Project 2 – Bayesian Networks

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In your second assignment, you are supposed to solve theoretical problems from the covered topics on Bayesian networks.

Instructions

You may obtain up to 10 points. Justify all your answers (unjustified answers will not yield any points). All solutions should be your own. Teamwork is not allowed. Submit a PDF with your answers to the BRUTE system. Also see the system for the strict submission deadline (approximately three weeks since publishing the assignment). One point will be subtracted for each day of delay (you will not receive negative points).

Problems

Problem 1 (6p) Consider the following game of chance.

> Three fair dice are rolled. The sum of values rolled on the the first two determines how we interpret the value rolled on the third. Value on the third dice may either be *high* or *low*. If the sum was less than 8, then only 6 on the third dice is considered high. If the sum was greater than 8, then the numbers 5 and 6 are considered high. Finally, if the sum was equal to 8, the numbers $4, 5$ and 6 are considered high.
>
> The player is only told whether the value on the first dice was a prime or not, and whether the value on the second dice was greater than 4 or not. Based on that information alone, the player must guess whether the third roll will be high or low.
>
> The participation in the game costs 1 coin. If the player guesses *high* correctly, they are given 2 coins (i.e., they gain 1 coin). If the player guesses *low* correctly, they are given 1 coin (i.e., they get their bet back and they gain nothing). In the case of an incorrect guess, the player gets nothing (i.e., they lose 1 coin).

(a) Design a *causal* Bayesian network modelling the game above.
Define random variables you use, draw the network structure (remember to justify present/absent edges) and compute the (conditional) probability tables.

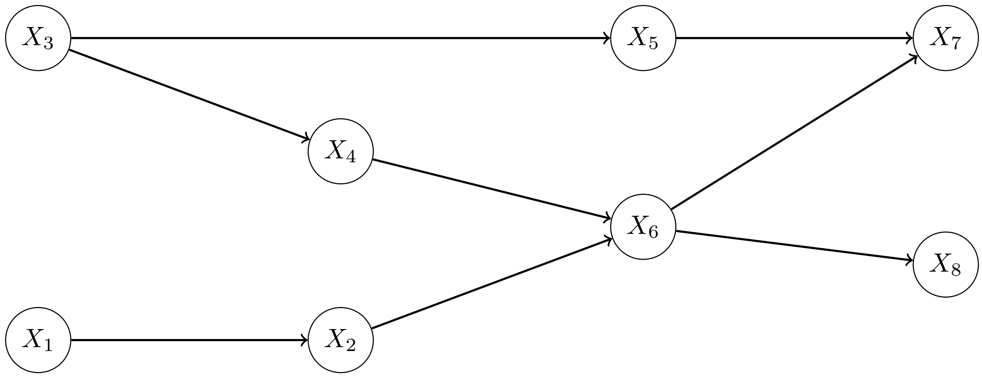
(b) Suppose the player adopts the strategy

$$\pi = \underset{d_3 \in \{high, low\}}{\arg\max} \; P(d_3 \mid d_1 \text{ is prime}, d_2 > 4),$$

where d_1 and d_2 denote the values rolled on the first and the second dice, respectively. Hence, for each possible pair of the partial observations, the player wants to guess the more likely outcome.

What will be the player's expected gain for each possible pair of observations?

Problem 2 (4p) Consider the network below



Let $\mathcal{R}$ denote a subset of the network's random variables. Find the largest $\mathcal{R}$ such that

(a) $X_4 \perp\!\!\!\perp \mathcal{R} \mid X_6$

(b) $X_5 \perp\!\!\!\perp \mathcal{R} \mid X_3$

(c) $X_1 \perp\!\!\!\perp \mathcal{R} \mid \{X_3, X_7\}$

(d) $X_2 \perp\!\!\!\perp \mathcal{R} \mid \varnothing$