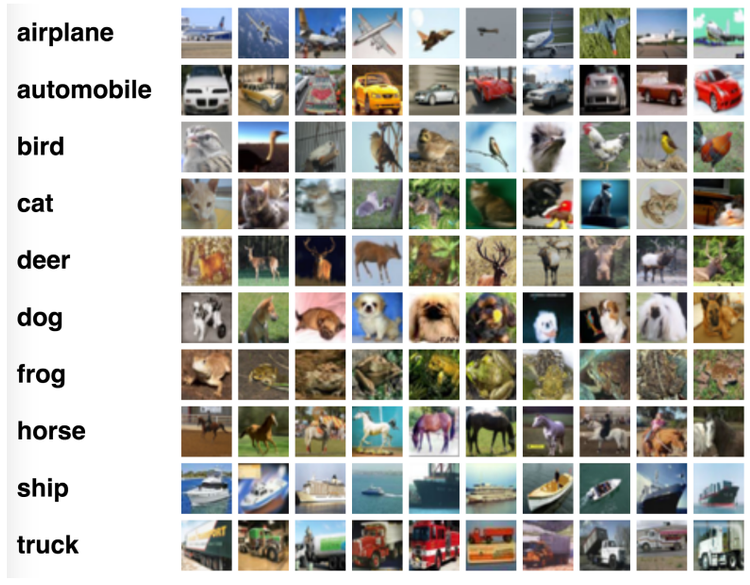
https://benchmarks.ai

Dataset		Error	
		Linear	NN
MNIST		8%	2%
CIFAR-10		63%	55%

https://benchmarks.ai

Dataset		Error Linear	Error NN	ConvNet
MNIST		8%	2%	0.2% [CVPR 2013]
CIFAR-10		63%	55%	1% [EfficientNet, 2018]

https://benchmarks.ai

Dataset		Error Linear	Error NN	ConvNet
MNIST		8%	2%	0.2% [CVPR 2013]
CIFAR-10		63%	55%	1% [EfficientNet, 2018]
		underfit	overfit	cortex inspired structure

https://benchmarks.ai

Competencies required for the test T1

- Ability to draw a computational graph.
- Compute edge gradients/jacobians.
- Perform one step of backpropagation in a vectorized form
-

$$\mathbf{w}^* = \arg\min_{\mathbf{w}} \left(\sum_i -\log(p(y_i|\mathbf{x}_i, \mathbf{w})) \right) + (-\log p(\mathbf{w}))$$

loss function prior/regulariser

- Class of function represented by a NN is too general.
- Naive regulariser helps a bit, but dimensionality/wildness is huge => curse-of-dimensionality, overfitting,…
- What is number of weights between two 1000-neuron layers?
- **Next lecture:** study animal cortex to find a stronger prior on the class of suitable functions.
- **Spoiler alert 2:**
  reduce very general class of functions ”neuron layer” to very specific sub-class of functions “convolution layer”

Competencies required for the test T1

- Ability to draw a computational graph.
- Compute edge gradients/jacobians.
- Perform one step of backpropagation in a vectorized form