The Catering Problem

Motivation

A catering company to cook n dishes, but has only one available oven. At most a single dish can be inside the oven at one time.
Each dish i has its earliest time when it can be put into the oven r_i (since it needs to be prepared before it is put into the oven), the latest time it should be taken from the oven d_i (since the customers do not want to wait too long), and the time it needs to stay in the oven p_i. The goal is to find the vector of times $\mathbf{s} = (s_0, \dots, s_{n-1})$ (denoting the times when each dish is put into the oven) such that the finish time of the last dish is minimal.

Input

You are given the following:

- number of dished n
- parameters r_i, d_i and p_i for each dish i

For the testing purposes, you can experiment with the following instance:

In [1]:
```
n = 5
params = {
    0: {'r': 20, 'd': 45, 'p': 15},
    1: {'r':  4, 'd': 30, 'p': 19},
    2: {'r':  5, 'd': 80, 'p': 20},
    3: {'r': 17, 'd': 70, 'p':  8},
    4: {'r': 27, 'd': 66, 'p':  7}
}

# Note: parameter d_1 can be obtained by params[1]["d"]
```

Output

You are expected to find the vector $\mathbf{s} = (s_0, \dots, s_{n-1})$ denoting the times when each dish should be put into the oven.

The optimal solution vector for the given instance is $\mathbf{s} = (23, 4, 53, 38, 46)$.

Exercise

Your task is to formulate the ILP model of the catering problem, solve it, and extract the vector $\mathbf{s}$. The example solution follows:

Hint: to ensure that any two dishes i and j are not overlapping in the oven, you need to ensure that one of the following constraints holds: $s_i + p_i \leq s_j$ or $s_j + p_j \leq s_i$. This might be perhaps done using big-M...

In [2]:

```
import gurobipy as grb  # import Gurobi module

# model -----------------------------------------------------
model = grb.Model()

# - ADD VARIABLES
# TODO

# - ADD CONSTRAINTS
# TODO

# - SET OBJECTIVE
# TODO

# call the solver -------------------------------------------
model.optimize()

# print the solution ----------------------------------------
print('\nSOLUTION:')
# TODO
```

```
Set parameter Username
Academic license - for non-commercial use only - expires 2024-02-16
Gurobi Optimizer version 9.5.1 build v9.5.1rc2 (win64)
Thread count: 6 physical cores, 12 logical processors, using up to 12 threads
Optimize a model with 0 rows, 0 columns and 0 nonzeros
Model fingerprint: 0xf9715da1
Coefficient statistics:
  Matrix range     [0e+00, 0e+00]
  Objective range  [0e+00, 0e+00]
  Bounds range     [0e+00, 0e+00]
  RHS range        [0e+00, 0e+00]
Presolve time: 0.00s
Presolve: All rows and columns removed
Iteration    Objective       Primal Inf.    Dual Inf.      Time
       0    0.0000000e+00   0.000000e+00   0.000000e+00      0s

Solved in 0 iterations and 0.01 seconds (0.00 work units)
Optimal objective  0.000000000e+00

SOLUTION:
```

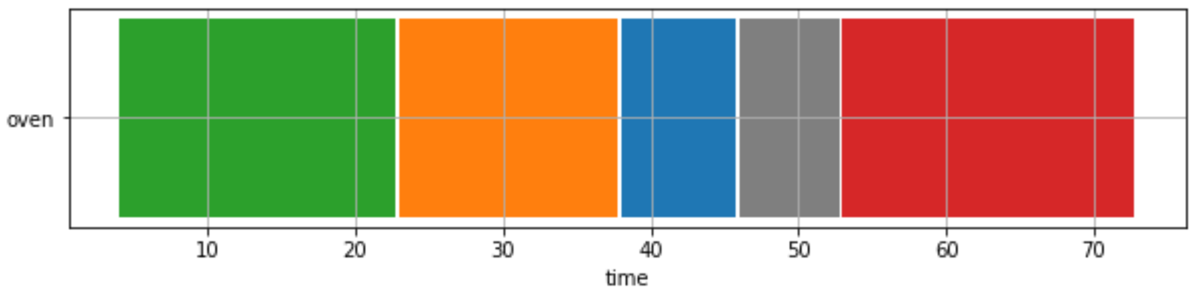
Solution visualization

In [3]:

```
import matplotlib.pyplot as plt

def plot_solution(s, p):
    """
    s: solution vector
    p: processing times
    """
    fig = plt.figure(figsize=(10,2))
    ax = plt.gca()
    ax.set_xlabel('time')
    ax.grid(True)
    ax.set_yticks([2.5])
    ax.set_yticklabels(["oven"])
    eps = 0.25  # just to show spaces between the dishes
    ax.broken_barh([(s[i], p[i]-eps) for i in range(len(s))], (0, 5),
                   facecolors=('tab:orange', 'tab:green', 'tab:red', 'tab:blue', 'tal

# TODO: plot your solution
plot_solution([23.0, 4.0, 53.0, 38.0, 46.0], [params[i]["p"] for i in range(n)])
```



In [ ]: