Electromagnetic Field Theory 1 (fundamental relations and definitions)

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Fundamental Question of Classical Electrodynamics

A specified distribution of elementary charges is in a state of arbitrary (but known) motion. At certain time we pick one of them and ask what is the force acting on it.

Rather difficult question – will not be fully answered

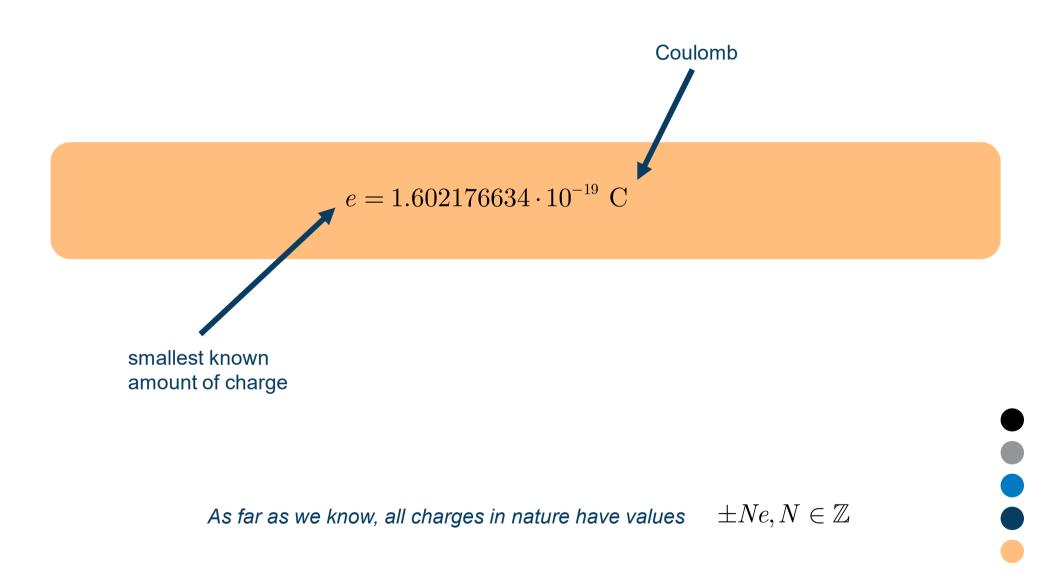
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Elementary Charge



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Amount of charge is conserved in every frame (even non-inertial).

Neutrality of atoms has been verified to 20 digits

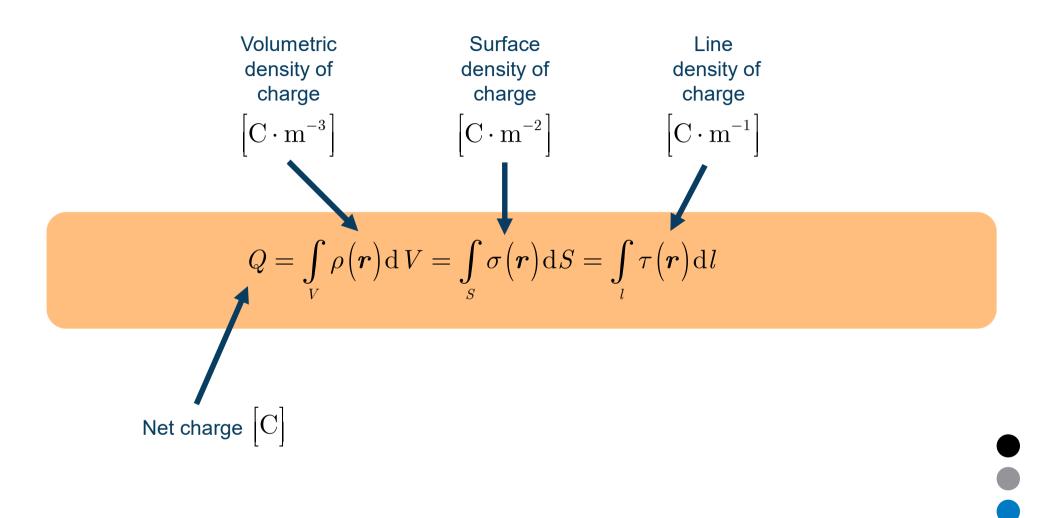
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Continuous approximation of charge distribution



Continuous approximation allows for using powerful mathematics

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Fundamental Question of Electrostatics

There exist a specified distribution of static elementary charges. We pick one of them and ask what is the force acting on it.

This will be answered in full details

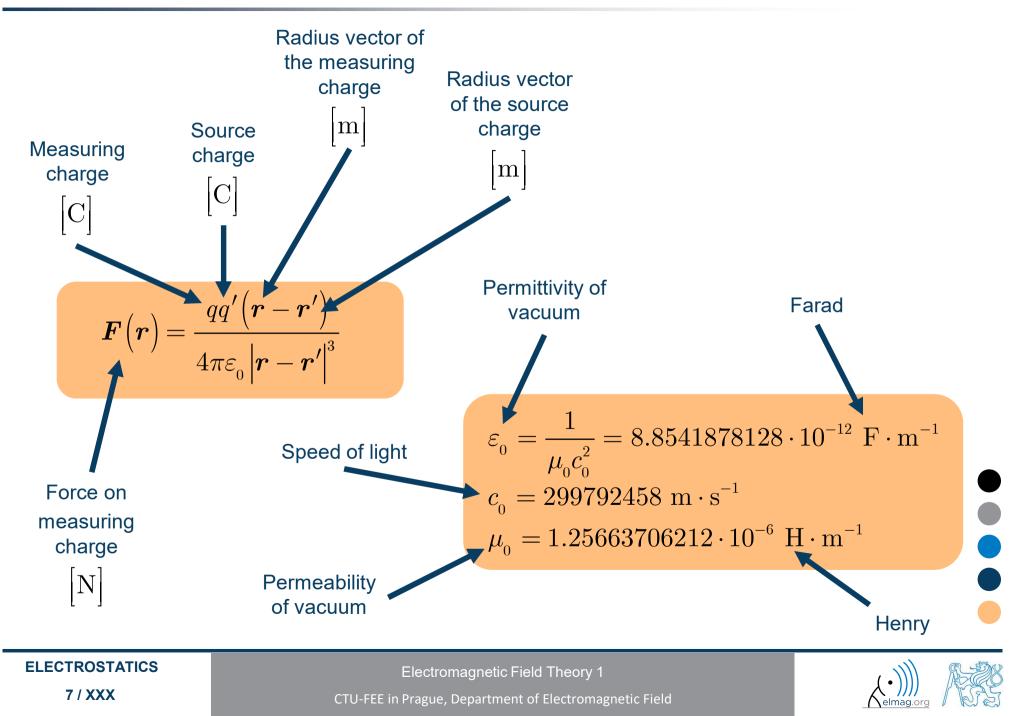
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Coulomb('s) Law



Coulomb('s) Law + Superposition Principle

$$oldsymbol{F}\left(oldsymbol{r}
ight)=rac{q}{4\piarepsilon_{_{0}}}{\displaystyle\sum_{_{n}}}rac{q_{n}^{\prime}\left(oldsymbol{r}-oldsymbol{r}_{n}^{\prime}
ight)}{\left|oldsymbol{r}-oldsymbol{r}_{n}^{\prime}
ight|^{^{3}}}$$

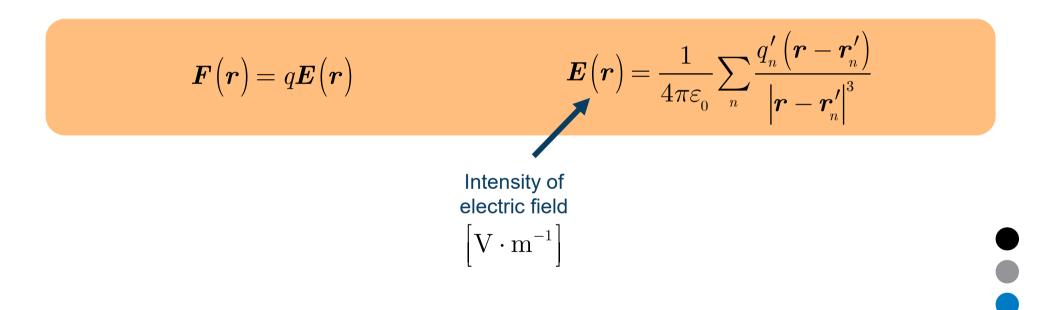
Entire electrostatics can be deduced from this formula

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Force is represented by field – entity generated by charges and permeating the space



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Continuous Distribution of Charge

Continuous description of charge allows for using powerful mathematics

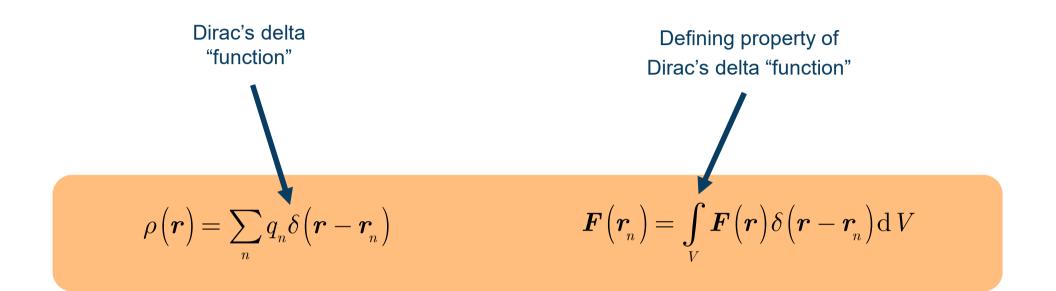


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Continuous Description of a Point Charge

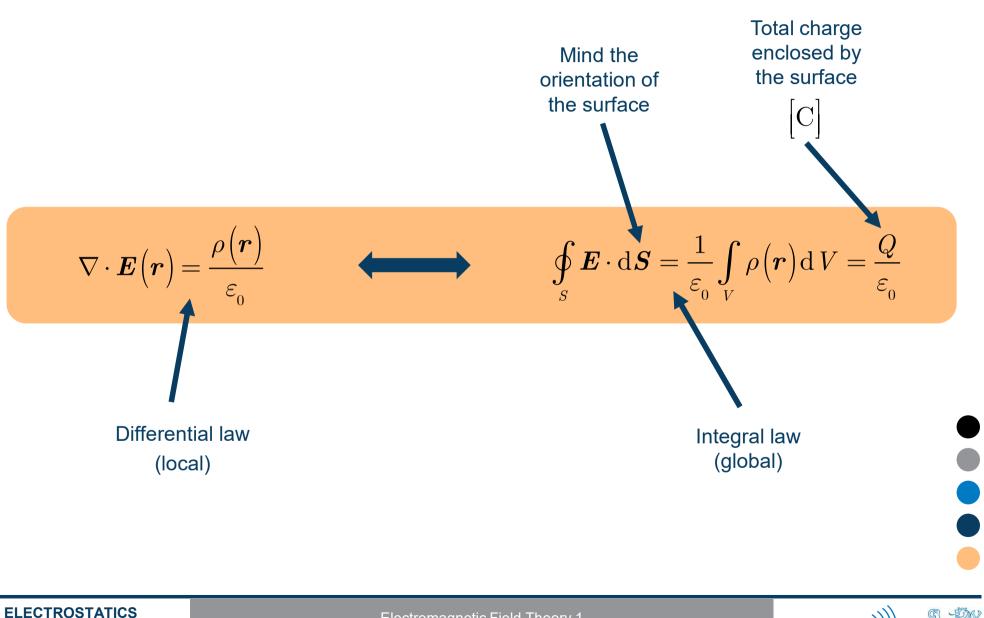




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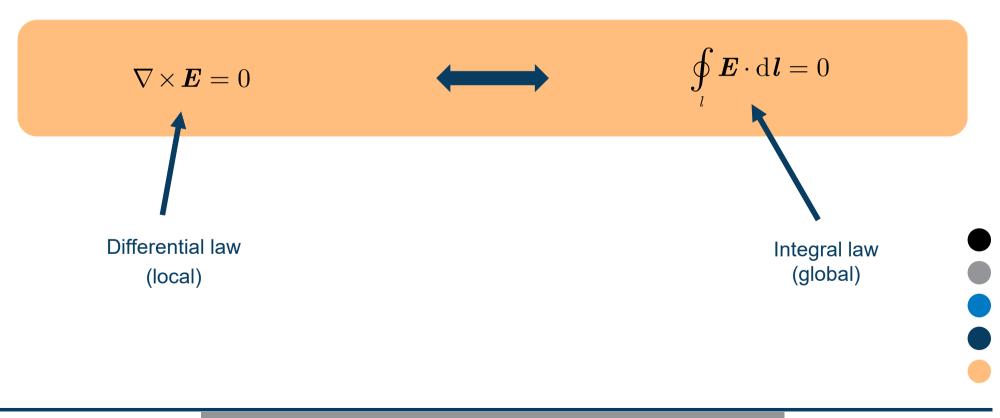
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Rotation of Electric Field



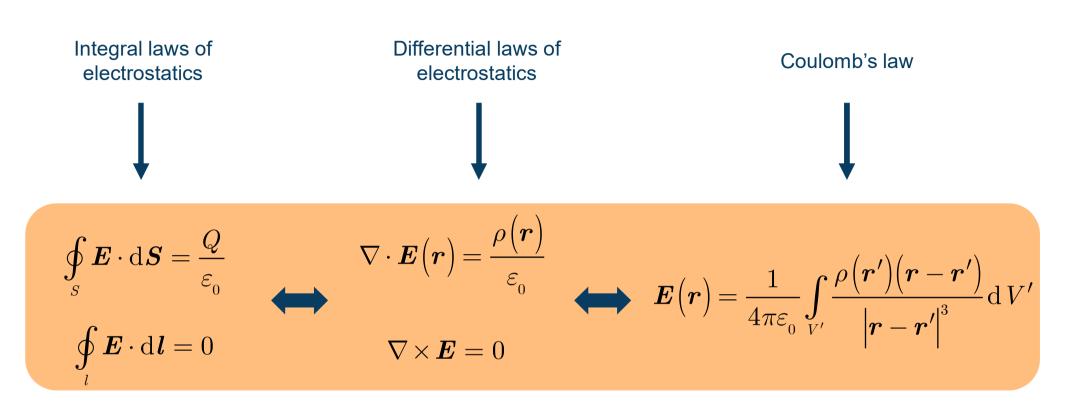
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Various Views on Electrostatics



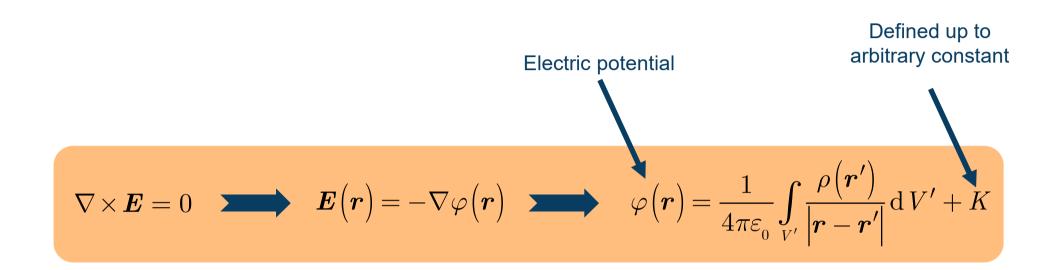
The physics content is the same, the formalism is different.



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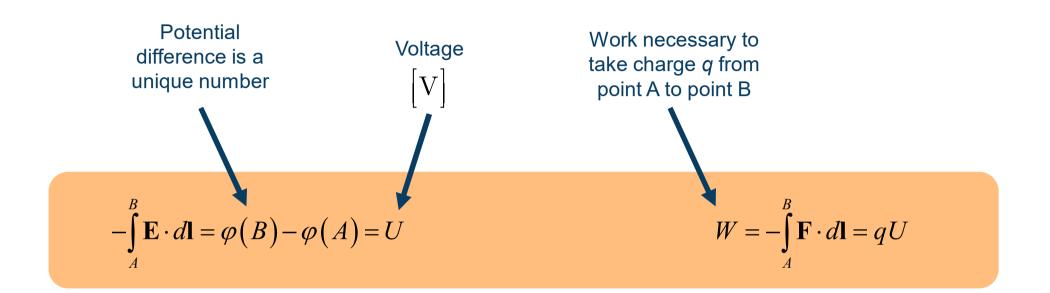
Scalar description of electrostatic field

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Voltage represents connection of abstract field theory with experiments

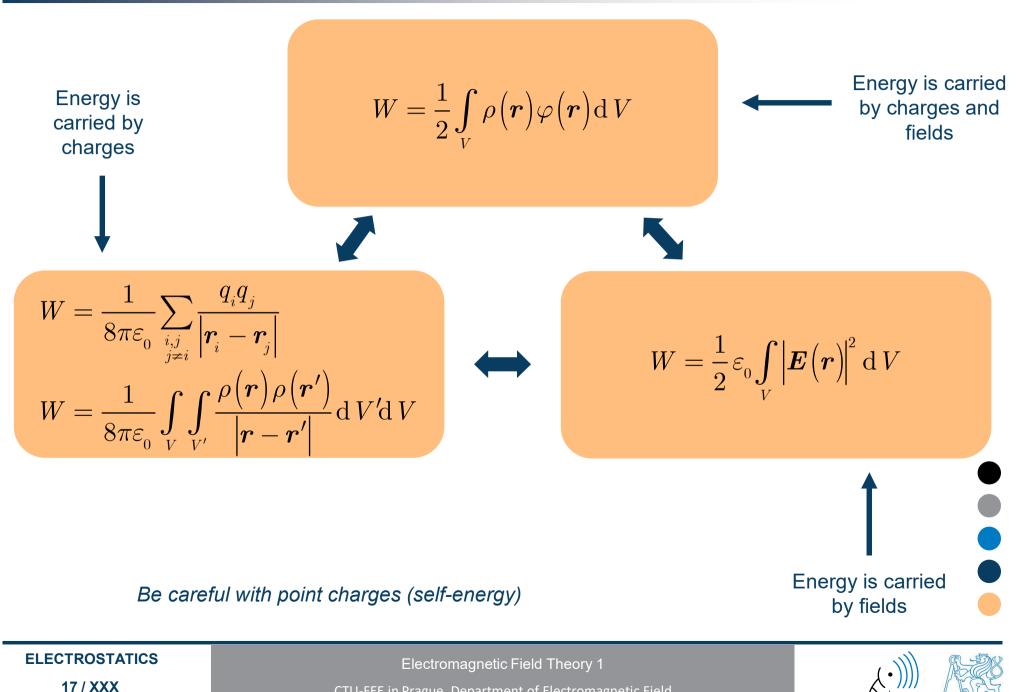


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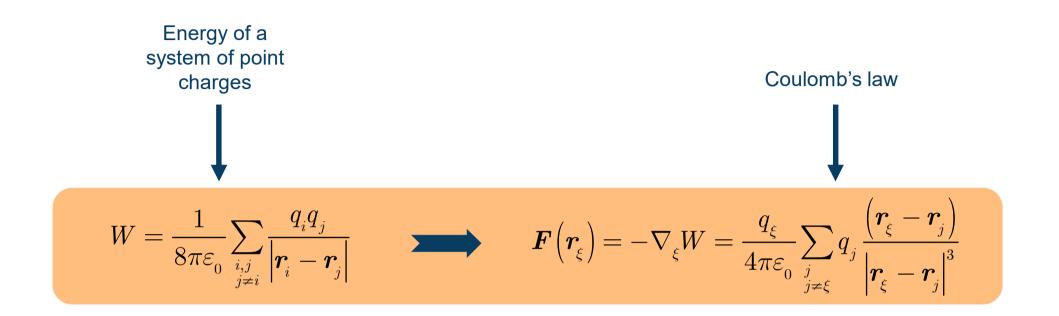
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Electrostatic Energy



Electrostatic Energy vs Force



Electrostatic forces are always acting so as to minimize energy of the system

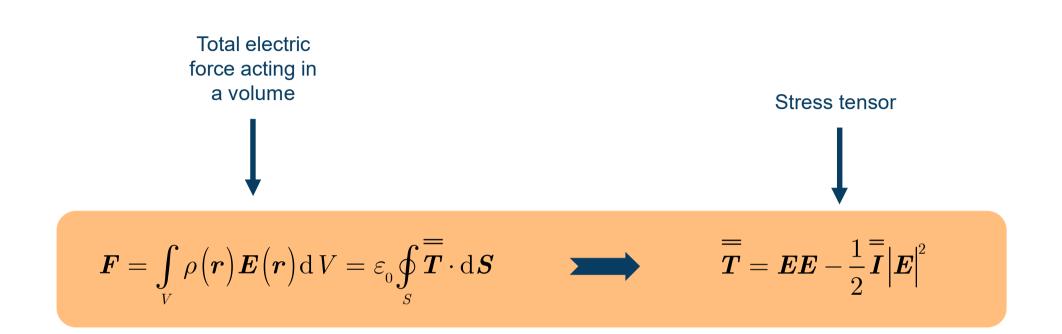


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Electric Stress Tensor



All the information on the volumetric Coulomb's force is contained at the boundary



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Ideal conductor contains unlimited amount of free charges which under action of external electric field rearrange so as to annihilate electric field inside the conductor.

In 3D, the free charge always resides on the external bounding surface of the conductor.

In 1D and 2D it is not so

Generally, free charges in conductors move so as to minimize the energy



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In an ideal conductor, wave functions of electrons in outer shells perceive flat potential background. In reaction to an external electric field, these wave functions are slightly modified so as to provide zero average charge density inside the conductor. Due to flat potential background, there is no counter interaction.

Long-range transport of charge does not truly happen in a solid conductor



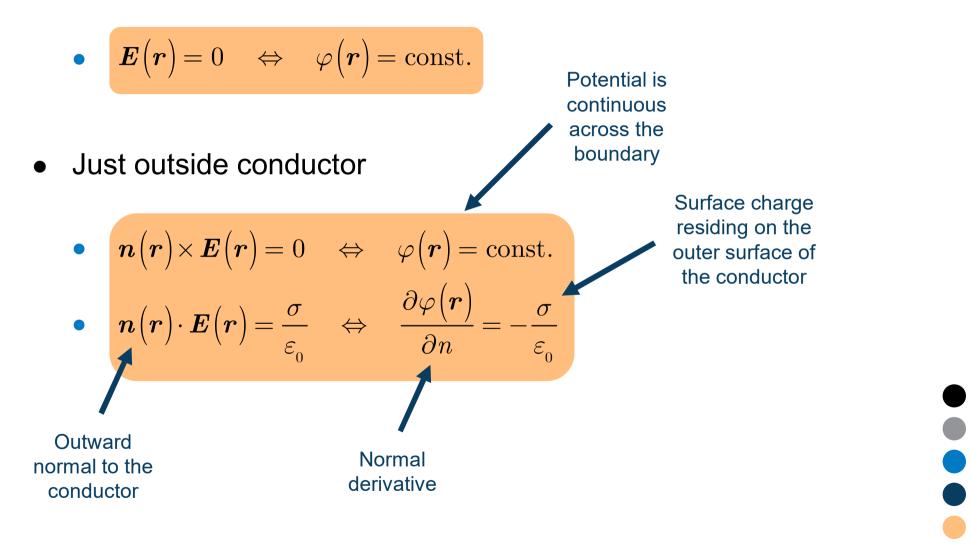
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Boundary Conditions on Ideal Conductor

• Inside conductor



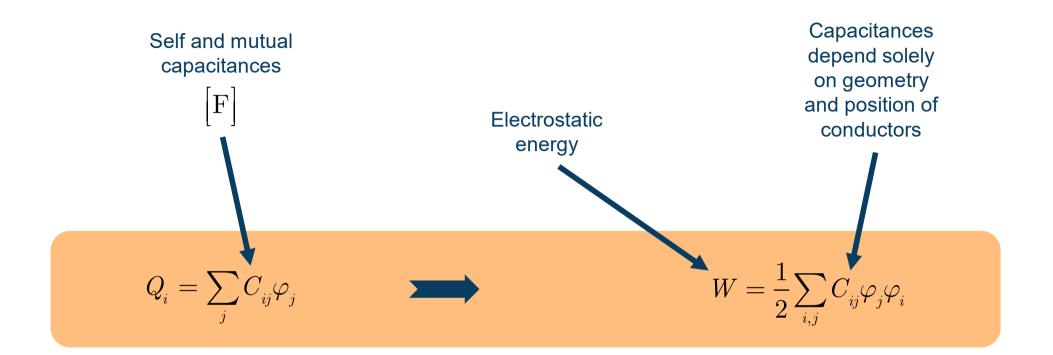
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Capacitance of a System of N conductors



Electrostatic system is fully characterized by capacitances (we know the energy)

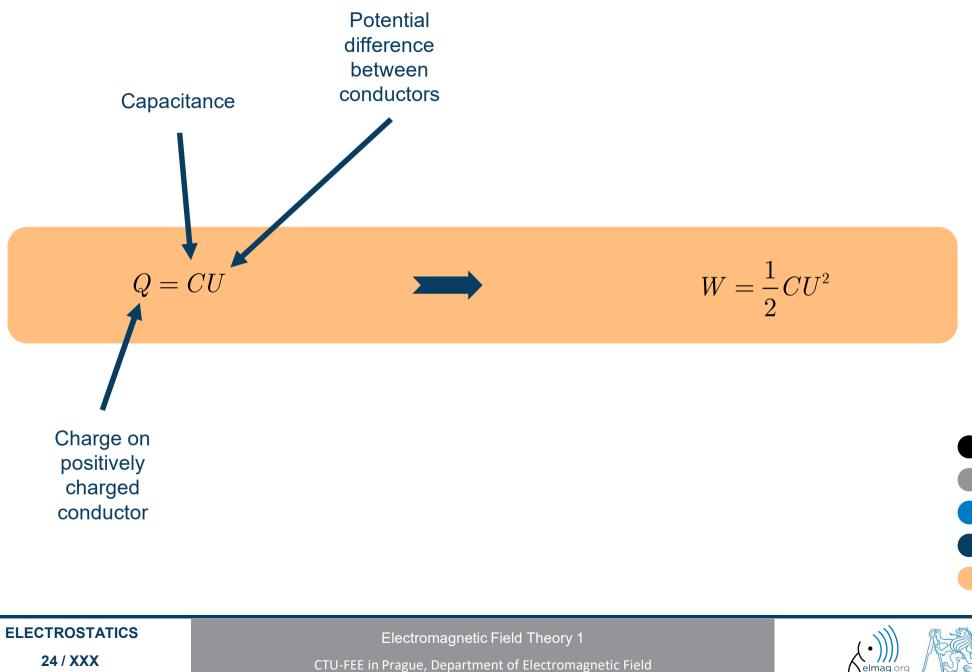


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Capacitance of a System of two conductors



Poisson('s) equation

$$\Delta \varphi \left(oldsymbol{r}
ight) = -rac{
ho \left(oldsymbol{r}
ight)}{arepsilon_{_{0}}}$$

The solution to Poisson's equation is unique in a given volume once the potential is known on its bounding surface and the charge density is known throughout the volume.



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Laplace('s) equation

$$\Delta arphi \left(oldsymbol{r}
ight) = 0$$

The solution to Laplace's equation is unique in a given volume once the potential is known on its bounding surface.

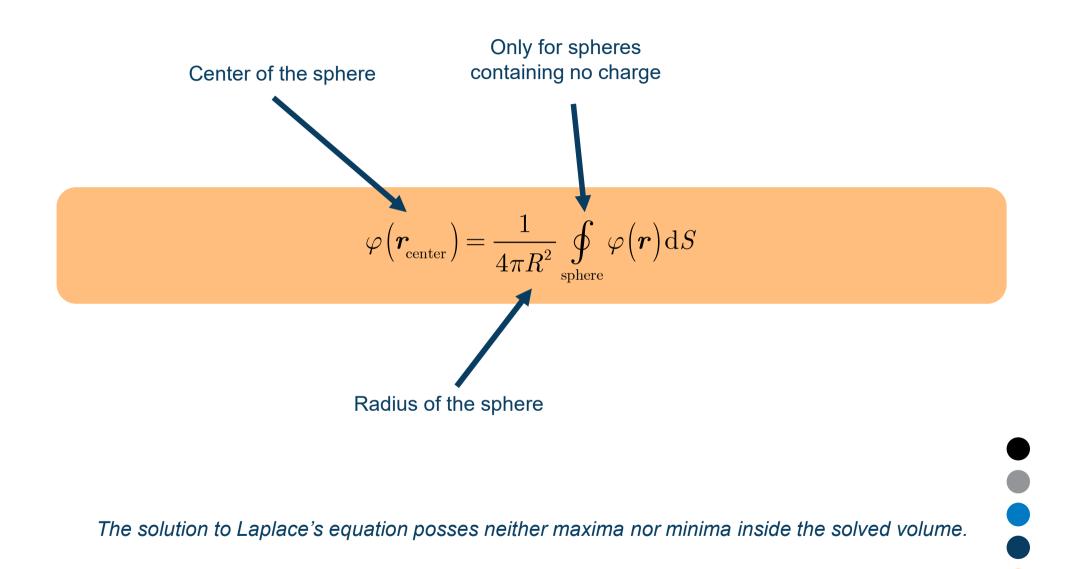


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Mean Value Theorem



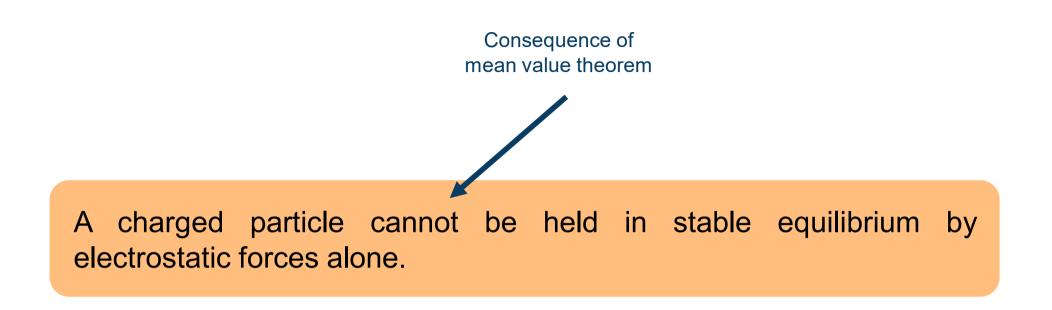


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Earnshaw('s) Theorem



Mind that the solution to Laplace's equation posses neither maxima nor minima inside the solved volume. This means that charged particle will always travel towards the boundary.



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When solving field generated by charges in the presence of conductors, it is sometimes possible to remove the conductor and mimic its boundary conditions by adding extra charges to the exterior of the solution volume. The uniqueness theorem claims that this is a correct solution.

Image method always works with planes and spheres.

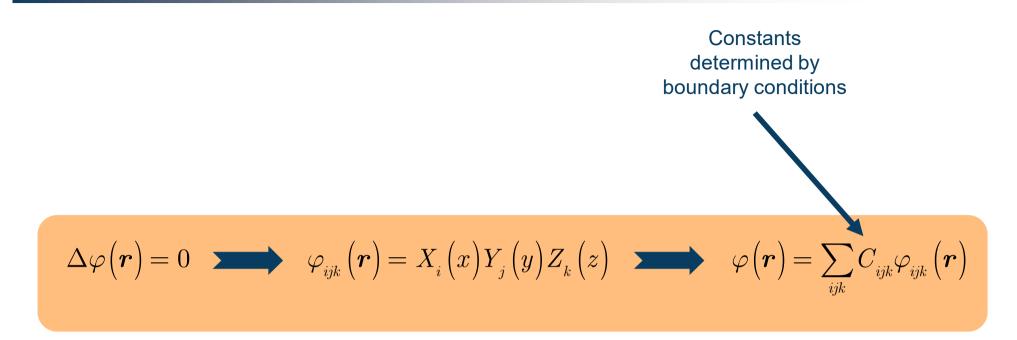


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Separation of Variables



Semi-analytical method for canonical problems

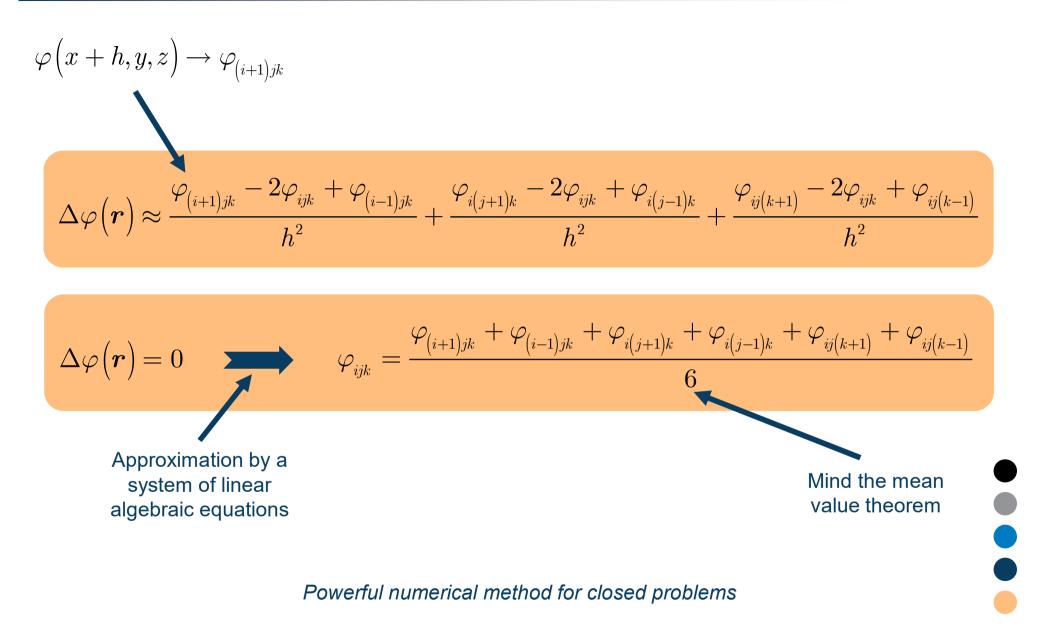


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Finite Differences



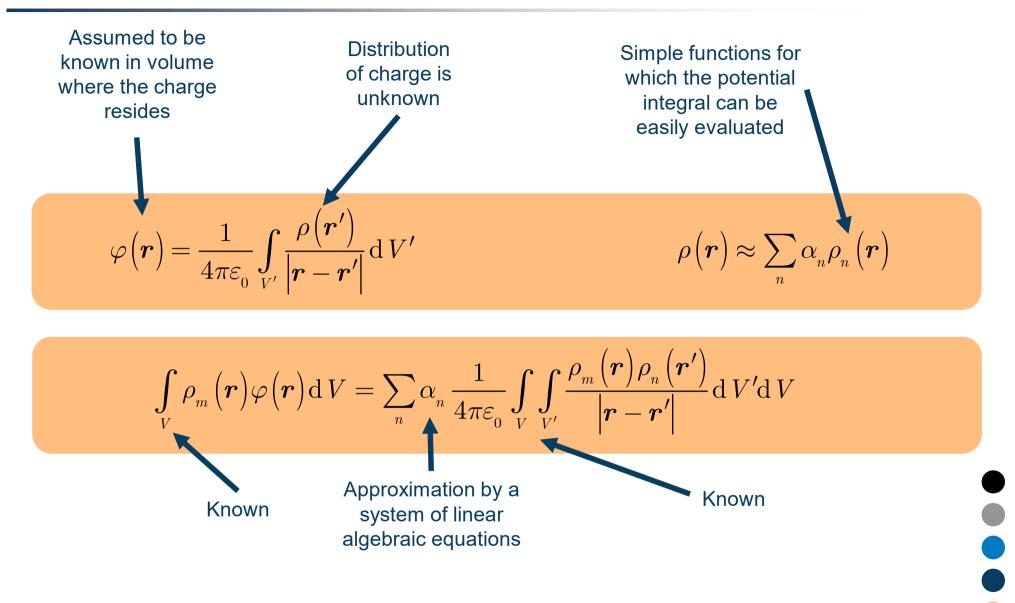
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Integral Equation & Method of Moments



Powerful numerical method for open problems

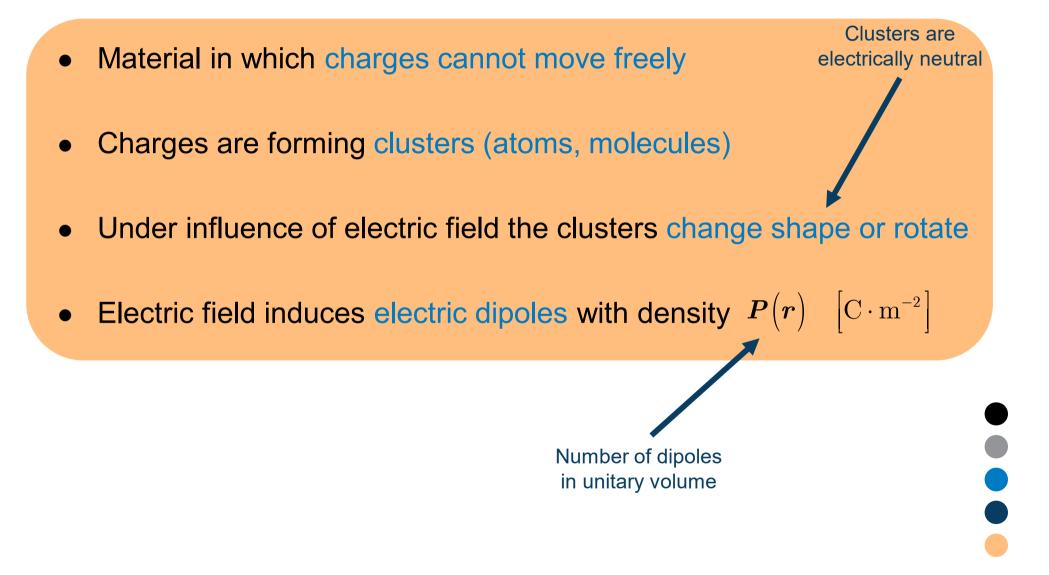
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Dielectrics



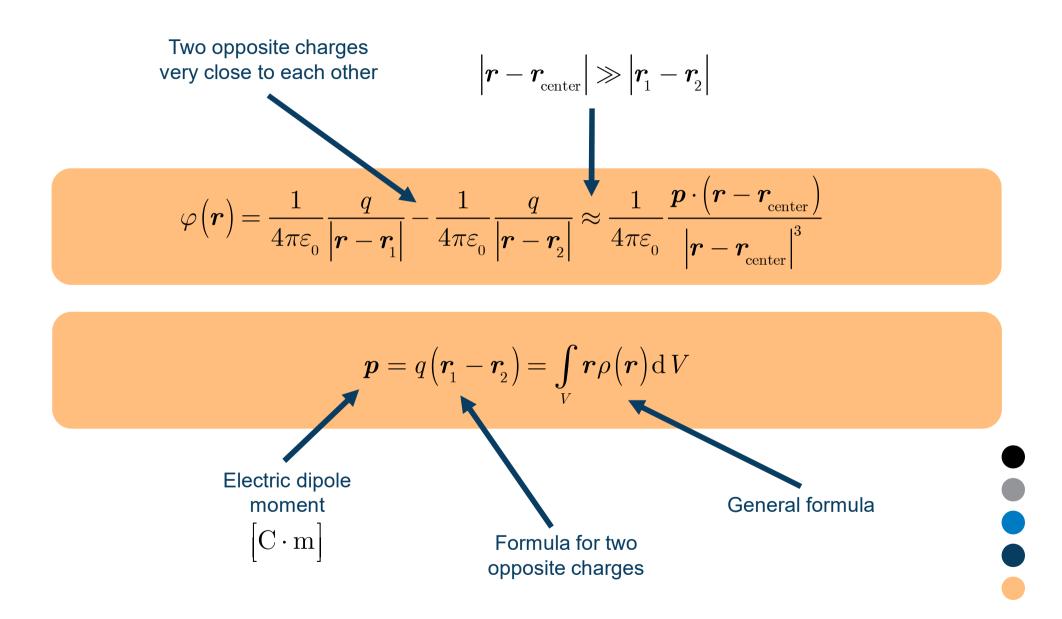


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Electric Field of a Dipole



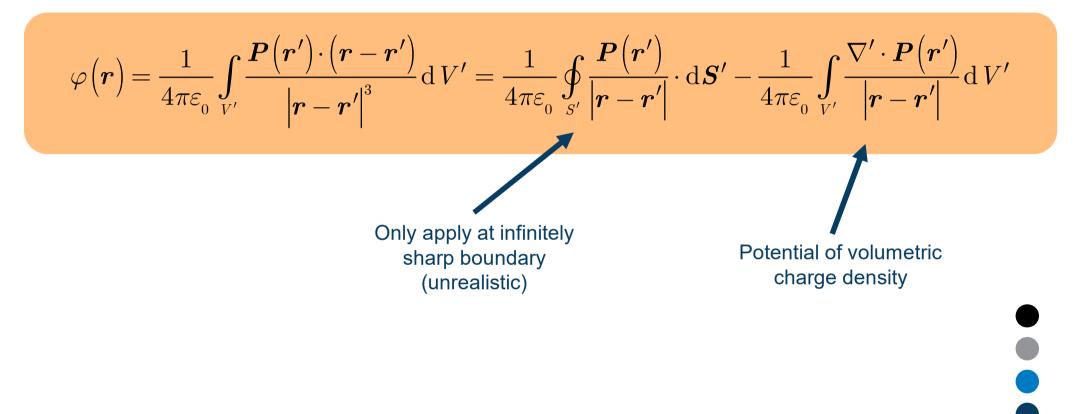


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Field Produced by Polarized Matter



This formula holds very well outside the matter and, curiously, it also well approximates the field inside

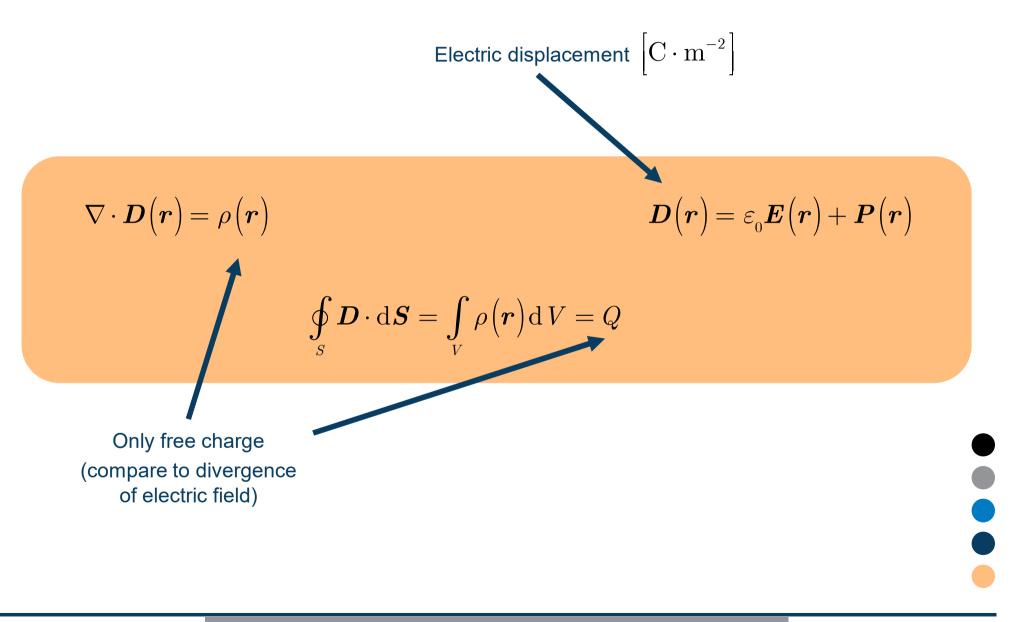


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Electric Displacement



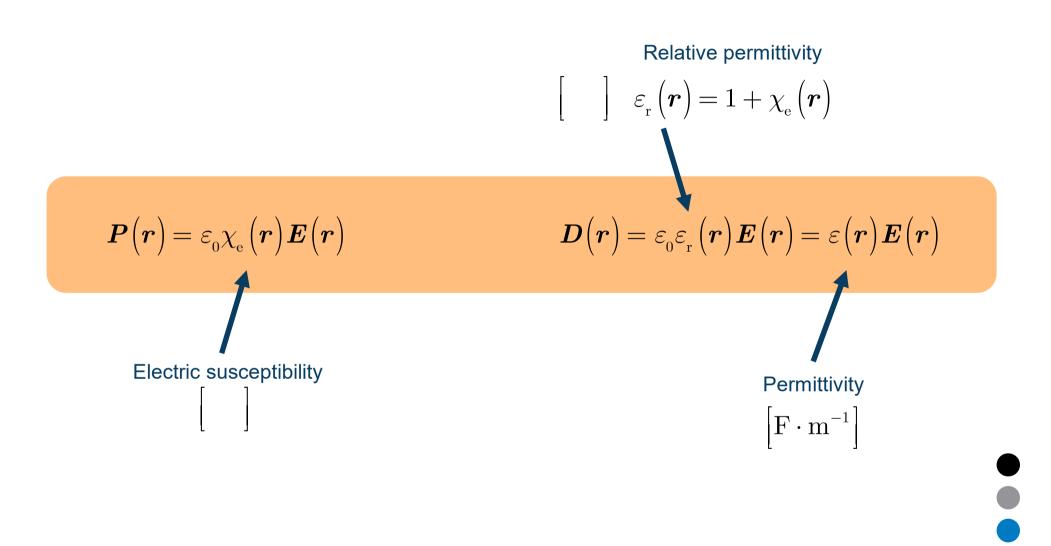
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Linear Isotropic Dielectrics



All the complicated structure of matter reduces to a simple scalar quantity

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Analogy with electric field in vacuum can only be used when entire space is homogeneously filled with dielectric.

$$\nabla \times \boldsymbol{D}(\boldsymbol{r}) = \nabla \times [\varepsilon(\boldsymbol{r})\boldsymbol{E}(\boldsymbol{r})] \neq 0$$

Inequality is due to boundaries

Analogy with vacuum can only be used when space is homogeneously filled with dielectric

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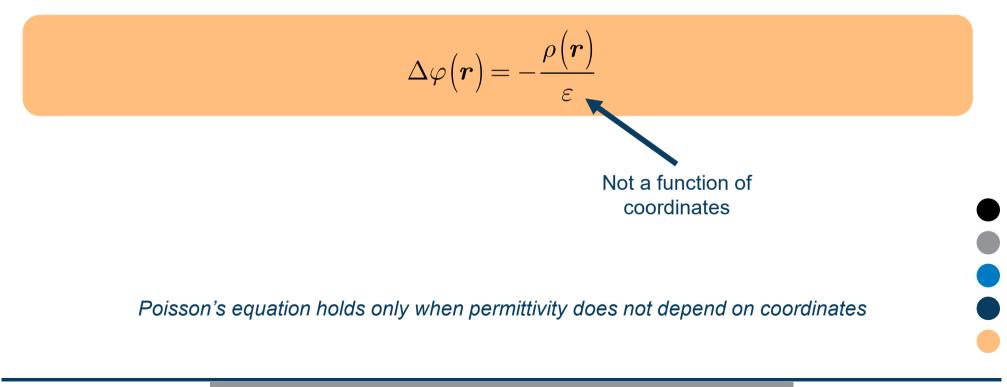


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$$\nabla \times \boldsymbol{E}(\boldsymbol{r}) = 0 \Leftrightarrow \boldsymbol{E}(\boldsymbol{r}) = -\nabla \varphi(\boldsymbol{r}) \qquad \qquad \qquad \qquad \nabla \cdot \left[\varepsilon(\boldsymbol{r}) \nabla \varphi(\boldsymbol{r})\right] = -\rho(\boldsymbol{r})$$



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$$\boldsymbol{n}(\boldsymbol{r}) \times \left[\boldsymbol{E}_{1}(\boldsymbol{r}) - \boldsymbol{E}_{2}(\boldsymbol{r})\right] = 0 \quad \Leftrightarrow \quad \varphi_{1}(\boldsymbol{r}) - \varphi_{2}(\boldsymbol{r}) = 0$$
$$\boldsymbol{n}(\boldsymbol{r}) \cdot \left[\varepsilon_{1}\boldsymbol{E}_{1}(\boldsymbol{r}) - \varepsilon_{2}\boldsymbol{E}_{2}(\boldsymbol{r})\right] = \sigma(\boldsymbol{r}) \quad \Leftrightarrow \quad \varepsilon_{1}\frac{\partial\varphi_{1}(\boldsymbol{r})}{\partial n} - \varepsilon_{2}\frac{\partial\varphi_{2}(\boldsymbol{r})}{\partial n} = -\sigma(\boldsymbol{r})$$
Normal pointing to region (1)

Both conditions are needed for unique solution

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Electrostatic Energy in Dielectrics

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Forces on Dielectrics



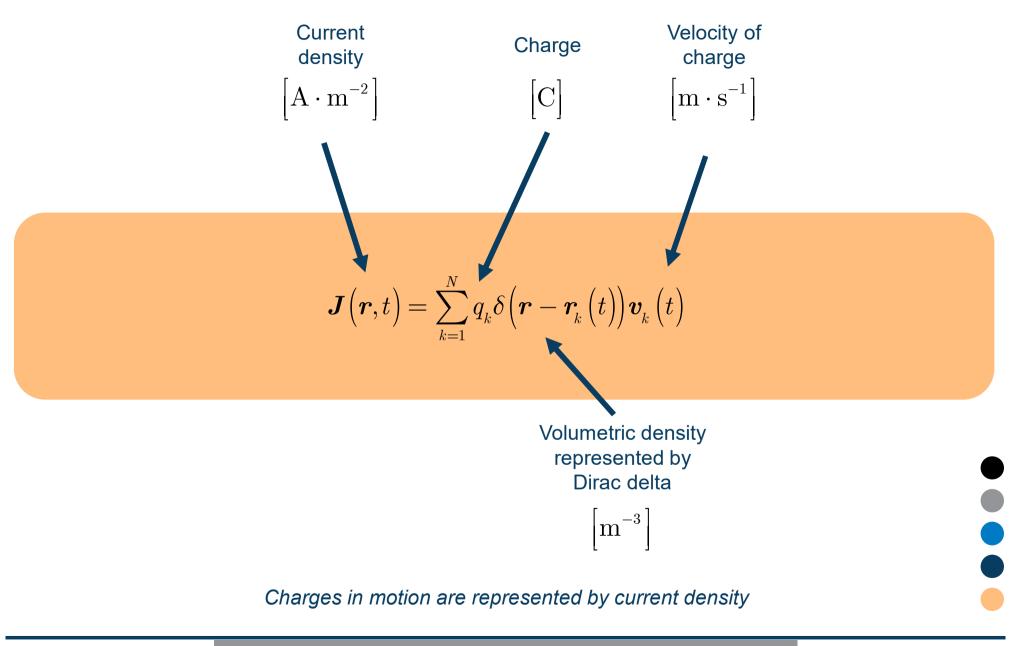
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Electric Current





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Local Charge Conservation

$$\nabla \cdot \boldsymbol{J}(\boldsymbol{r},t) = -\frac{\partial}{\partial t} \sum_{k=1}^{N} q_k \delta(\boldsymbol{r} - \boldsymbol{r}_k(t)) = -\frac{\partial \rho(\boldsymbol{r},t)}{\partial t}$$

Charge is conserved locally at every space-time point

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Global Charge Conservation

When charge leaves a given volume, it is always accompanied by a current through the bounding envelope

$$\oint_{S} \boldsymbol{J}(\boldsymbol{r},t) \cdot d\boldsymbol{S} = -\frac{\partial Q(t)}{\partial t}$$

Charge can neither be created nor destroyed. It can only be displaced.

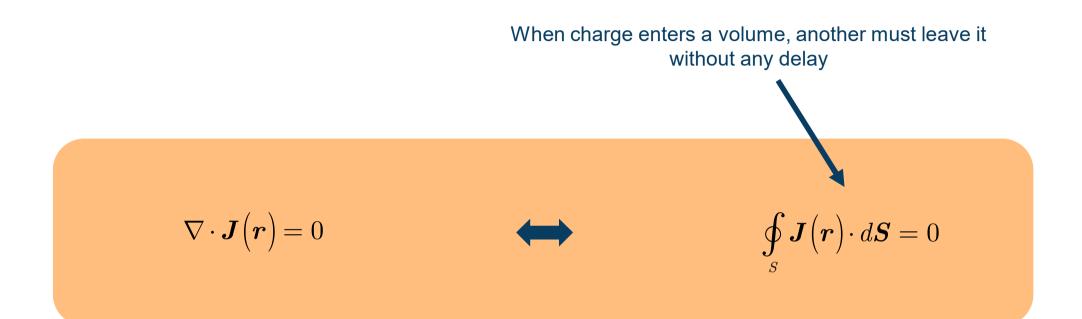
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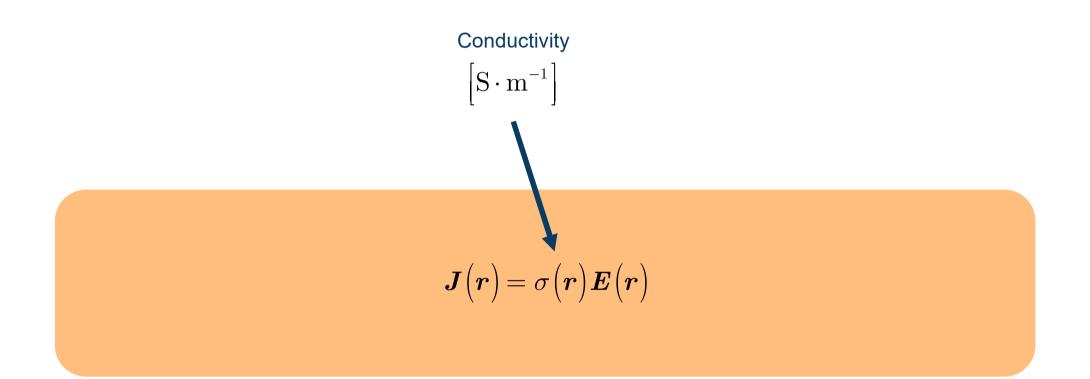
Stationary Current



There is no charge accumulation in stationary flow

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This simple linear relation holds for enormous interval of electric field strengths



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Stationary flow of charges cannot be caused by electrostatic field. The motion forces are non-conservative, are called electromotive forces, and are commonly of chemical, magnetic or photoelectric origin.





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Boundary Conditions for Stationary Current

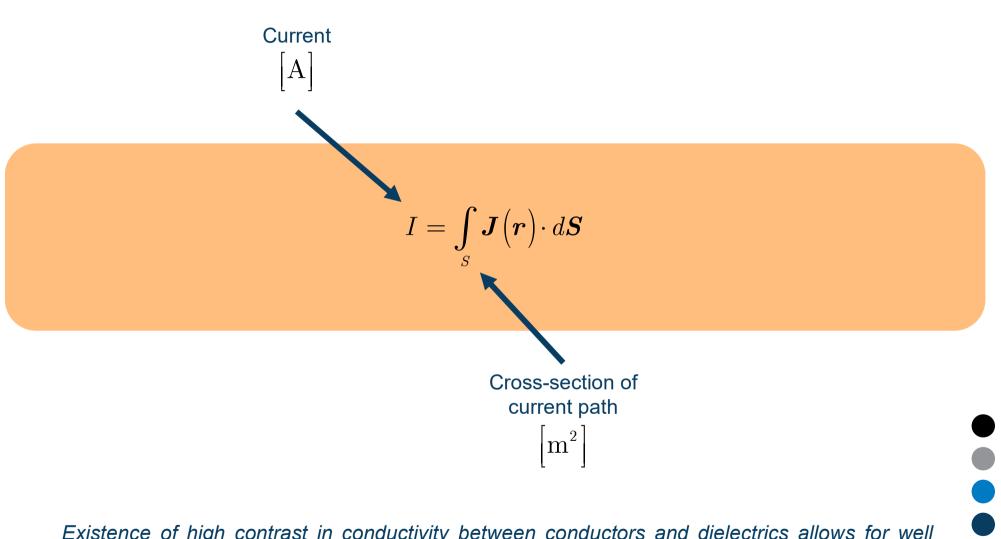
$$\begin{split} \boldsymbol{n}(\boldsymbol{r}) \times \left[\boldsymbol{E}_{1}\left(\boldsymbol{r}\right) - \boldsymbol{E}_{2}\left(\boldsymbol{r}\right)\right] &= 0 \quad \Leftrightarrow \quad \varphi_{1}\left(\boldsymbol{r}\right) - \varphi_{2}\left(\boldsymbol{r}\right) = 0 \\ \boldsymbol{n}(\boldsymbol{r}) \cdot \left[\varepsilon_{1}\boldsymbol{E}_{1}\left(\boldsymbol{r}\right) - \varepsilon_{2}\boldsymbol{E}_{2}\left(\boldsymbol{r}\right)\right] &= \sigma\left(\boldsymbol{r}\right) \quad \Leftrightarrow \quad \varepsilon_{1}\frac{\partial\varphi_{1}\left(\boldsymbol{r}\right)}{\partial n} - \varepsilon_{2}\frac{\partial\varphi_{2}\left(\boldsymbol{r}\right)}{\partial n} = -\sigma\left(\boldsymbol{r}\right) \\ \boldsymbol{n}\left(\boldsymbol{r}\right) \cdot \left[\sigma_{1}\boldsymbol{E}_{1}\left(\boldsymbol{r}\right) - \sigma_{2}\boldsymbol{E}_{2}\left(\boldsymbol{r}\right)\right] &= 0 \quad \Leftrightarrow \quad \sigma_{1}\frac{\partial\varphi_{1}\left(\boldsymbol{r}\right)}{\partial n} - \sigma_{2}\frac{\partial\varphi_{2}\left(\boldsymbol{r}\right)}{\partial n} = 0 \end{split}$$

Charge conservation forces the continuity of current across the boundary



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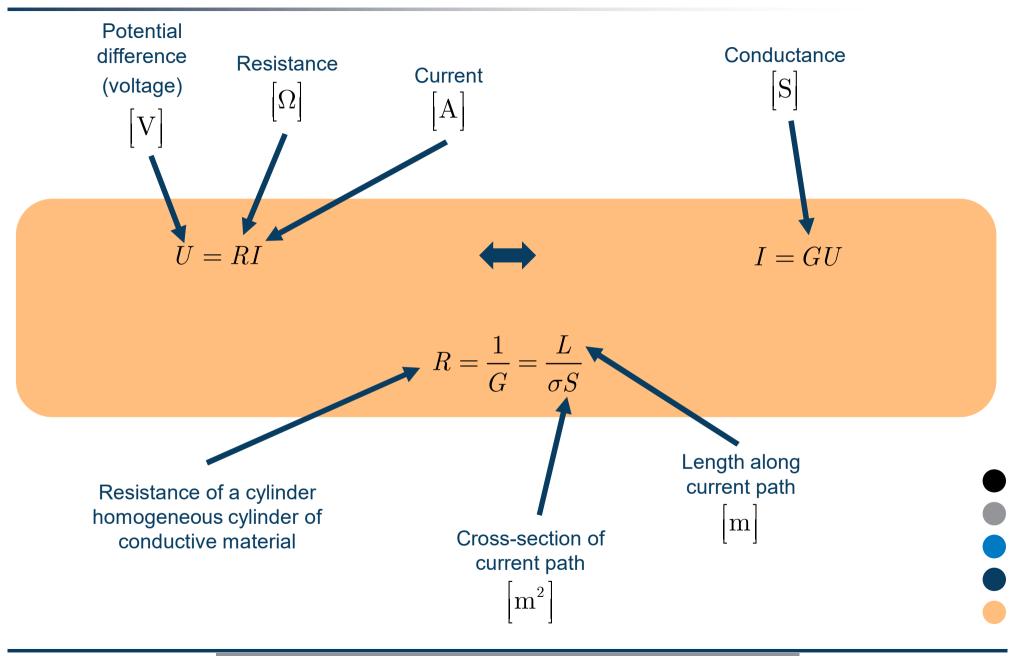




Existence of high contrast in conductivity between conductors and dielectrics allows for well defined current paths.

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Resistance (Conductance)





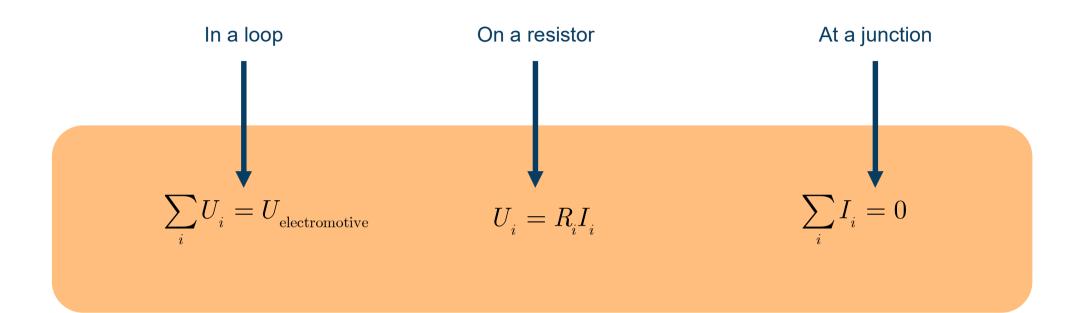
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Resistive Circuits and Kirchhoff('s) Laws

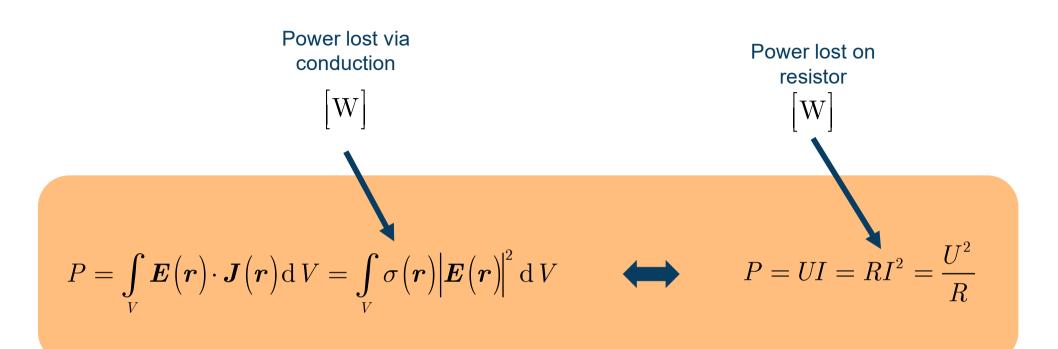


Kirchhoff's laws are a consequence of electrostatics and law's of stationary current flow

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Electric field within conducting material produces heat

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Fundamental Question of Magnetostatics

There exist a specified distribution of stationary current. We pick a differential volume of it and ask what is the force acting on it.

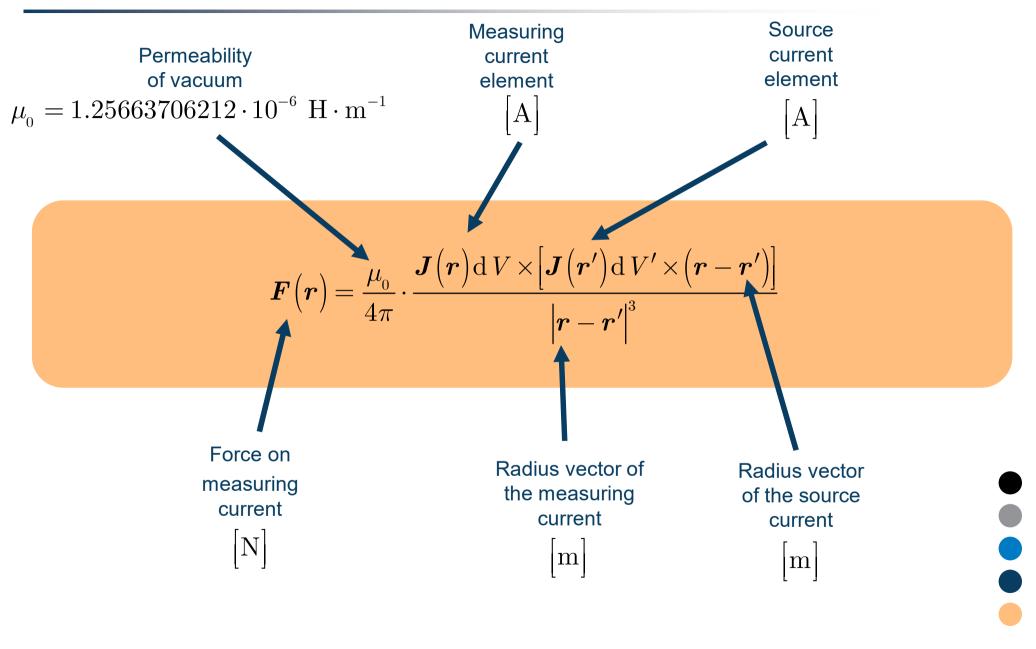


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Biot-Savart('s) Law



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Biot-Savart('s) Law + Superposition Principle

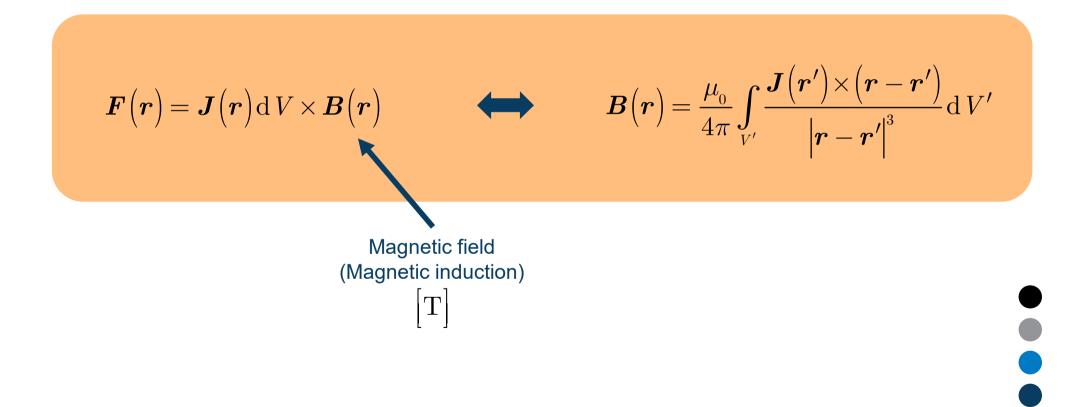
$$\boldsymbol{F}(\boldsymbol{r}) = \boldsymbol{J}(\boldsymbol{r}) d V \times \frac{\mu_0}{4\pi} \int_{V'} \frac{\boldsymbol{J}(\boldsymbol{r}') \times (\boldsymbol{r} - \boldsymbol{r}')}{|\boldsymbol{r} - \boldsymbol{r}'|^3} d V'$$

Entire magnetostatics can be deduced from this formula

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Divergence of Magnetic Field

$$abla \cdot \boldsymbol{B}(\boldsymbol{r}) = 0$$
 \longleftrightarrow
 $\boldsymbol{\delta}_{S} \boldsymbol{B}(\boldsymbol{r}) \cdot d\boldsymbol{S} = 0$

There are no point sources of magnetostatic field

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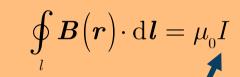
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Curl of Magnetic Field – Ampere('s) Law

$$abla imes oldsymbol{B}ig(oldsymbol{r}ig) = \mu_{_0}oldsymbol{J}ig(oldsymbol{r}ig)$$





Total current captured within the curve



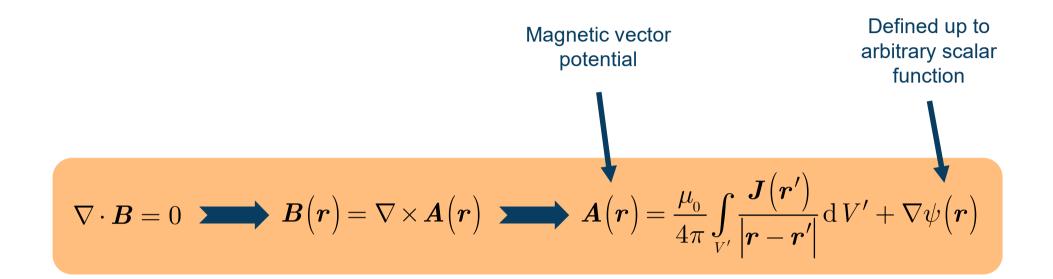


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Magnetic Vector Potential



Reduced description of magnetostatic field

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Poisson('s) equation

$$\Delta oldsymbol{A}ig(oldsymbol{r}ig) = -\mu_{_0}oldsymbol{J}ig(oldsymbol{r}ig)$$

The solution to Poisson's equation is unique in a given volume once the potential is known on its bounding surface and the current density is known through out the volume.

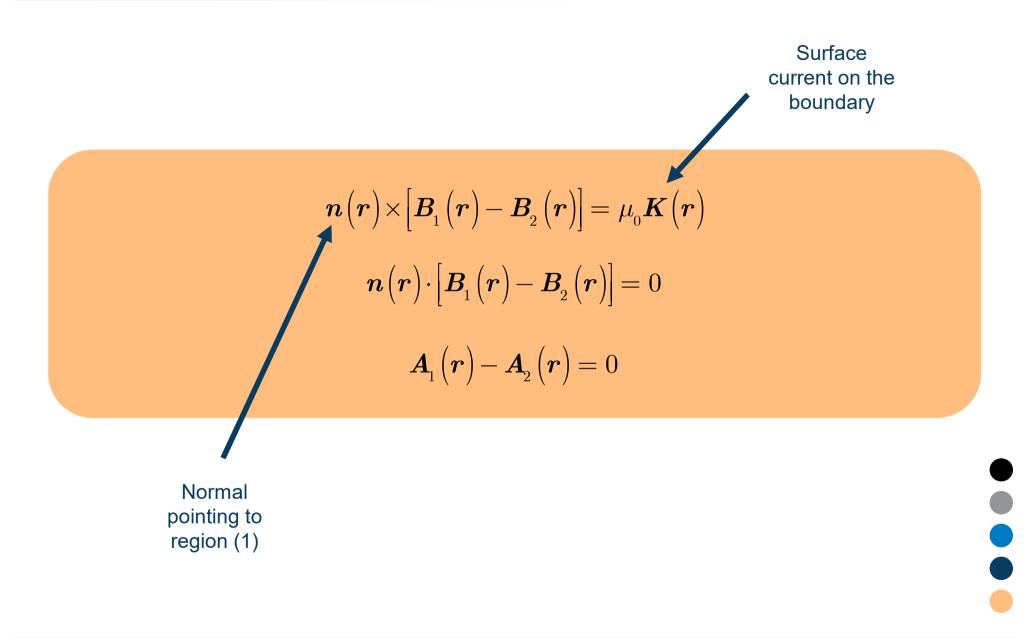


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Boundary Conditions





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Magnetostatic Energy

For now it is just a formula that works – it must be derived with the help of time varying fields

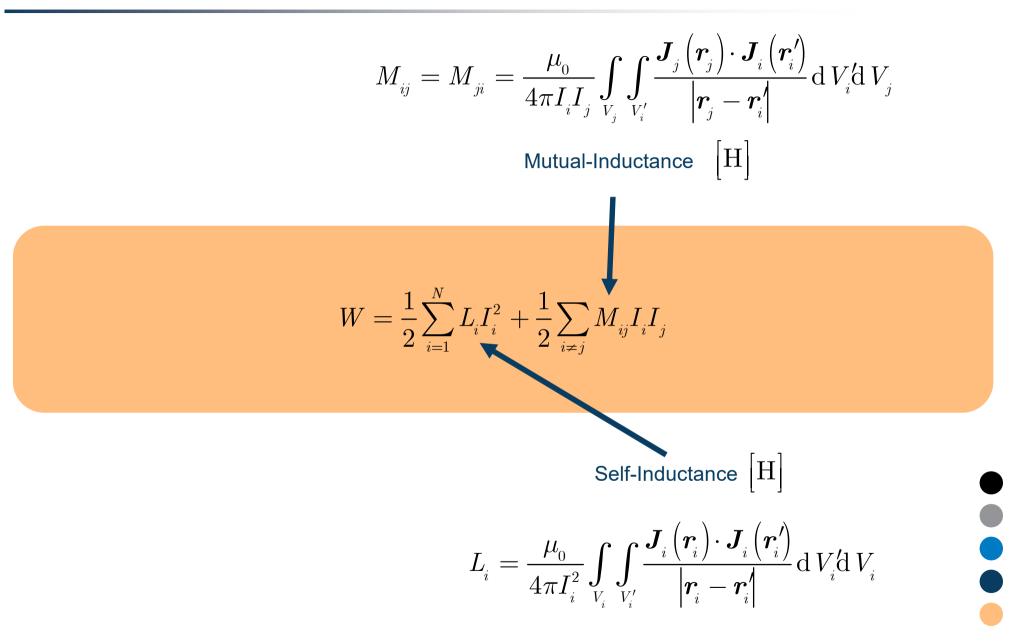
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Magnetostatic Energy – Current Circuits



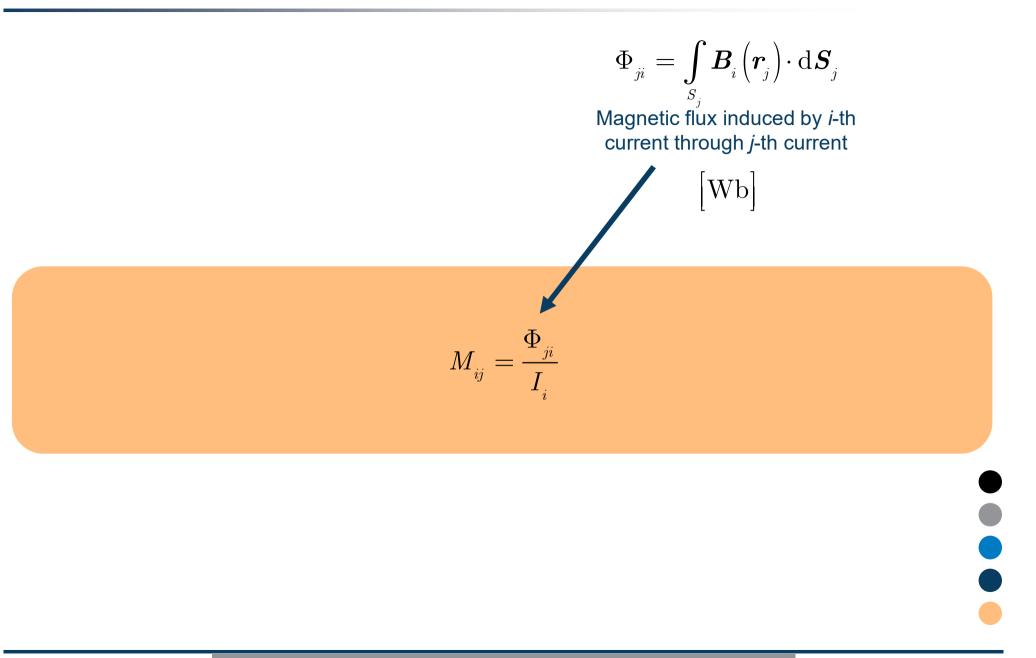


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Mutual Inductance – Thin Current Loop





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- Material response is due to magnetic dipole moments
- Magnetic moment comes from spin or orbital motion of an electron
- Magnetic field tends to align magnetic moments
- Magnetic field induces magnetic dipoles with density $oldsymbol{M}(oldsymbol{r}) = |\mathrm{A}\cdot\mathrm{m}^{-1}|$

Number of dipoles in unitary volume

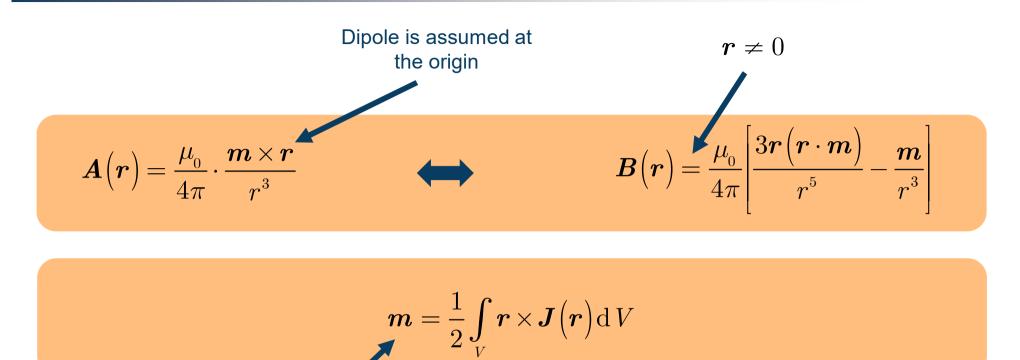


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Electromagnetic Field Theory 1

Magnetic Field of a Dipole



 $\begin{array}{c} \text{Magnetic dipole} \\ \text{moment} \\ \left[A \cdot m^2 \right] \end{array}$

Magnetic dipole approximates infinitesimally small current loop

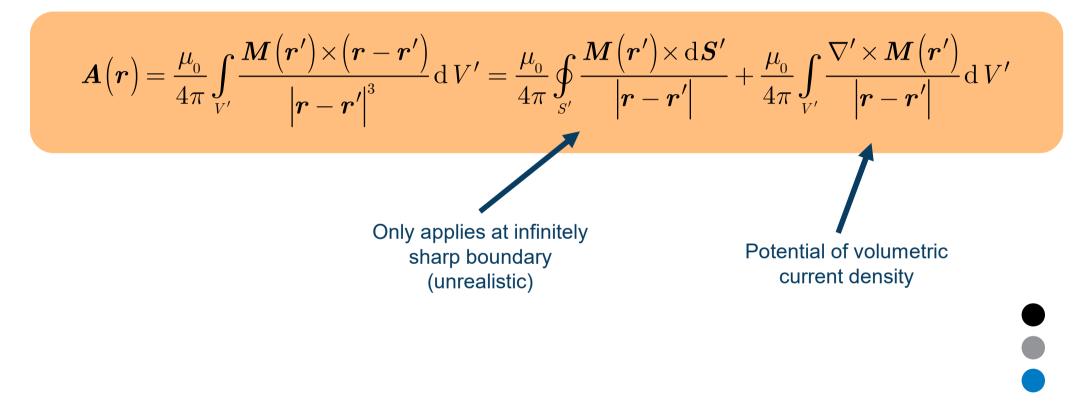


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Field Produced by Magnetized Matter



This formula holds very well outside the matter and, curiously, it also well approximates the field inside

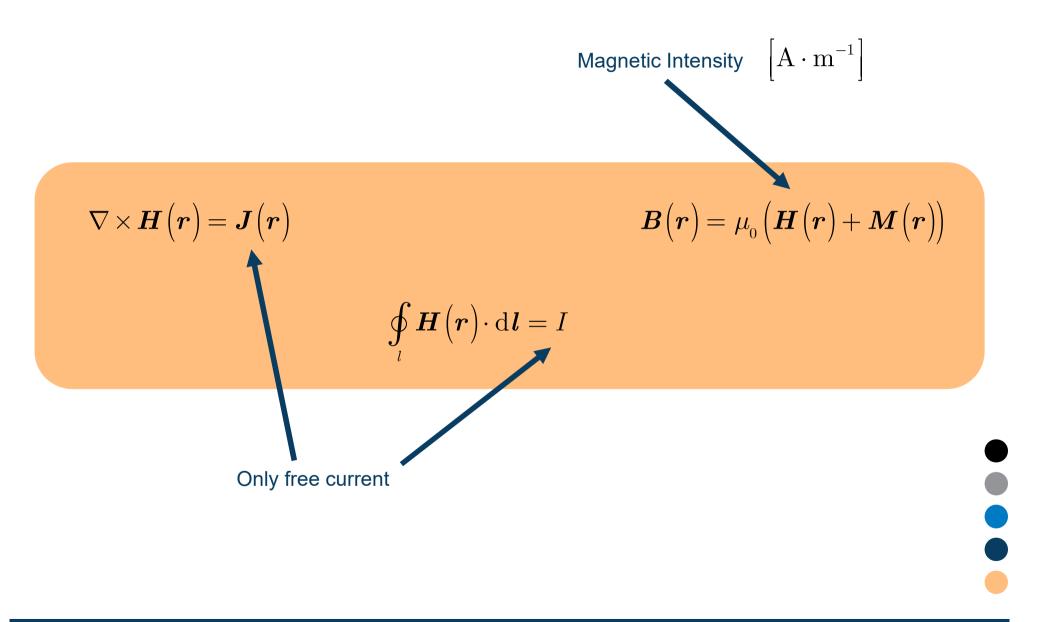


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Magnetic Intensity



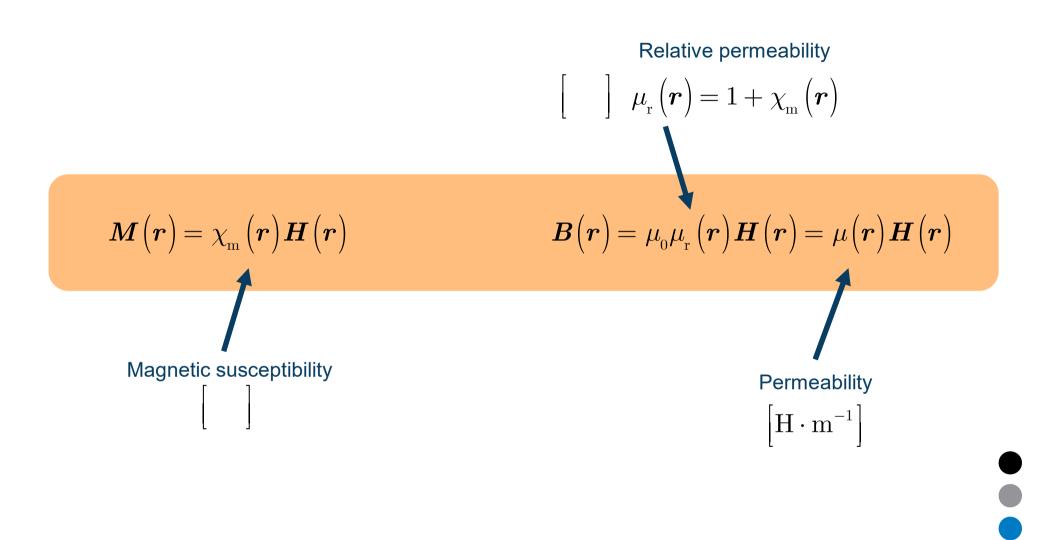


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Linear Isotropic Magnetic Materials



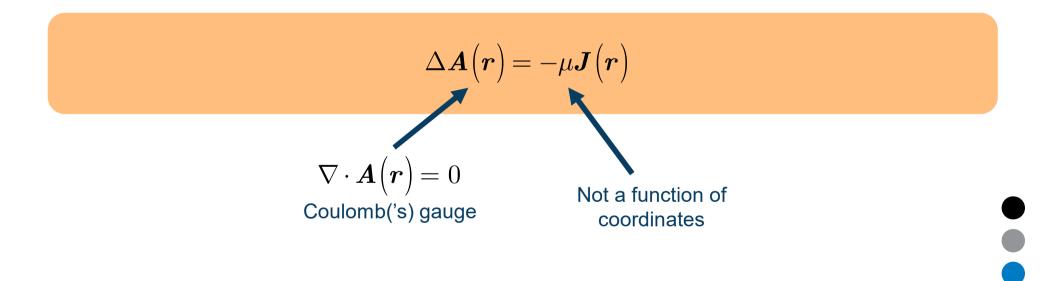
All the complicated structure of matter reduces to a simple scalar quantity



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Poisson's equation holds only when permittivity does not depend on coordinates

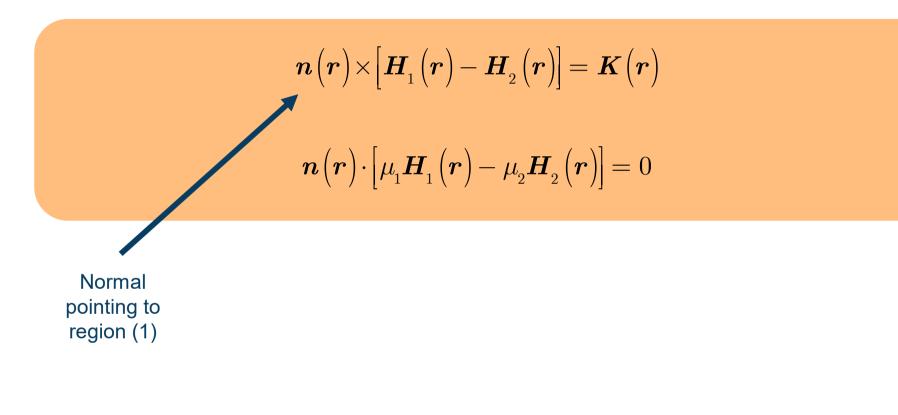


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Magnetic Material Boundaries



Both conditions are needed for unique solution

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Magnetostatic Energy in Magnetic Material

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• Paramagnetic – small positive susceptibility (small attraction – linear)

• Diamagnetic – small negative susceptibility (small repulsion – linear)

• Ferromagnetic – "large positive susceptibility" (large attraction – nonlinear)



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Ferromagnetic Materials

- Spins are ordered within domains
- Magnetization is a non-linear function of field intensity
- Magnetization curve Hysteresis, Remanence
- Susceptibility can only be defined as local approximation
- Above Curie('s) temperature ferromagnetism disappears

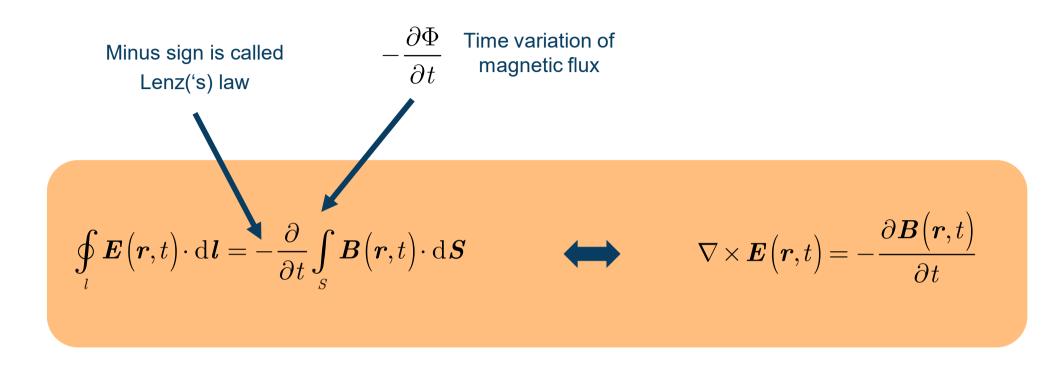
Exact calculations are very difficult – use simplified models (soft material, permanent magnet)



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Time variation in magnetic field produces electric field that tries to counter the change in magnetic flux (electromotive force)

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The current created by time variation of magnetic flux is directed so as to oppose the flux creating it.

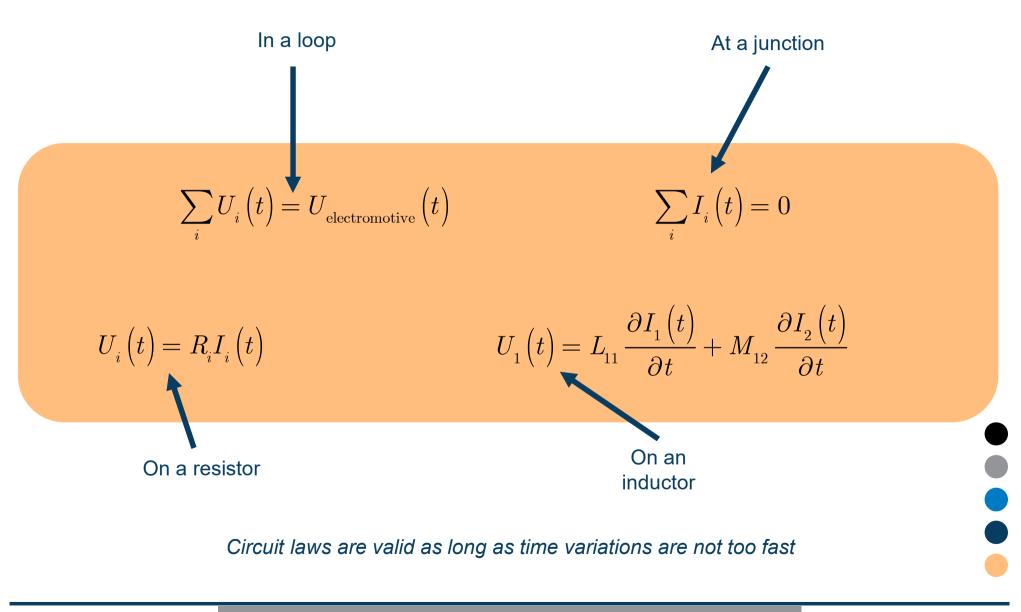


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Time Varying RL Circuits



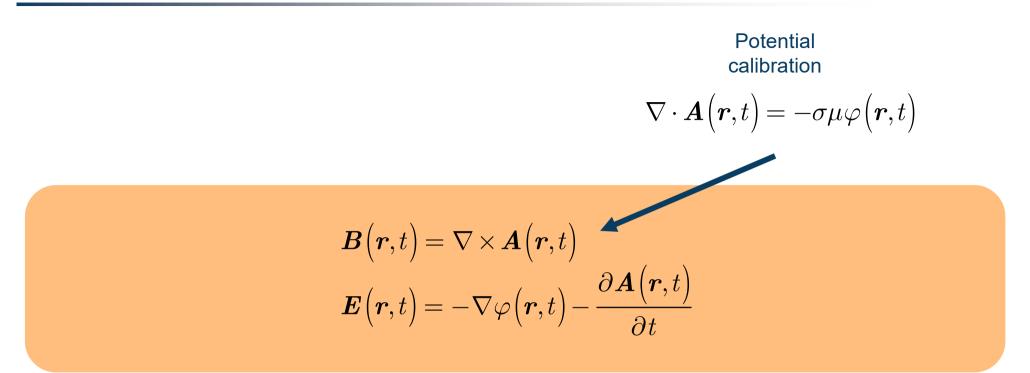
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Time Varying Potentials



In time varying fields scalar potential becomes redundant

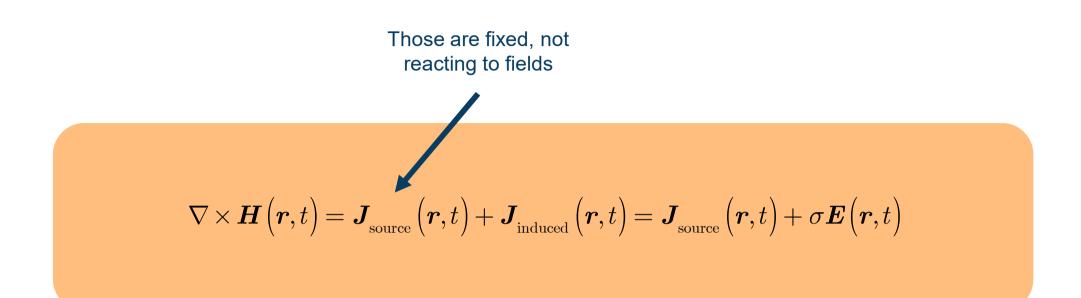
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Electromagnetic Field Theory 1



Source and Induced Currents





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Diffusion Equation

$$\begin{split} \Delta \boldsymbol{A}(\boldsymbol{r},t) &- \sigma \mu \frac{\partial \boldsymbol{A}(\boldsymbol{r},t)}{\partial t} = -\mu \boldsymbol{J}_{\text{source}}\left(\boldsymbol{r},t\right) \\ \Delta \boldsymbol{H}\left(\boldsymbol{r},t\right) &- \sigma \mu \frac{\partial \boldsymbol{H}\left(\boldsymbol{r},t\right)}{\partial t} = -\nabla \times \boldsymbol{J}_{\text{source}}\left(\boldsymbol{r},t\right) \\ \Delta \boldsymbol{E}\left(\boldsymbol{r},t\right) &- \sigma \mu \frac{\partial \boldsymbol{E}\left(\boldsymbol{r},t\right)}{\partial t} = \frac{1}{\varepsilon} \nabla \rho_{\text{source}}\left(\boldsymbol{r},t\right) + \mu \frac{\partial \boldsymbol{J}_{\text{source}}\left(\boldsymbol{r},t\right)}{\partial t} \end{split}$$

Material parameters are assumed independent of coordinates

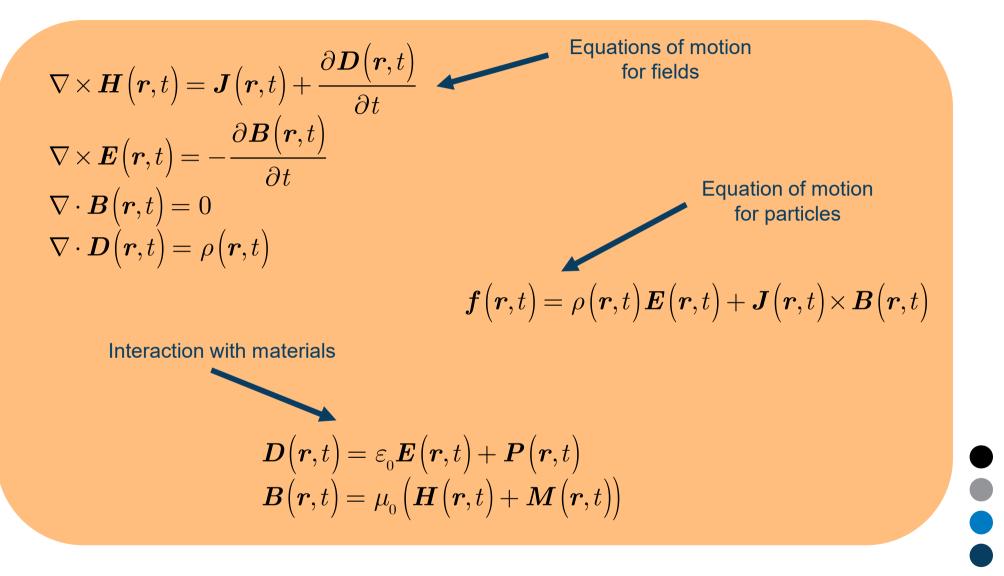


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Electromagnetic Field Theory 1

Maxwell('s)-Lorentz('s) Equations



Absolute majority of things happening around you is described by these equations

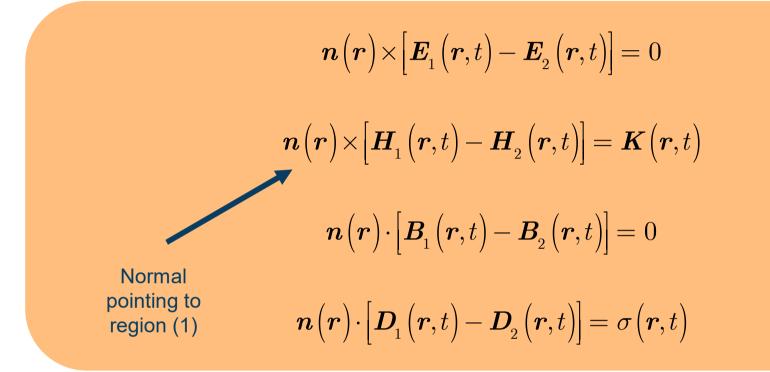
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Boundary Conditions

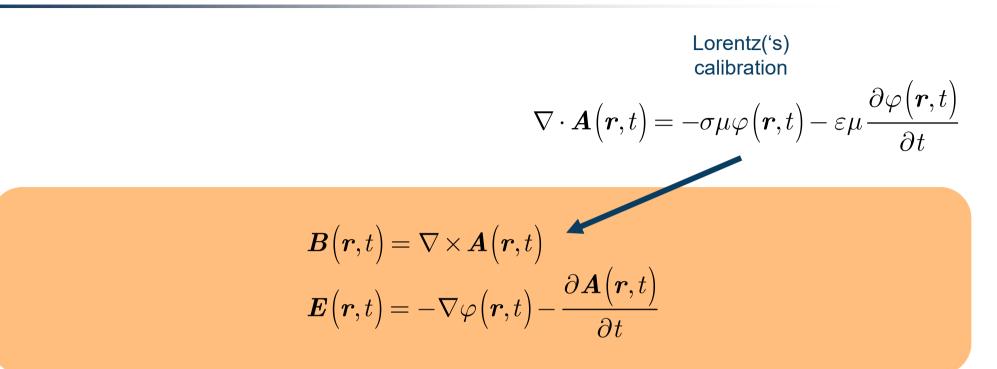


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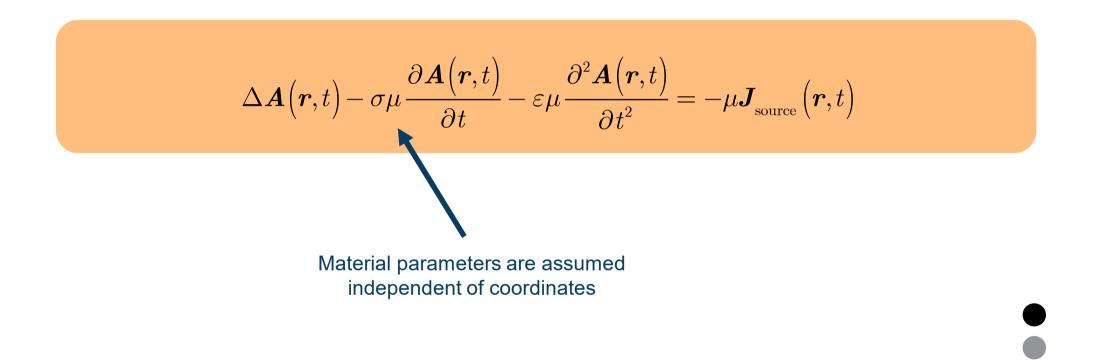
Electromagnetic Potentials



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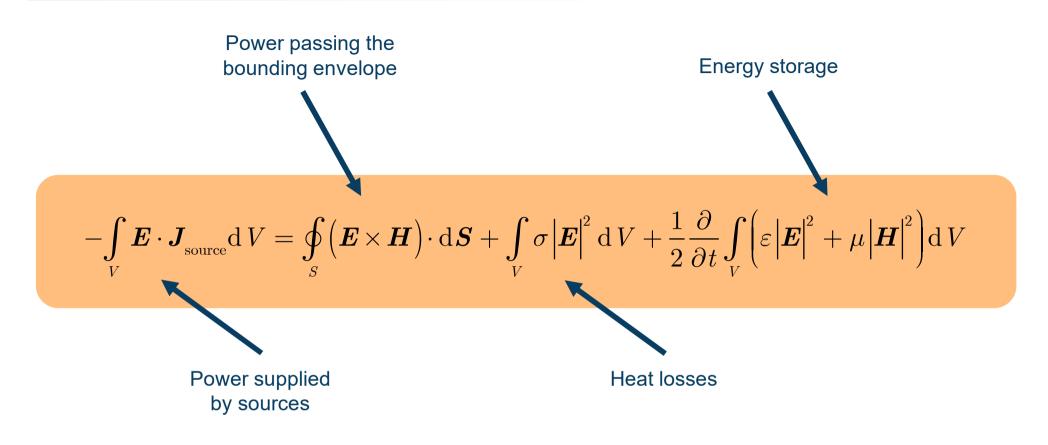


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Poynting('s)-Umov('s) Theorem



Energy balance in an electromagnetic system

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Electromagnetic Field Theory 1



Linear Momentum Carried by Fields



 $oldsymbol{p} = rac{1}{c_0^2} \int\limits_V ig(oldsymbol{E} imes oldsymbol{H} ig) \mathrm{d}\, V$

This formula is only valid in vacuum. In material media things are more tricky.

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Angular Momentum Carried by Fields

$$oldsymbol{L} = rac{1}{c_0^2} \int\limits_V oldsymbol{r} imes \left(oldsymbol{E} imes oldsymbol{H}
ight) \mathrm{d} \, V$$

This formula is only valid in vacuum. In material media things are more tricky.

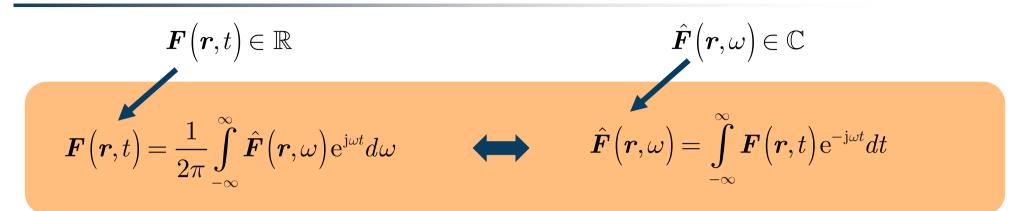
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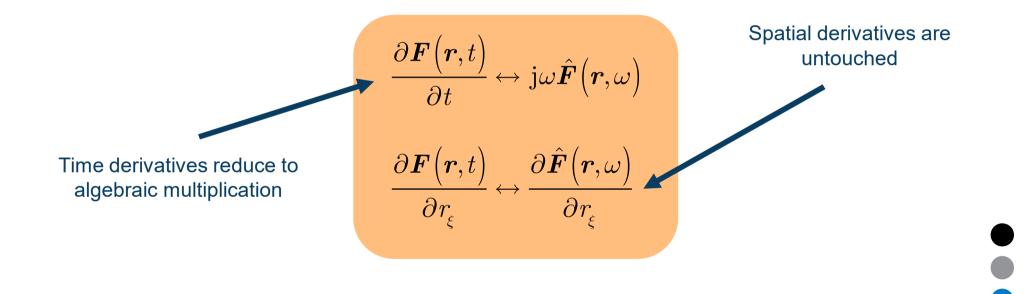
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Electromagnetic Field Theory 1



Frequency Domain





Frequency domain helps us to remove explicit time derivatives

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$$\hat{oldsymbol{F}}\left(oldsymbol{r},-\omega
ight)=\hat{oldsymbol{F}}^{*}\left(oldsymbol{r},\omega
ight)=rac{1}{\pi}\int_{0}^{\infty}\mathrm{Re}\Big[\hat{oldsymbol{F}}\left(oldsymbol{r},\omega
ight)\mathrm{e}^{\mathrm{j}\omega t}\Big]d\omega$$

Reduced frequency domain representation

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Electromagnetic Field Theory 1



Maxwell('s) Equations – Frequency Domain

$$egin{aligned} &
abla imes \hat{oldsymbol{H}}ig(oldsymbol{r},\omegaig) &= \hat{oldsymbol{J}}ig(oldsymbol{r},\omegaig) + \mathrm{j}\omegaarepsilon\hat{oldsymbol{E}}ig(oldsymbol{r},\omegaig) \ &
abla imes \hat{oldsymbol{E}}ig(oldsymbol{r},\omegaig) &= -\mathrm{j}\omega\mu\hat{oldsymbol{H}}ig(oldsymbol{r},\omegaig) \ &
abla imes \hat{oldsymbol{H}}ig(oldsymbol{r},\omegaig) &= 0 \ &
abla \cdot \hat{oldsymbol{E}}ig(oldsymbol{r},\omegaig) &= rac{\hat{eta}ig(oldsymbol{r},\omegaig)}{arepsilon} \end{aligned}$$

We assume linearity of material relations

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Wave Equation – Frequency Domain

$$\Delta \hat{\boldsymbol{A}}(\boldsymbol{r},\omega) - j\omega\mu(\sigma + j\omega\varepsilon)\hat{\boldsymbol{A}}(\boldsymbol{r},\omega) = -\mu\hat{\boldsymbol{J}}_{source}(\boldsymbol{r},\omega)$$
Helmholtz('s) equation

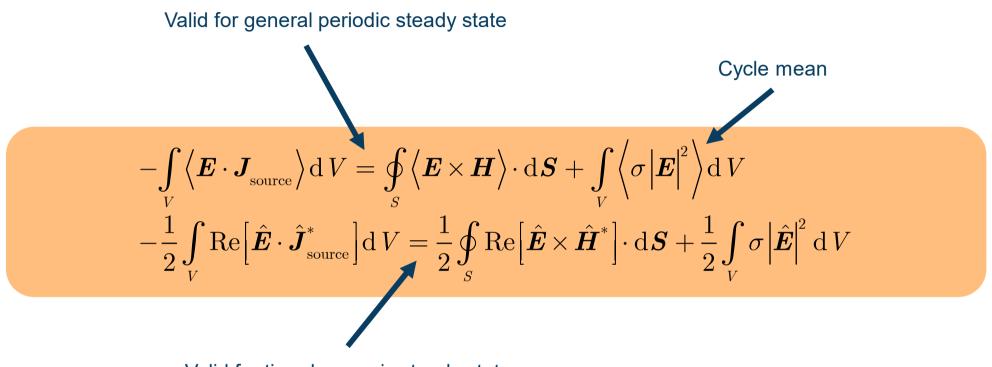


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Heat Balance in Time-Harmonic Steady State



Valid for time-harmonic steady state



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Electromagnetic Field Theory 1

Plane Wave

Unitary vector representing the direction of propagation

Electric and magnetic fields are mutually orthogonal

Electric and magnetic fields are orthogonal to propagation direction

$$\hat{\boldsymbol{E}}(\boldsymbol{r},\omega) = \boldsymbol{E}_{0}(\omega) e^{-jkn\cdot\boldsymbol{r}}$$

$$\hat{\boldsymbol{E}}(\omega) e^{-jkn\cdot\boldsymbol{r}}$$

$$\hat{\boldsymbol{H}}(\boldsymbol{r},\omega) = rac{k}{\omega\mu} \Big[\boldsymbol{n} imes \boldsymbol{E}_{_{0}}(\omega) \Big] \mathrm{e}^{-\mathrm{j}k\boldsymbol{n}\cdot\boldsymbol{r}}$$

 $oldsymbol{n}\cdotoldsymbol{E}_{_{0}}ig(\omegaig)=0$

$$oldsymbol{n}\cdotoldsymbol{H}_{_{0}}ig(\omegaig)=0$$

$$\mathbf{k}^2 = -\mathbf{j}\omega\mu \Big(\sigma + \mathbf{j}\omega\varepsilon \Big)$$

Wave-number

The simplest wave solution of Maxwell('s) equations

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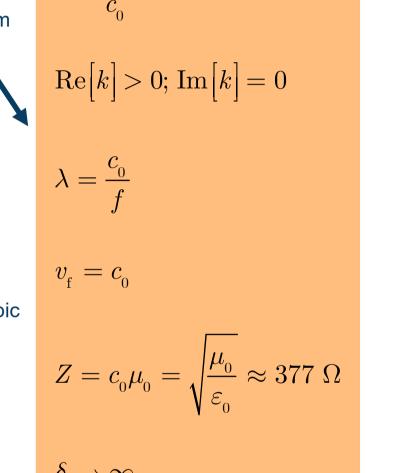
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Plane Wave Characteristics

 $k = \sqrt{-j\omega\mu(\sigma + j\omega\varepsilon)}$ $k = \frac{\omega}{c_0}$ Vacuum $\operatorname{Re}[k] > 0; \operatorname{Im}[k] < 0$ $\lambda = \frac{2\pi}{\operatorname{Re}[k]}$ $v_{\rm f} = \frac{\omega}{{
m Re}[k]}$ General isotropic material $Z = \frac{\omega \mu}{k}$ $\delta \to \infty$ $\delta = -\frac{1}{\mathrm{Im}[k]}$



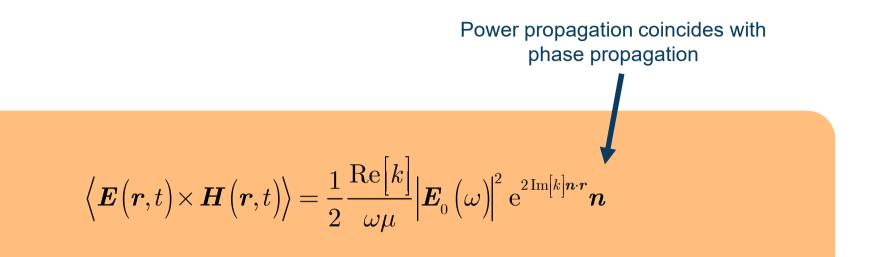


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Cycle Mean Power Density of a Plane Wave





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