

Lecture 1: MATLAB Environment, Basic Math Operators

B0B17MTB, BE0B17MTB – MATLAB

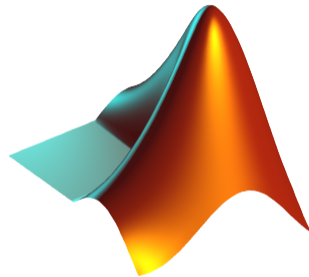
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1. MATLAB Environment
2. Scalars, Vectors, Matrices
3. Basic Math Operations
4. Exercises





The MATLAB Environment

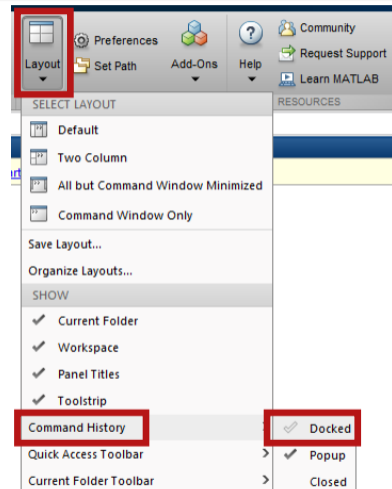
The screenshot shows the MATLAB R2019a academic user interface. The interface is divided into several panes:

- 1**: Command Window, where MATLAB code is entered and executed.
- 2**: Workspace, which displays the current variables in the workspace.
- 3**: Command History, which shows a list of previously executed commands.
- 4**: Current Folder, which shows the current directory and its contents.
- 5**: Details, which provides information about the selected file in the Current Folder pane.
- 6**: File Explorer, which allows for navigating through the file system.
- 7**: The bottom status bar, which displays the current file name and path.
- 8**: The top right corner, which contains the search bar and the user's name.



The MATLAB Environment – Panels

1. Command Window
2. Workspace
3. Command History – *not activated, to activate* →
4. Current Folder
5. Current Folder – Details
6. Current Working Directory
7. Status (“Busy” when MATLAB is executing your code)
8. Search in documentation





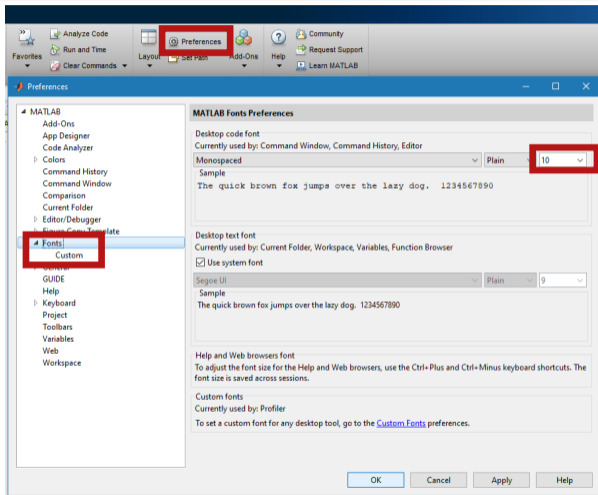
Preferences

▶ Command:

```
>> preferences
```

▶ Ribbon menu:

- ▶ Change font size.





MATLAB Online

- ▶ matlab.mathworks.com
- ▶ Runs in a web browser.
- ▶ Requires (CTU) log in.
- ▶ Slower than regular MATLAB.

```

MATLAB
https://matlab.mathworks.com
HOME PLOTS APPS
New Script New Live Script New Open Download Find Files Import Data Save Workspace Clear Workspace New Variable Open Variable Favorites Clear Commands Simulink Layout Parallel Add-Ons Help Feedback Learn MATLAB
FILE VARIABLE CODE SIMULINK ENVIRONMENT RESOURCES
MATLAB Drive
Current Folder
Name Git
final
Published (my site)
Workspace
Name Value Size Cl
a [1;1;1;1;1;1;1;1;1;1] 10x1 dou
>> a = ones(10, 1)
a =
1
1
1
1
1
1
1
1
1
1
1
1
>> |
  
```

Documentation



```
>> doc % opens documentation window
```

```
>> help % MATLAB help
```

```
>> demo % tutorials
```



The Help Structure

► Command:

```
>> help sin
```

► Output:

```
sin    Sine of argument in radians.  
      sin(X) is the sine of the elements of X.
```

```
See also asin, sind, sinpi.
```

```
Reference page for sin
```




The Documentation Structure I.

► Command:

```
>> doc sin
```

1. Documentation page
2. Search field
3. Documentation contents
4. Bookmarks of this page

The screenshot shows the MATLAB Help interface for the 'sin' function. The search field at the top right is labeled '1'. The main title 'sin' is labeled '2'. The left-hand navigation menu is labeled '3', and the 'ON THIS PAGE' section within that menu is labeled '4'. The main content area includes sections for 'Syntax' ($Y = \sin(X)$), 'Description' (explaining that $\sin(X)$ returns the sine of the elements of X), and 'Examples' (showing a plot of the sine function over the domain $-\pi \leq x \leq \pi$).



The Documentation Structure II.

- ▶ Check the origin of the function.
 - ▶ Several functions with the same name may exist.
- ▶ Functions types by origin:
 - ▶ MATLAB core functions – most of them build-in, some are available for editing (not recommended!).
 - ▶ Functions from installed toolboxes.
 - ▶ User-created functions.
- ▶ Calling priority for functions will be discussed later.
- ▶ During this course, **always open a function from core installation.**

The screenshot shows a search bar with 'sin|' entered. Below it, a list of search results is displayed under the heading 'Functions'. Each result includes the function name, a brief description, and the source toolbox. A search icon is visible in the top right corner of the search bar.

Function	Description	Source
<code>sin</code>	Sine of argument in radians	MATLAB
<code>sin</code>	Symbolic sine function	Symbolic Math Toolbox
<code>sin</code>	Sine of fixed-point values	Fixed-Point Designer
<code>sind</code>	Sine of argument in degrees	MATLAB
<code>sinh</code>	Hyperbolic sine of argument in radians	MATLAB

» 136 more



Workspace Browser

- ▶ List of variables.
- ▶ Deleting/modification of existing variables.
- ▶ Saving/loading.
- ▶ Values, Class and Memory information.
- ▶ Other information can be added: size, min, max, ...
- ▶ All information can be obtained using MATLAB functions that we learn later, *e.g.*, min, max, max, length.
- ▶ Fast data plotting option (in ribbon).

The screenshot shows the MATLAB Workspace Browser window with the following data:

Name	Value	Class	Bytes
1	1	double	8
A	[-1,1;-2]	double	32
5	5	double	8
B	[1,2,3;4,5,6;7,8,9]	double	72
c	[1,0,0]	double	24
d	[0;1;0]	double	24

The Variable Editor for 'A' shows a 2x2 double matrix:

	1	2	3	4	5	6
1	-1	1				
2	1	-2				
3						
4						
5						



MATLAB Commands

- ▶ Matlab is **cAsE sEnSiTiVe!**
 - ▶ Almost entirely, with certain exceptions (properties of graphics objects, ...).
 - ▶ Pay attention to typos and variable names (see later).
 - ▶ New versions of MATLAB offer certain options.

```
>> AA = [1 1 1]
>> Aa
```

- ▶ Beware of different syntax in Mathematica.
 - ▶ Following syntax is incorrect both in MATLAB and Mathematica:

```
>> Sin(pi/2) % function names start with lower case
>> cos[pi/2] % function input is in parentheses ()
```

- ▶ Will be discussed in the next lectures.



Naming Conventions

- ▶ Names of variables can have max. 63 characters starting with letter
(`>> namelengthmax`)
 - ▶ Letters and numbers are allowed, other symbols (colon “:”, hyphen “-” and others) are not.
 - ▶ Underscore is allowed in the variable name “_” (not at the beginning, though!).
- ▶ Lowercase letters in the names of scalars and variables (`a = 17.59;`).
- ▶ Matrix names usually start with a capital letter (`A = [. .];`).
- ▶ Iteration variables, variables used in `for` cycles usually named `m`, `n`, `k`, etc.
 - ▶ It is advisable to avoid `i` and `j` (complex unit).
- ▶ Avoid, if possible, standalone letter “`l`” (to be confused with one “`1`”) and predefined variables in MATLAB environment (see later).
- ▶ Choose names corresponding to the meaning of each particular variable.
- ▶ Avoid using names of existing functions or scripts (overloading can occur).
- ▶ The same conventions are valid for names of functions and scripts.



Variable Names

- ▶ Examples of valid variable names:

```
a, A, b, c, x1, x2, M_12, test1, matrix_A, fx, fX
```

- ▶ Examples of invalid variable names:

```
1var      % starts with a number (not possible in MATLAB)  
matrix A  % contains space  
coef.a    % possible only if coef is of type 'struct'  
Test-1    % algebraic expressing: ans = Test - 1  
f(y)      % makes sense when using symbolic expressions
```

- ▶ Examples of valid numbers in MATLAB,

```
3, -66, +0.0015, .015, 1e2, 1.6025e-10, 05.1
```



Functions who, whos

- ▶ Function `who` lists all variables in MATLAB Workspace.
 - ▶ Wide variety of options.
- ▶ Functions `whos` lists the variable names + dimension, size and data type of the variables or displays content of a file.
 - ▶ Wide variety of options.

```
>> whos('-file', 'matlab.mat');
```

```
>> a = 15; b = true; c = 'test'; d = 1 + 5j;  
>> who  
>> whos  
>> Ws = whos;
```



Workspace – Output Deletion

- ▶ To clean (erase) command window:

```
>> clc
```

- ▶ To clean one (or more) variable(s):

```
>> clear          % whole Workspace is deleted  
>> clear XX      % variable XX is deleted  
>> clear XX YY   % variables XX and YY are deleted  
>> clear z*      % everything starting with 'z' is deleted
```

- ▶ clear has a number of other options (graphics, I/O)



Command History Window

- ▶ Command History window stores all commands from the Command Window.
- ▶ Command History is accessible through \uparrow or \downarrow .
- ▶ it is possible to filter out past commands by, *e.g.*:
 - » `A = [+ \uparrow .`
- ▶ It is possible to copy-and-paste entire Command History:
`SHIFT / CTRL / CTRL + A \rightarrow CTRL + C.`



Matrices in MATLAB

- ▶ Matrix is a basic data structure in MATLAB.
- ▶ There are following variables types depending on size:
 - ▶ scalar: 1×1
 - ▶ vector: $M \times 1$ or $1 \times N$
 - ▶ matrix: $M \times N$
 - ▶ array (multidimensional matrices):
 $M \times N \times P \times Q \times R \times \dots$
- ▶ Matrices can be complex.
- ▶ It can contain text as well (beware the length).

- ▶ M -by- N matrix:

$$\begin{array}{c}
 a_{i,j} \\
 \downarrow \\
 M \text{ rows} \\
 i \text{ changes}
 \end{array}
 \begin{array}{c}
 \xrightarrow{N \text{ columns}} \\
 j \text{ changes}
 \end{array}
 \begin{bmatrix}
 a_{1,1} & a_{1,2} & a_{1,3} & \dots \\
 a_{2,1} & a_{2,2} & a_{2,3} & \dots \\
 a_{3,1} & a_{3,2} & a_{3,3} & \dots \\
 a_{4,1} & a_{4,2} & a_{4,3} & \dots \\
 \vdots & \vdots & \vdots & \ddots
 \end{bmatrix}$$



Matrix Creation

- ▶ Following techniques are available:
 - ▶ element-by-element entering (suitable for small matrices only),
 - ▶ colon notation “:” to define elements of series,
 - ▶ generation by built-in functions,
 - ▶ generation of matrices in m-files,
 - ▶ import and export from/to external files(.mat, .txt, .xls, ...).



Matrix Construction Element-by-element I.

- ▶ Test following commands to construct matrices by element enumeration.
 - ▶ Suitable for small matrices only.

```
>> a1 = -1
>> a2 = [-1] % brackets are redundant
```

```
>> v1 = [-1 0 1]
>> v2 = [-1; 0; 1]
```

```
>> M1 = [-1 0 1; -2 0 2]
>> M2 = [-1 -2; 0 0 ; 1 2]
>> M3 = [[-1 -2]; [0 0]] % inner brackets are redundant
```

$$a_1 = a_2 = -1$$

$$\mathbf{v}_1 = \begin{bmatrix} -1 & 0 & 1 \end{bmatrix}$$

$$\mathbf{v}_2 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

$$\mathbf{M}_1 = \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \end{bmatrix}$$

$$\mathbf{M}_2 = \begin{bmatrix} -1 & -2 \\ 0 & 0 \\ 1 & 2 \end{bmatrix}$$

$$\mathbf{M}_3 = \begin{bmatrix} -1 & -2 \\ 0 & 0 \end{bmatrix}$$



Matrix Construction Element-by-element II.

- ▶ Construct following matrices:
 - ▶ Matrix values are defined inside square brackets `[]`,
 - ▶ semicolon “;” separates individual rows of a matrix.

$$\mathbf{A} = \begin{bmatrix} -1 & 1 \\ 1 & -2 \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$



Matrix Construction Element-by-element II.

- ▶ Construct following matrices:
 - ▶ Matrix values are defined inside square brackets [],
 - ▶ semicolon “;” separates individual rows of a matrix.

$$\mathbf{A} = \begin{bmatrix} -1 & 1 \\ 1 & -2 \end{bmatrix} \quad \mathbf{B} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

```
>> A = [-1 1; 1 -2]
>> B = [1 2 3; 4 5 6; 7 8 9]
```



Matrix Construction

- ▶ Semicolon placed at the end of a command suppresses display of the output in Command Window.

```
>> a = 1  
>> b = 5;
```

- ▶ When there is more than one command on the same line, comma is used to separate each of the commands.

```
>> a = 1, b = 5  
>> a = 1; b = 5;
```

- ▶ Note: it is possible to copy and paste code including “»”
- ▶ Row vs. column vector:

```
>> c = [1 0 0]  
>> d = [0; 0; 1]
```



Basic Math Operators I.

- ▶ Operator types:
 - ▶ arithmetic:
 - ▶ matrix,
 - ▶ vector,
 - ▶ relational,
 - ▶ logical and other (to be mentioned later ...).
- ▶ Other operations using MATLAB functions:
 - ▶ complex conjugate,
 - ▶ sum, determinant, square root,
 - ▶ and hundreds of other functions ...

+	addition
-	subtraction
*	multiplication
^	power
.'	transpose
\	left matrix division
/	right matrix division
.	dot notation



Operator Precedence in MATLAB

- ▶ According to the following table:
 - ▶ see MATLAB → Language Fundamentals → Operators and Elementary Operations → Arithmetic

1	parentheses	()								
2	transpose, power	'	.	^	.	^				
3	(matrix) power with unary and logical operations	.	^-	.	^+	.	^~			
4	unary plus, unary minus, logical negation	+	-	~						
5	multiplication, division	*	.	*	/	\	.	/	.	\
6	addition, subtraction	+	-							
7	colon operator	:								
8	relation operators	<	>	<=	>=	==	~=			
9	logical AND (element-wise)	&								
10	logical OR (element wise)									
11	logical AND (short-circuit)	&&								
12	logical OR (short-circuit)									



Basic Math Operators II.

- ▶ Type in the following commands:
 - ▶ Zero can be omitted with a decimal number beginning with zero (not recommended).

```
>> a3 = -2/4  
>> a4 = -0.5  
>> a5 = -.5
```

- ▶ What is the difference between a_3 , a_4 and a_5 ?
- ▶ Beware the precedence of operators:

```
>> 3*5*6  
>> a1 = 15  
>> a2 = 10;  
>> a2/a3  
>> a2/a3*a4  
>> a2/(a3*a4)
```

- ▶ Explain the difference between a_2/a_3*a_4 and $a_2/(a_3/a_4)$.
- ▶ Verify the rules of operator precedence from the previous slide.



Lengthy commands in MATLAB

- ▶ It is suitable to structure command blocks for clarity:
 - ▶ next line: SHIFT + ENTER

```
>> A = [1 1 1]; B = [2 2 2]; % SHIFT + ENTER  
C = [2 3 2];
```

- ▶ Three dots notation:
 - ▶ For continuation of the same command on the next line.
 - ▶ Compare results:

```
>> A1 = [ 1 1 ...  
2 3]
```

```
>> A2 = [ 1 1  
2 3]
```



Basic Math Functions I.

- ▶ Math functions in MATLAB are generally divided in three groups:
 - ▶ **Scalar:**
 - ▶ Function operates over individual elements of a matrix,
 - ▶ *e.g.:* sin, sqrt, log, factorial.
 - ▶ **Vector:**
 - ▶ Function operates over individual rows/columns of a matrix,
 - ▶ *e.g.:* sum, max.
 - ▶ **Matrix:**
 - ▶ Function operates over a whole matrix,
 - ▶ *e.g.:* det, trace.



Basic Math Functions II.

- ▶ Using MATLAB help, calculate the following expression: $a \sin^2(\alpha) + a \cos^2(\alpha) - a$
 - ▶ Use numerical values your own choice.

- ▶ Verify following logarithmic identity: $\log_{10}(a) + \log_{10}(b) - \log_{10}(ab) = 0$

- ▶ Find sum of all elements in individual rows of the following matrix:

$$T = \begin{bmatrix} \frac{1}{2} & \frac{1}{3} & \frac{1}{4} & \frac{1}{5} \\ 6 & 7 & 8 & 9 \\ 0.2 & 0.3 & 0.4 & 0.5 \end{bmatrix}$$



Basic Math Functions II.

- ▶ Using MATLAB help, calculate the following expression: $a \sin^2(\alpha) + a \cos^2(\alpha) - a$
 - ▶ Use numerical values your own choice.

```
>> a = 3.24; alpha = pi/7;
>> a*sin(alpha)^2 + a*cos(alpha)^2 - a
```

- ▶ Verify following logarithmic identity: $\log_{10}(a) + \log_{10}(b) - \log_{10}(ab) = 0$

```
>> a = 14; b = 28;
>> log10(a) + log10(b) - log10(a*b)
```

- ▶ Find sum of all elements in individual rows of the following matrix:

$$T = \begin{bmatrix} \frac{1}{2} & \frac{1}{3} & \frac{1}{4} & \frac{1}{5} \\ 6 & 7 & 8 & 9 \\ 0.2 & 0.3 & 0.4 & 0.5 \end{bmatrix}$$

```
>> T = [1/2 1/3 1/4 1/5; 6 7 8 9; 0.2 0.3 0.4 0.5];
>> sum(T, 2)
```



Basic Math Functions III.

- ▶ Assume following vectors $\mathbf{u} = (1, 2, 3)$ and $\mathbf{v} = (3, 2, 1)$.

- ▶ Calculate:

$$\begin{array}{cc} \mathbf{u}\mathbf{v}^T & \mathbf{v}\mathbf{u}^T \\ \mathbf{v}^T\mathbf{u} & \mathbf{u}^T\mathbf{v} \\ \mathbf{u} \cdot \mathbf{v} & \mathbf{u} \times \mathbf{v} \end{array}$$

- ▶ Following functions are needed:
 - ▶ transpose (.'') of a matrix,
 - ▶ dot scalar product,
 - ▶ cross product.
- ▶ What is the result of the above mentioned operations?

$$\mathbf{A} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$$

$$\mathbf{A}^T = \begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{bmatrix}$$



Basic Math Functions III.

- ▶ Assume following vectors $\mathbf{u} = (1, 2, 3)$ and $\mathbf{v} = (3, 2, 1)$.

- ▶ Calculate:

scalar $\mathbf{u}\mathbf{v}^T$ $\mathbf{v}\mathbf{u}^T$ scalar
 matrix $\mathbf{v}^T\mathbf{u}$ $\mathbf{u}^T\mathbf{v}$ matrix
 scalar $\mathbf{u} \cdot \mathbf{v}$ $\mathbf{u} \times \mathbf{v}$ vector

- ▶ Following functions are needed:

- ▶ transpose (.') of a matrix,
- ▶ dot scalar product,
- ▶ cross product.

- ▶ What is the result of the above mentioned operations?

```

>> u = [1 2 3]; v = [3 2 1];
>> u*transpose(v), v * transpose(u)
>> v.'*u, u.'*v
>> dot(u, v), cross(u, v)
  
```

$$\mathbf{A} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$$

$$\mathbf{A}^T = \begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{bmatrix}$$

$$\begin{matrix} 10 & & 10 \\ \begin{bmatrix} 3 & 6 & 9 \\ 2 & 4 & 6 \\ 1 & 2 & 3 \end{bmatrix} & & \begin{bmatrix} 3 & 2 & 1 \\ 6 & 4 & 2 \\ 9 & 6 & 3 \end{bmatrix} \\ 10 & & \begin{bmatrix} -4 & 8 & 4 \end{bmatrix} \end{matrix}$$



Matrix Division in MATLAB

- ▶ Two cases are distinguished:
 - ▶ **left** division (`\` - `mldivide`),
 - ▶ **right** division (`/` - `mrdivide`).
- ▶ Solution of a linear system of equations:
 - ▶ **A** is an invertible (regular) matrix,
 - ▶ **b** is a column (row) vector.

$$\mathbf{Ax} = \mathbf{b}$$

$$\mathbf{x} = \mathbf{A}^{-1}\mathbf{b}$$

```
>> x = A \ b
```

$$\mathbf{xA} = \mathbf{b}$$

$$\mathbf{x} = \mathbf{bA}^{-1}$$

```
>> x = b / A
```



Basic Math Functions IV.

- ▶ Find the sum of diagonal elements (trace of a matrix) of the matrix \mathbf{T} with elements coming from normal distribution with mean equal to 10 and standard deviation equal to 4.
- ▶ Find determinant of matrix \mathbf{U} .

$$\mathbf{U} = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 0 \\ 0 & -2 & -1 \end{bmatrix}$$

- ▶ Solve the linear system of equations:

$$x_1 + 2x_2 + 3x_3 = 6$$

$$\mathbf{Ax} = \mathbf{b}$$

$$4x_1 + 5x_2 + 6x_3 = 15$$

$$\mathbf{x} = \mathbf{A}^{-1}\mathbf{b}$$

$$7x_1 + 8x_2 + x_3 = 16$$

```
>> T = 10 + 4*randn(7, 7);
```

```
>> U = [1 2 3; 0 2 0; ...  
0 -2 -1];
```



Basic Math Functions IV.

- ▶ Find the sum of diagonal elements (trace of a matrix) of the matrix \mathbf{T} with elements coming from normal distribution with mean equal to 10 and standard deviation equal to 4.
- ▶ Find determinant of matrix \mathbf{U} .

$$\mathbf{U} = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 0 \\ 0 & -2 & -1 \end{bmatrix}$$

- ▶ Solve the linear system of equations:

$$x_1 + 2x_2 + 3x_3 = 6$$

$$\mathbf{Ax} = \mathbf{b}$$

$$4x_1 + 5x_2 + 6x_3 = 15$$

$$\mathbf{x} = \mathbf{A}^{-1}\mathbf{b}$$

$$7x_1 + 8x_2 + x_3 = 16$$

```
>> T = 10 + 4*randn(7, 7);
```

```
>> trace(T)
```

```
>> U = [1 2 3; 0 2 0; ...  
0 -2 -1];
```

```
>> det(U)
```

```
>> A = [1 2 3; 4 5 6; 7 8 1];  
>> b = [6 15 16].';  
>> x = inv(A)*b;  
>> % OR  
>> x = A \ b;
```



Predefined Values in MATLAB

- ▶ MATLAB contains several predefined values:
 - ▶ `eps` – precision of single/double numbers (Determines the shortest distance between two single/double numbers).
 - ▶ `ans` – *answer* – most recent answer.
 - ▶ `NaN` – *not a number* (every expression containing NaN is NaN)
 - ▶ NaN can be used advantageously in some cases.
 - ▶ `Inf` – *infinite number* (variable `Inf` can be used in calculation)
 - ▶ Pay attention to `Inf` propagation throughout your code (use allowed operations only).
 - ▶ `i`, `j` – complex unit.
 - ▶ They are all basically functions (without input parameter).
 - ▶ Check results of the following expressions:

```

>> t1 = 10/0    % t1 = Inf
>> t2 = 0/0    % t2 = NaN
>> t3 = t1*5    % t3 = Inf
>> t4 = t1 + t2 % t4 = NaN
  
```

- ▶ `pi`, `intmin`, `intmax`, `realmin`, `realmax`, ... (functions)



Format of Command Line Output

- ▶ Up to now we have been using basic setup.
- ▶ MATLAB offers number of other formatting options
 - ▶ Use format `style`.
 - ▶ Output format does not change neither the computation accuracy nor the accuracy of stored results (`eps`, `realmax`, `realmin`, ... still apply).

<code>style</code>	format description
<code>short</code>	fixed 4 decimal points are displayed
<code>long</code>	15 decimal points for double precision, 7 decimal points for single precision
<code>shortE</code>	floating-point format (scientific notation)
<code>longE</code>	-//-
<code>bank</code>	two decimal points only (eur – cents)
<code>rat</code>	MATLAB attempts to display the results as a fraction
<code>compact</code>	suppressed the display of blank lines
and others...	note: omitting <code>style</code> parameter restores default setup



Format of Command Line Output

- ▶ Try following output format settings:
 - ▶ Each format is suitable for different type of problems.

```
>> s = [-5 1/2 1/3 10*pi sqrt(2)];  
>> format long; s  
>> format rat; s  
>> format bank; s  
>> format hex; s  
>> format +; s  
>> format; s
```

- ▶ There exist other formats with slight differences.
 - ▶ Check `>> doc format`
- ▶ Later, we will learn how to use formatted conversion into strings (commands `sprintf` and `fprintf`).



Complex Numbers I.

- ▶ More entry options in MATLAB.

```
>> C1 = 1 + 1j % preferred
>> C2 = 1 + 5i % preferred
>> C3 = 1 + 5*i % NO!
>> C4 = 1 + 5*sqrt(-1)
>> C5 = complex(1, 2)
>> C6 = 1e1i
>> C7 = exp(1j*pi/4)
```

- ▶ `cart2pol` and `pol2cart`, among others, can be used as well...

- ▶ Frequently used functions:

<code>real, imag</code>	real and imaginary part of a complex number
<code>conj</code>	complex conjugate
<code>abs</code>	absolute value of a complex number
<code>angle</code>	angle in complex plane [rad]
<code>complex</code>	constructs complex number from real and imaginary components
<code>isreal</code>	checks if the input is a complex number (more on that later)
<code>i, j</code>	complex unit
<code>cplxpair</code>	sort complex numbers into complex conjugate pairs



Complex Numbers II.

- ▶ Create complex number $z = 1 + 1j$ and its complex conjugate $s = z^*$.
- ▶ Switch between Cartesian and polar form (find $|z|$ and φ).

$$z = \operatorname{Re}\{z\} + j\operatorname{Im}\{z\} = a + jb$$

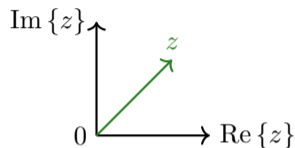
$$z = |z| e^{j\varphi}, |z| = \sqrt{a^2 + b^2}$$

$$z = |z| (\cos \varphi + j \sin \varphi)$$

- ▶ Verify Moivre's theorem:

$$z^n = (|z| e^{j\varphi})^n$$

$$z^n = |z|^n (\cos(n\varphi) + j \sin(n\varphi))$$





Complex Numbers II.

- ▶ Create complex number $z = 1 + 1j$ and its complex conjugate $s = z^*$.
- ▶ Switch between Cartesian and polar form (find $|z|$ and φ).

$$z = \operatorname{Re}\{z\} + j\operatorname{Im}\{z\} = a + jb$$

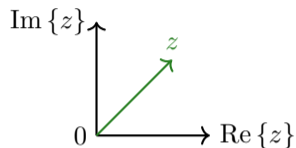
$$z = |z|e^{j\varphi}, |z| = \sqrt{a^2 + b^2}$$

$$z = |z|(\cos \varphi + j \sin \varphi)$$

- ▶ Verify Moivre's theorem:

$$z^n = (|z|e^{j\varphi})^n$$

$$z^n = |z|^n (\cos(n\varphi) + j \sin(n\varphi))$$



```
>> z = 1 + 1j;
>> s = conj(z);
>> % OR
>> s = z'
```

```
>> Z = abs(z)
>> phi = angle(z)
>> phi_deg = phi/pi * 180
>> zm = Z*(cos(phi) + 1j*sin(phi))
```

Exercises

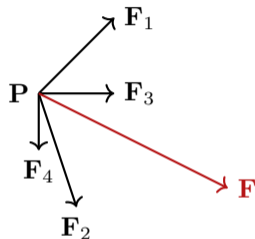


Exercise I.

- ▶ Following forces were localized at point \mathbf{P} in xy plane:

$$\begin{aligned}\mathbf{F}_1 &= [2, 2] & \mathbf{F}_3 &= [2, 0] \\ \mathbf{F}_2 &= [1, -3] & \mathbf{F}_4 &= [2, -1.5]\end{aligned}$$

- ▶ What is the direction of the resultant force \mathbf{F} ?
- ▶ Normalize the resulting vector.



$$\mathbf{n}_F = \frac{\mathbf{F}}{|\mathbf{F}|} = \frac{\mathbf{F}}{\sqrt{F_x^2 + F_y^2 + F_z^2}}$$



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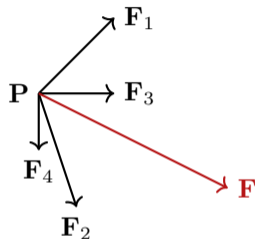
- ▶ What is the direction of the resultant force \mathbf{F} ?

```
>> F = [2 2] + [1 -3] + [2 0] + [2 -1.5]
```

- ▶ Normalize the resulting vector.

$$\mathbf{n}_F = \frac{\mathbf{F}}{|\mathbf{F}|} = \frac{\mathbf{F}}{\sqrt{F_x^2 + F_y^2 + F_z^2}}$$

```
>> Fn = F / norm(F)
>> Fn = F / vecnorm(F, 2, 2)
```





Exercise II.

- ▶ Type-in following commands:

```
>> clear, clc;  
>> w1 = [1 2 3 4]  
>> w2 = [-2 -3 -4]  
>> w3 = [-2; -3; -4]  
>> w4 = w1^2, w5 = w2 - w1
```

- ▶ Compare differences.
 - ▶ What is the cause of error in calculation of w4 and w5?
- ▶ Try also:

```
>> w3*3, w1 - 3  
>> w1 + [5 5 5 5]  
>> w6 = 5*w1 - [3 5 6] - w2
```

- ▶ Calculate the norm (magnitude) of vector w1.
 - ▶ Try more options.
- ▶ How to modify the calculation in the case of a complex vector?



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- ▶ Try also:

```
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>> w6 = 5*w1 - [3 5 6] - w2
```

- ▶ Calculate the norm (magnitude) of vector w_1 .

- ▶ Try more options.

```
>> norm(w1)
>> sqrt(w1*w1.')
```

- ▶ How to modify the calculation in the case of a complex vector?

```
>> norm(w1)
>> sqrt(dot(abs(w1), abs(w1)))
```



Exercise III.

- ▶ Calculate roots of the quadratic function:

$$-2x^2 - 5x = 3.$$

- ▶ First, rearrange the terms of the function.

$$2x^2 + 5x + 3 = 0 \Rightarrow a = 2, b = 5, c = 3$$

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-5 \pm \sqrt{25 - 24}}{4}$$

$$x_1 = -1, x_2 = -\frac{3}{2}$$

- ▶ MATLAB provides particular function for calculation of roots a function, try to search it out.



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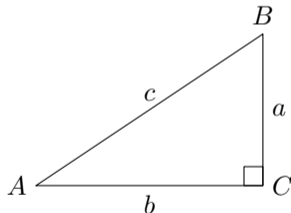
```
>> a = 2; b = 5; c = 3;
>> D = b^2 - 4*a*c;
>> x1 = (-b + sqrt(D))/(2*a);
>> x2 = (-b - sqrt(D))/(2*a);
```

```
>> roots([-2 -5 -3])
>> roots([2 5 3])
```




Exercise IV.

- ▶ Think over how many ways there are to calculate the length of hypotenuse when two legs of a triangle (a , b) are given.
 - ▶ Make use of various MATLAB operators and functions.
 - ▶ Consider also the case where the legs are complex numbers.

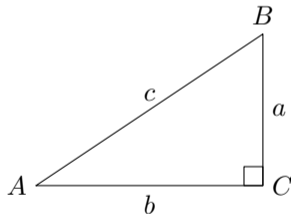




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 - ▶ Make use of various MATLAB operators and functions.
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```
>> a = 3; b = 4;  
>> c = hypot(a, b)  
>> c = sqrt(a^2 + b^2)  
>> c = sqrt([a, b]*[a; b])  
>> c = sqrt(sum([a^2, b^2]))  
>> c = sqrt(dot([a, b], [a, b]))
```





Exercise V.

- ▶ Create an arbitrary vector \mathbf{v} and rotate it around arbitrary angle α in xz plane using rotation matrix \mathbf{R} .

$$\mathbf{v}' = \mathbf{R}\mathbf{v}$$
$$\mathbf{R} = \begin{bmatrix} \cos \alpha & 0 & -\sin \alpha \\ 0 & 1 & 0 \\ \sin \alpha & 0 & \cos \alpha \end{bmatrix}$$



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$$\mathbf{R} = \begin{bmatrix} \cos \alpha & 0 & -\sin \alpha \\ 0 & 1 & 0 \\ \sin \alpha & 0 & \cos \alpha \end{bmatrix}$$

```
>> v = [1; 2; 3];  
>> a1 = pi/3;  
>> R = [cos(a1) 0 -sin(a1); ...  
0 1 0; ...  
sin(a1) 0 cos(a1)];  
>> v2 = R * v
```



Exercise VI.

- ▶ Use the following code and round the resulting number to:

```
>> r = 1 + 10*rand(1)
```

- ▶ nearest integer,
 - ▶ nearest integer greater than r ,
 - ▶ nearest integer lower than r ,
 - ▶ zero,
 - ▶ zero with precision of 2 decimal digits.
- ▶ Find remainder after r is divided by 0.1.
 - ▶ *modulus* vs. *remainder after division*



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 - ▶ *modulus* vs. *remainder after division*

```
>> round(r)
>> ceil(r)
>> floor(r)
>> fix(r)
>> fix(100*r)/100
```

```
>> rem(r, 0.1)
```



Exercise VII.

- Find out the magnitude of a complex vector (avoid indexing).
 - Use `abs` and `sqrt`.

$$\mathbf{Z} = [1 + 1j \quad \sqrt{2}]$$

$$\|\mathbf{Z}\| = ?, \quad \mathbf{Z} \in \mathbb{C}^2$$

- Alternatively, use following functions:
 - `norm`
 - `dot` (*dot product*)
 - `hypot` (*hypotenuse*)

```
Z = [1+1j, sqrt(2)]
Zz = abs(Z)
sqrt(Zz*Zz.')
```

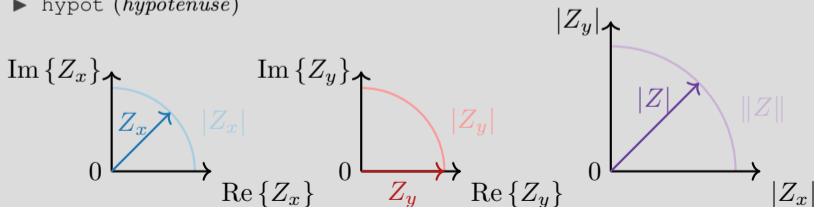
% or

```
sqrt(sum(Zz.^2))
```

% or with indexing

```
sqrt(Zz(1)^2 + Zz(2)^2)
```

```
norm(Z)
sqrt(dot(Z, Z))
hypot(Z(1), Z(2))
```



Questions?

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