Homework 1 (A8B17CAS)

Problem Set 1

October 17, 2023

1 Assignment

For all the following problems, consider N as a positive integer. Please, do not use the `for`/`while` cycle and/or `if`/`switch` branching.

Problem 1-A Create a matrix $\mathbf{A} \in \mathbb{R}^{N\times 5}$:

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 1 & 1 & 0/(N-1) \\ 0 & 1 & 1 & 2 & 1/(N-1) \\ 0 & 1 & 1 & 3 & 2/(N-1) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 1 & 1 & N & (N-1)/(N-1) \end{bmatrix}. \tag{1}$$

Do not enter the numbers element-wise, use the MATLAB functions instead.

(**1 point**)

Problem 1-B Calculate the norm of the vectors arranged one below the other in matrix $\mathbf{B} \in \mathbb{R}^{N\times 3}$ and normalize them to unitary size. To solve the problem and to verify the solution, use the following matrix:

```
B = reshape((1:3*N), 3, []).'
```

(**1 point**)

Problem 1-C Find all the elements in the general matrix $\mathbf{C} \in \mathbb{R}^{N\times N}$ greater than or equal to $x = N/2$, return them to vector $\mathbf{u}$ and replace these values in the original matrix $\mathbf{C}$ by 0. The following matrix $\mathbf{C}$ is used to validate the solution:

```
C = magic(N)
```

(**2 points**)

Problem 1-D Create a matrix $\mathbf{D} \in \mathbb{R}^{N\times N}$ defined as

$$D_{mn} = 2N + 1 - (m + n), \tag{2}$$

where N denotes the size of matrix $\mathbf{D}$, m denotes the row index, and n denotes the column index. Try to find as simple solution as possible.

(**2 points**)

Problem 1-E Create a matrix $\mathbf{E} \in \mathbb{C}^{2(N+1)\times 2(N+1)}$:

$$\mathbf{E} = \begin{bmatrix} \mathbf{e+0} & \mathbf{e-1} & \mathbf{e-2} & \cdots & \mathbf{e-N} \\ \mathbf{e+1} & \mathbf{e+0} & \mathbf{e-1} & \cdots & \mathbf{e-N+1} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \mathbf{e+N} & \mathbf{e+N-1} & \mathbf{e+N-2} & \cdots & \mathbf{e+0} \end{bmatrix}, \tag{3}$$

such that matrix $\mathbf{e} \in \mathbb{C}^{2\times 2}$ is a complex matrix

$$\mathbf{e} = \begin{bmatrix} 1 & -\mathrm{j} \\ \mathrm{e} & \pi \end{bmatrix}, \tag{4}$$

and the remaining matrices are as follows:

$$\mathbf{0} = 0 \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, \tag{5}$$

up to

$$\mathbf{N} = N \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}. \tag{6}$$

A hint: Take a look at MATLAB function `repelem`. Remember from the class how to set the Euler's Number $\mathrm{e} = \exp(1)$.

(**2 points**)

Problem 1-F Evaluate matrix $\mathbf{F}$, which is so-called Kronecker tensor product

$$\mathbf{F} = \mathbf{f} \otimes \mathbf{p} \tag{7}$$

of matrices $\mathbf{f}$ and $\mathbf{p}$, respectively, where

$$\mathbf{f} = \begin{bmatrix} 1 & 0 & 1 & 0 & \cdots & 1 & 0 \\ 0 & 1 & 0 & 1 & \cdots & 0 & 1 \\ 1 & 0 & 1 & 0 & \cdots & 1 & 0 \\ 0 & 1 & 0 & 1 & \cdots & 0 & 1 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 1 & 0 & 1 & 0 & \cdots & 1 & 0 \\ 0 & 1 & 0 & 1 & \cdots & 0 & 1 \end{bmatrix} \in \mathbb{R}^{2N \times 2N}, \tag{8}$$

and

$$\mathbf{p} = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}, \tag{9}$$

so that

$$\mathbf{F} = \begin{bmatrix} 1 & -1 & 0 & 0 & \cdots & 1 & -1 & 0 & 0 \\ -1 & 1 & 0 & 0 & \cdots & -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 & \cdots & 0 & 0 & 1 & -1 \\ 0 & 0 & -1 & 1 & \cdots & 0 & 0 & -1 & 1 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ 1 & -1 & 0 & 0 & \cdots & 1 & -1 & 0 & 0 \\ -1 & 1 & 0 & 0 & \cdots & -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 & \cdots & 0 & 0 & 1 & -1 \\ 0 & 0 & -1 & 1 & \cdots & 0 & 0 & -1 & 1 \end{bmatrix} \in \mathbb{R}^{4N \times 4N}. \tag{10}$$

A hint: Take a look at MATLAB function `kron`.

(**2 points**)

2 Instructions

The deadline for all assignments is

- November 13, 23:59.

Write your solutions into m-file called `homework1`. Each problem is solved within one of the MATLAB code "cell"s (use syntax: %%). Alternatively, you can solve each problem (A-F) individually in one m-file. They are called `homework1A-F` then. Upload all files via BRUTE system. When uploading more files, add them to a ZIP archive.