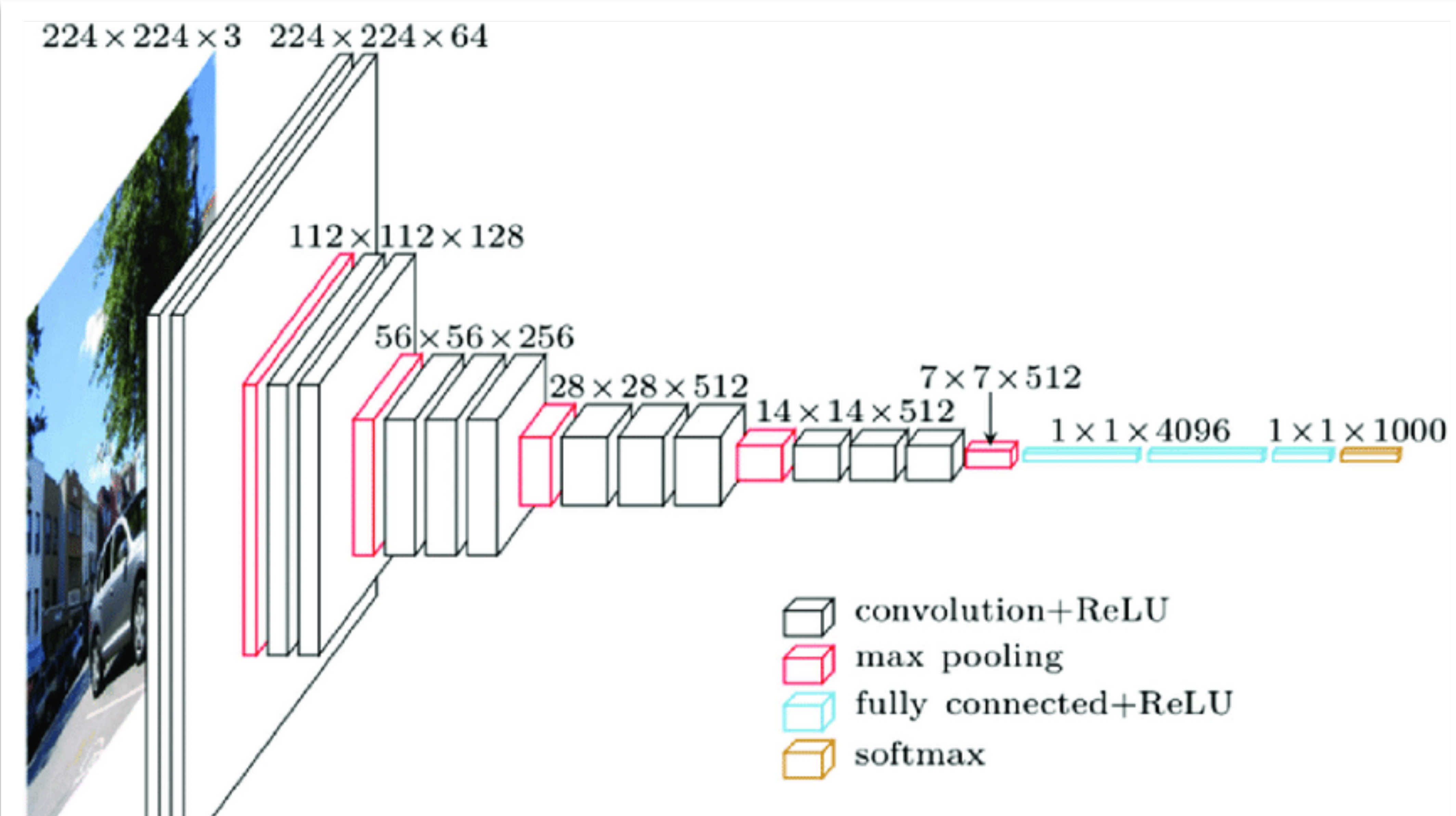
Deep Learning (BEV033DLE)
Lecture 5
Convolutional Neural Networks

Czech Technical University in Prague

- Introduction, CNN for Classification
  - Correlation filters, translation equivariance, convolution and cross-correlation
  - Multi-channel, stride, 1x1
  - Pooling, receptive field
- More Kinds of Convolutions
  - Dilation, transposed, ...
- Hierarchy of Parts

CNN for Classification



[VGG network, Image: Davi Frossard]

- We will go step-by-step though this diagram
- Understand its design principles
- Consider everything about convolutions in more detail

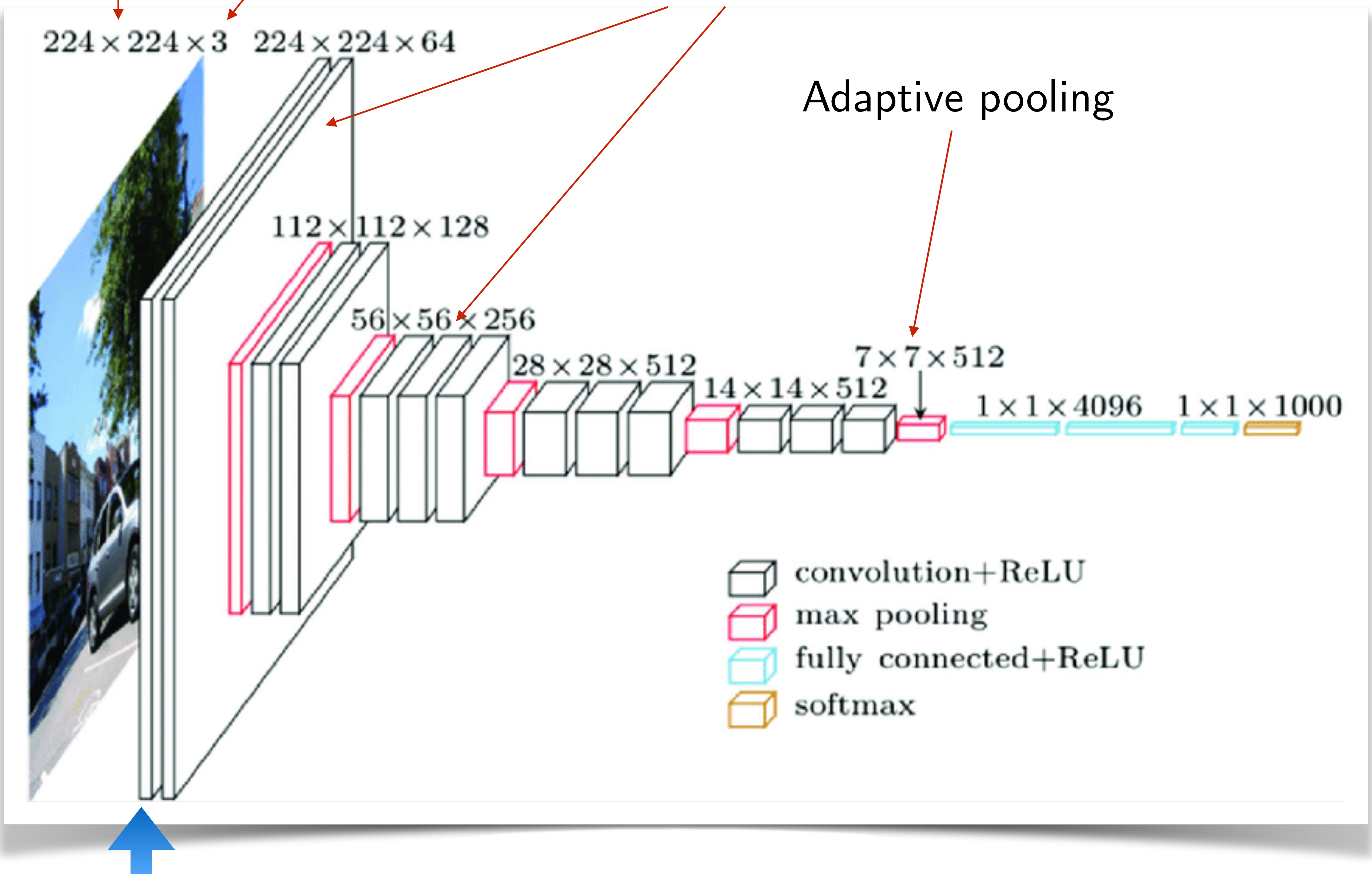
Classification CNN

Spatial size of the input image

channels

features

Adaptive pooling

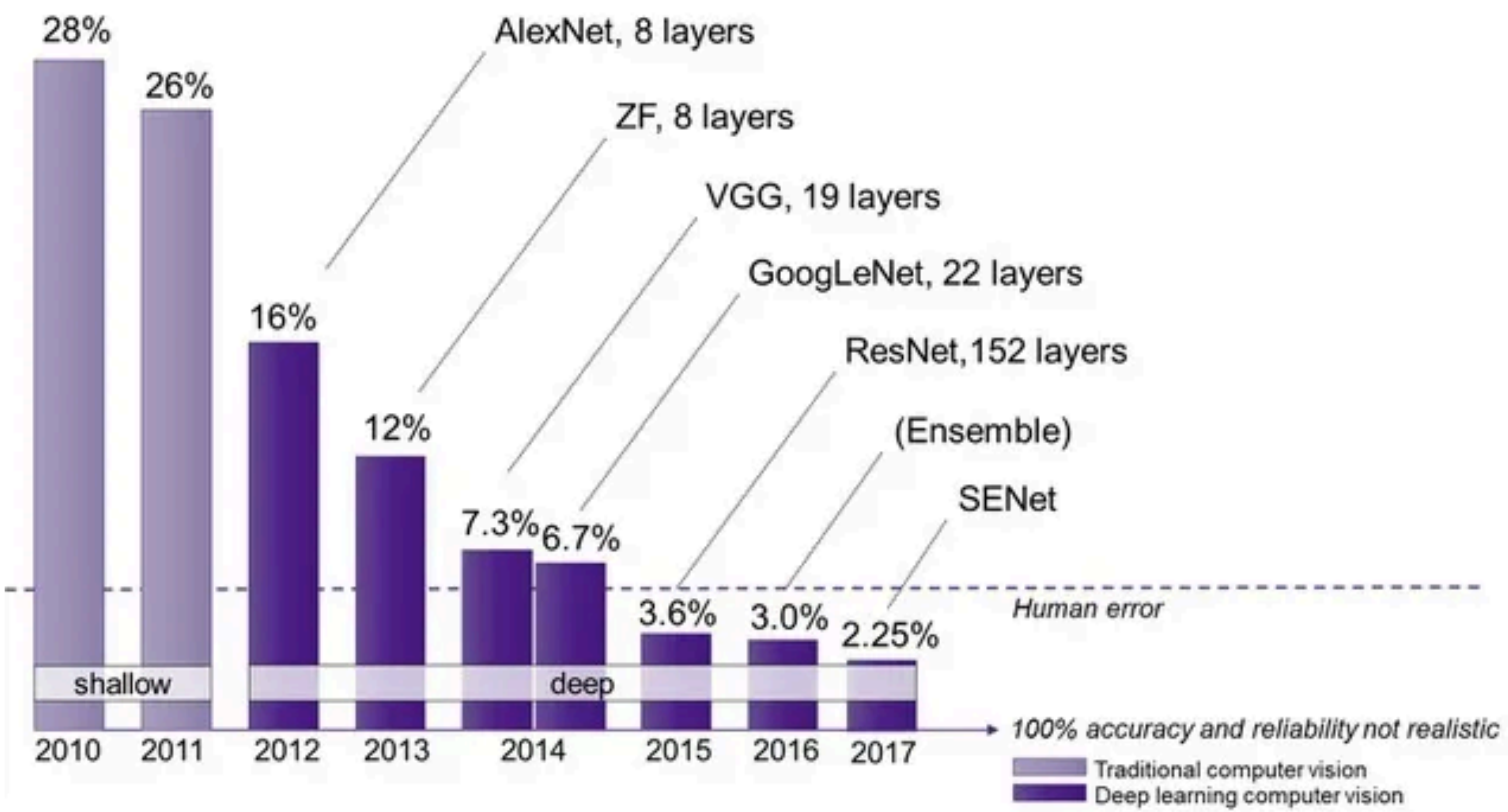


Result of $\mathrm{conv}(K \times K, 3 \rightarrow 64)$ followed by ReLU

Big Picture: ImageNet Classification

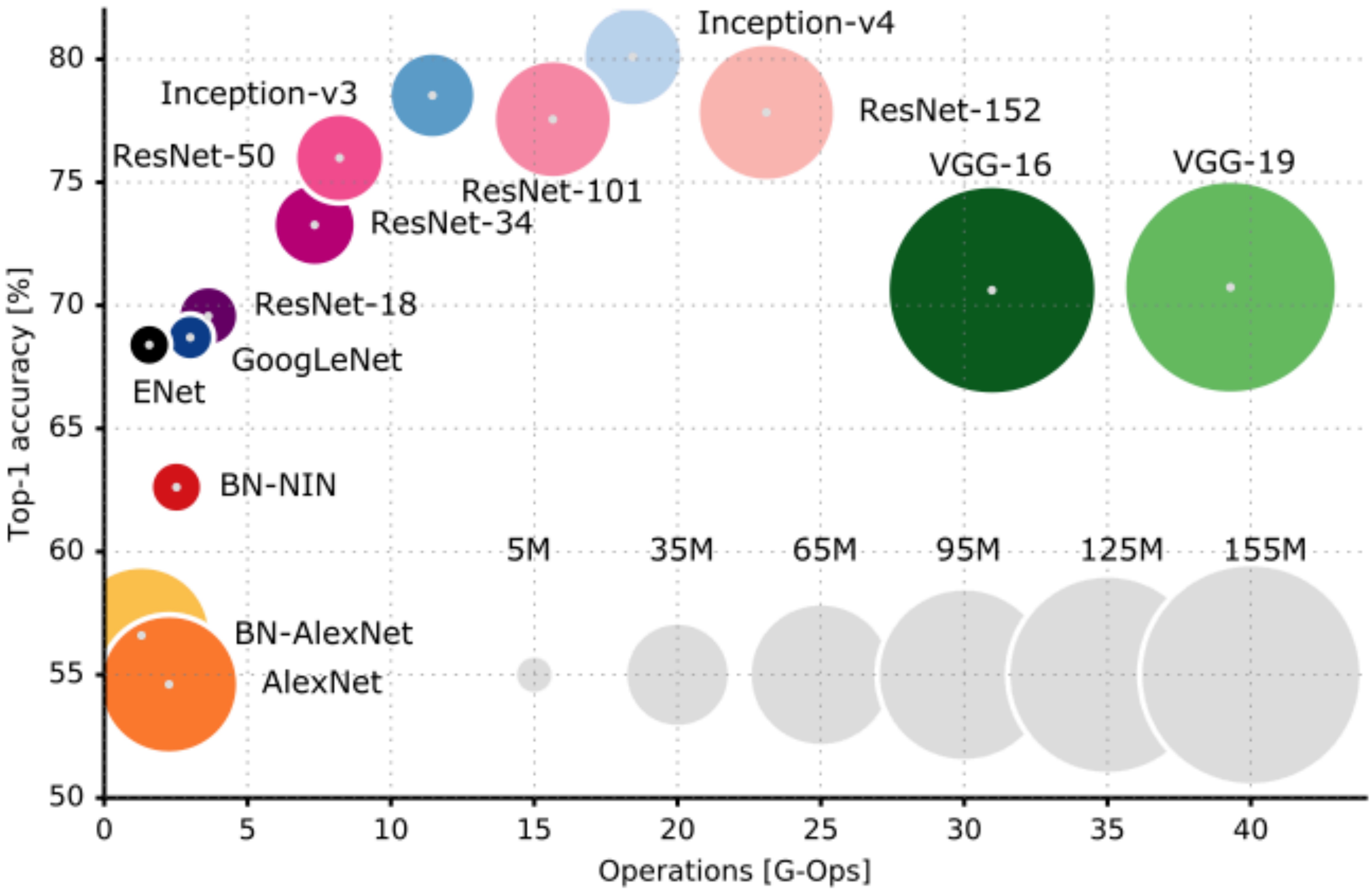
- The ImageNet Large Scale Visual Recognition Challenge (2010): 1000 classes, 1.4M images
- Deep Learning Revolution

Top-5 error rate



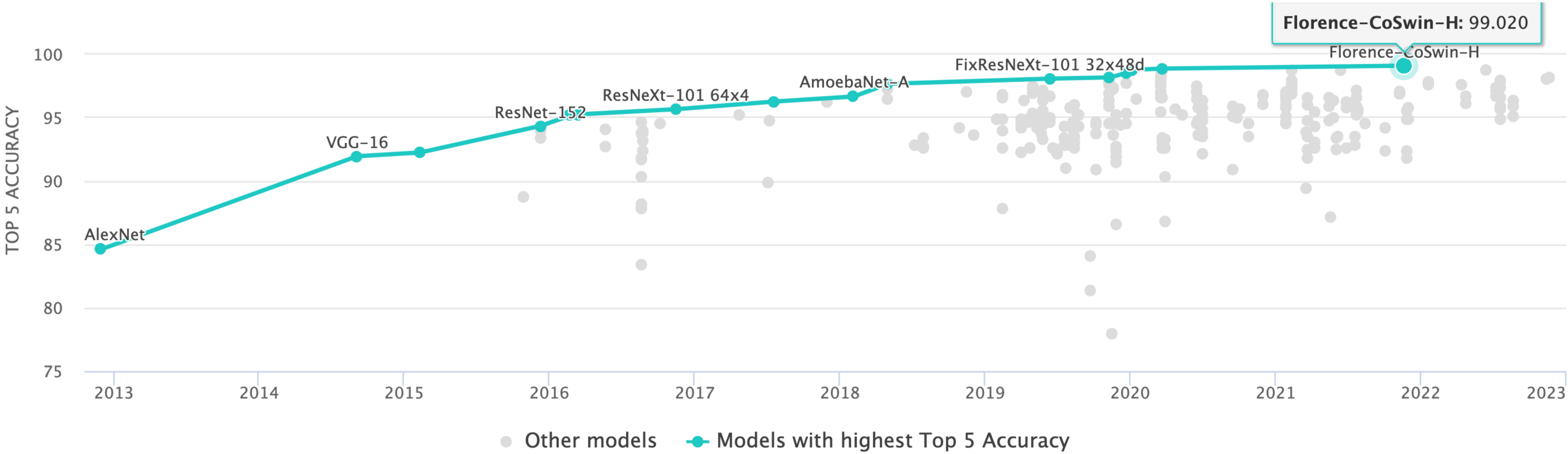
Big Picture: ImageNet Classification

- Efficiency Matters



Big Picture: ImageNet Classification

✦ State of the Art (IamgeNet 14M images)



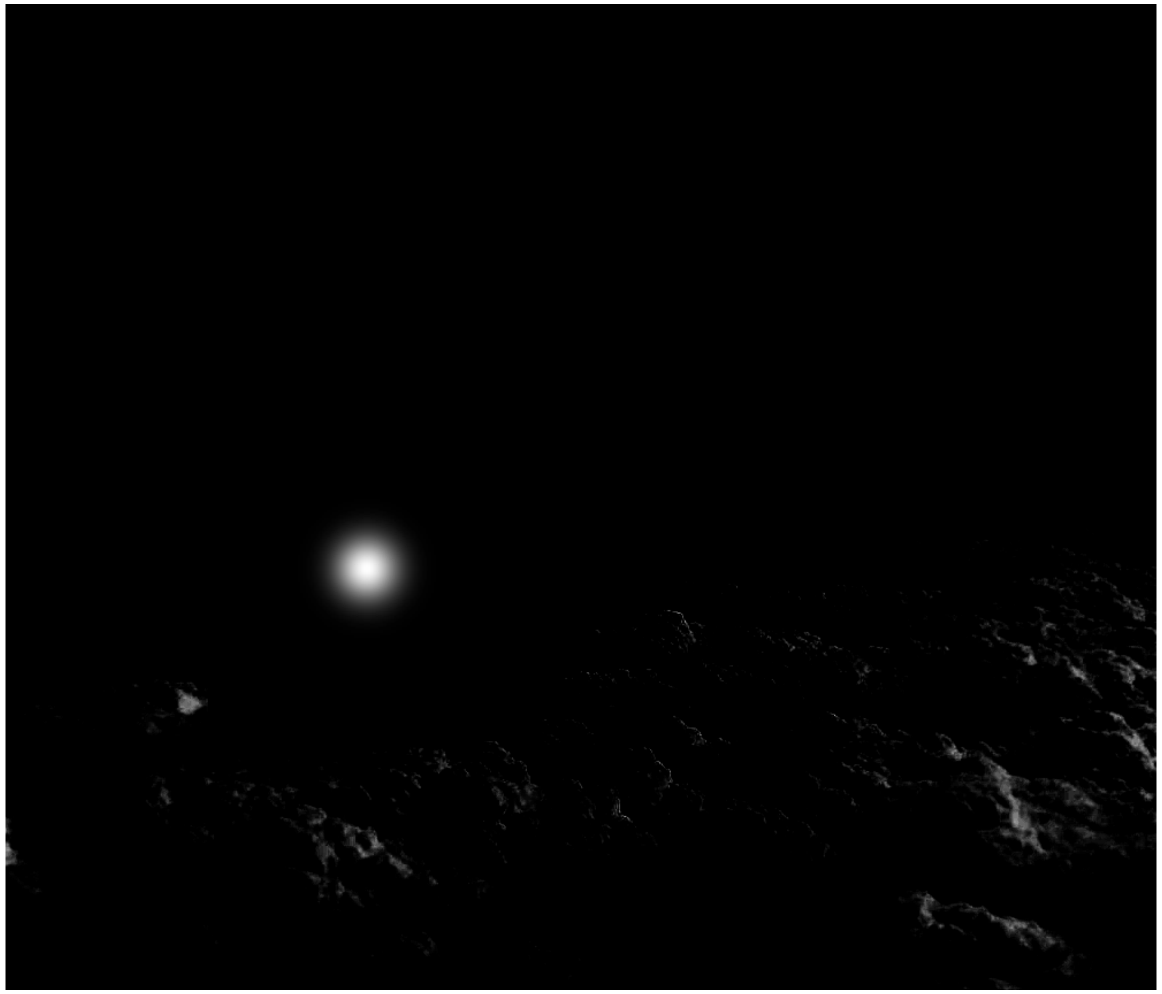
[paperswithcode.com]

Introduction

Template

Image

Quality of Match

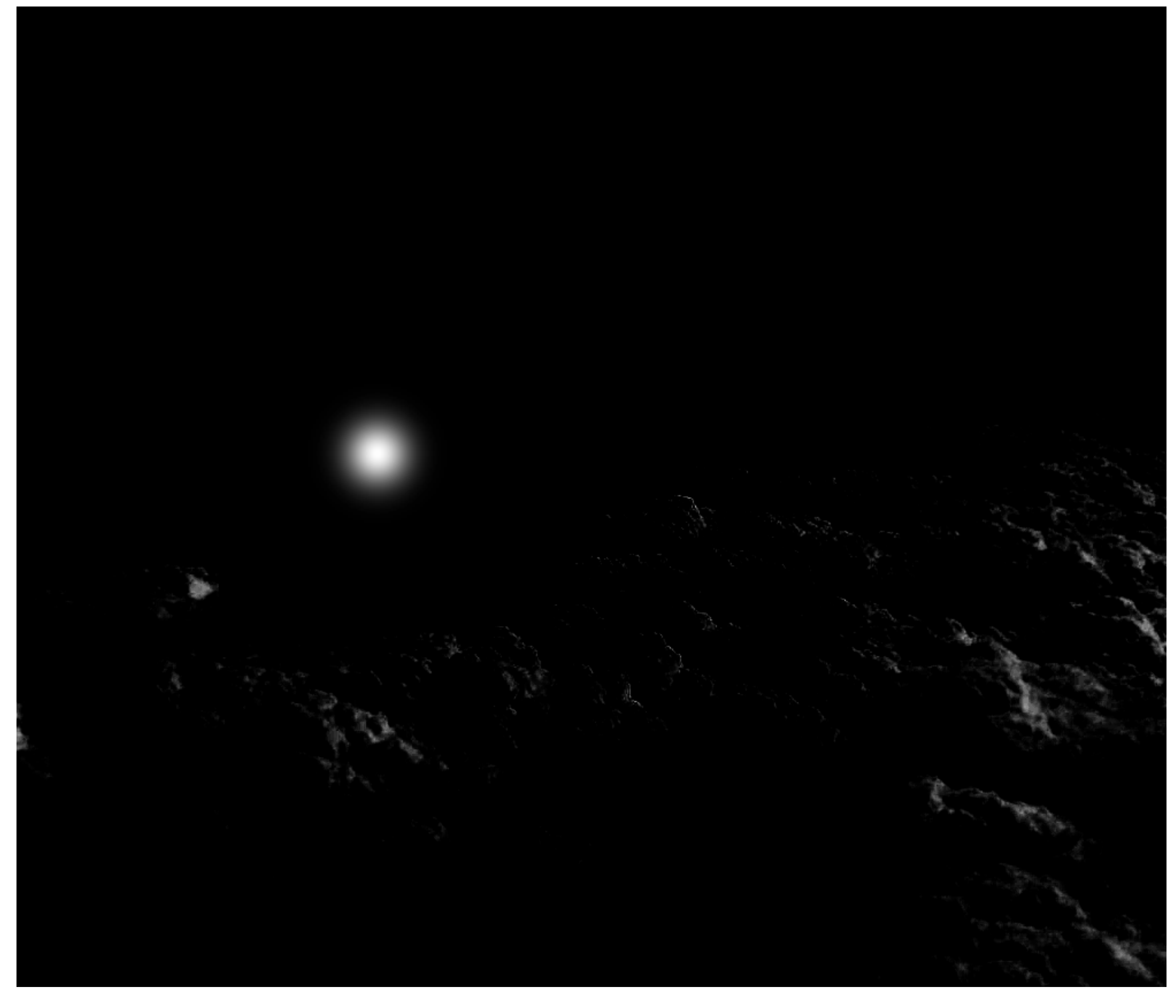


Introduction

Template

Image

Quality of Match



- Translational equivariance idea: when the input shifts, the output shifts
  - Would be hard to achieve if the image was given as a general vector — we are using 2D grid structure and require that all locations are treated equally

Basics

- Convolution and Correlation (1D)
  - Convolution $y = w * x$: $y_j = \sum_{k=-h}^{h} w_k x_{j-k} = \sum_{k=-h}^{h} w_{-k} x_{j+k}$
  - Cross-correlation $y = w \star x$: $y_j = \sum_{k=-h}^{h} w_k x_{j+k}$

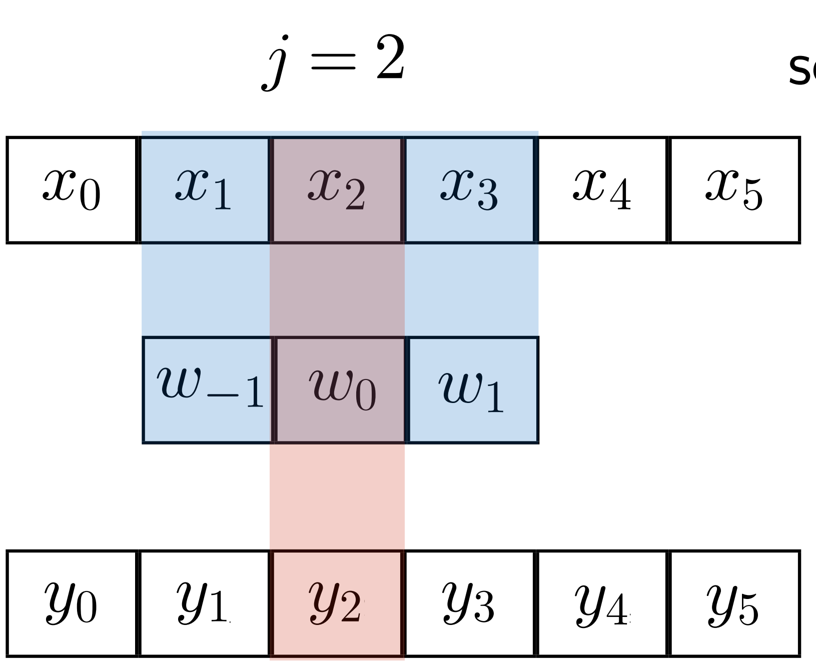
flip of the weight matrix

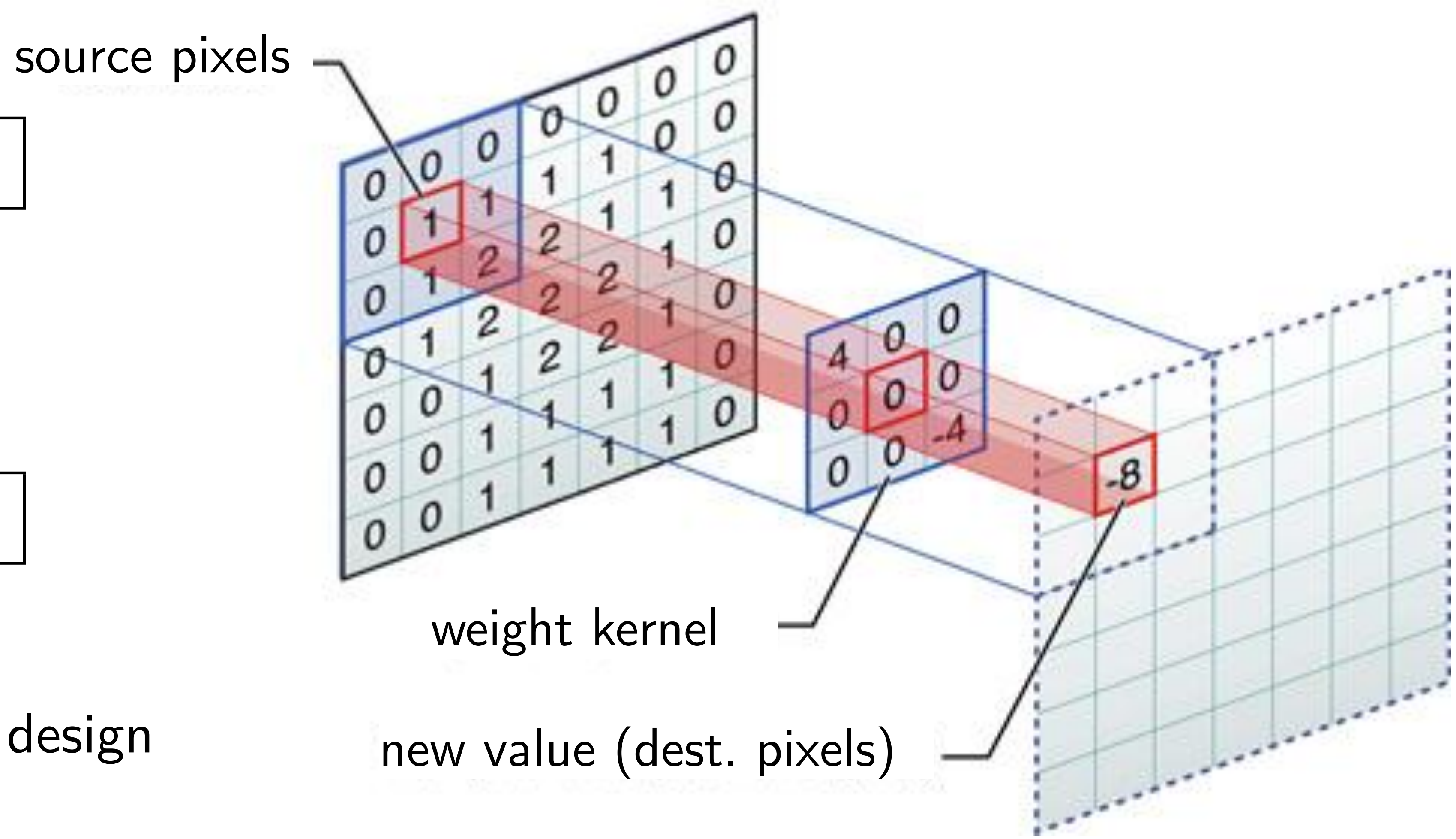
output

weight kernel

input

Easily convertible, more convenient to consider cross-correlation in Deep Learning

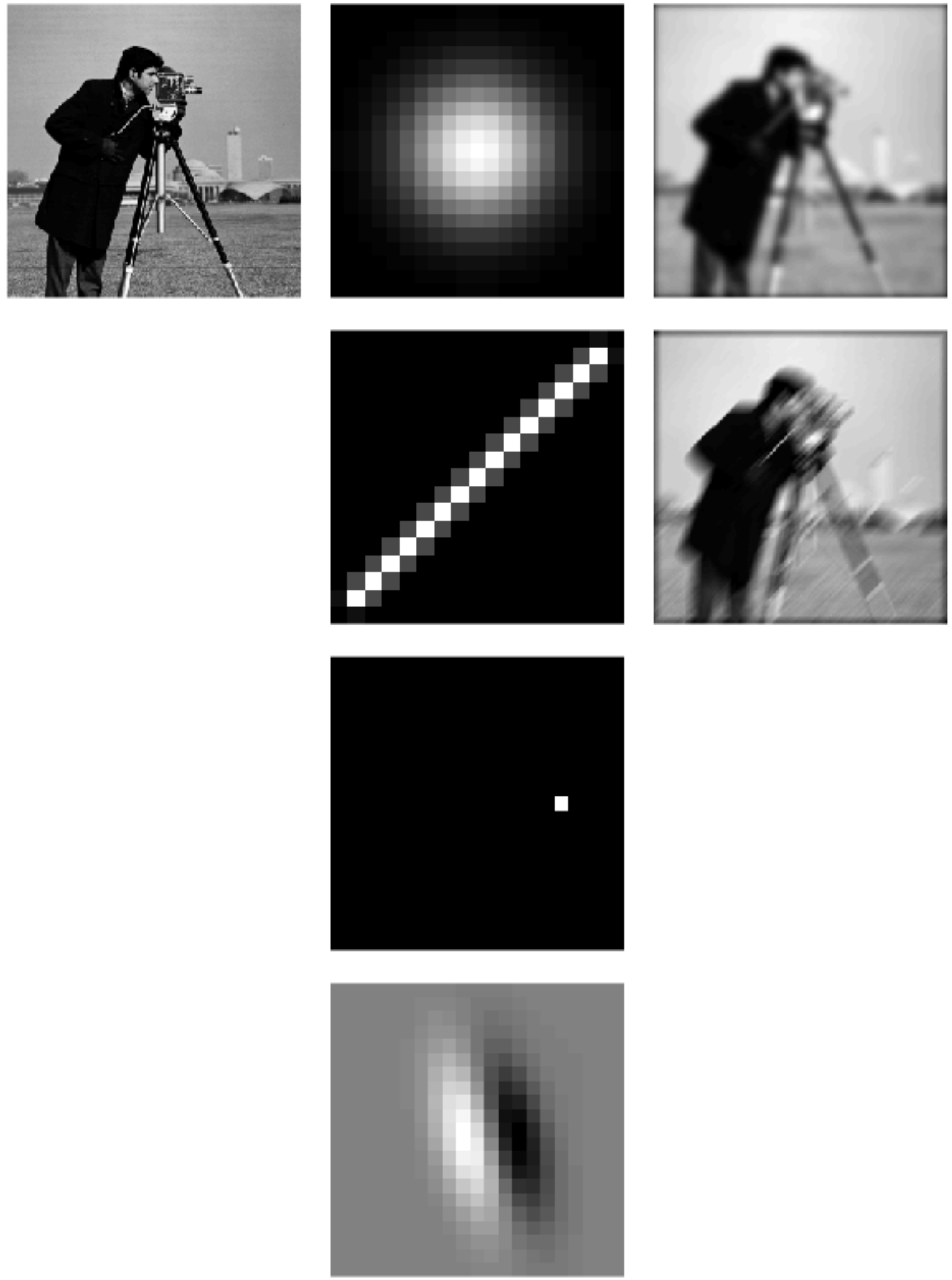




- Translation equivariance by design

Examples (Cross-Correlation)

Input | Kernel | Output

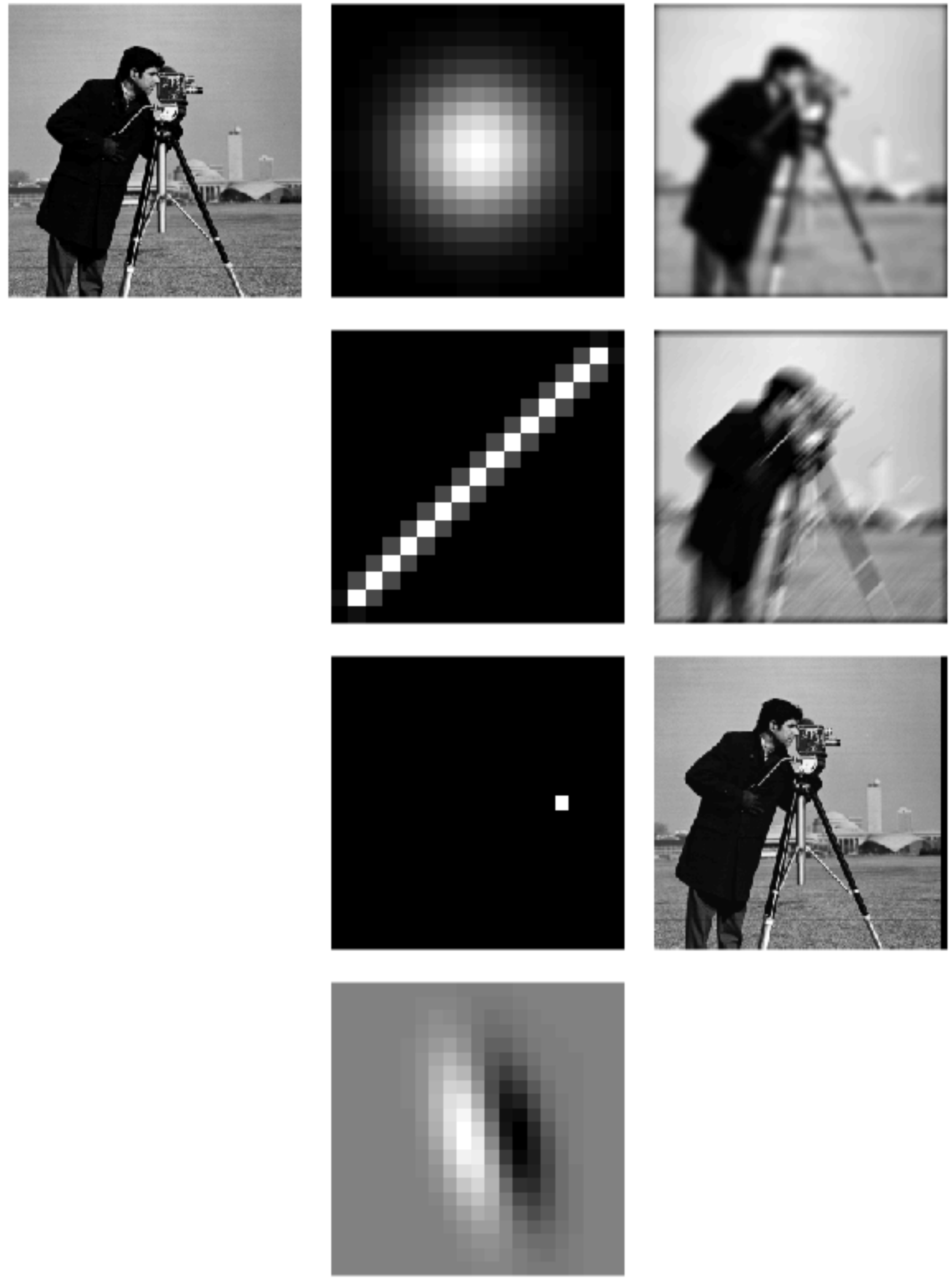


Blur

Motion Blur

Examples (Cross-Correlation)

Input Kernel Output

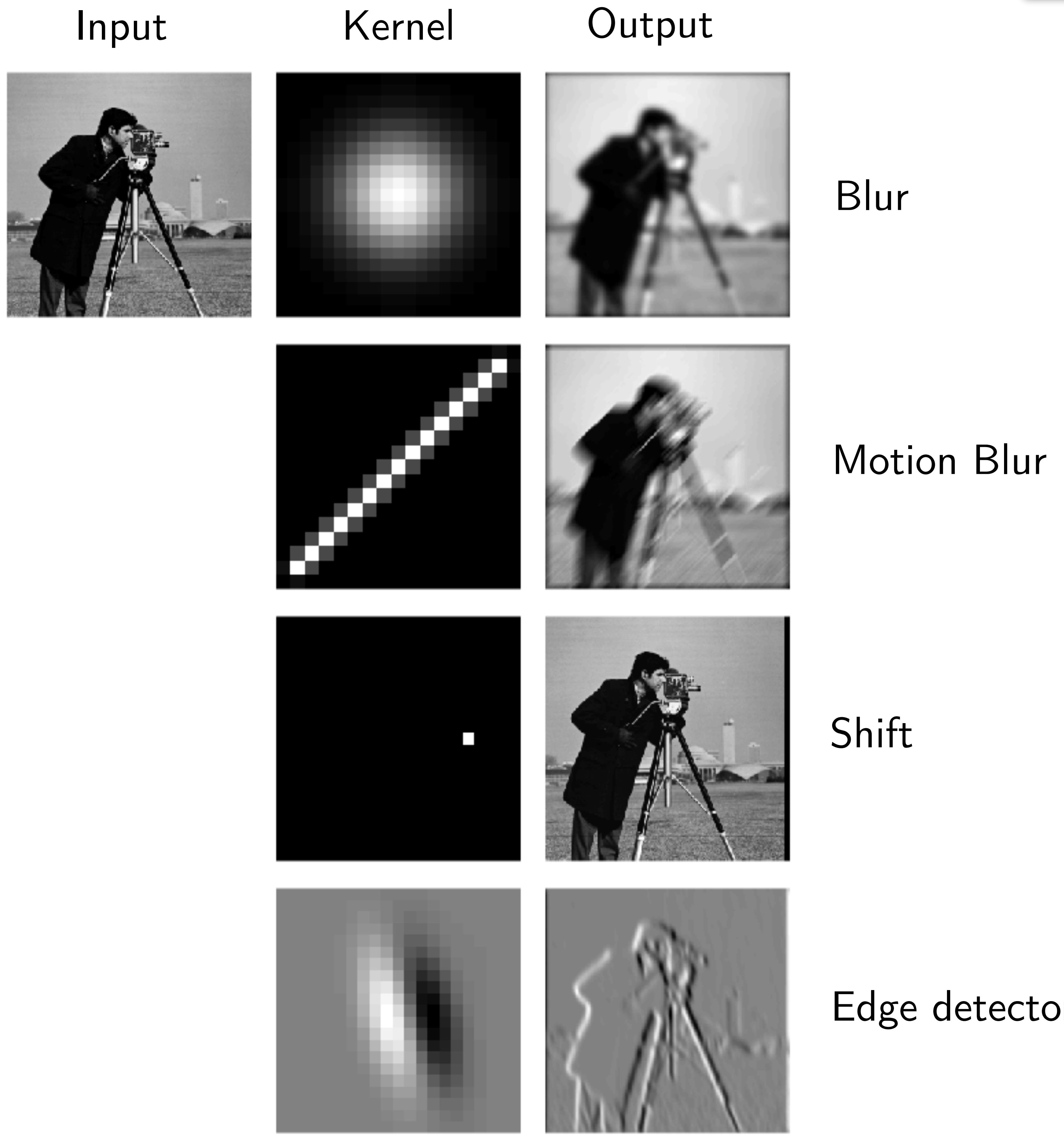


Blur

Motion Blur

Shift

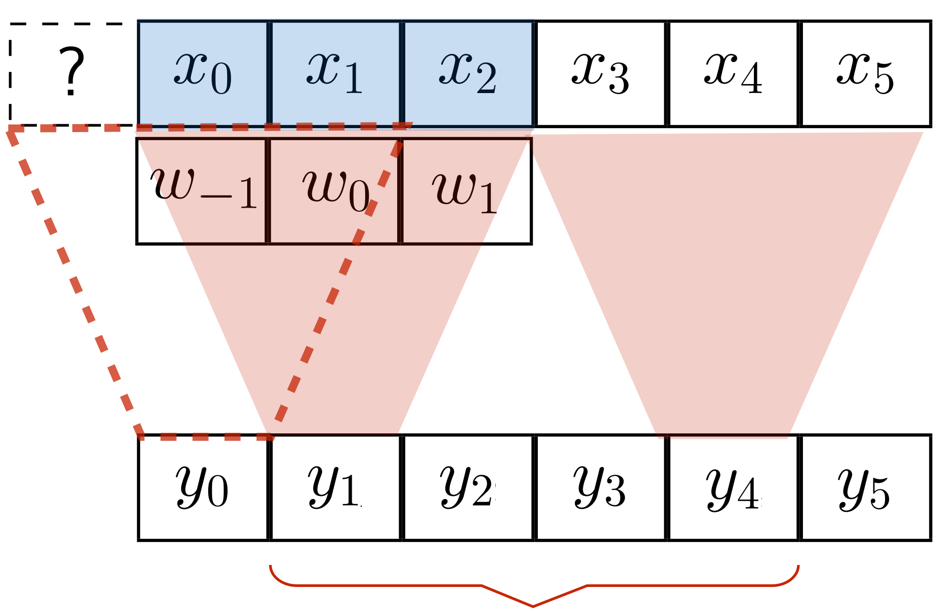
Examples (Cross-Correlation)



Properties

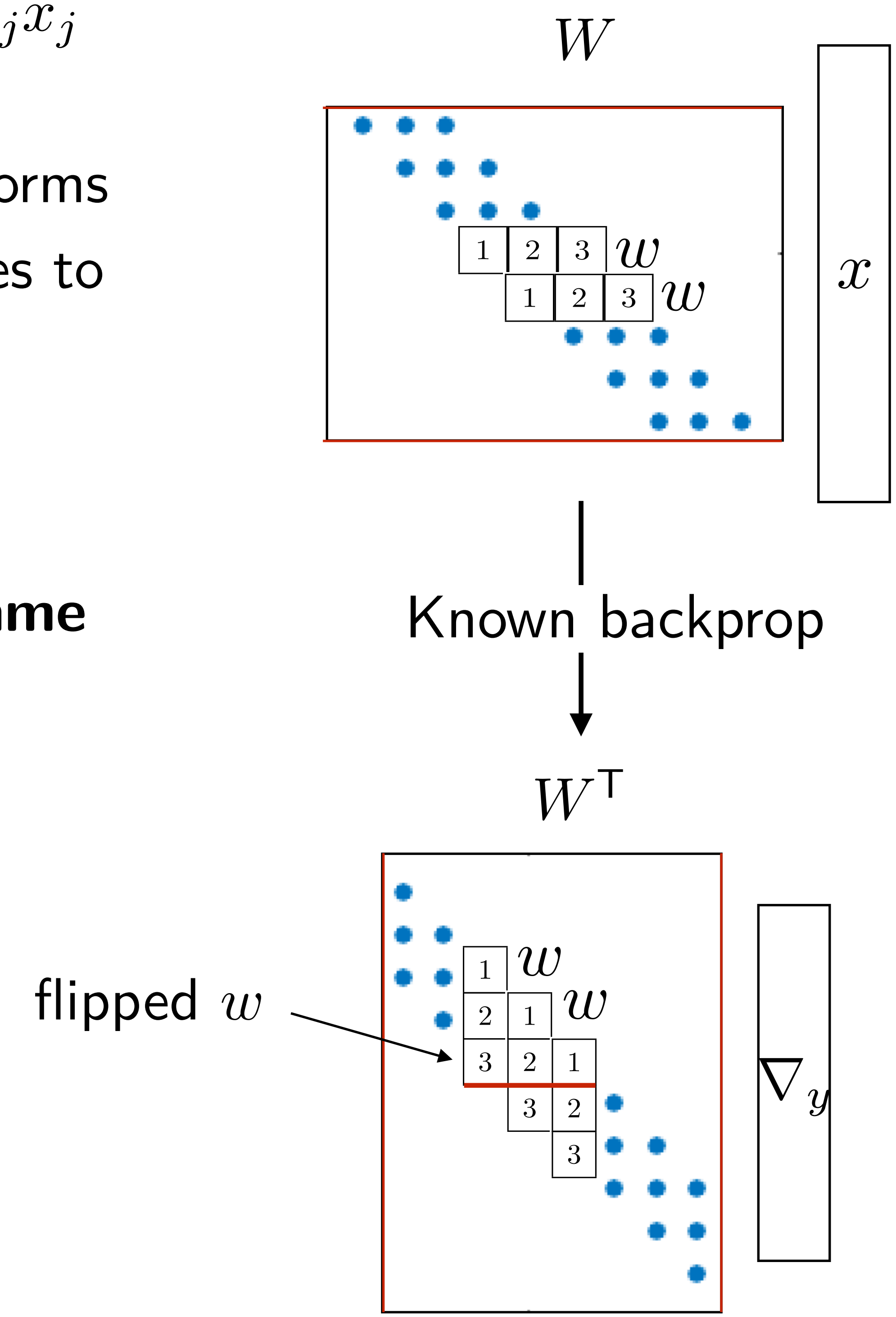
- Cross-Correlation:
  - $y_i = \sum_{k=-h}^{h} w_k x_{i+k}$
- As matrix-vector product: $y_i = \sum_j w_{j-i} x_j = \sum_j W_{ij} x_j$
  - Relation: $j = i + k \Rightarrow k = j - i$
  - **Compact representation** of certain linear transforms
  - Everything that applies to linear transforms applies to convolution and cross-correlation
- **Valid** range for i:

  $0 \leq j \leq n \Rightarrow 0 \leq i - h,\ i + h \leq n \Rightarrow h \leq i \leq n - h.$
  - Optionally may **pad** input with zeros to obtain **same** range as unpadded input



valid range

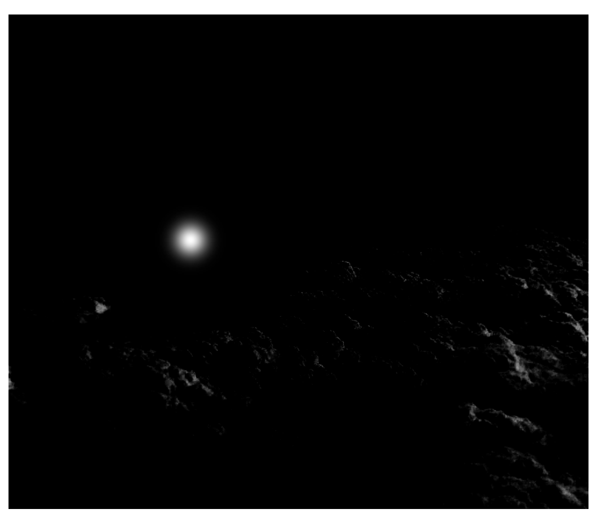
Correlation (valid)



= Convolution (with padding)

Properties

- As a binary operation $y = w * x$
  - Everything that applies to linear operators, eg. associativity: $u * (w * x) = (u * w) * x$
  - Commutativity for convolutions: $w * x = x * w$:
    $\sum_k w_k x_{i-k} = \sum_j x_j w_{i-j}$
  - No commutativity for cross-correlation. But $u \star w \star x = w \star u \star x$
- Examples:
  - edge_filter(blur(image)) = blur(edge_filter(image)) = (blur(edge_filter))(image)
  - filter(translation(image)) = translation(filter(image))
    equivariance w.r.t. translation



When the image shifts, the output shifts
Great prior knowledge for learning

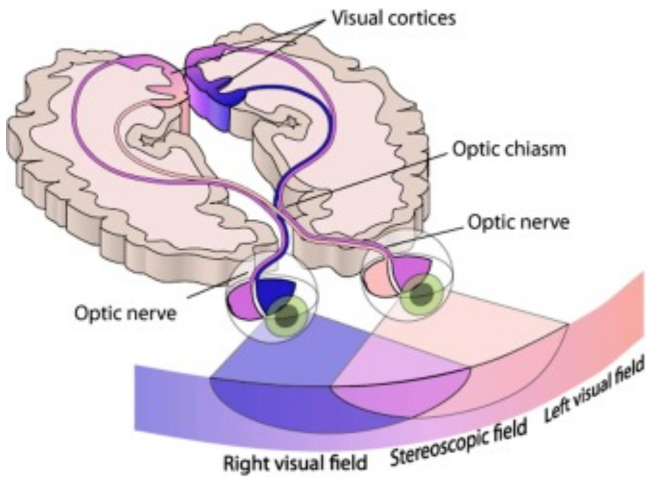
- In fact, linearity + translation equivariance = convolution

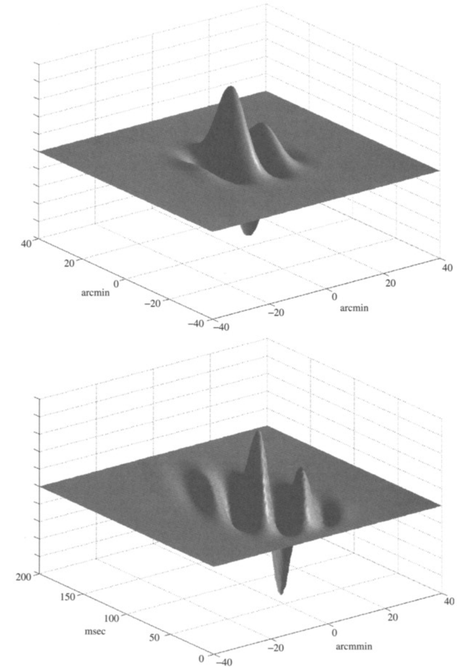
First Convolutional Layer

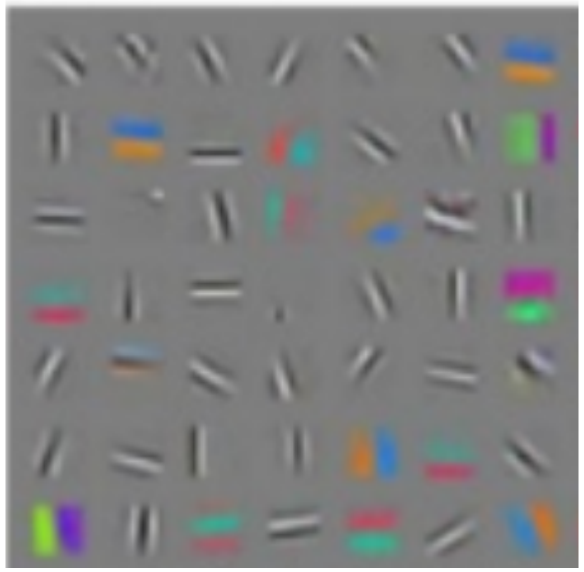
✦ Large filters are not very useful:

- Think of viewpoint changes, object deformations, variations within a category
- Small filters capture elementary, non task-specific, features

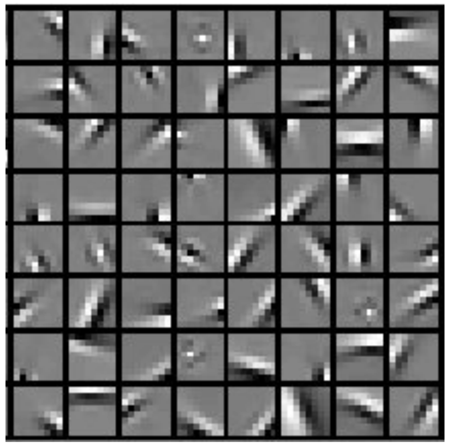
Gabor Filters - Mathematical model for V1 cells:



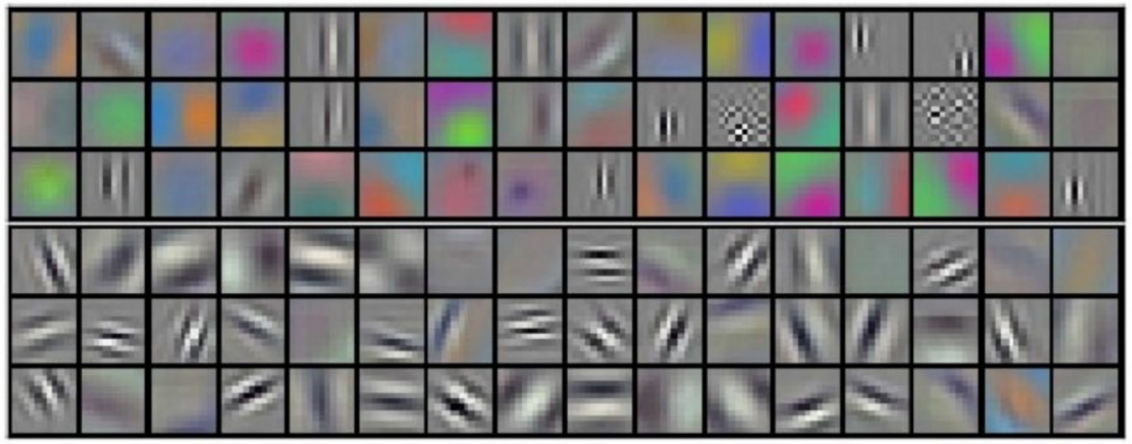




PCA of Image Patches

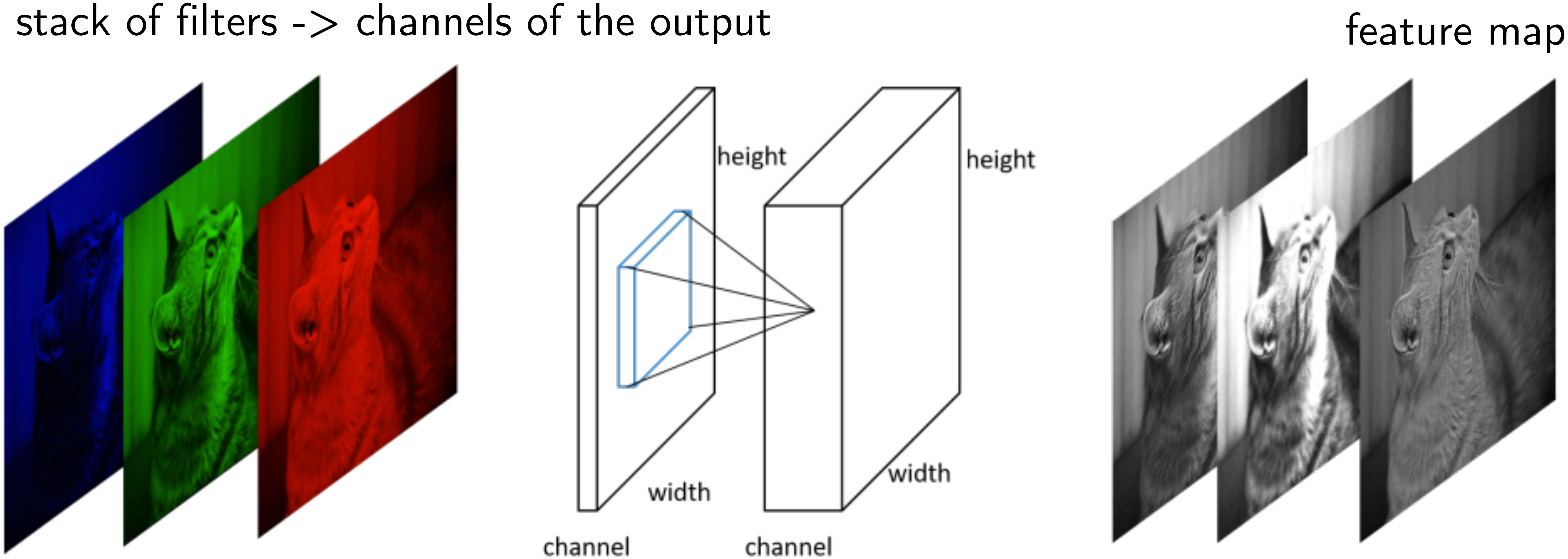


CNN first layer filters (learned)



Multi-Channel Convolution

- We just discussed:
  - color input images -> convolution kernel needs to have 3 channels
  - stack of filters -> channels of the output



- Multi-channel cross-correlation:

$$\sum_{c}\sum_{\Delta i}\sum_{\Delta j} x_{c,i+\Delta_i,j+\Delta j}\; w_{o,c,\Delta i,\Delta j} = y_{o,i,j}$$

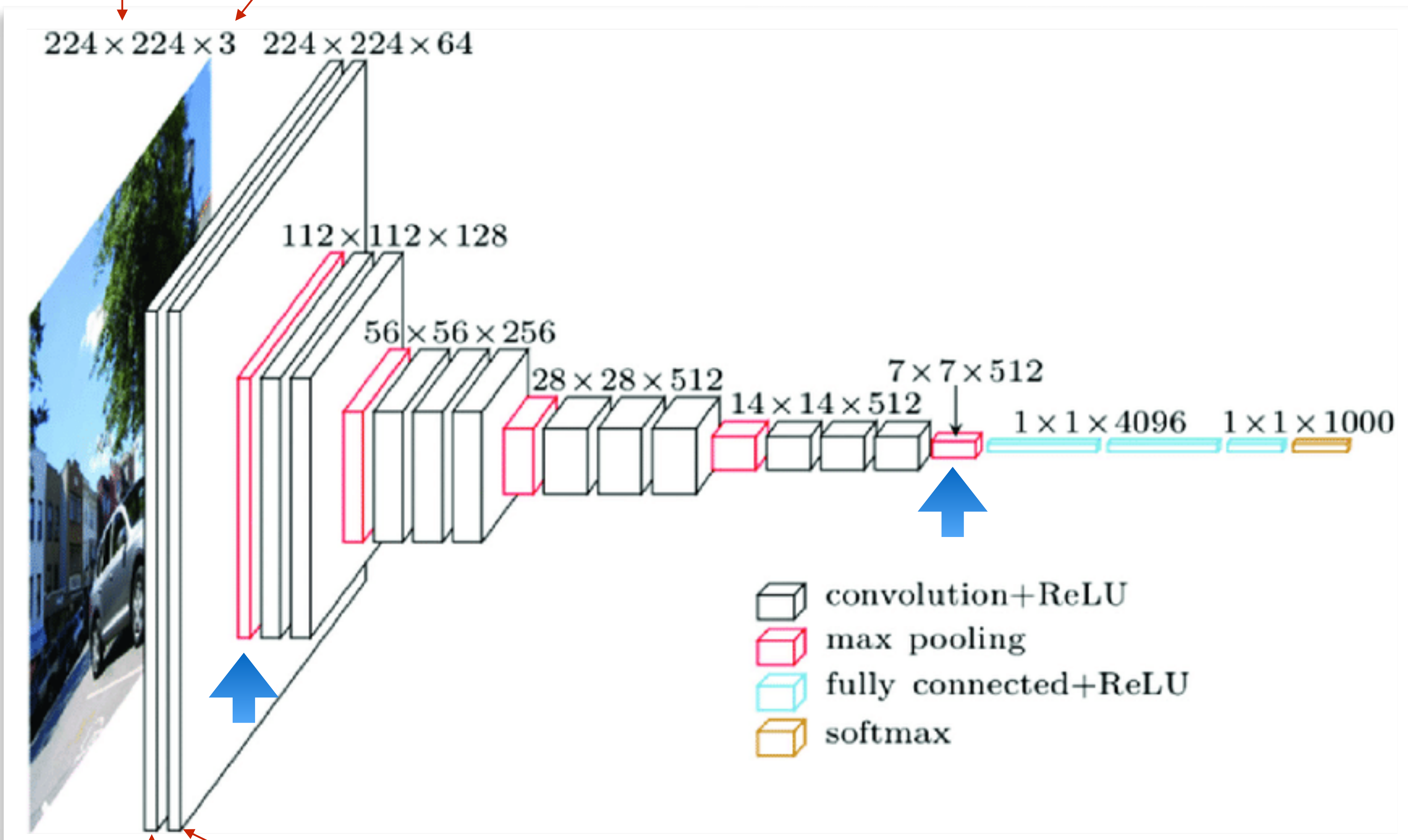
input channel — filter spatial dimensions — output channel

- input is 3D tensor, weight is 4D tensor, output is 3D tensor
- Essentially: a cross-correlation on spatial dims and fully connected on channel dims

Classification CNN

Spatial size of the input image

channels



$\mathrm{conv}(3\times3,\, 64\to64)$

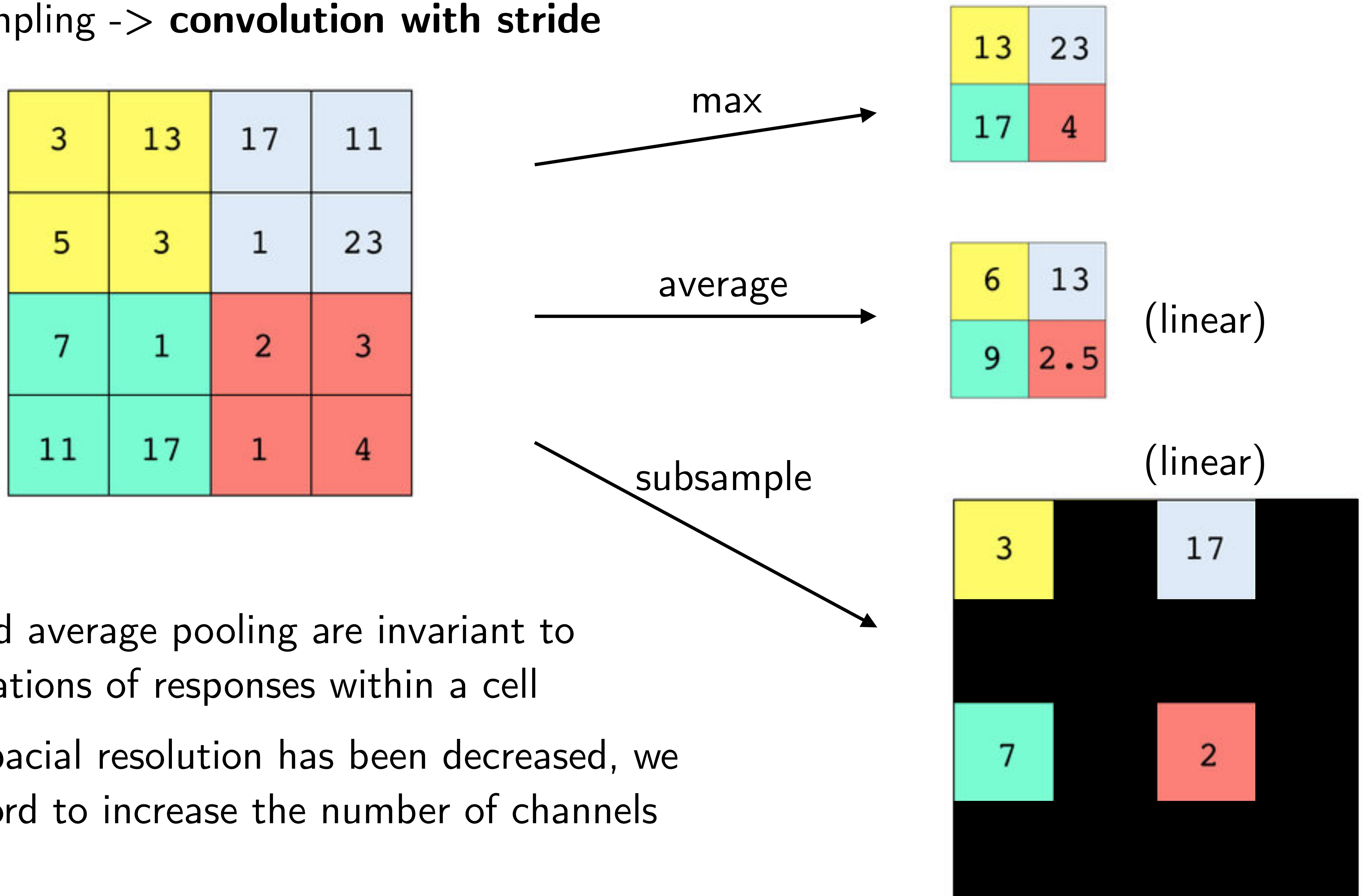
Result of $\mathrm{conv}(K\times K,\, 3\to64)$ followed by ReLU

- Eventually want to classify -> need to reduce spatial dimensions

Pooling

✦ Following approaches are used to reduce the spatial resolution:

- **max pooling**
- **average pooling**
- subsampling -> **convolution with stride**

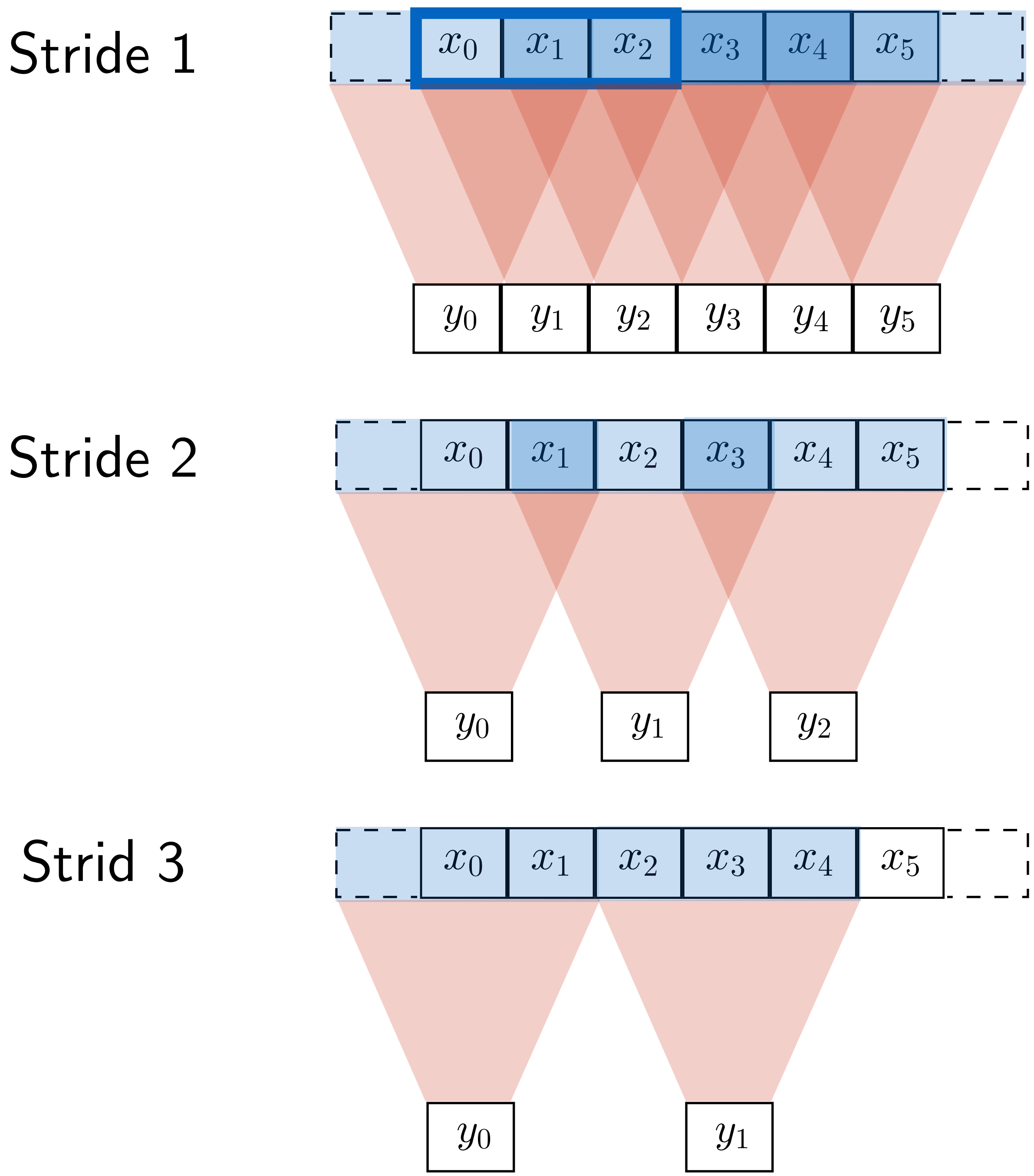


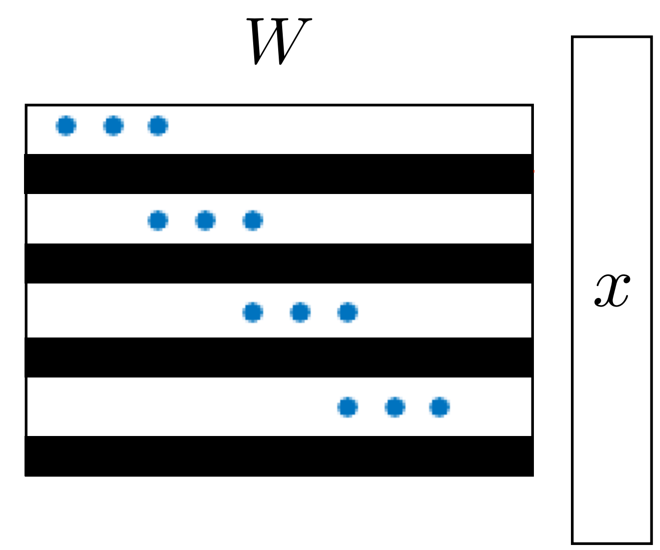
✦ max and average pooling are invariant to permutations of responses within a cell

✦ Once spacial resolution has been decreased, we can afford to increase the number of channels

Convolution with Stride

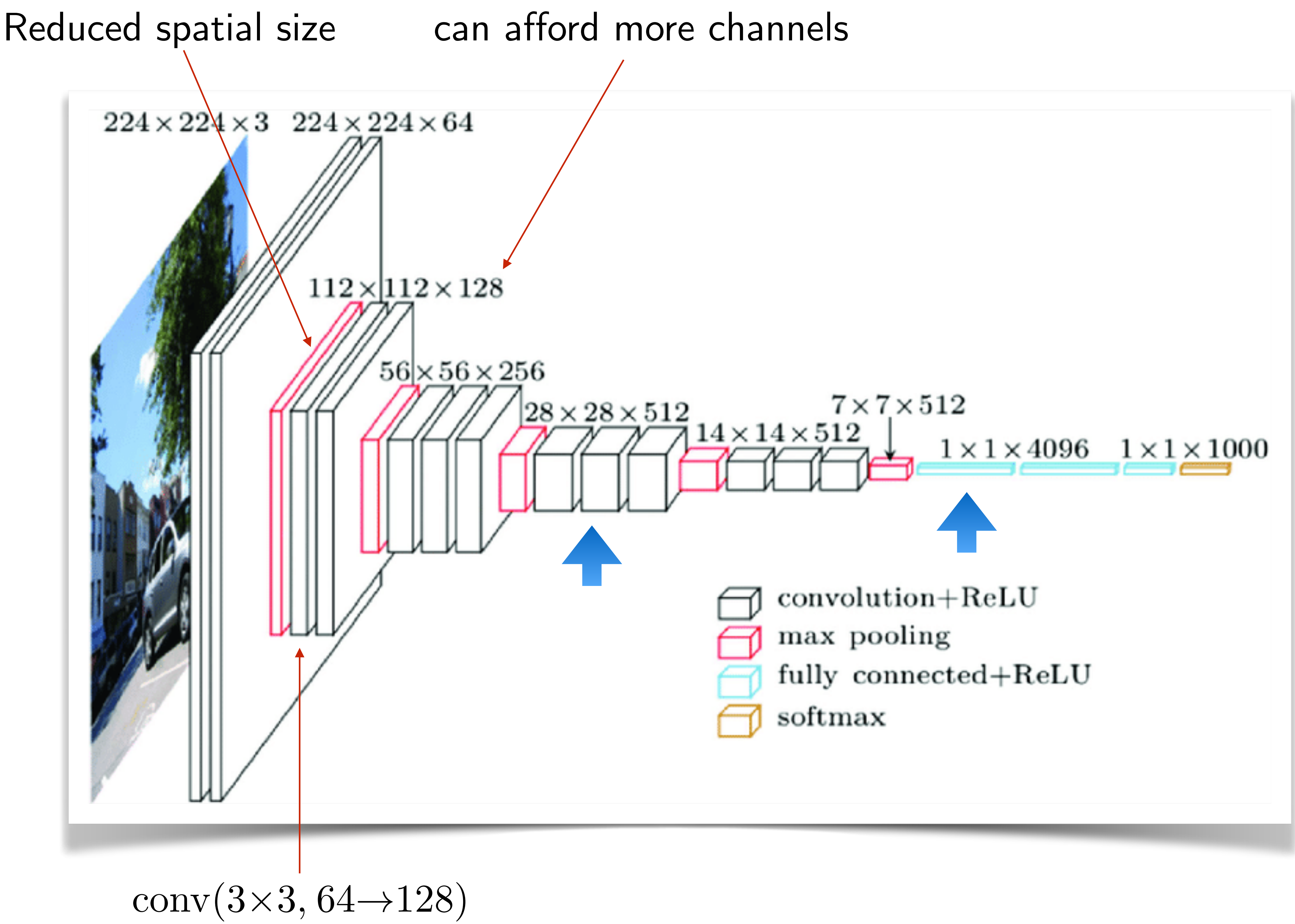
- Full convolution + subsampling is equivalent to calculating the result at the required locations only, stepping with a **stride**





All variants and detail: [Dumoulin, Visin (2018): A guide to convolution arithmetic for deep learning]

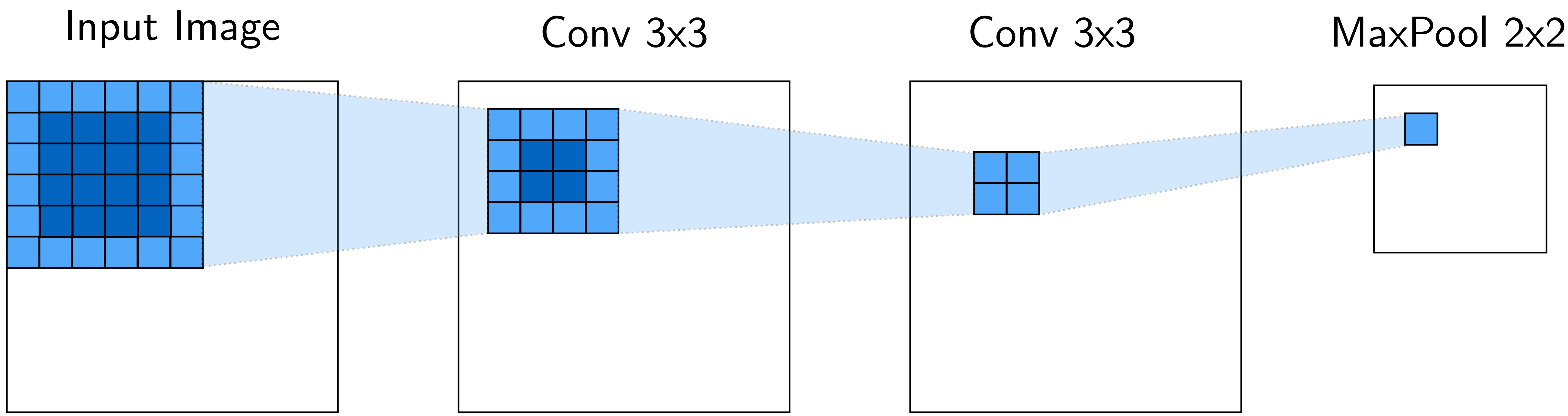
Classification CNN



- Combining convolutions and spatial pooling allows to aggregate information from a larger area

Receptive Field

✦ Receptive Filed = pixels in the input which **can** contribute to the specific output



✦ Small convolutions are not sufficient to create large enough receptive fields. Example:

- Want to classify images of size 256x256
- Each 3x3 convolution increases the receptive filed by 2 pixels
- Would take 128 convolutional layers

✦ Need pooling / strides / larger filters

✦ Effective receptive filed (with non-negligible expected contribution) is usually smaller

Weight Kernel Sizes

- With pooling we reduced the size of feature maps. What about filter kernels?
  - First layer: $(7 \times 7,\ 3 \to 64) \approx 10^3$ – can afford large filter size
  - Second layer: $(3 \times 3,\ 64 \to 64) \approx 3 \cdot 10^4$ – small filter size preferable
  - Layers with more channels: $(3 \times 3,\ 256 \to 256) \approx 5 \cdot 10^5$ – become expensive
- Need further efficient parametrization techniques
  - Depth-wise separable convolutions:
    spatial convolution same for all channels composed with a general linear transform on the channels (1x1 convolution)
  - Something in between:
    $\mathrm{conv}(K \times K, S \to S)$ composed with $\mathrm{conv}(1 \times 1, C \to S)$, $S < C$

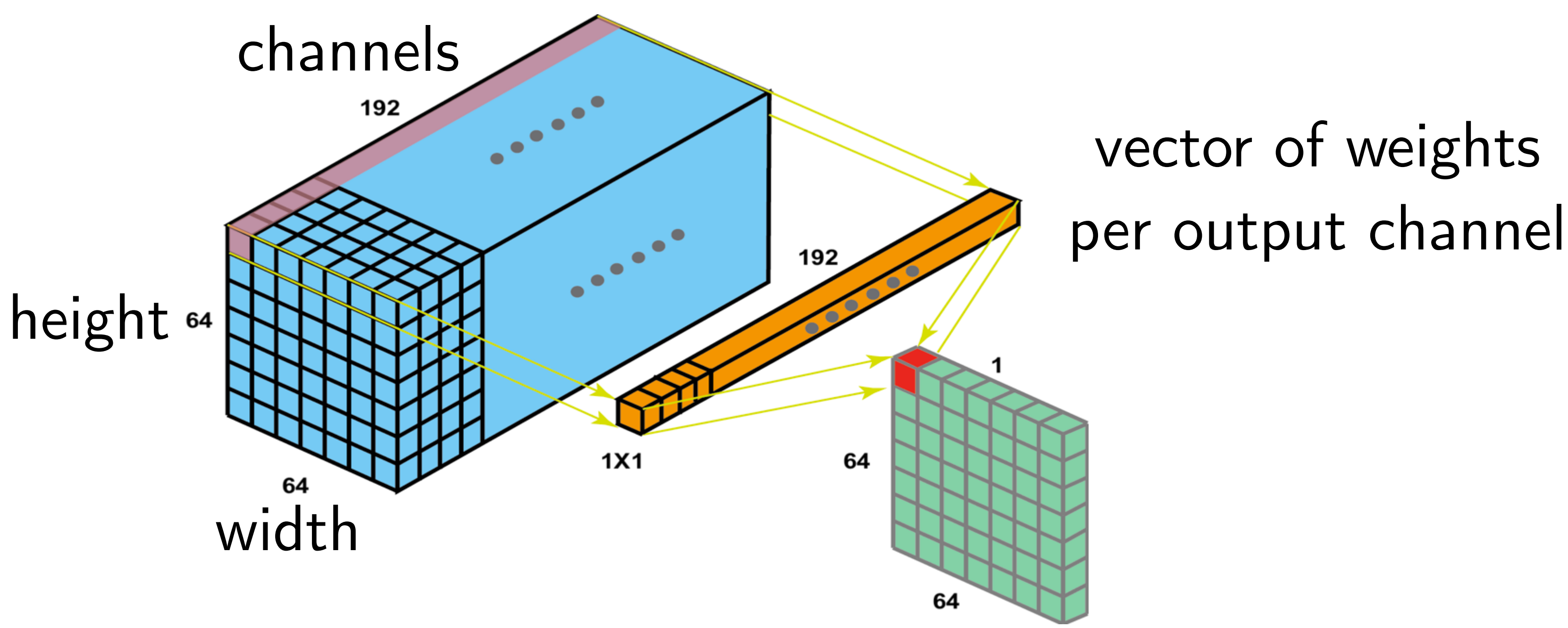
1x1 Convolution

- Kernel size 1×1:

$$y_{o,i,j} = \sum_c \sum_{\Delta i=0} \sum_{\Delta j=0} w_{o,c,\Delta i,\Delta j}\ x_{c,i+\Delta i,j+\Delta j}$$

$$= \sum_c w_{o,c,0,0}\ x_{c,i,j}$$

- For all i,j a linear transformation on channels with a matrix $w_{o,c,0,0}$



Example 3×3, $256\to256$, is too expensive, simplify:

Input: WxHx256

$\mathrm{conv}(1\times1, 256\to 64)$

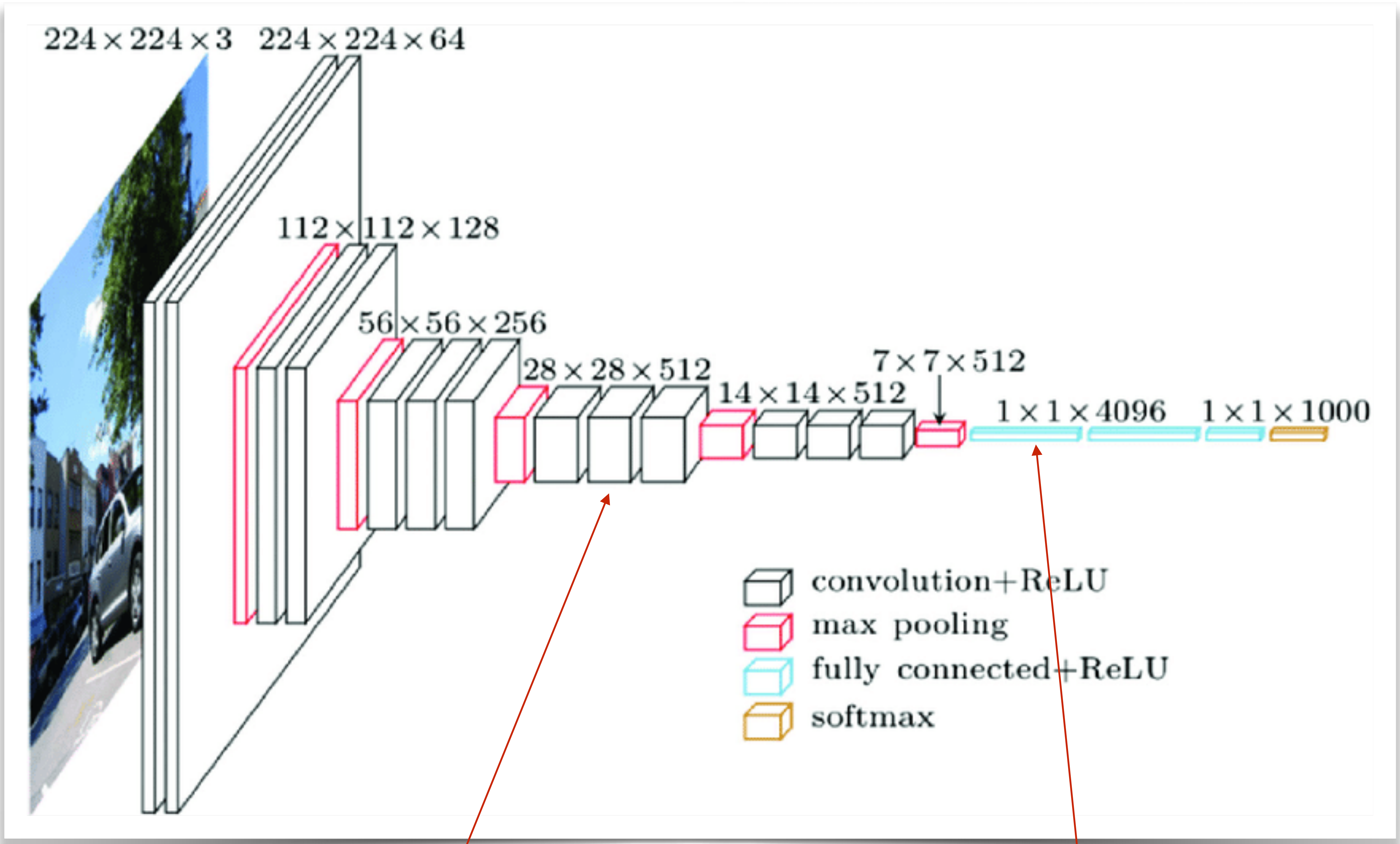
$\mathrm{conv}(3\times3, 64\to 64)$

$\mathrm{conv}(1\times1, 64\to 256)$

Output: W'xH'x256

- Useful to perform operations along channels dimension:
  - Increase /decrease number of channels
  - Normalization operations
  - In combination with purely spatial convolution = separable transform

Classification CNN



Could use efficient module here

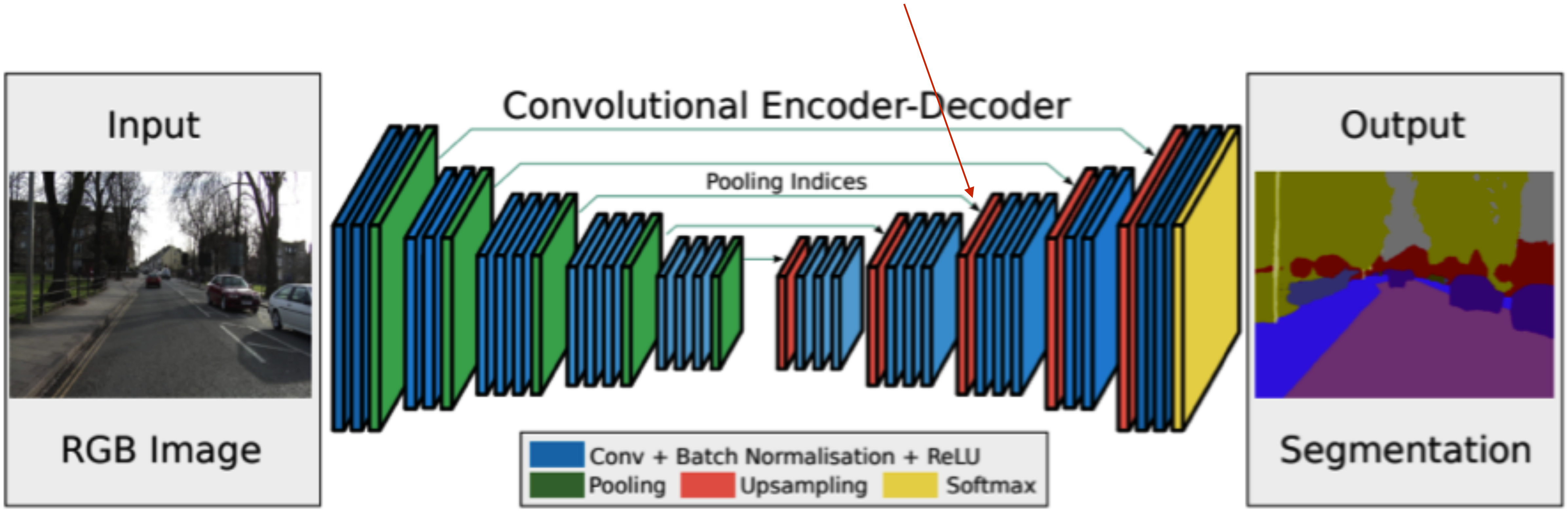
1x1 convolution for input of size 1x1 is equivalent to fully connected

- Second last layer has 4096*4096 =16M parameters!

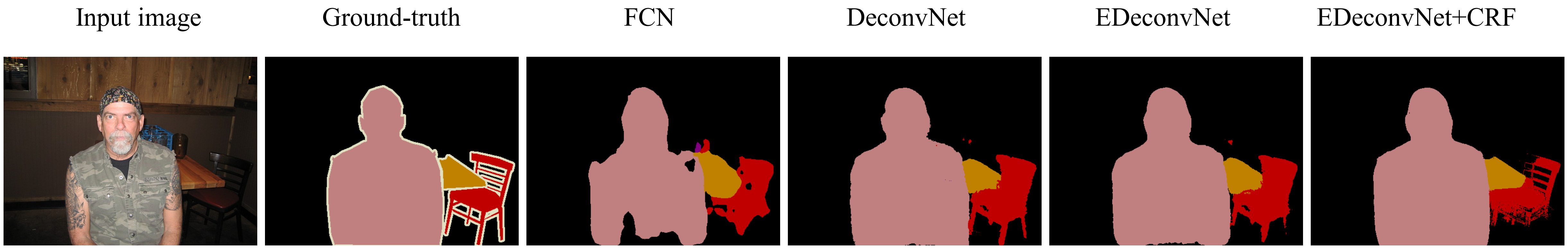
More Convolutions in DL

Deconvolution

Semantic Segmentation Architectures need unpooling / upsampling



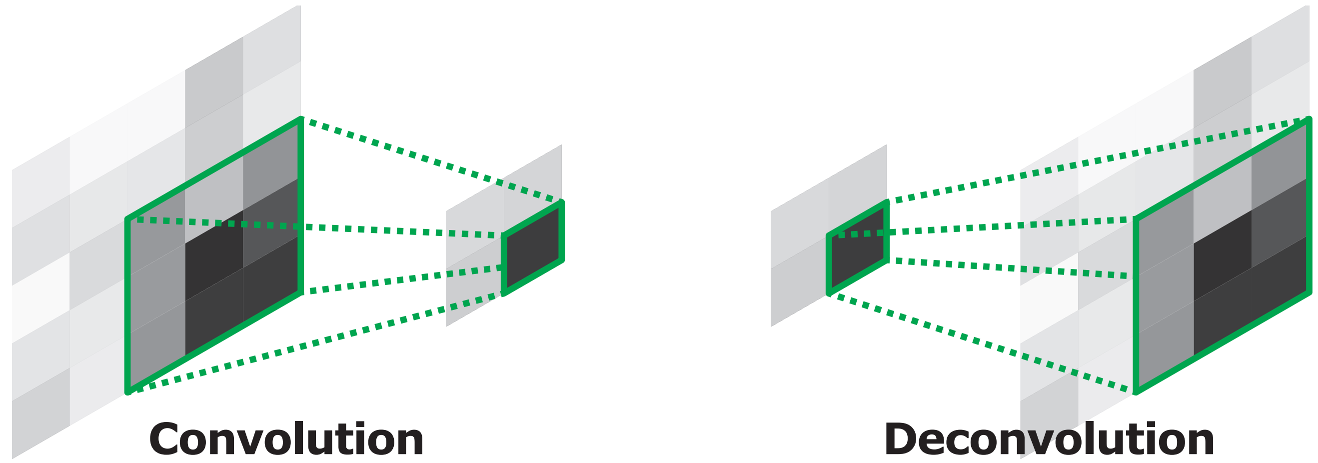
We will look at up-sampling with "transposed" convolution ("deconvolution")



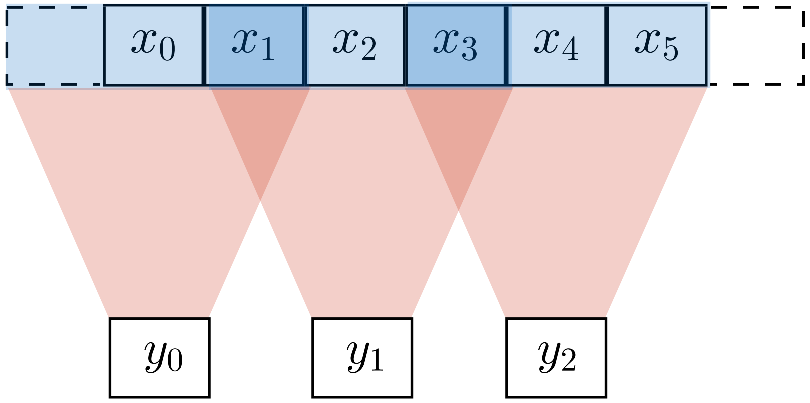
[Noh et al. (2015) Learning Deconvolution Network for Semantic Segmentation]

Transposed Convolution

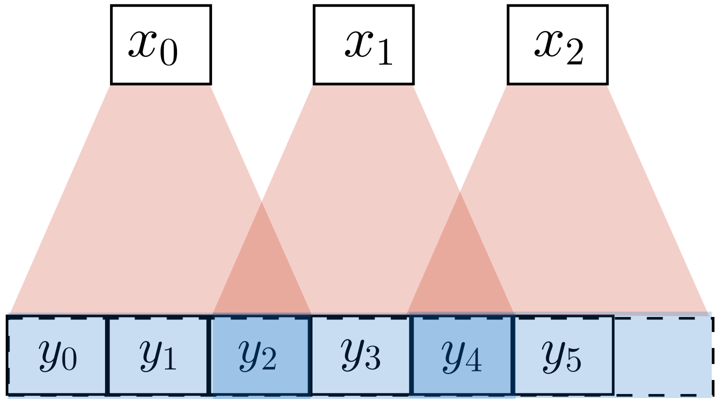
- Deconvolution = Transposed strided convolution = backprop of strided convolution



Stride 2 Convolution



Stride 2 Deconvolution

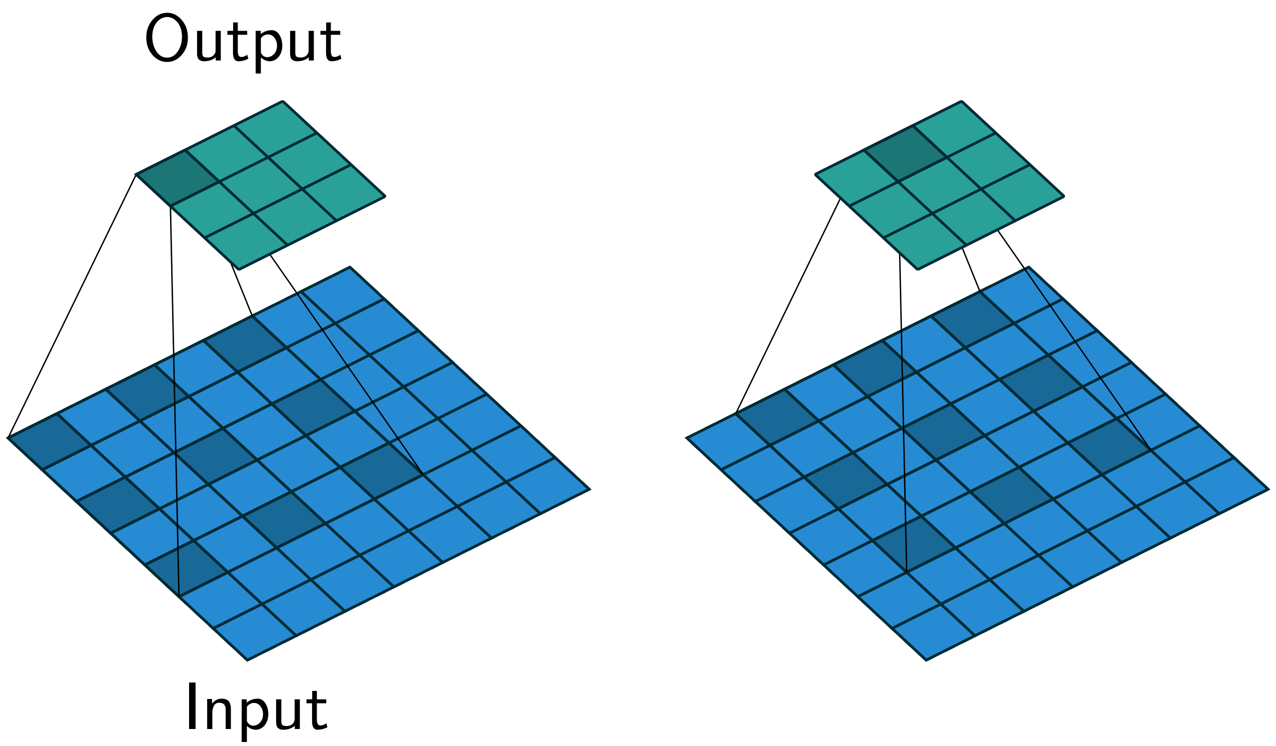


Sparse Convolutions

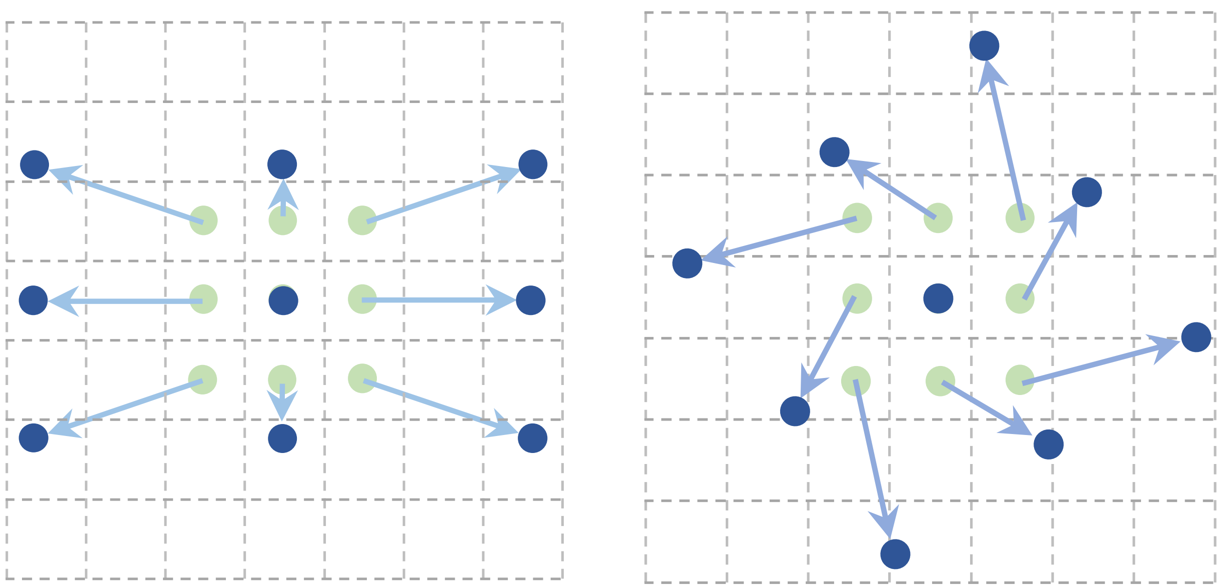
✦ Want to increase receptive field size

- without decreasing spatial resolution and having too many layers
- Can increase kernel size, but it was also costly
- Can use a sparse mask for the kernel

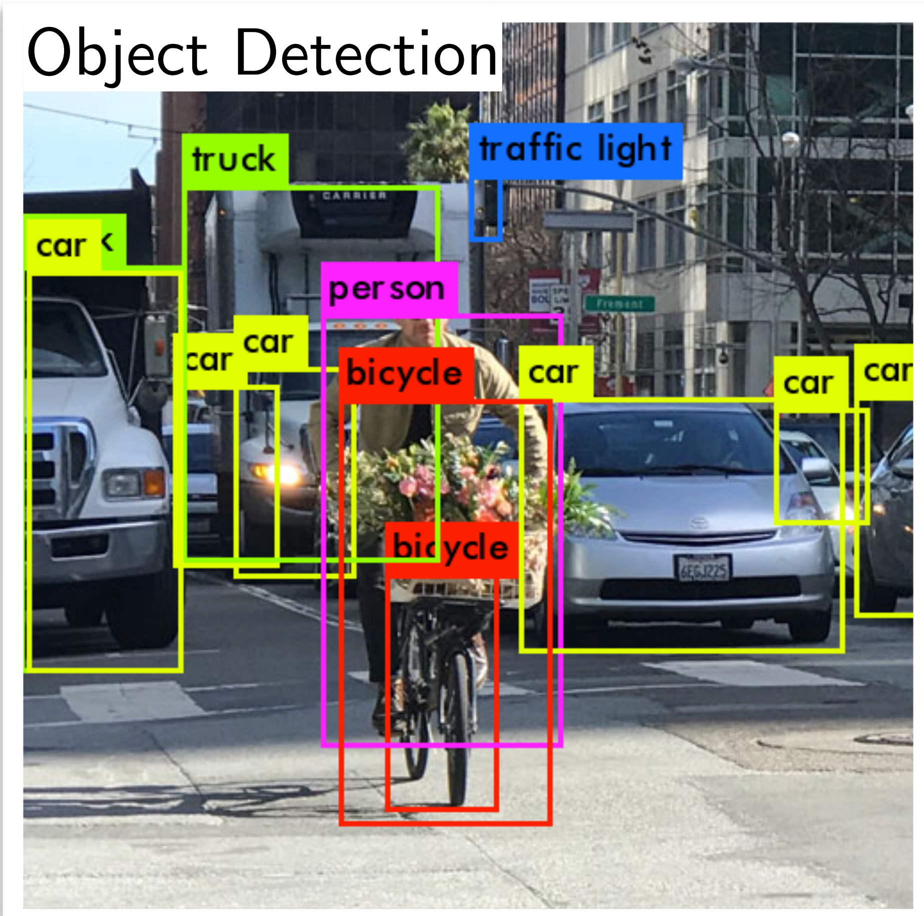
Dilated convolutions



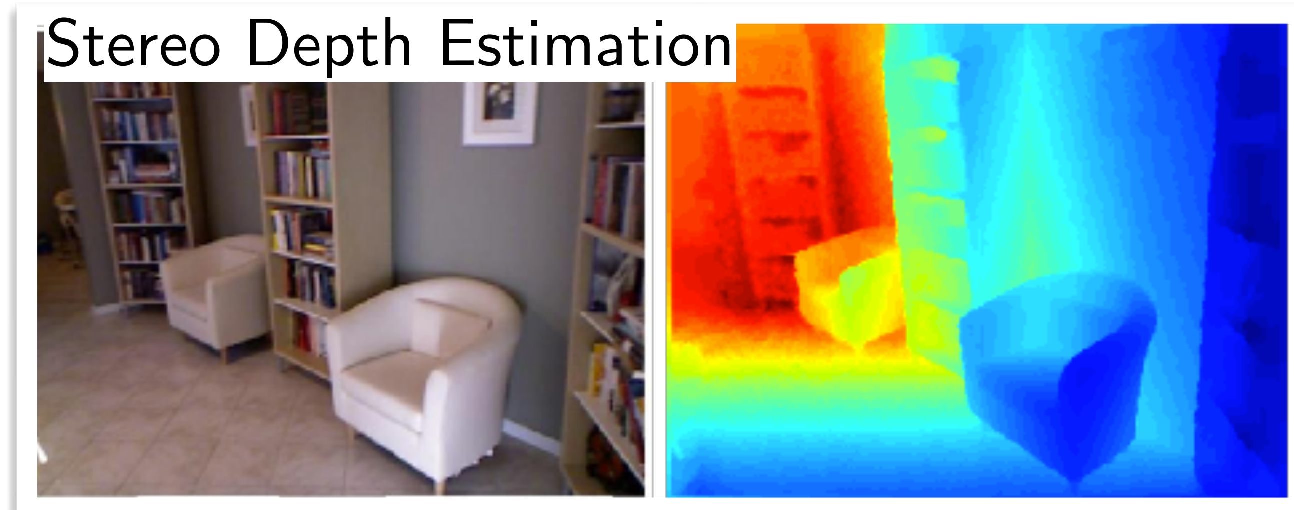
Can even learn sparse locations — **deformable** convolutions



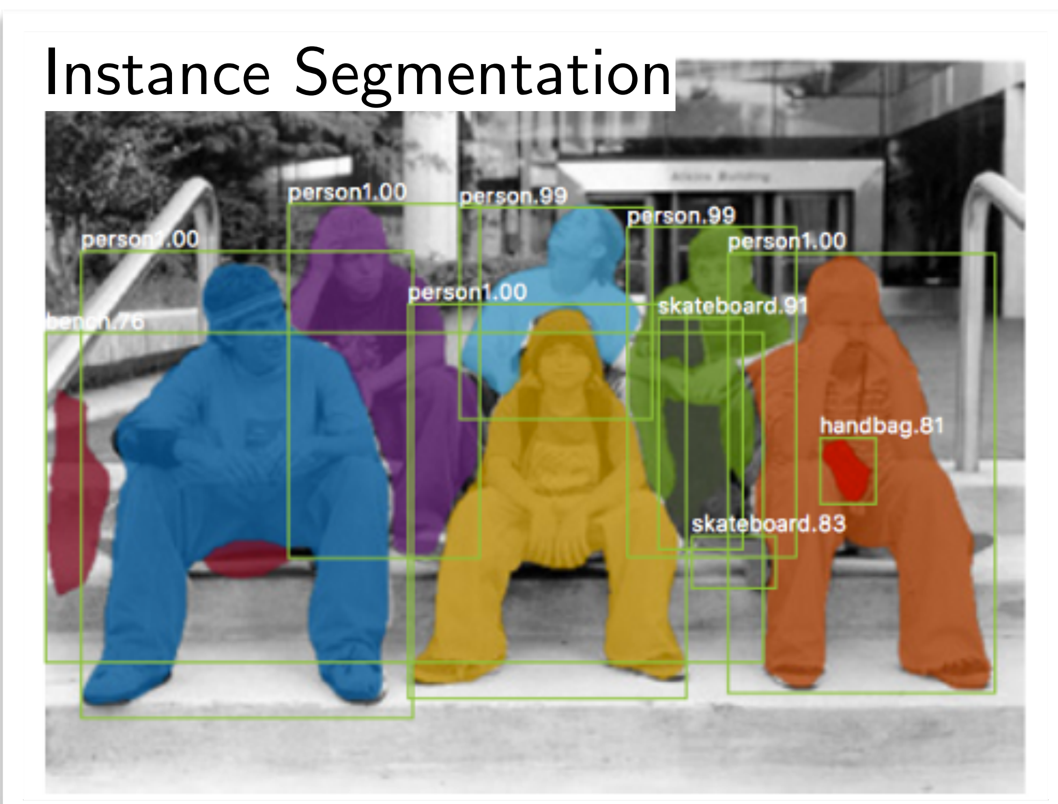
Many More Examples and Smart Architectures



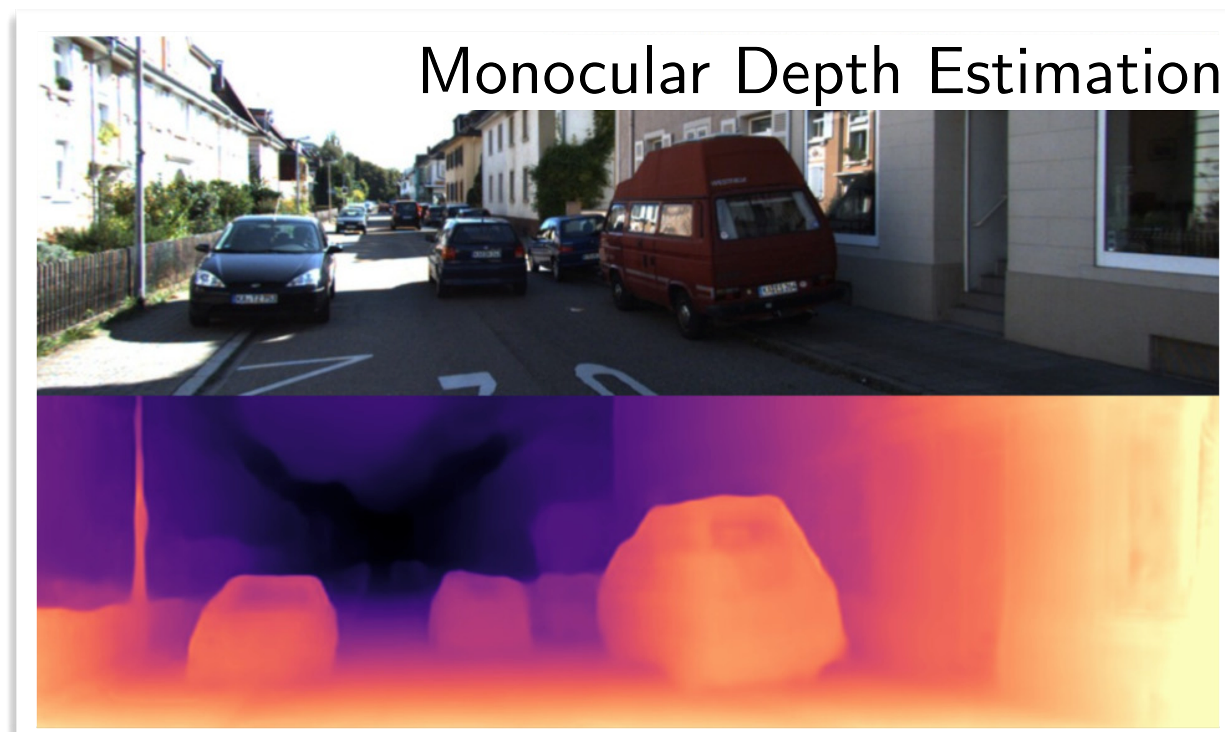
Object Detection



Stereo Depth Estimation



Instance Segmentation



Monocular Depth Estimation

In Other Courses

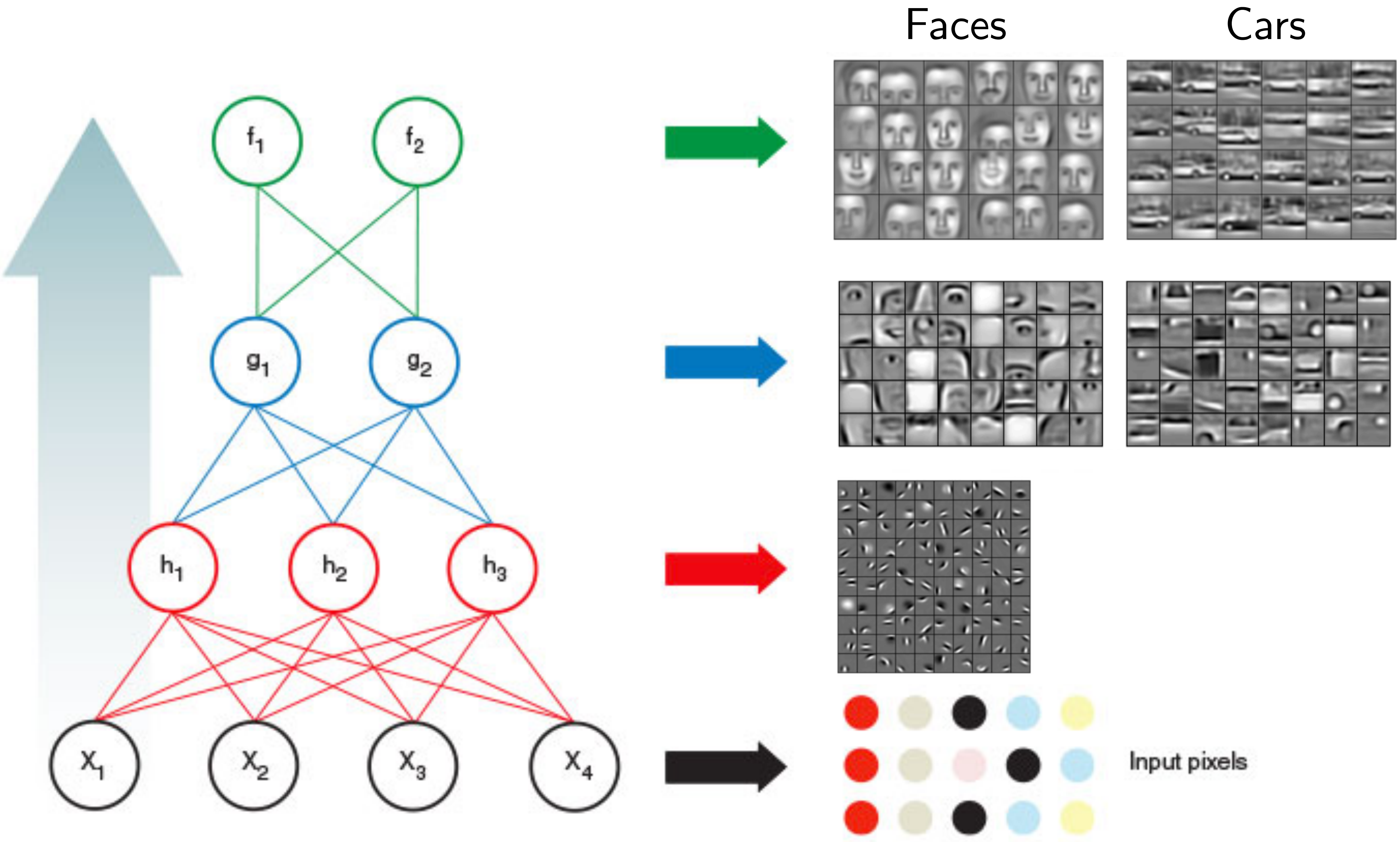
- Computer Vision Methods (BE4M33MPV, Spring)
  - Lectures 1, 2: overview of vision architectures, examples
  - Lecture 9: deep retrieval

- Vision for Robotics (B3B33VIR, Fall)
  - Lecture 6,8: (architectures)
  - Lecture 7: self-supervision, weak supervision
  - Lecture 9: Convolutions in 1D, 2D, 3D, graphs
  - Lecture 10, 11: Deep reinforcement learning
  - Lecture 12: Generative adversarial networks

Hierarchy of Parts, Self-organisation

Hierarch of Parts Phenomenon

- In networks trained for different complex problems
  - some intermediate layers activations correspond object parts



Hierarch of Parts Phenomenon

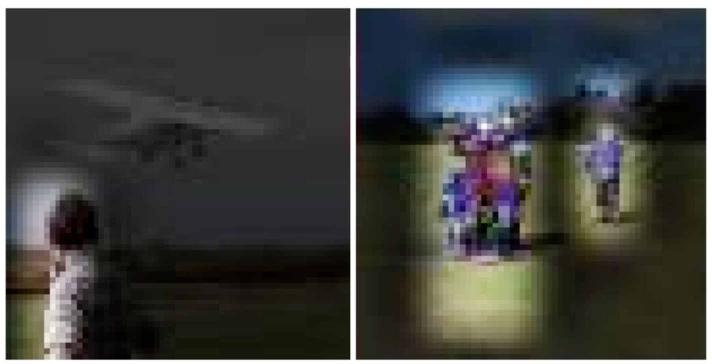
✦ In networks trained for different complex problems

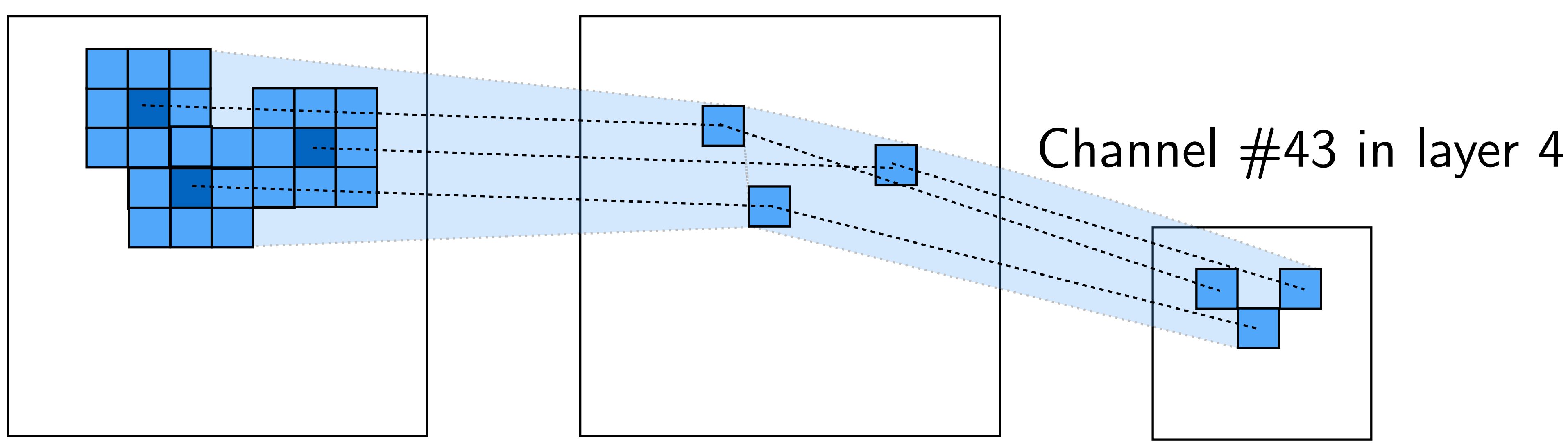
- some intermediate layers activations correspond object parts

lamps in places net

wheels in object net

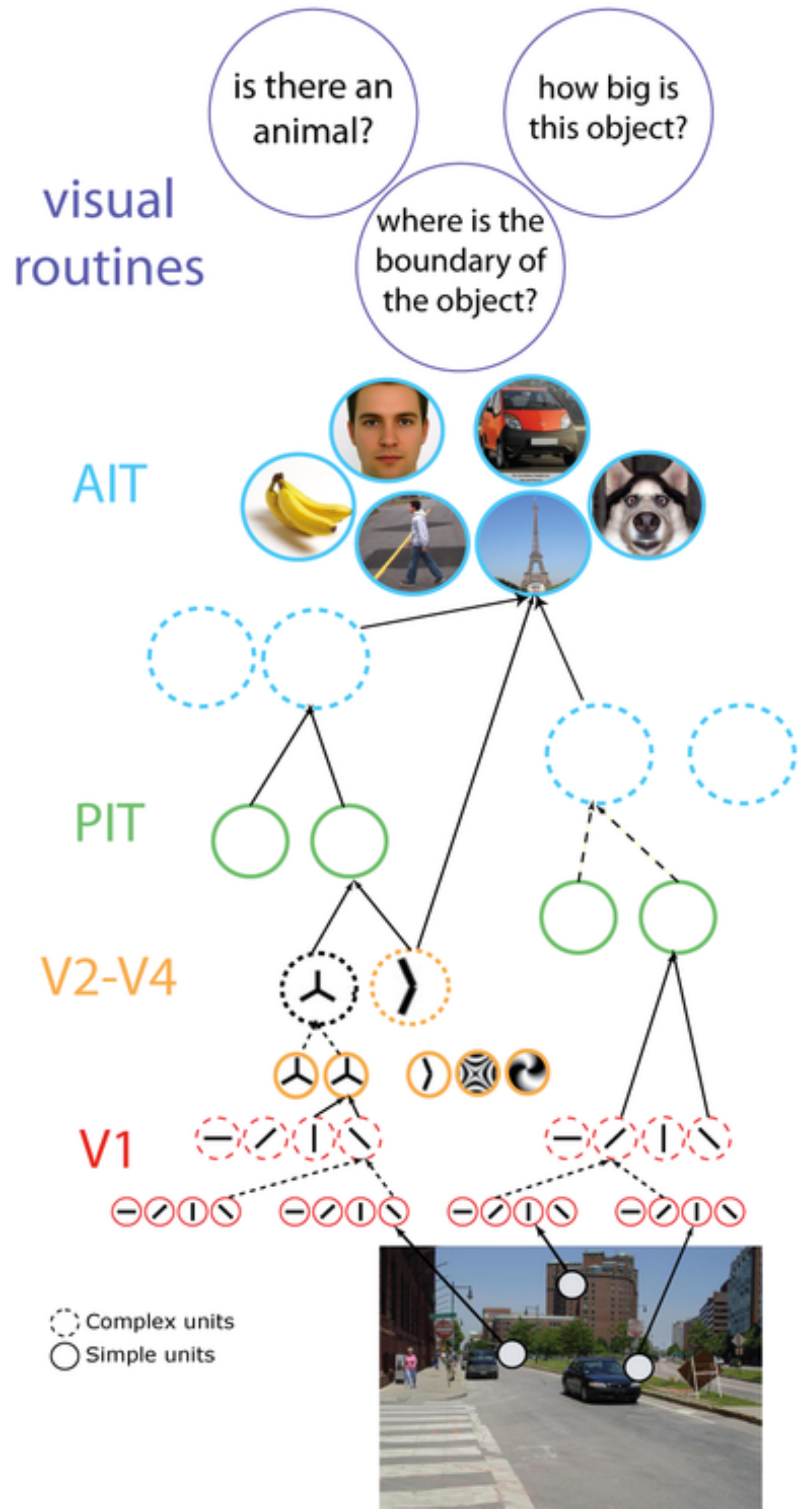
people in video net



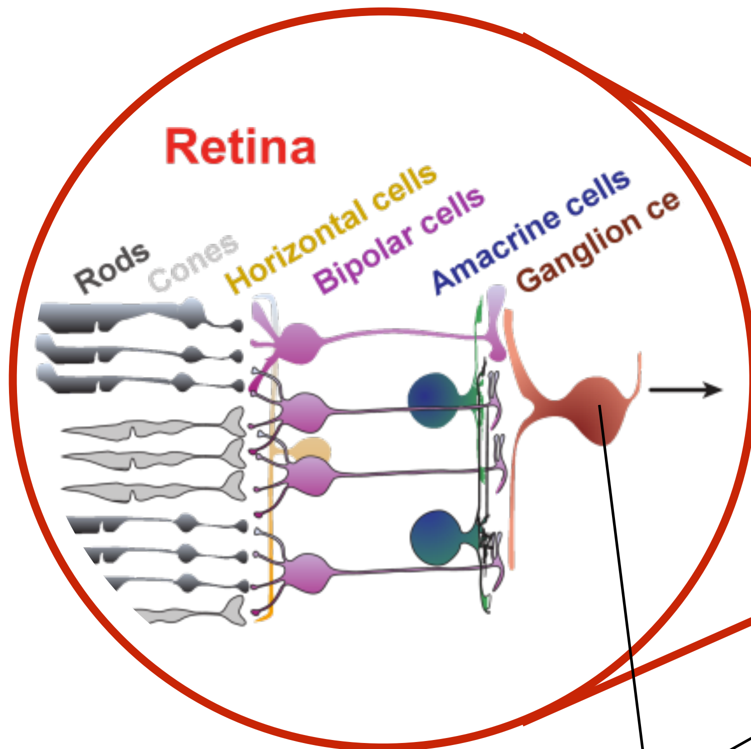


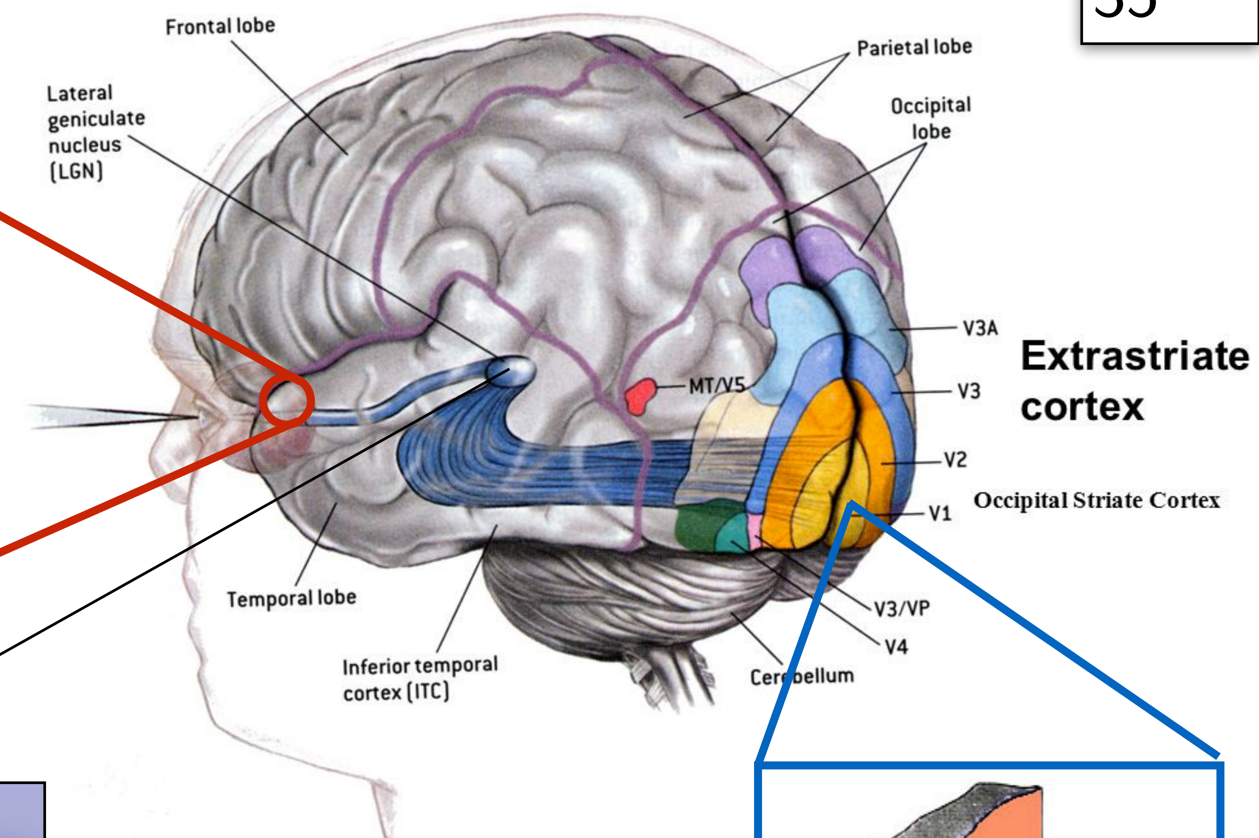
Computational Model of the Brain

- Complex tasks build upon the capabilities of simpler tasks

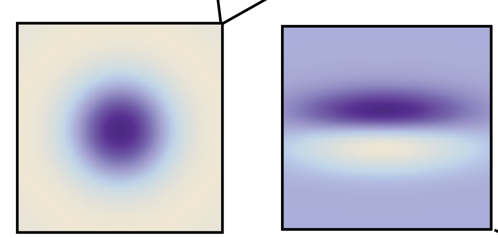


Parallels with Visual Cortex





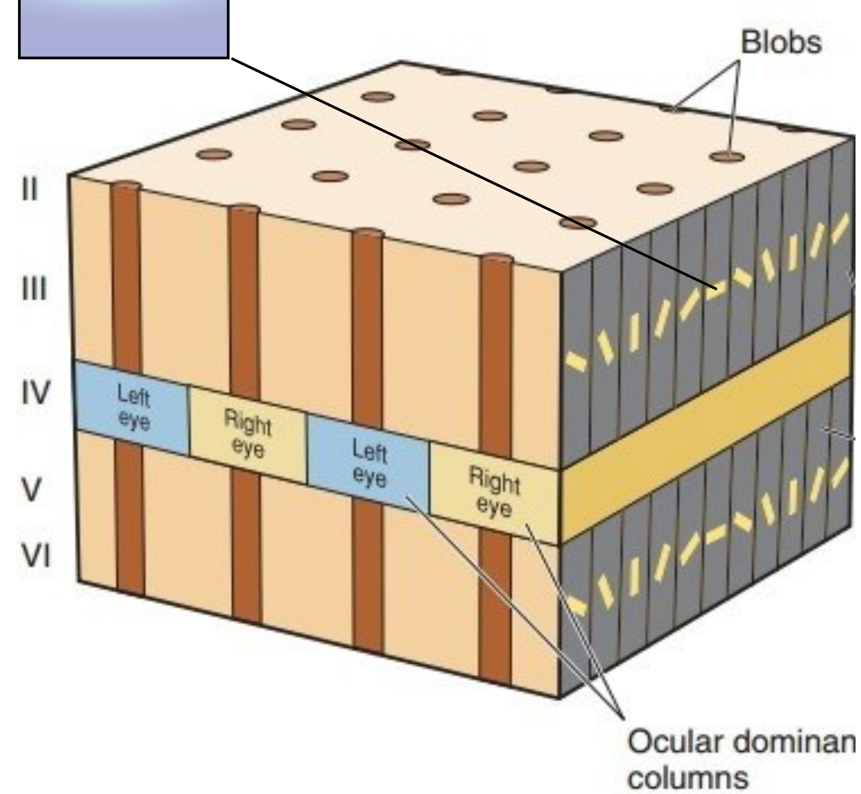
✦ LGN:
no orientation preference
space-time separable

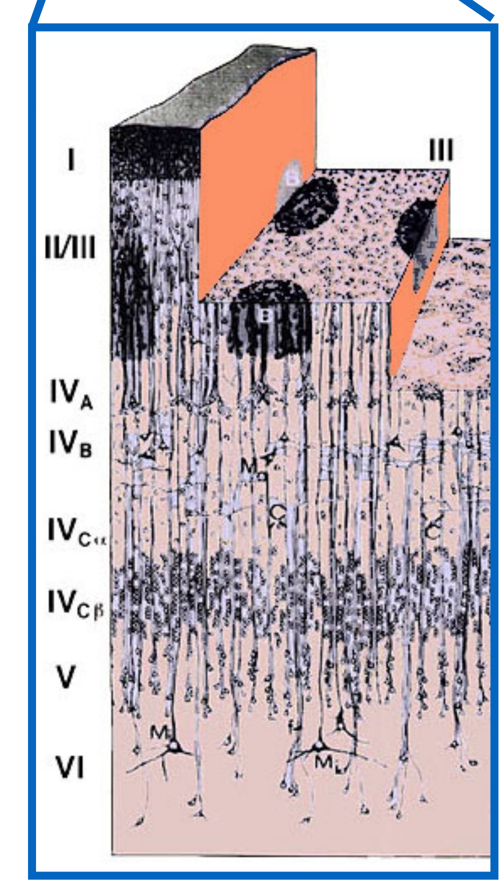


✦ V1 packing in 2D problem:

- location in the view (retinotopy)
- orientation
- ocular dominance
- motion

✦ feedback connections





50000 neurons / mm^3