Lecture 2: Complex Numbers, Editor, Matrix Creation

A8B17CAS

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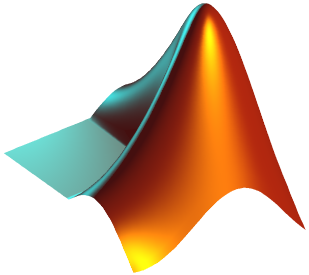
Outline



Warm Up: Adding Forces (Superposition)

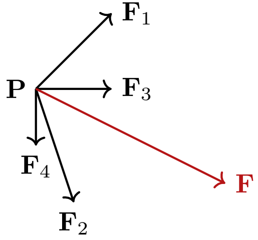
- Following forces were localized at point **P** in xy plane:

$$\begin{array}{ll} \mathbf{F}_1 = [2, 2] & \mathbf{F}_3 = [2, 0] \\ \mathbf{F}_2 = [1, -3] & \mathbf{F}_4 = [2, -1.5] \end{array}$$

  - What is the direction of the resultant force **F**?

- Normalize the resulting vector.

$$\mathbf{n}_{\mathrm{F}} = \frac{\mathbf{F}}{|\mathbf{F}|} = \frac{\mathbf{F}}{\sqrt{F_x^2 + F_y^2 + F_z^2}}$$



Exercise

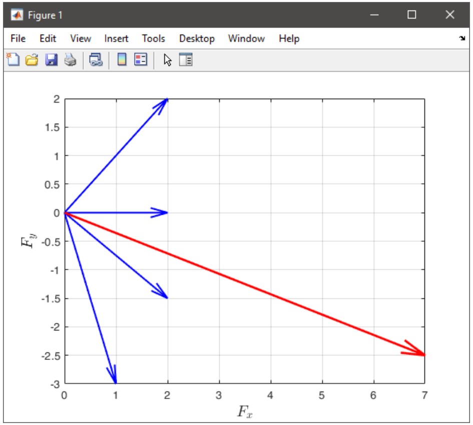
Bonus: Visualization of the Forces and Their Superposition

```
clear;
clc;

% User input
Fs = [2 2; 1 -3; 2 0; 2 -1.5];
F  = sum(Fs, 1) % resulting force

%% Graphical output
Z = zeros(size(Fs, 1), 1);

figure('color', 'w');
quiver(Z, Z, Fs(:,1), Fs(:,2), 0, ...
   'LineWidth', 1.5, 'Color', 'b');
hold on; % allows to have more graphs in a figure
quiver(0, 0, F(1), F(2), 0, ...
   'LineWidth', 2, 'Color', 'r');
grid on; % enable grid
% add labels (LaTeX interpreter possible)
option = {'Interpreter', 'LaTeX', 'FontSize', 14};
xlabel('$F_x$', option{:});
ylabel('$F_y$', option{:});
```



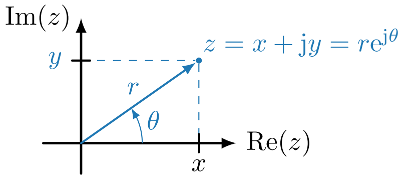
Complex Numbers I.

- Cartesian complex plane (real and imaginary parts)

$$z = x + \mathrm{j}y$$

- Polar complex plane (modulus and argument)

$$z = |z|\,\mathrm{e}^{\mathrm{j}\phi}$$



```
>> C1 = 3 + 2j % prefered
>> C2 = 3 + 2i % prefered
>> C3 = 3 + 2*i % slow
>> C4 = 3 + 2*sqrt(-1)
>> C5 = complex(3, 2)
```

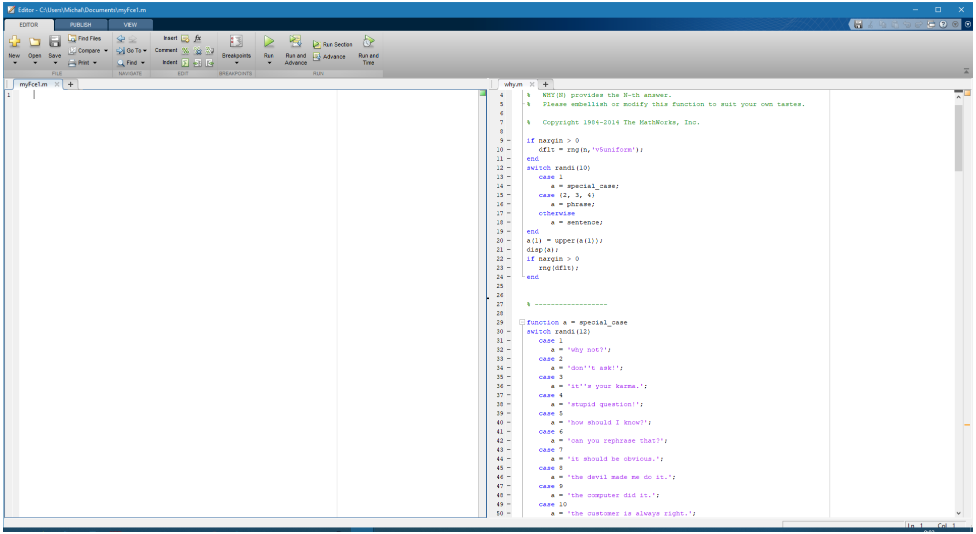
Complex Numbers II.

$$z = x + \mathrm{j}y = |z|\,\mathrm{e}^{\mathrm{j}\phi}$$

- Frequently used functions:

x, y	`real`, `imag`	real and imaginary parts of a complex number
z^*	`conj`	complex conjugate
$\|z\|$	`abs`	absolute value of a complex number
ϕ	`angle`	angle in complex plane [rad]
$x, y \rightarrow z$	`complex`	constructs complex number from real and imaginary parts
	`isreal`	checks if the input is a complex number (more on that later)
j	`i`, `j`	complex unit
	`cplxpair`	sort complex numbers into complex conjugate pairs

Matlab Editor



```
%   WHY(N) provides the N-th answer.
%   Please embellish or modify this function to suit your own tastes.

%   Copyright 1984-2014 The MathWorks, Inc.

if nargin > 0
   dflt = rng(n,'v5uniform');
end
switch randi(10)
   case 1
      a = special_case;
   case {2, 3, 4}
      a = phrase;
   otherwise
      a = sentence;
end
a(1) = upper(a(1));
disp(a);
if nargin > 0
   rng(dflt);
end


% ------------------

function a = special_case
switch randi(12)
   case 1
      a = 'why not?';
   case 2
      a = 'don''t ask!';
   case 3
      a = 'it''s your karma.';
   case 4
      a = 'stupid question!';
   case 5
      a = 'how should I know?';
   case 6
      a = 'can you rephrase that?';
   case 7
      a = 'it should be obvious.';
   case 8
      a = 'the devil made me do it.';
   case 9
      a = 'the computer did it.';
   case 10
      a = 'the customer is always right.';
```

Script Execution, m-files

- To execute a script:
  - F5 function key in MATLAB Editor,
  - Current folder → select script → context menu → Run,
  - Current folder → select script → F9,
  - from the command line:

```
>> script_name
```

- Scripts are stored as so called m-files, `.m`
- Caution: If you have MATHEMATICA installed, the `.m` files may be launched by MATHEMATICA.

Useful Shortcuts

F5	run a script
F9	run selected code
%%	add cell
CTRL+SHIFT	run actual cell of code
CTRL+R	comment selected code
CTRL+T	uncomment selected code

Script Commenting

- MAKE COMMENTS!!
  - Important/complicated parts of code.
  - Description of functionality, ideas, change of implementation.
- Typical single-line comment:

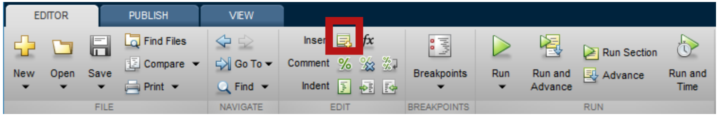
```
% create matrix, sum all members
matX = [1, 2, 3, 4, 5];
sumX = sum(matX); % sum of matrix
```

- Cell mode enables to separate script into more blocks:

```
matX = [1, 2, 3, 4, 5];
%% CELL mode (must be enabled in Editor)
sumX = sum(matX);
```

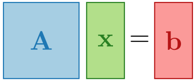
Cell Mode in MATLAB Editor

- Cells enable to separate the code into smaller, logically compacted parts.
  - Separator `%%`.
  - The separation is visual only, but it is possible to execute a single cell: shortcuts CTRL+ENTER and CTRL+SHIFT+ENTER.



Solving System of Linear Equations in MATLAB

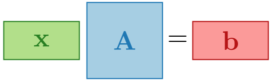
- Two cases are distinguished:
  - left division (`\` – `mldivide`),
  - right division (`/` – `mrdivide`).
- Solution of a linear system of equations:
  - $\mathbf{A}$ is an invertible (regular) matrix,
  - $\mathbf{b}$ is a column (row) vector.



$$\mathbf{A}\mathbf{x} = \mathbf{b}$$

$$\mathbf{x} = \mathbf{A}^{-1}\mathbf{b}$$

```
>> x = A \ b
```



$$\mathbf{x}\mathbf{A} = \mathbf{b}$$

$$\mathbf{x} = \mathbf{b}\mathbf{A}^{-1}$$

```
>> x = b / A
```

Linear Equations

- Find the sum of diagonal elements (trace of a matrix) of the matrix **T** with elements coming from normal distribution with mean equal to 10 and standard deviation equal to 4.
- Find determinant of matrix **U**.

$$\mathbf{U} = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 0 \\ 0 & -2 & -1 \end{bmatrix}$$

- Solve the linear system of equations:

$$\begin{aligned} x_1 + 2x_2 + 3x_3 &= 6 \\ 4x_1 + 5x_2 + 6x_3 &= 15 \\ 7x_1 + 8x_2 + x_3 &= 16 \end{aligned}$$

$$\mathbf{Ax} = \mathbf{b}$$

$$\mathbf{x} = \mathbf{A}^{-1}\mathbf{b}$$

```
>> T = 10 + 4*randn(7, 7);
```

```
>> U = [1 2 3; 0 2 0; ...
0 -2 -1];
```

Exercise

Predefined Values in MATLAB

- MATLAB contains several predefined values:
  - `eps` – precision of single/double numbers (Determines the shortest distance between two single/double numbers).
  - `ans` – *answer* – most recent answer.
  - `NaN` – *not a number* (every expression containing `NaN` is `NaN`)
    - `NaN` can be used advantageously in some cases.
  - `Inf` – *infinite number* (variable `Inf` can be used in calculation)
    - Pay attention to `Inf` propagation throughout your code (use allowed operations only).
  - `i`, `j` – complex unit.
    - They are all basically functions (without input parameter).
  - Check results of the following expressions:

```
>> t1 = 10/0   % t1 = Inf
>> t2 = 0/0    % t2 = NaN
>> t3 = t1*5   % t3 = Inf
>> t4 = t1 + t2 % t4 = NaN
```

Format of Command Line Output

MATLAB offers number of other formatting options

- Use `>> format style`.
- Output format does not change neither the computation accuracy (`single`/`double`) nor the accuracy of stored results (`eps`, `realmax`, `realmin`, ...still apply).

`style`	format description
`short`	fixed 4 decimal points are displayed
`long`	15 decimal points for double precision, 7 decimal points for single precision
`shortE`	floating-point format (scientific notation)
`longE`	—//—
`compact`	suppressed the display of blank lines
and others	check `>> doc format`

- Omitting `style` parameter restores default setup!

Format of Command Line Output

- Try following output format settings:
  - Each format is suitable for different type of problems.

```
>> clc;
>> s = [-5, 1/2, 1/3, 10*pi, sqrt(2), cos(pi/2)];
>> format compact
>> format long; s
>> format longE; s
>> format short; s
>> format shortE; s
>> format +; s
>> format; s
```

- Later, we will learn how to use formatted conversion into strings (commands `sprintf` and `fprintf`).

Exercise

Entering Matrices Using ":" (Colon) Operator and `linspace`

Large vectors and matrices require automated input.

- For equidistantly spaced values from `a` to `b` with an increment `x`:

```
A = a:x:b
```

```
>> A = 1:4:13
A =
   1   5   9   13
```

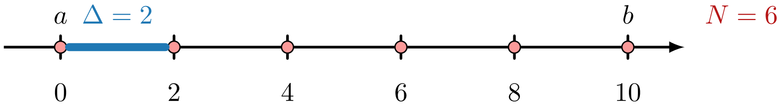
- `b` doesn't have to be an element of the series.
- increment `x` can be negative.

```
>> A = 10:-4:1
A =
   10   6   2
```

- For `N` equidistantly spaced valued from `a` to `b`:

```
A = linspace(a, b, N)
```

```
>> A = linspace(0, 20, 5)
A =
   0    5    10    15    20
```



Entering Matrices I.

- Using the colon operator ":" create
  - the following vector

$$\mathbf{v} \quad = \quad [25 \;\; 20 \;\; \ldots \;\; -5]^{\mathrm{T}}$$

  - the following matrix
    - Caution, the third column can't be created using colon operator ":" only.

$$\mathbf{T} = \begin{bmatrix} -4 & 1 & \dfrac{\pi}{2} \\ -5 & 2 & \dfrac{\pi}{4} \\ -6 & 3 & \dfrac{\pi}{6} \end{bmatrix}$$

Exercise

Entering Matrices II.

- Create a vector of 100 evenly spaced points in the interval $[-1.15, 75.4]$.
- Create a vector of 201 evenly spaced points in the interval $[-100, 100]$ sorted in descending order.
- Create a vector with spacing of -10 in the interval $[100, -100]$ sorted in descending order.
  - Try both options using `linspace` and colon ":".

Exercise

Entering Matrices Using Functions I.

- Special types of matrices of given sizes are needed quite often.
  - MATLAB offers a number of functions to serve the purpose., *e.g.*, `zeros`, `ones`, `NaN`, `inf`, `eye`, `rand`, `randn`, `randi`, `true`, `false`.
- Example: matrix filled with zeros
  - Will be used frequently.

```
zeros(m)                % matrix of size [m x m]
zeros(m, n)             % matrix of size [m x n]
zeros(m, n, p, ..)        % matrix of size [m x n x p x ..]
zeros([m, n])           % matrix of size [m x n]




% see documentation for other options
```

Entering Matrices Using Functions III.

- Create following matrices
  - use MATLAB functions
  - begin with matrices you find easy to cope with.

$$\mathbf{M}_1 = \begin{bmatrix} \mathrm{NaN} & \mathrm{NaN} \\ \mathrm{NaN} & \mathrm{NaN} \end{bmatrix}$$

$$\mathbf{M}_2 = \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix}$$

$$\mathbf{M}_3 = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & -5 \end{bmatrix}$$

$$\mathbf{M}_4 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Exercise

Entering Matrices

- Quite often, there are several options how to create a given matrix.
  - It is possible to use an output of one function as an input of another function in MATLAB:
- Consider:
  - clarity,
  - simplicity,
  - speed,
  - convention.

```
plot(diag(randn(10, 1), 1))
```

- *e.g.* band matrix with “1” on main diagonal and with “2” and “3” on secondary diagonals.

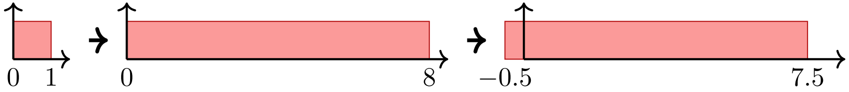
```
N = 10;
A = diag(ones(N, 1)) + diag(2 * ones(N - 1, 1), 1) + diag(3 * ones(N - 1, 1), -1)
```

  - Can be done using `for` cycle as well (see later in the semester).
  - Some other idea?

Generation of a Random Matrix

- Create matrix **M** of size `size(M) = [3 4 2]` containing random numbers coming from uniform distribution on the interval $[-0.5, 7.5]$.

$$I(x) = (I_{\max} - I_{\min})\,\mathrm{rand}(\dots) + I_{\min}$$



Exercise

Transpose and Matrix Conjugate

- Pay attention to situations where the matrix is complex, $\mathbf{A} \in \mathbb{C}^{M \times N}$.
- There are two operations:

transpose	$\mathbf{A}^{\mathrm{T}} = [A_{ij}]^{\mathrm{T}} = [A_{ji}]$	`transpose(A)`	`A.'`
transpose + conjugate	$\mathbf{A}^{\mathrm{H}} = [A_{ij}]^{\mathrm{H}} = [\mathbf{A}^*]^{\mathrm{T}}$	`ctranspose(A)`	`A'`

```
>> A = magic(2) + 1j * magic(2)'
A =
   1.0000 + 1.0000i   3.0000 + 4.0000i
   4.0000 + 3.0000i   2.0000 + 2.0000i
```

```
>> A.'
ans =
   1.0000 + 1.0000i   4.0000 + 3.0000i
   3.0000 + 4.0000i   2.0000 + 2.0000i
```

```
>> A'
ans =
   1.0000 - 1.0000i   4.0000 - 3.0000i
   3.0000 - 4.0000i   2.0000 - 2.0000i
```

Matrix Operations I.

There are other useful functions apart from transpose (`transpose`), *e.g.*,

- vector to a diagonal matrix, matrix diagonal to a vector (`diag`),
- upper/lower triagular matrix (`triu`, `tril`),
- array replication (`repmat`),
- array reshape (`reshape`),
- array flip (`flip`),
- array rotation (`rot90`),
- circular shift (`circshift`),
- block-diagonal matrix from individual matrices (`blkdiag`),
- arranging two (or more) matrices side by side (`cat`),
- and many others…

Always check the documentation (» doc …).

Matrix Operations II.

- Convert matrix $\mathbf{A}$ into the form of matrices $\mathbf{A}_1$ to $\mathbf{A}_4$.

```
A = [1 pi; exp(1) -1i]
```

$$\mathbf{A} = \begin{bmatrix} 1 & \pi \\ \mathrm{e} & -\mathrm{i} \end{bmatrix}$$

- Use `repmat`, `reshape`, `triu`, `tril` and `conj`.

$$\mathbf{A}_1 = \begin{bmatrix} 1 & \pi & 1 & \pi & 1 & \pi \\ \mathrm{e} & -\mathrm{i} & \mathrm{e} & -\mathrm{i} & \mathrm{e} & -\mathrm{i} \end{bmatrix}$$

$$\mathbf{A}_2 = \begin{bmatrix} 1 & \pi & \mathrm{e} & -\mathrm{i} \end{bmatrix}$$

$$\mathbf{A}_3 = \begin{bmatrix} 1 & \pi \\ \mathrm{e} & +\mathrm{i} \\ 1 & \pi \\ \mathrm{e} & +\mathrm{i} \\ 1 & \pi \\ \mathrm{e} & +\mathrm{i} \end{bmatrix}$$

$$\mathbf{A}_4 = \begin{bmatrix} 1 & \pi & 0 & 0 & 0 & 0 \\ \mathrm{e} & -\mathrm{i} & \mathrm{e} & 0 & 0 & 0 \\ 0 & \pi & 1 & \pi & 0 & 0 \\ 0 & 0 & \mathrm{e} & -\mathrm{i} & \mathrm{e} & 0 \\ 0 & 0 & 0 & \pi & 1 & \pi \\ 0 & 0 & 0 & 0 & \mathrm{e} & -\mathrm{i} \end{bmatrix}$$

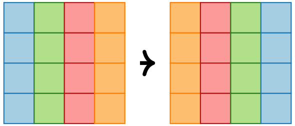
Exercise

Matrix Operations III.

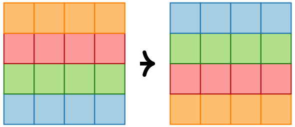
- Create the following matrix (use advanced techniques)

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & 3 & 1 & 2 & 3 \\ 0 & 2 & 4 & 0 & 2 & 4 \\ 0 & 0 & 5 & 0 & 0 & 5 \end{bmatrix}$$

- Create matrix **B** by swapping columns in matrix **A**.



- Create matrix **C** by swapping rows in matrix **B**.



Exercise

Matrix Operations IV. – Tensor Products

Kronecker tensor product

```
K = kron(A, B)
```

- Convolution kernel `A` is applied to a mask `B`.

Example:

$$\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix} \otimes \frac{1}{2}\begin{bmatrix}1 & -1\end{bmatrix} = \frac{1}{2}\begin{bmatrix}0 & 0 & 1 & -1\\ 1 & -1 & 0 & 0\end{bmatrix}$$

```
kron([0 1; 1 0], [1/2, -1/2])
```

Tensor product

```
C = tensorprod(A, B, dimA, dimB)
```

- Inner product

$$\sum_n \cdots \sum_k \sum_j A_{jk\cdots n} B_{jk\cdots n} = c$$

- Outer product

$$[A_{jk\cdots n}][B_{pq\cdots t}] = [C_{jk\cdots npq\cdots t}]$$

- Tensor product

$$\sum_j \cdots \sum_p \sum_j A_{jk\cdots n} B_{pq\cdots t} = [C_{k\cdots nq\cdots t}]$$

Size of Matrices and Other Structures I.

- It is often needed to know sizes of matrices and arrays.
- Function `size` returns vector giving the size of a matrix/array.

```
A = randn(3, 5);
d = size(A)  % d = [3 5]
```

- Function `length` returns largest dimension of an array.

```
length(A) = max(size(A))
```

```
A = randn(3, 5, 8);
e = length(A)  % e = 8
```

- Function `ndims` returns number of dimensions of a matrix/array.

```
ndims(A) = length(size(A))
```

```
m = ndims(A)  % m = 3
```

- Function `numel` returns number of elements of a matrix/array.

```
numel(A) = prod(size(A))
```

```
n = numel(A)  % n = 120
```

- Functions `height` and `width` return number of rows and columns, respectively.

Size of Matrices and Other Structures II.

- Create an arbitrary 3D array.
  - You can make use of the following commands:

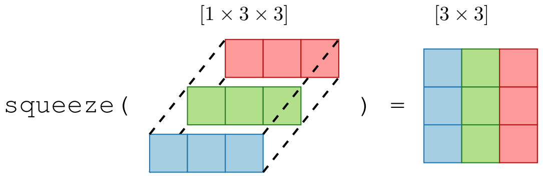
```
A = rand(2 + randi(10), 3 + randi(5));
A = cat(3, A, rot90(A,2))
```

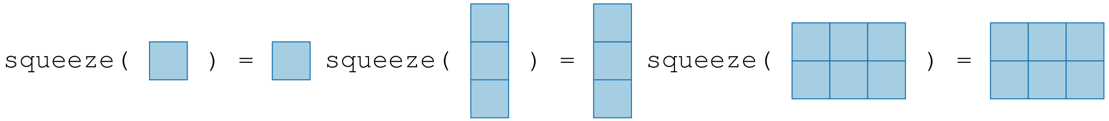
- And now:
  - Find out the size of `A`.
  - Find the number of elements of `A`.
  - Find out the number of elements of `A` in the “longest” dimension.
  - Find out the number of dimensions of `A`.

Exercise

Squeeze

- Function `squeeze` removes dimension of an array with length 1.
  - If the input is scalar, vector or array without any dimension of the length 1, the output is identical to the input.





Function `gallery`

- Function enabling to create a vast set of matrices that can be used for MATLAB code testing.
- Most of the matrices are special-purpose.
  - Function `gallery` offers significant coding time reduction for advanced MATLAB users.
- See: `doc gallery`
- Try for instance:

```
gallery('pei', 5, 4)
gallery('leslie', 10)
gallery('clement', 8)
```

Questions?

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