

# Humanoid robots - Walking (& Balancing)

Introduction to the guest lecture  
by Prof. Sergej Čelikovský

Doc. Mgr. Matěj Hoffmann, Ph.D.

# Motivation



A Compilation of Robots Falling Down at the DARPA Robotics Challenge

<https://youtu.be/g0TaYhjpOfo>

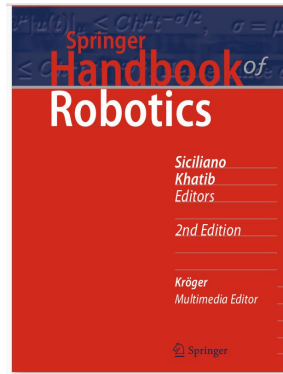
# HUMANOID ROBOTS MODELING AND CONTROL

DRAGOMIR N. NENCHEV AND ATSUSHI KONNO  
WITH CONTRIBUTION BY TEPPEI TSUJITA



## 5. Balance Control

- 5.1 Overview 203
- 5.2 Dynamic Postural Stability 205
- 5.3 Inverted Pendulum-on-Foot Stability Analysis 207
  - 5.3.1. The Extrapolated CoM and the Dynamic Stability Margin 207
  - 5.3.2. Extrapolated CoM Dynamics 209
  - 5.3.3. Discrete States With Transitions 210
  - 5.3.4. Dynamic Stability Region in 2D 211
- 5.4 ZMP Manipulation–Type Stabilization on Flat Ground 212
  - 5.4.1. The ZMP Manipulation–Type Stabilizer 214



Siciliano  
Khatib  
Editors

2nd Edition

Kröger  
Multimedia Editor



Pierre–Brice Wieber, Russ Tedrake, Scott Kuindersma

The promise of legged robots over wheeled robots is to provide improved mobility over rough terrain. Unfortunately, this promise comes at the cost of a significant increase in complexity. We now have a good understanding of how to make legged robots walk and run dynamically, but further research is still necessary to make them walk and run efficiently in terms of energy, speed, reactivity, versatility, and robustness. In this chapter, we will discuss how legged robots are usually modeled, how their stability analysis is approached, how dynamic motions are generated and controlled, and finally summarize the current trends in trying to improve their performance. The main problem is avoiding to fall. This can prove difficult since legged robots have to rely entirely on available contact forces to do so. The temporality of leg motions appears to be a key aspect in this respect, as current control solutions include continuous anticipation of future motion (using some form of model predictive control), or focusing more specifically on limit cycles and orbital stability.

- 48.1 A Brief History of Legged Robots ..... 1204
- 48.2 The Dynamics of Legged Locomotion ..... 1204
  - 48.2.1 Lagrangian Dynamics ..... 1205
  - 48.2.2 Newton and Euler Equations of Motion ..... 1205
  - 48.2.3 Contact Models ..... 1207

## 48. Modeling and Control of Legged Robots

- 48.3 Stability Analysis – Not Falling Down ..... 1209
  - 48.3.1 Fixed Points ..... 1210
  - 48.3.2 Limit Cycles ..... 1210
  - 48.3.3 Viability ..... 1211
  - 48.3.4 Controllability ..... 1212
  - 48.3.5 Robust or Stochastic Stability ..... 1213
  - 48.3.6 Input–Output Stability ..... 1214
  - 48.3.7 Stability Margins ..... 1214
- 48.4 Generation of Dynamic Walking and Running Motions ..... 1214
  - 48.4.1 Early Offline Motion Generation Schemes ..... 1215
  - 48.4.2 Online Motion Generation: A Model Predictive Control Point of View ..... 1215
  - 48.4.3 Motion in Constrained Environments ..... 1219
  - 48.4.4 Motion Generation with Limited Computing Resources ..... 1220
  - 48.5 Motion and Force Control ..... 1222
    - 48.5.1 Towards More Efficient Walking ..... 1225
    - 48.5.1 Gait Generation for Dynamic Walking ..... 1225
    - 48.5.2 Orbital Trajectory Stabilization and Control ..... 1226
    - 48.5.3 Different Contact Behaviors ..... 1227
    - 48.5.1 Wall Climbing ..... 1227
    - 48.5.2 Tethered Walking ..... 1227
    - 48.5.3 Legs with Wheels ..... 1228
    - 48.5.4 Wheels with Legs ..... 1228
  - 48.6 Conclusion ..... 1228
  - References ..... 1228

1203

Part E 148

- 5.4.2. Velocity-Based ZMP Manipulation–Type Stabilization in 3D 215
- 5.4.3. Regulator-Type ZMP Stabilizer 217
- 5.4.4. ZMP Stabilization in the Presence of GRF Estimation Time Lag 219
- 5.4.5. Torso Position Compliance Control (TPCC) 220
- 5 Capture Point-Based Analysis and Stabilization 222
  - 5.5.1. Capture Point (CP) and Instantaneous Capture Point (ICP) 222
  - 5.5.2. ICP-Based Stabilization 223
  - 5.5.3. ICP Stabilization in the Presence of GRF Estimation Time Lag 224
  - 5.5.4. ICP Dynamics and Stabilization in 3D 225
- 6 Stability Analysis and Stabilization With Angular Momentum Component 226
  - 5.6.1. Stability Analysis Based on the LRWP Model 226
  - 5.6.2. Stability Analysis in 3D: the Divergent Component of Motion 228
  - 5.6.3. DCM Stabilizer 231
  - 5.6.4. Summary and Conclusions 232
- 7 Maximum Output Admissible Set Based Stabilization 233
- 8 Balance Control Based on Spatial Momentum and Its Rate of Change 235
  - 5.8.1. Fundamental Functional Dependencies in Balance Control 235
  - 5.8.2. Resolved Momentum Control 237
  - 5.8.3. Whole-Body Balance Control With Relative Angular Momentum/Velocity 237
  - 5.8.4. SNS-Based Stabilization of Unstable Postures 242
  - 5.8.5. An Approach to Contact Stabilization Within the Resolved Momentum Framework 244
  - 5.8.6. Spatial Momentum Rate Stabilization Parameterized by the CMP/VRP 246
- 5.8.7. CRB Motion Trajectory Tracking With Asymptotic Stability 247
- 5.9 Task-Space Controller Design for Balance Control 248
  - 5.9.1. Generic Task-Space Controller Structure 249
  - 5.9.2. Optimization Task Formulation and Constraints 250
- 5.10 Noninteractive Body Wrench Distribution Methods 253
  - 5.10.1. Pseudoinverse-Based Body-Wrench Distribution 253
  - 5.10.2. The ZMP Distributor 254
  - 5.10.3. Proportional Distribution Approach 255
  - 5.10.4. The DCM Generalized Inverse 256
  - 5.10.5. The VRP Generalized Inverse 262
  - 5.10.6. Joint Torque–Based Contact Wrench Optimization 264
- 5.11 Noninteractive Spatial Dynamics–Based Motion Optimization 266
  - 5.11.1. Independent Motion Optimization With CRB Wrench-Consistent Input 266
  - 5.11.2. Stabilization With Angular Momentum Damping 267
  - 5.11.3. Motion Optimization With Task-Based Hand Motion Constraints 270
  - 5.11.2. Noninteractive Whole-Body Motion/Force Optimization 271
    - 5.12.1. Multicontact Motion/Force Controller Based on the Closed-Chain Model 271
    - 5.12.2. Motion/Force Optimization Based on the Operational-Space Formulations 275
  - 5.11.3. Reactive Balance Control in Response to Weak External Disturbances 278
  - 5.11.3. Gravity Compensation-Based Whole-Body Compliance With Passivity 279
  - 5.13.2. Whole-Body Compliance With Multiple Contacts and Passivity 280

### Part VI Humanoid Balance ..... 1313

- Introduction to Humanoid Balance ..... 1315
- Jerry E. Pratt, Christian Ott, and Sang-Ho Hyon
- Human Sense of Balance ..... 1323
- Thomas Mergner and Robert J. Peterka
- Torque-Based Balancing ..... 1361
- Christian Ott and Sang-Ho Hyon
- Angular Momentum-Based Balance Control ..... 1387
- Sung-Hee Lee, Andreas Hofmann, and Ambarish Goswami
- Stepping for Balance Maintenance Including Push-Recovery ..... 1419
- Jerry E. Pratt, Sylvain Bertrand, and Twan Koolen
- Feedback Control of Inverted Pendulums ..... 1467
- Shuuji Kajita
- Technical Implementations of the Sense of Balance ..... 1489
- Michael Bloesch and Marco Hutter
- Balancing via Position Control ..... 1519
- Youngjin Choi, Yonghwan Oh, and Gihoo Jang
- Optimization-Based Control Approaches to Humanoid Balancing ..... 1541
- Aurélien Ibanez, Philippe Bidaud, and Vincent Padois

### Part VII Humanoid Motion Planning, Optimization, and Gait Generation ..... 1569

- Introduction: Motion Planning, Optimization, and Biped Gait Generation ..... 1571
- Eiichi Yoshida and Katja Mombaur
- Whole-Body Motion Planning ..... 1575
- Eiichi Yoshida, Fumio Kanehiro, and Jean-Paul Laumond

# Honda Asimo - Fully actuated walking



All New Honda Asimo 2018 at the USA Science and Engineering Festival

[https://youtu.be/1urL\\_X\\_vp7w](https://youtu.be/1urL_X_vp7w)

# Passive dynamic walking



McGeer and Passive Dynamic Bipedal Walking

<https://youtu.be/WOPED7I5Lac>

McGeer, T. (1990). Passive dynamic walking. *Int. J. Robotics Res.*, 9(2), 62-82.



Pneumatic passive-based biped

Martijn Wisse  
Jan van Frankenhuyzen  
2004

Delft Biorobotics Laboratory

TU Delft

STW

Collins, S., Ruina, A., Tedrake, R., & Wisse, M. (2005). Efficient bipedal robots based on passive-dynamic walkers. *Science*, 307(5712), 1082-1085.

# Passive dynamic walker

Tad McGeer

School of Engineering Science  
Simon Fraser University  
Burnaby, British Columbia, Canada V5A 1S6

## Passive Dynamic Walking

### Abstract

There exists a class of two-legged machines for which walking is a natural dynamic mode. Once started on a shallow slope, a machine of this class will settle into a steady gait quite comparable to human walking, without active control or energy input. Interpretation and analysis of the physics are straightforward; the walking cycle, its stability, and its sensitivity to parameter variations are easily calculated. Experiments with a test machine verify that the passive walking effect can be readily exploited in practice. The dynamics are most clearly demonstrated by a machine powered only by gravity, but they can be combined easily with active energy input to produce efficient and dextrous walking over a broad range of terrain.

### 1. Static vs. Dynamic Walking

Research on legged locomotion is motivated partly by fundamental curiosity about its mechanics, and partly by the practical utility of machines capable of traversing uneven surfaces. Increasing general interest in robotics over recent years has coincided with the appearance of a wide variety of legged machines. A brief classification will indicate where our own work fits in. First one should distinguish between *static* and *dynamic* machines. The former maintain static equilibrium throughout their motion. This requires at least four legs and, more commonly, six. It also imposes a speed restriction, since cyclic accelerations must be limited in order to minimize inertial effects. Outstanding examples of static walkers are the Odex series (Russell 1983) and the Adaptive Suspension Vehicle (Waldron 1986). Dynamic machines, on the other hand, are more like people; they can have fewer legs than static machines, and are potentially faster.

The International Journal of Robotics Research,  
Vol. 9, No. 2, April 1990.  
© 1990 Massachusetts Institute of Technology.

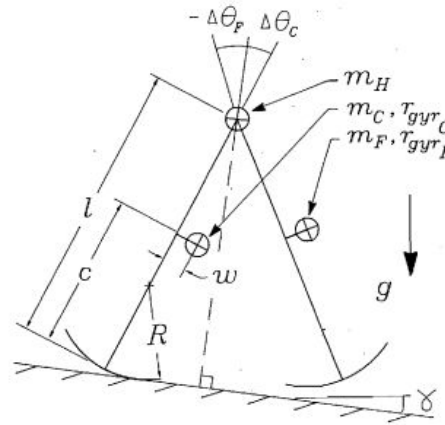
### 2. Dynamics vs. Control

Our interest is in dynamic walking machines, which for our purposes can be classified according to the role of active control in generating the gait. At one end of the spectrum is the biped of Mita et al. (1984), whose motion is generated entirely by linear feedback control. At the end of one step, joint angles are commanded corresponding to the end of the next step, and the controller attempts to null the errors. There is no explicit specification of the trajectory between these end conditions. Yamada, Furusho, and Sano (1985) took an approach that also relies on feedback, but in their machine it is used to track a fully specified trajectory rather than just to close the gap between start and end positions. Meanwhile the stance leg is left free to rotate as an inverted pendulum, which, as we shall discuss, is a key element of passive walking. Similar techniques are used in biped walkers by Takamishi et al. (1985), Lee and Liao (1988), and Zheng, Shen, and Sias (1988).

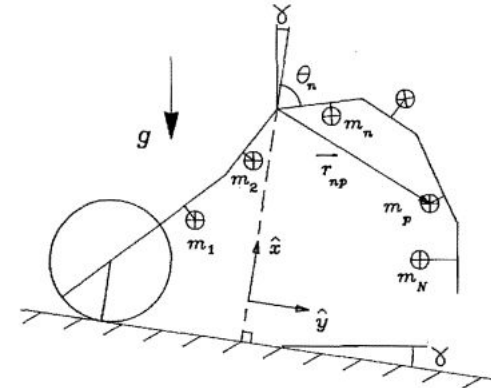
By contrast the bipeds of Miura and Shimoyama (1984) generate their gait by feedforward rather than feedback; joint torque schedules are precalculated and played back on command. Again the stance leg is left free. However, the "feedforward" gait is unstable, so small feedback corrections are added to maintain the walking cycle. Most significantly, these are *not* applied continuously (i.e., for tracking of the nominal trajectory). Instead the "feedforward" step is treated as a processor whose output (the end-of-step state) varies with the input (the start-of-step state). Thus the feedback controller responds to an error in tracking by modifying initial conditions for subsequent steps, and so over several steps the error is eliminated. In this paper you will see analysis of a similar process. Raibert (1986) has developed comparable concepts but with a more pure implementation, and applied them with great success to running machines having from one to four legs.

All of these machines use active control in some form to generate the locomotion pattern. They can be

Fig. 2. General arrangement of a 2D biped. It includes legs of arbitrary mass and inertia, semicircular feet, and a point mass at the hip.



N-LINK 2-D CHAIN WITH ROLLING SUPPORT



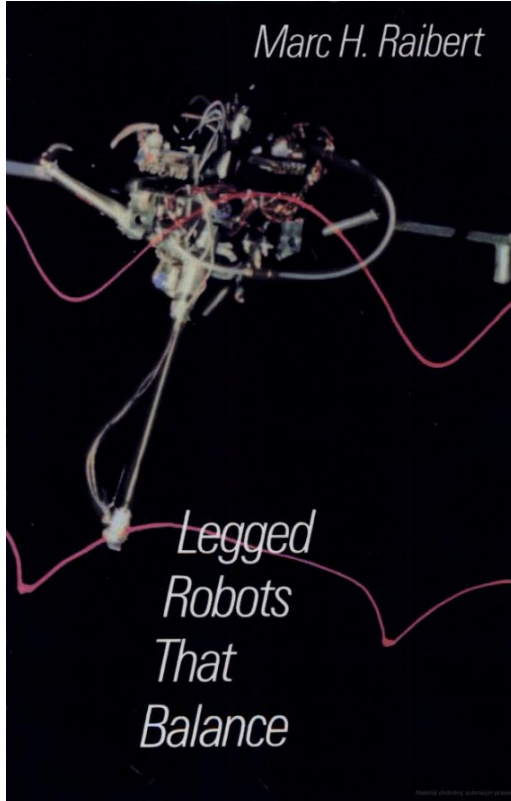
# Boston dynamics - Atlas



Atlas Gets a Grip | Boston Dynamics - 2023

[https://youtu.be/-e1\\_QhJ1EhQ](https://youtu.be/-e1_QhJ1EhQ)

# Where it started....



<b>Chapter 1. Introduction</b>	1
Why Study Legged Machines?	1
Dynamics and Balance Improve Mobility	4
Research on Legged Machines	6
Research on Active Balance	11
Introduction to Running Machines	14
Additional Readings	26
<b>Chapter 2. Hopping on One Leg in the Plane</b>	29
A Planar Machine That Hops on One Leg	30
Control of Running Decomposed into Three Parts	37
Hopping Experiments	48
Improvements and Limitations	52
Summary	55
<b>Chapter 3. Hopping in Three Dimensions</b>	57
Balance in Three Dimensions	58
3D One-Legged Hopping Machine	62
Control System for 3D One-Legged Machine	67

viii	<b>Chapter</b>	
	Hopping Experiments in Three Dimensions	71
	Summary	78
	Appendix 3A. Kinematics of 3D One-Legged Machine	80
	<b>Chapter 4. Biped and Quadruped Running</b>	83
	One-Foot Gaits	84
	Virtual Legs	92
	Quadruped Trotting Experiments Using Virtual Legs	96
	Discussion of Quadruped Experiments	102
	Summary	105
	Appendix 4A. Equations for Virtual Leg	107
	Appendix 4B. Kinematics for Four-Legged Machine	109
	<b>Chapter 5. Symmetry in Running</b>	115
	Mechanics of Symmetry	117
	Symmetry in Animal Running	124
	Scissor Symmetry	130



The MIT Press, Cambridge, Massachusetts  
Copyright © 1986 by Marc H. Raibert  
All rights reserved. Published 1986  
Printed in the United States of America

Robots from MIT's Leg Lab  
<https://youtu.be/XFXj81mvInc>



# Do we need modeling?

- Or can we do with machine learning / deep learning like in grasping?
- Marc Raibert, CEO Boston Dynamics, IROS, Kyoto, October 2022:
  - In everything you have seen from Boston Dynamics till now, there is **zero machine learning / deep learning**.
  - Whenever we had to choose whether to put machine learning or a bunch of engineers on the problem, so far we always went for the engineers.
- How are Boston Dynamics robots controlled?
  - Principles originate in the early Raibert's work - modeling and engineering.
  - Heavy use of Model Predictive Control (MPC).

# Resources

- Books / book sections
  - [Chapter 5 - Balance control in Nenchev, D. N., Konno, A., & Tsujita, T. (2018). Humanoid robots: Modeling and control. Butterworth-Heinemann.]
- Articles
  - McGeer, T. (1990). Passive dynamic walking. *Int. J. Robotics Res.*, 9(2), 62-82.