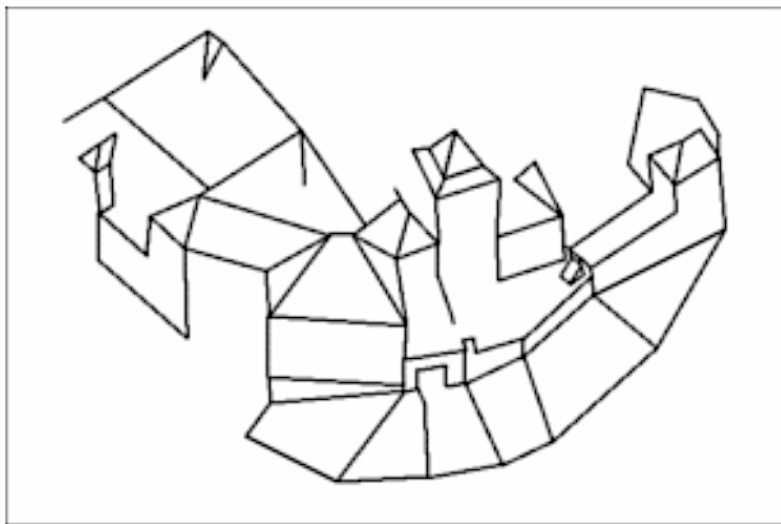
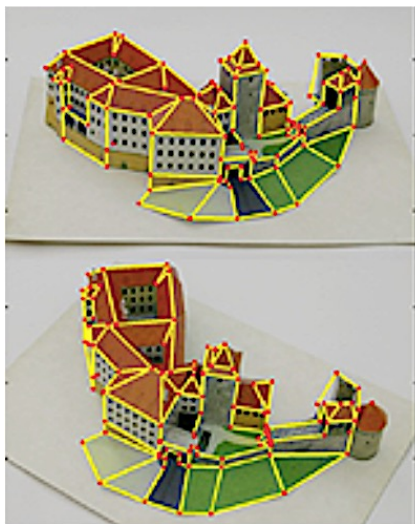


GVG – Geometry of Computer Vision & Graphics



Tomas Pajdla, Torsten Sattler

2023

CIIRC – Czech Institute of Informatics Robotics and Cybernetics, CTU in Prague



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Geometry of Computer Vision and Graphics 2023

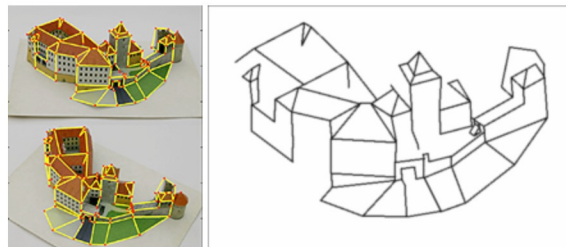


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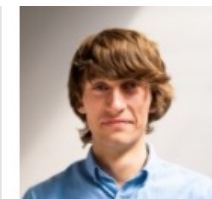
We will explain Euclidean, Affine, and Projective geometry basics, introduce a model of the perspective camera, and explain how images change when moving a camera. We will show how to compute camera poses and the 3D scene geometry from images. We will demonstrate the theory in practical panorama construction tasks, finding the camera pose, adding a virtual object to a real scene, and reconstructing a 3D model of a scene from its images. We will build on our previous knowledge of linear algebra and provide fundamentals of geometry for computer vision, computer graphics, augmented reality, image processing, and object recognition.

[Tomas Pajdla](#)

[Torsten Sattler](#)

[Martin Matousek](#)

[Viktor Korotynskiy](#)

[Vojtech Panek](#)


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e l i s
European Laboratory for Learning and Intelligent Systems

Basic & Applied Research

- Research**
We apply elements of
- Algebra
 - Geometry
 - Statistics
 - Optimization in
 - Computer Vision
 - Robotics
 - Machine Learning



3D Reconstruction



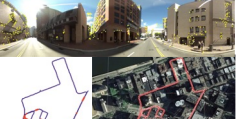
Camera Geometry



Robotics



Visual localization



Tomas Pajdla
AAG Leader
Vision
Robotics
Mathematics



Josef Sivic
IMPACT Leader
Vision
Robotics
Machine Learning



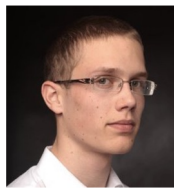
Torsten Sattler
Researcher
3D Vision
Learning



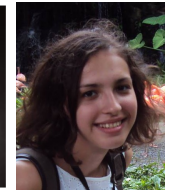
Jiri Sedlar
Postdoc
Vision
Robotics



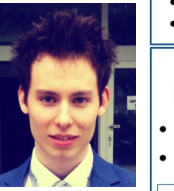
Michal Polic
PhD Student
3D
Vision



Pavel Trutman
PhD Student
Polynomial
Optimization



Diana Sungatullina
PhD Student
Vision
Learning



Viktor Korotynskiy
PhD Student
Algebraic
Geometry

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Continental Camera calibration

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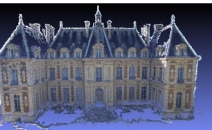
Continental Camera calibration

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3D Reconstruction



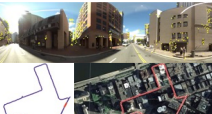
Camera Geometry



Robotics



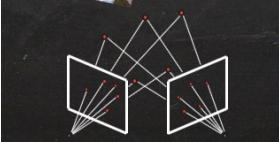
Visual localization



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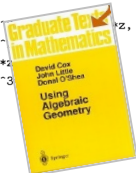
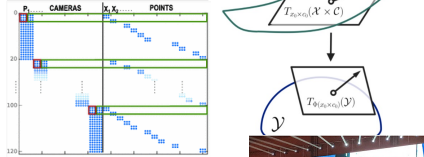
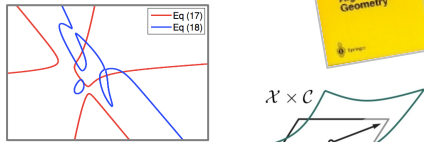
Theory

Camera Geometry
Many Minimal reconstruction Problems in RANSAC based optimization

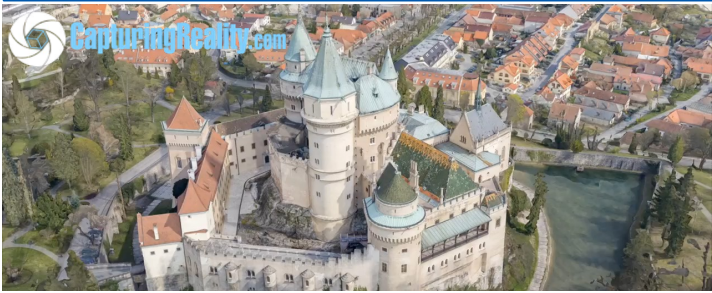


Macaulay2 program over the finite field $\mathbb{Z}/13$

```
R = ZZ/13[x,y,z, MonomialOrder=>GRewLex];
I = ideal(8*x^2*y^2 + 5*x*y^3 + 3*x^5 + 2*y^3*z^2 + 13*y^4 + 8*x^3 + 12*y^3 + x^2 + 7*x^2*y^4 + 18*x*y^3*z^2 + y^3);
G = gens gb I
```



Applications



- ### Research
- We apply elements of
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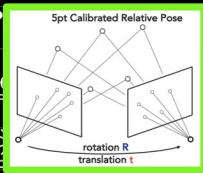
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- ### Industry
- We collaborate with
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 - Leica
 - Photogrammetry
 - Magik Eye
 - 3D sensing
 - Continental
 - Camera calibration

30 Minimal Problems

| | | | | | | | | | | |
|---------------|----------------|-------|-----|-----|-----|-----|-----|-----|----|------|
| # views | 6 | 5 | 5 | 5 | 4 | 4 | 4 | | | |
| Configuration | | | | | | | | | | |
| # solutions | $\approx 10^6$ | 11296 | 20 | | | | | | | 1728 |
| # views | 3 | 3 | | | | | | | | 3 |
| Configuration | | | | | | | | | | |
| # solutions | 360 | 552 | 480 | 264 | 432 | 328 | 480 | 240 | 64 | 216 |
| # views | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 2 | 2 | 2 |
| Configuration | | | | | | | | | | |
| # solutions | 312 | 224 | 40 | 144 | 144 | 144 | 64 | 20 | 16 | 12 |



CCCV 2019
Best Student Paper Award
Timothy Darr, Katarina Galic, Annes Leebin, Tamas Pajdla

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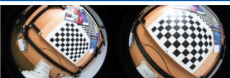
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3D Reconstruction



Camera Geometry



Robotics



Visual localization



Rendering of RAW 3D reconstruction

3D Mapping

Reconstruction by Capturing Reality (www.capturingreality.com)
Spin-off of M Bujnak and M Jancosek (PhD students of T Pajdla)
Based on 3D reconstruction techniques developed at the CTU in Prague



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in Prague

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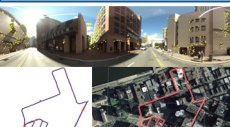
DAIMLER
Omni-Vision

Leica
Photogrammetry

Magik Eye
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Camera calibration

T Pajdla pajdla@cvut.cz



Visual Effects



"See You Up There", dir. Albert Dupontel, prod. ADCB Films, Manchester Films



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3D Reconstruction



Camera Geometry



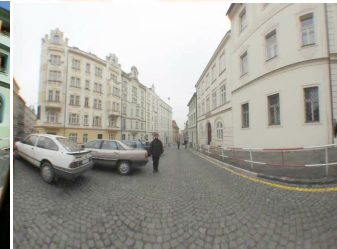
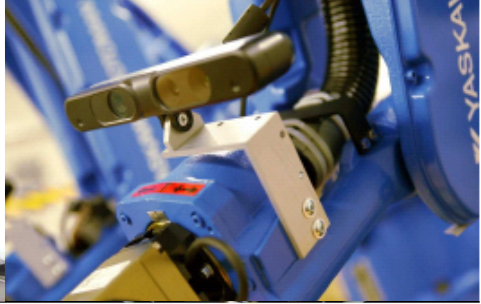
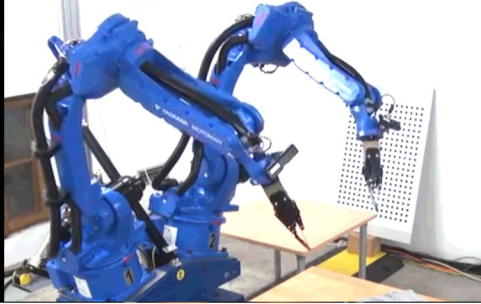
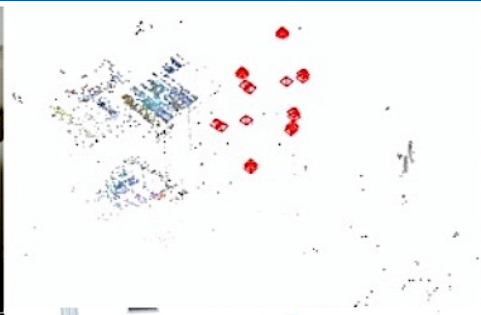
Robotics



Visual localization



Robotics & Machine Perception



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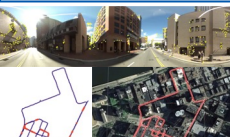
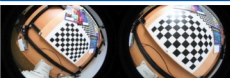
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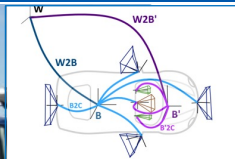
Leica Photogrammetry

Magik **Eye** 3D sensing

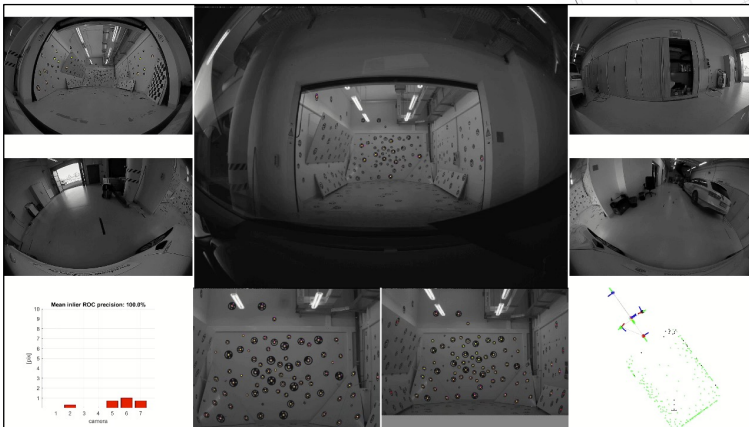
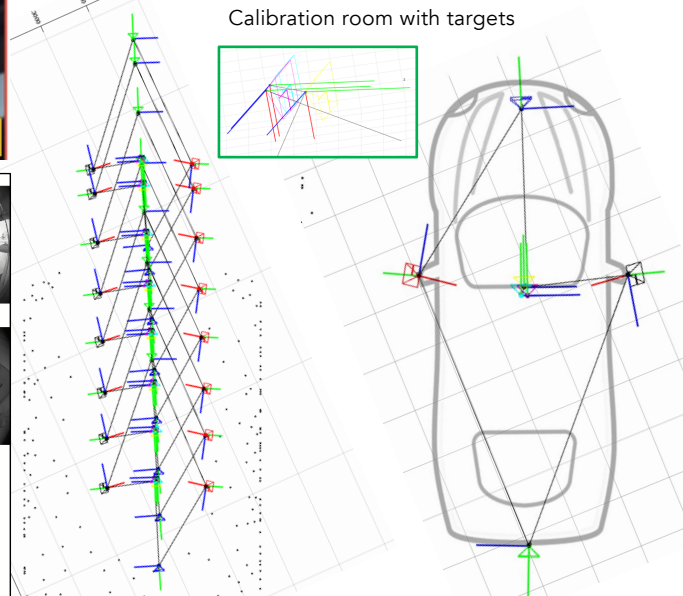
Continental Camera calibration



Cameras on Cars



Calibration room with targets



4 x Continental wide angle + 1 x 3-focal Mobileye

Many poses + Bundle Adjustment

Result: Cameras on car

Research

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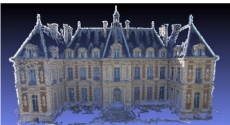
Continental
Camera calibration

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3D Reconstruction



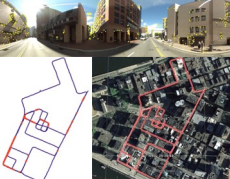
Camera Geometry



Robotics



Visual localization



UP-Drive

Automated Urban Parking and Driving

An H2020 European Project

PROJECT CONSORTIUM RESOURCES MEDIA

Autonomous Driving



ETH Zürich



Czech Technical University
in Prague

SLAM by ETH on cams calibrated by CTU



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Tomas Pajdla - pajdla@cvut.cz

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Geometry of Computer Vision and Graphics 2023

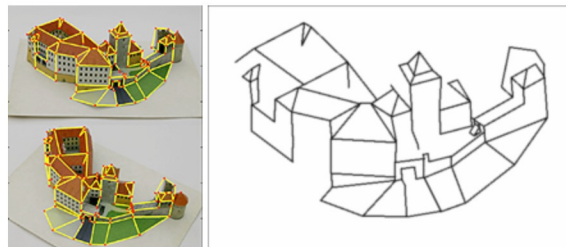


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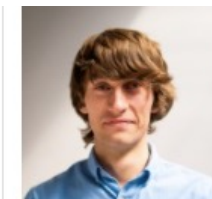
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[Tomas Pajdla](#)










































[Torsten Sattler](#)

[Martin Matousek](#)

[Viktor Korotynskiy](#)













[Vojtech Panek](#)


Tomas Pajdla, Torsten Sattler


| # | Date | Lecture  T Pajdla. Elements of Geometry for Computer Vision and Computer Graphics |
|----|-------|---|
| 01 | 20.2. | TS: Geometry of CV & CG  S  V, LA [Sec. 2.1]  S  V, Image coordinate system [Sec. 5]  V |
| 02 | 27.2. | TS: Mathematical model of the perspective camera [Sec. 6], Kronecker product [Sec. 2.5]  S  V1  V2  V3 |
| 03 | 06.3. | TP: Camera calibration and pose [Sec. 7.1]  S  V1  V2 |
| 04 | 13.3. | TP: Calibrated camera pose computation [Sec. 7.2, 7.3], Vector product [Sec. 2.2, 2.3]  S  V |
| 05 | 20.3. | TP: Homography [Sec. 8.1-8.5]  S  V1  V2 |
| 06 | 27.3. | TS: Image based camera localization  S  V1  V2  V3 |
| 07 | 03.4. | TP: Projective plane [Sec. 9.1-9.2]  S  V |
| — | 10.4. | <i>Easter Monday</i> |
| 08 | 17.4. | TP: Vanishing points & line [Sec. 9.4, 9.5] projective space [Sec. 10] camera autocalibration [Sec. 11]  S  V |
| 09 | 24.4. | TP: Dual space [Sec. 2.4] lines under homography [Sec. 9.3]  S0  S1  V |
| 10 | 04.5. | TS: Epipolar geometry [Sec. 12.1-12.2]  Slides  S0  S1  V |
| 11 | 09.5. | TP: 3D reconstruction with a calibrated camera [Sec. 12.3, 12.4]  S0,  S1,  V |
| 12 | 15.5. | TP: Calibrated camera motion computation [Sec. 12.5]  S0,  S1,  V |
| 13 | 22.5. | TS: 3D Reconstruction pipelines  S  V |

Martin Matoušek, Viktor Korotynskiy, Vojtěch Pánek, Diana Sungatullina:

- solving of algebraic problems related to vision geometry; this is without computer, i.e. 'pen-and-paper'
- solving of practical tasks (home-works) on a real data with computer

| Week | Date | Pen & Paper | Solution | Test | Assignment | Deadline |
|------|-------|---|----------|--------|-------------------------------|-------------------|
| 01 | 20.2. | Basic elements of LA | | Test-a | HW-01 Image Coordinate System | |
| 02 | 27.2. | Projection matrix  Lab-02 | | | HW-02 Projection Matrix | |
| 03 | 06.3. | Camera internal calibration  Lab-03 | | | HW-03 Camera calibration | HW-01 |
| 04 | 13.3. | Calibrated camera pose  Lab-04 | | | HW-04a Calibrated pose I | HW-02 |
| 05 | 20.3. | Preparation for Test 1  Lab-05 | | | HW-04b Calibrated pose II | HW-03 |
| 06 | 27.3. |  Test 1 Example | | Test 1 | HW-05 Homography | HW-04a |
| 07 | 03.4. | Homography  Lab-07 | | | HW-06 Panorama | HW-04b |
| 08 | 10.4. | <i>Easter monday</i> | | | | |
| 09 | 17.4. | Projective space  Lab-08 | | | HW-07 Autocalibration | HW-05 |
| 10 | 24.4. | Lines under homography  Lab-09 | | | | HW-06 |
| 11 | 04.5. |  Test 2 Example | | Test 2 | HW-08 Epipolar geometry | HW-07 |
| 12 | 09.5. | Epipolar geometry  Lab-12 | | | HW-09a 3D reconstruction I | |
| 13 | 15.5. | 3D Reconstruction  Lab-13 | | | HW-09b 3D reconstruction II | HW-08 |
| 14 | 22.5. |  Test 3 Example | | Test 3 | | HW-09a, HW-09b |


Assessment

1. All homework must be submitted via  BRUTE and accepted.
2. At least 50% of points in total for the homework.
3. At least 50% of points in total from the tests.
4. Regular submission of homework **ends on May 24, 2021**. Later submissions are possible only by an agreement with the assistants.
5. All the above conditions have to be fulfilled, and the results have to be recorded in the Submission system before the exam.






Exam

The exam consists of a written and an oral part. It is required to achieve at least 50% of points from the written exam to be admitted to the oral exam. The grade depends on the exam (40%), tests (30%), and homework (30%).

Exam content:

1. **Linear algebra [4,5,6,7]:** linear space, basis, coordinates, linear dependence/independence, matrices, rank, determinant, eigenvalues and eigenvectors, solving systems of linear equations, Frobenius theorem and linear independence, linear function, affine function, linear mapping and its matrix, computing roots of a polynomial via eigenvalues of its companion matrix, dual space, dual basis, change of the dual basis corresponding to a change of a basis, vector product and derived linear mappings, SVD, dual space, and dual basis.
2.  **Course material**

Rules

1. **Lecture:** It is very **difficult** to pass the course without attending  **online** lectures.
2. **Labs:** It is **impossible** to pass the course without attending  **online** labs.
3. **Homework:** Homework is assigned at a lab where it can be discussed with teaching assistants. Students work out homework **individually** ( **rules**  **in Czech**). The deadline for submitting homework via  BRUTE is on Monday at 6:00 in the morning two weeks after the assignment. Late submissions are penalized (10% for each commenced **day** of delay but not more than 50% of points).
4. **Assessment:** see above.
5. **Tests:** Students take tests **individually**.

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Geometry of Computer Vision and Graphics 2023

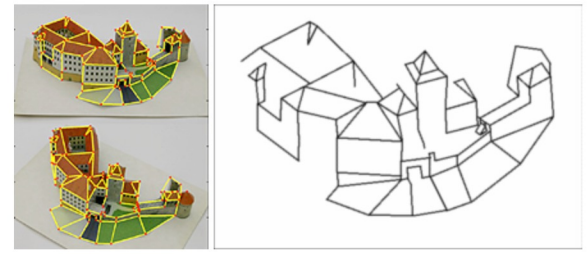


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- Rules
- Literature
- Contacts

BRUTE (cw.felk.cvut.cz/brute) – homework, tests, quizzes, points, feedback

Forum is the communication channel (limited emails & no messages in MS Teams)

1. Login to **BRUTE** at cw.felk.cvut.cz/brute
2. Check that you see **GVG in 2023L** in BRUTE
3. Check that you see News in the **Forum**

Questions ...