## B3M33HRO HW3 <br> Multisensorial Calibration



Figure 1: Drawing of the robot with base frame, two joints (green circles), and end-effector $E E$ (red circle).

## 1 Points

You can get 7 points in total. Individual points are distributed as follows:

- Theoretical part (Sec. 2) - 1 point
- Submit in PDF or as a photo/scan of a hand-written text.
- Computational part (Sec. 3)-4 points
- Submit a photo/scan of all your calculations on paper or PDF written with LaTex * with sub-steps (not all sub-steps are required, but everything must be clear)
- Only solution is not sufficient
- Programming part (Sec. 4) - 2 points
- Submit the source code, e.g., .m file for Matlab or .py for Python
- Only Matlab (recommended), Python or C++ are allowed


## 2 Theoretical part

Imagine the robot in Fig. 1. Your task is to calibrate Denavit-Hartenberg parameters ( DH ) of the links $J_{1}$ and $J_{2}$. Describe shortly the scenario of how you would calibrate the parameters. Use at least two different approaches

One-two paragraphs are enough. You can attach a drawing if you want.
To help you get started, try to think about these questions:

- What do you need to measure?
- How can you measure that?
- How many times do you need to measure something?


## 3 Computational part

The setup in Fig. 1 is now extended with a laser tracker to precisely measure the location of the end-effector w.r.t to the base frame of the robot.

You want to estimate two parameters $s$ and $c$ in transformation $\mathbf{R}_{1}$ from the base link to the second joint $\left(J_{2}\right)$. You know the transformation $\mathbf{R}_{2}$ from $J_{2}$ to the end-effector $E E$ and position of the end-effector measured by the laser $\mathbf{O}_{\text {measured }}$. You also know that $R_{1}$ and $R_{2}$ together form the transformation from end-effector to the base as $R_{\text {base }}^{e e}=R_{1} R_{2}$.

Every student has a different assignment. You will be provided with matrices $\mathbf{R}_{1}, \mathbf{R}_{2}$ and $\mathbf{O}_{\text {measured }}$. Use the script generateHW3.p provided in hw3.zip and generate your own assignment as:
(i) In Matlab, go to the folder with generateHW3.p
(ii) In the Matlab command line, run $[R 1, R 2, O]=$ generateHW3(your_CTU_username).

- your_CTU_username is the username that you use to $\log$ in to the CTU services.
- Please, insert the username in quotes, e.g., generateHW3("rustlluk")
(iii) If you do not have Matlab and do not plan to use Matlab for programming part of this assignment, please write an email to your lab tutor to generate your assignment.

Fulfill the following tasks. Write with sub-steps on paper, or rewrite in LaTex.

- Compute the position of the end-effector $\mathbf{O}_{e e}$ in the base frame;
- Do not use $\mathbf{O}_{\text {measured }}$ here! Use $R_{\text {base }}^{e e}$ and the position of the end-effector in its own frame.
- Hint: The solution will contain only $s, c$ or numbers.
- Define function $\mathbf{g}=\mathbf{0}$ for Levenberg-Marquardt algorithm, that is defined as

$$
\begin{equation*}
\boldsymbol{x}_{k+1}=\boldsymbol{x}_{k}-\left(\boldsymbol{g}^{\prime}\left(\boldsymbol{x}_{k}\right)^{T} \boldsymbol{g}^{\prime}\left(\boldsymbol{x}_{k}\right)+\mu_{k} \boldsymbol{I}\right)^{-1} \boldsymbol{g}^{\prime}\left(\boldsymbol{x}_{k}\right)^{T} \boldsymbol{g}\left(\boldsymbol{x}_{k}\right) \tag{1}
\end{equation*}
$$

such that $\mathbf{O}_{e e}$ equals the position of the end effector $\mathbf{O}_{\text {measured }}$ measured by external device, i.e., $\mathbf{O}_{e e}=$ $\mathbf{O}_{\text {measured }}$;

- Compute the initial value of the cost function

$$
\begin{equation*}
f\left(s_{0}, s_{c}\right)=\mathbf{g}^{T} \mathbf{g} \tag{2}
\end{equation*}
$$

- Perform one step of the Levenberg-Marquardt algorithm (compute $\mathbf{x}_{1}$ ) with initial parameters $\mathbf{x}_{0}=$ $\left(s_{0}, c_{0}\right)=\left(\begin{array}{cc}0 & 0\end{array}\right)^{T}$ and $\mu=10 ;$
- Select new $\mu$ for the next iteration.


## 4 Programming part

Replicate your results in Sec. 3 with code. Matlab is recommended, but Python or C++ solution will also be accepted. No other programming language will be accepted. Submit your source code.

