Problem solving by search

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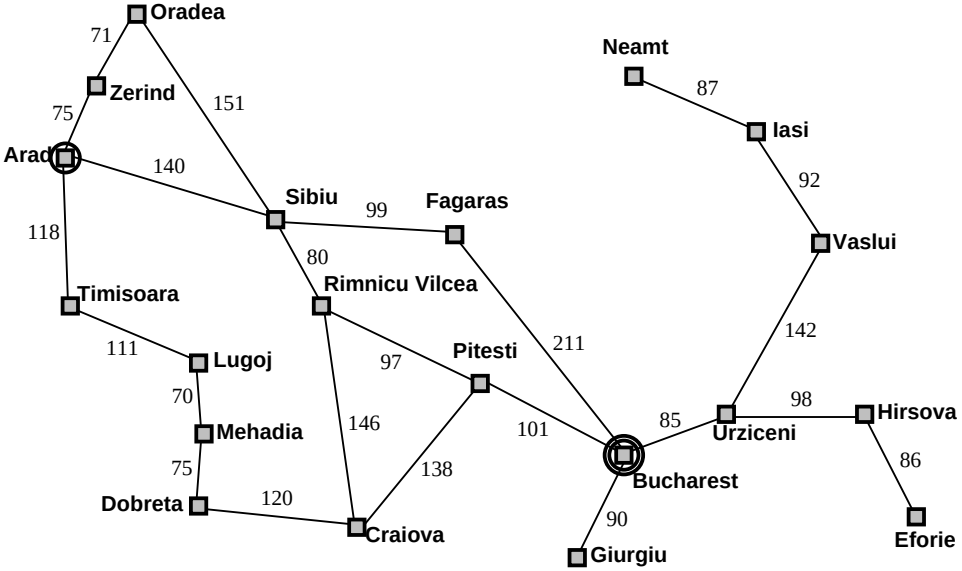
Outline

- Search problem. *What do you want to solve?*
- State space graphs. *How do you formalize/represent the problem? Problem abstraction.*
- Search trees. *Visualization of the algorithm run.*
- Strategies: which tree branches to choose?
- Strategy/Algorithm properties. *Memory, time, ...*
- Programming infrastructure.

Example: Traveling in Romania



Traveling Example: State and Actions

Goal:

be in Bucharest

Problem formulation:

states: position in a city (cities)

actions: at a crossing, select a road

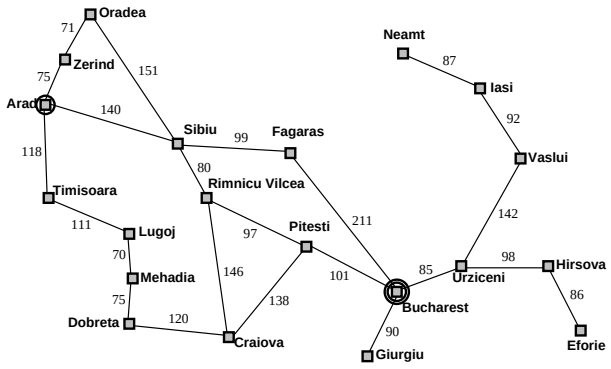
Solution:

Sequence of cities (path)

(action sequence [2])

Cost:

Energy, time, tolls, ...



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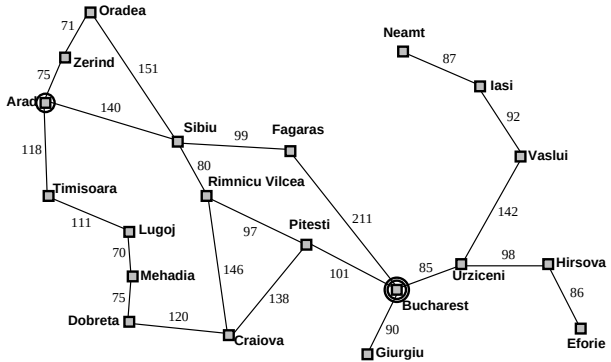

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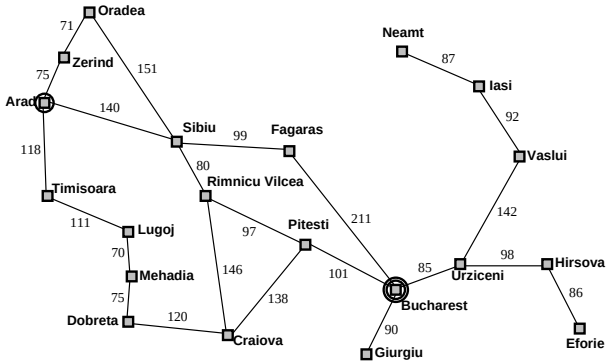

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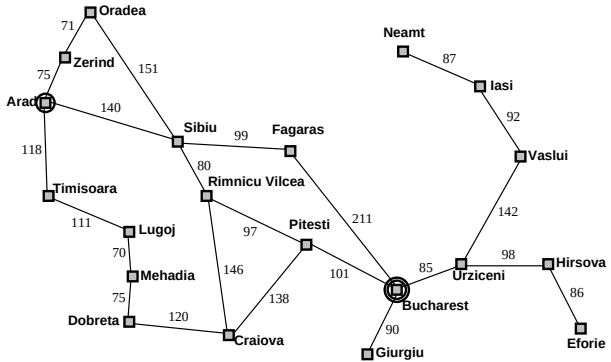

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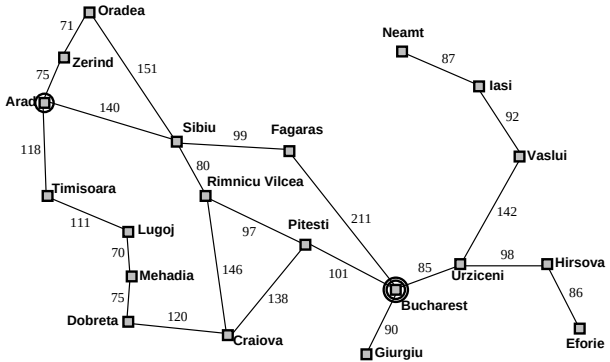

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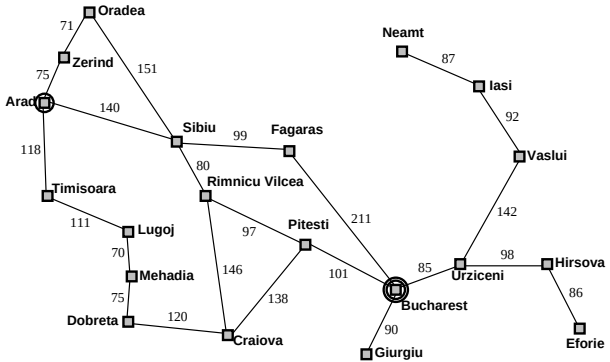
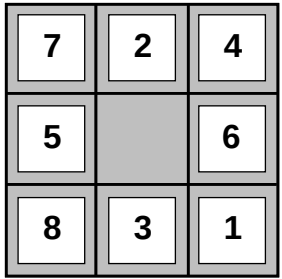
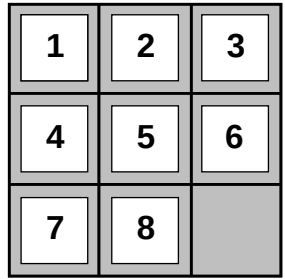


Example: The 8-puzzle



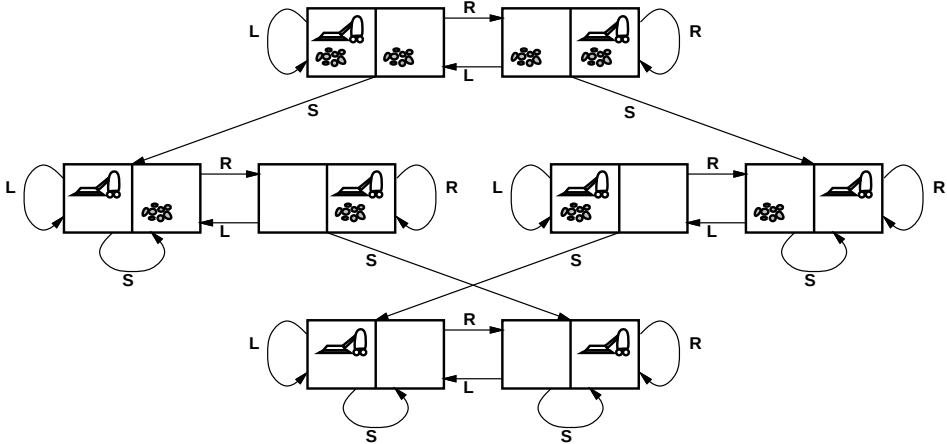
Start State



Goal State

states?
actions?
solution?
cost?

Example: Vacuum cleaner



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actions?
solution?
cost?

A Search Problem

- State space (including Start/Initial state): position, board configuration,
- Actions : drive to, Up, Down, Left ...
- Transition model : Given state and action return state (and cost)
- Goal test : Are we done?

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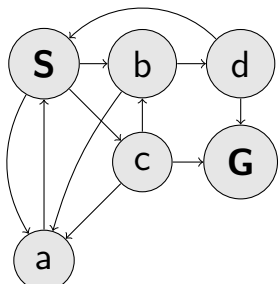
State Space Graphs

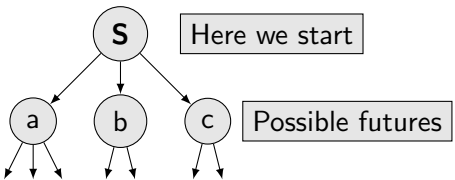
State space graph: a representation of a search problem

- Graph Nodes – states – are abstracted world configurations
- Arcs represent action results
- Goal test – a set of goal nodes

Each state occurs only *once* in a state (search) space.

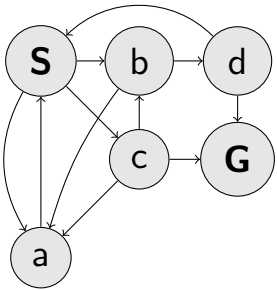


Search Trees



- A "what if" tree of plans and their outcomes
- Start node is the root
- Children are successors
- Nodes show/contains states, but correspond to *plans* that achieve those states

State Space Graphs vs. Search Trees



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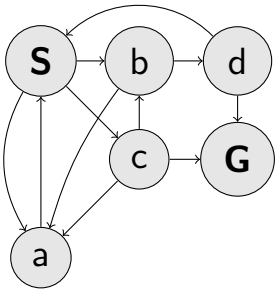

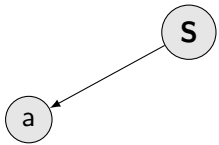

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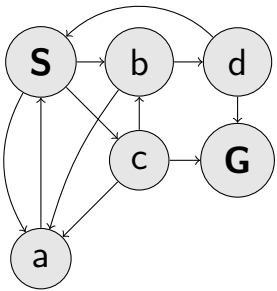

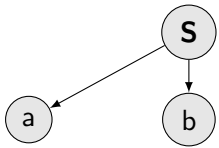

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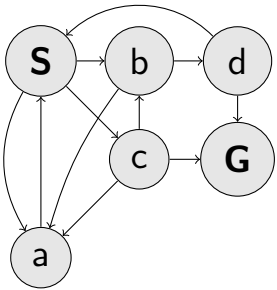

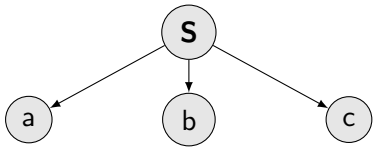

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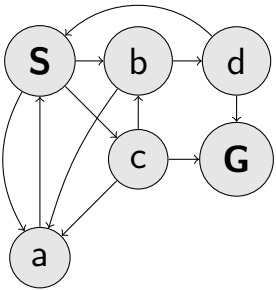

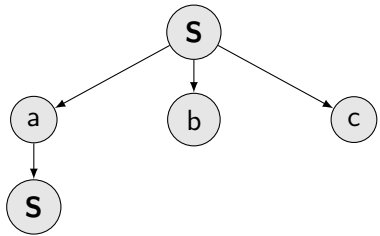

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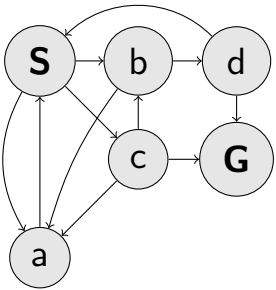

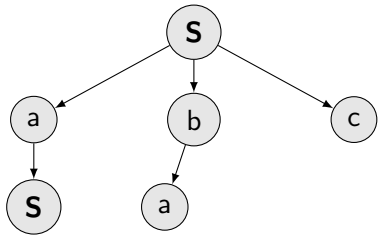

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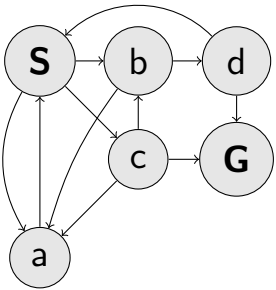

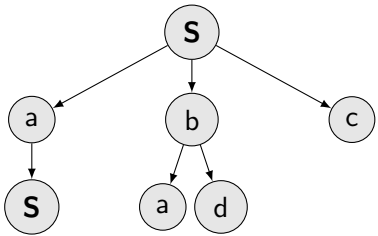

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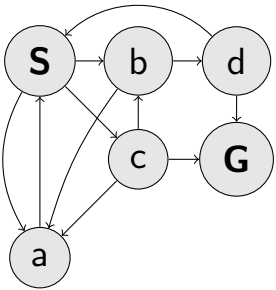

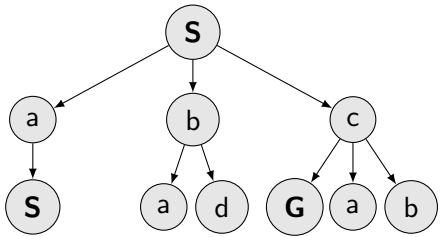

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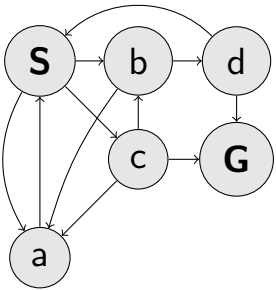

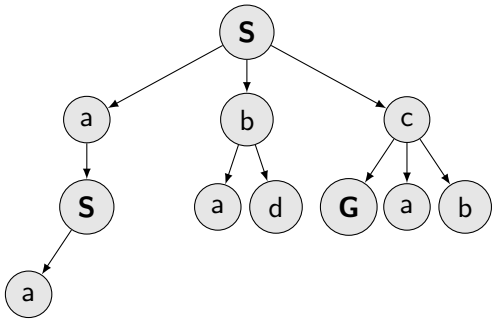

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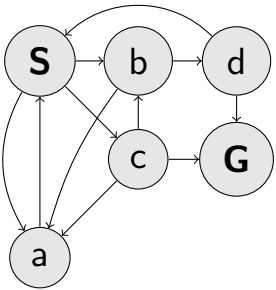

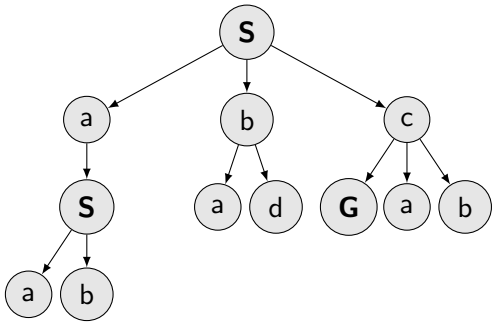

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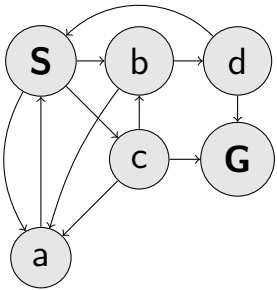

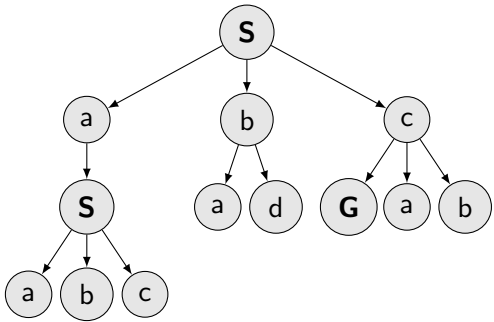

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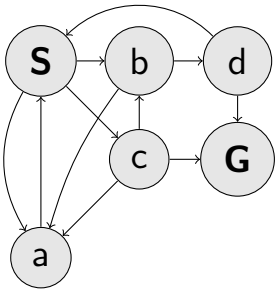

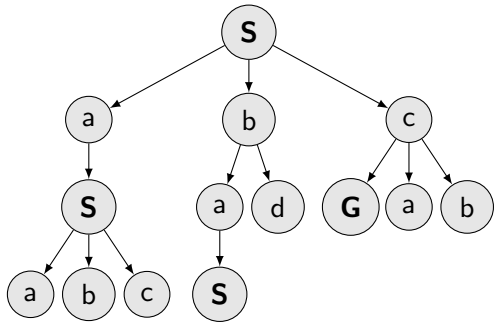

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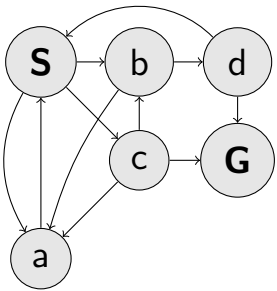

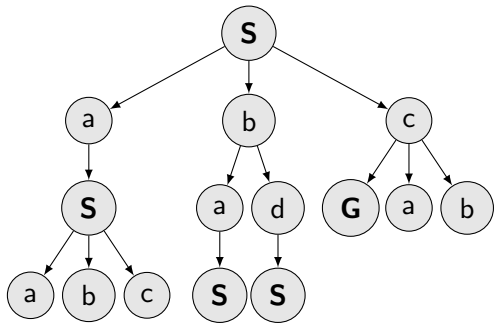

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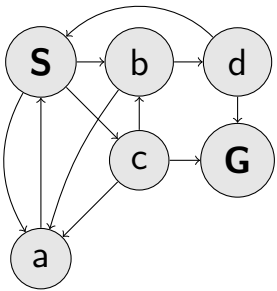

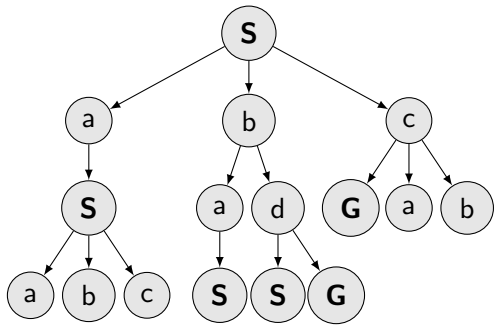

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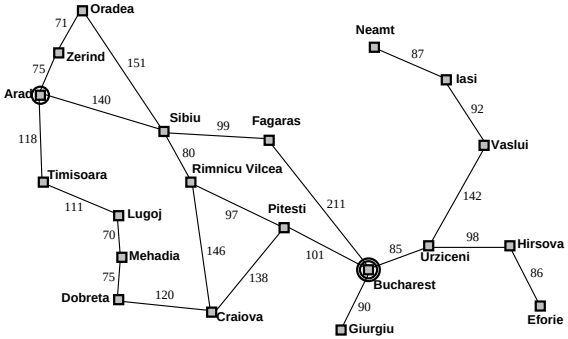
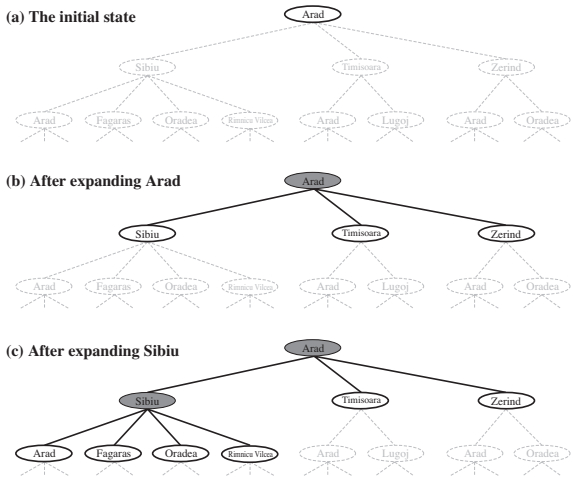
From problem/transition graph to search tree (Romania)

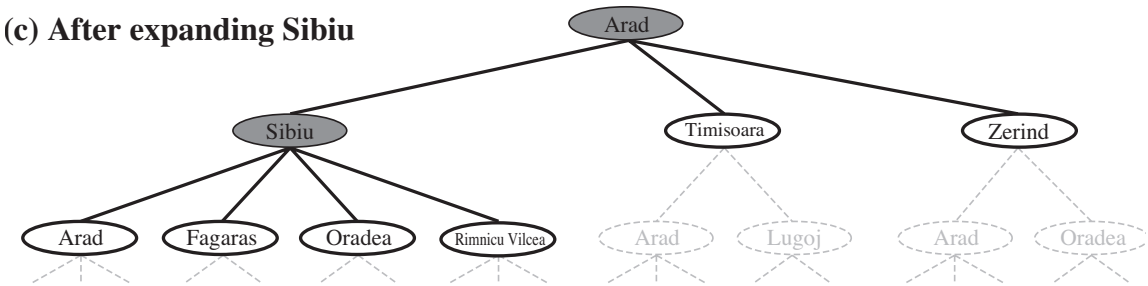




Problem/transition graph is revealed incrementally.

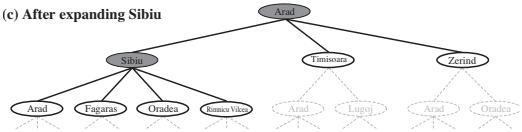
The revealing strategy can be visualized as a search tree.

Search elements - unvisited, dead, alive states



- Expand plans - possible ways (tree nodes).
- Manage/Maintain fringe (or frontier) of plans under consideration.
- Expand new nodes *wisely(?).*

Tree search algorithm



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initialize by using the initial state of the problem

loop

if no candidates for expansion **then return** failure

else choose a leaf node for expansion

end if

if the node contains a goal state **then return** the solution

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Expand the node and add the resulting nodes to the tree

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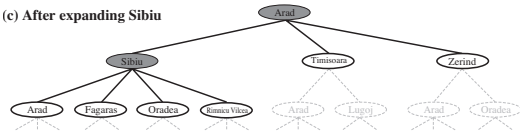

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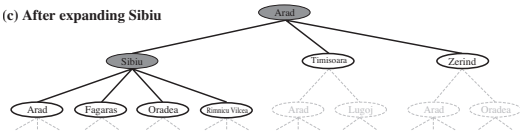

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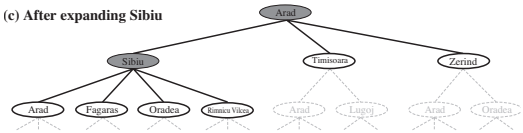

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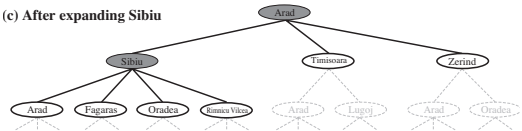

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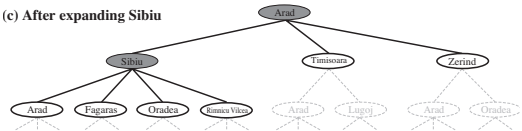

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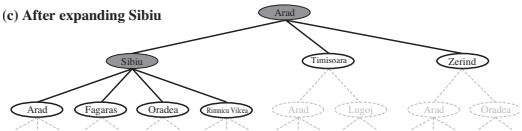

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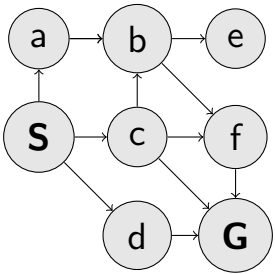

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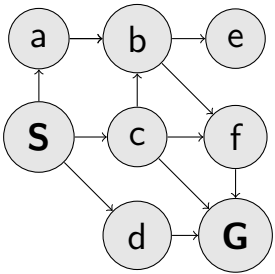

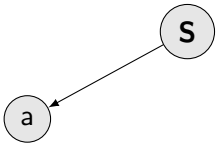

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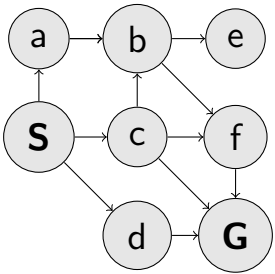

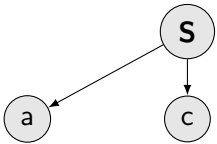

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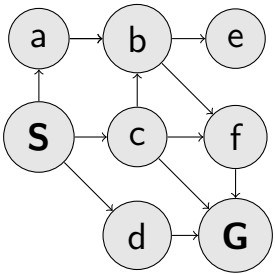

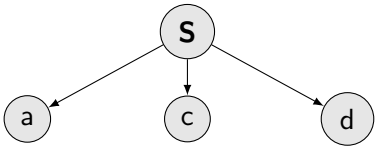

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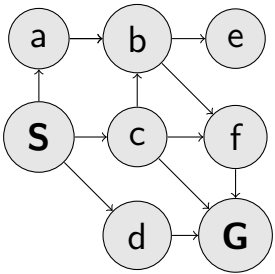

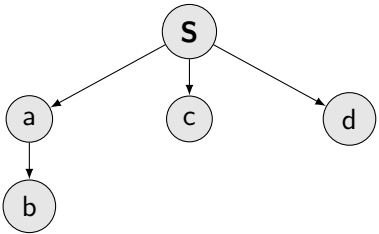

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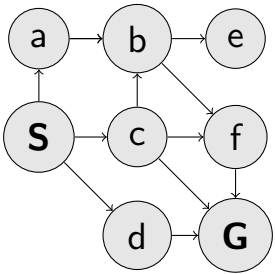

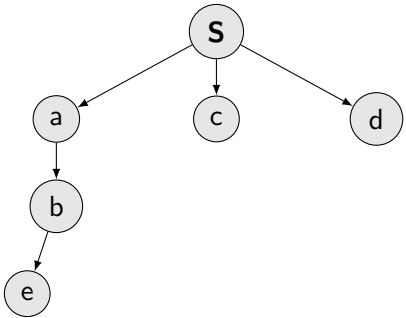

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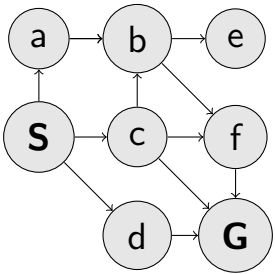

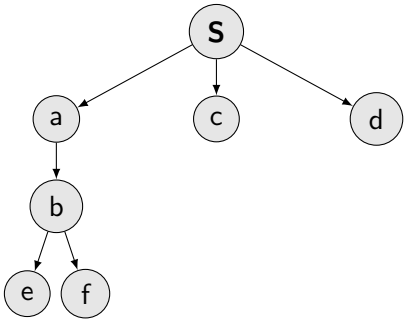

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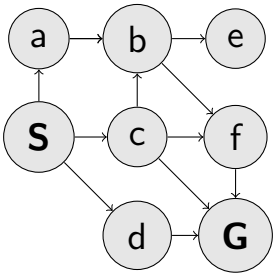

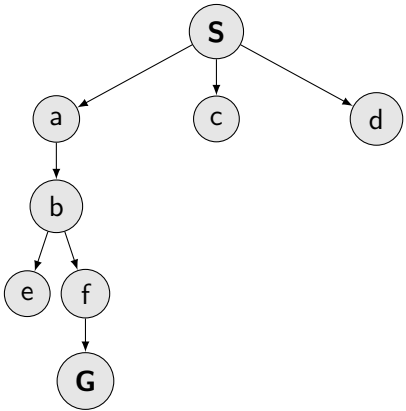

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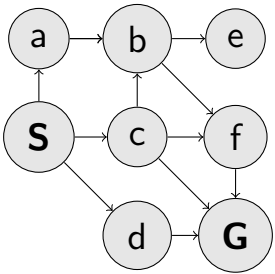

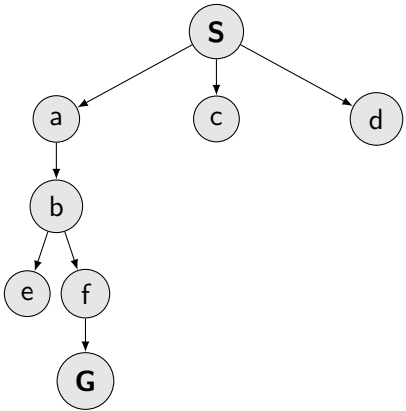

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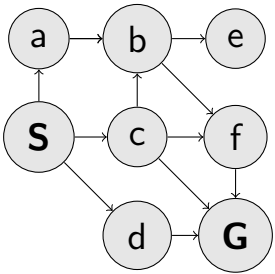

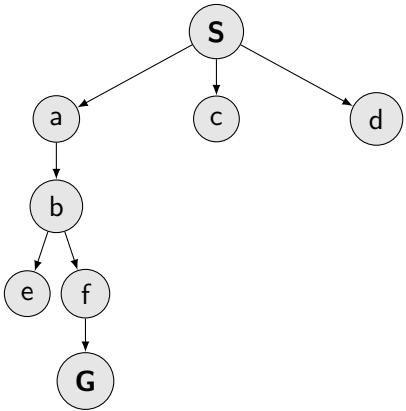

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Search (algorithm) properties

- Guaranteed to find a solution (if exists)? Complete?
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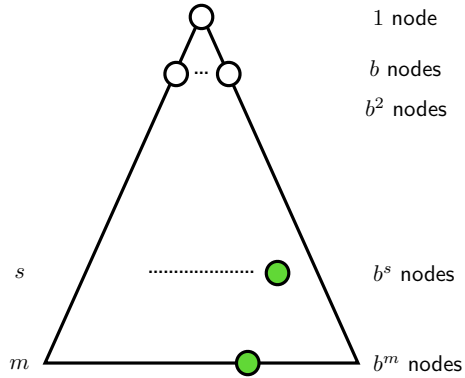
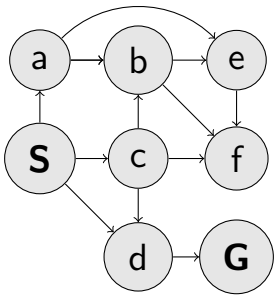
Strategies

How to traverse/build a search tree?

- Depth of the tree d.
- Max-Depth of the tree m. Can be ∞.
- Branching factor b.
- s denotes the shallowest Goal .
- How many nodes in the whole tree?



Depth-First Search (DFS)



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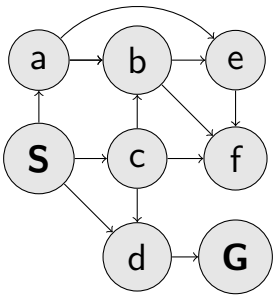

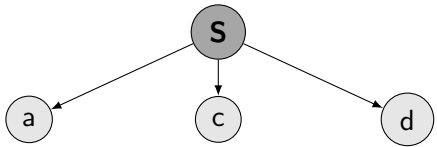

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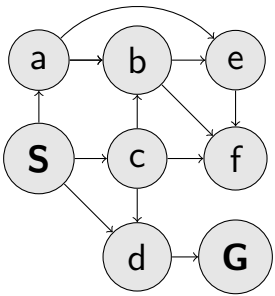

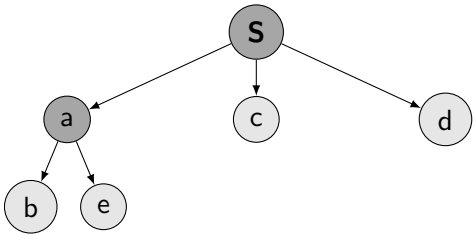

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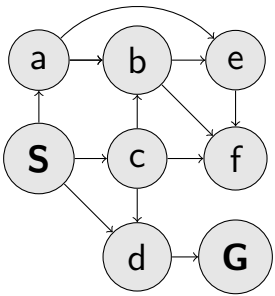

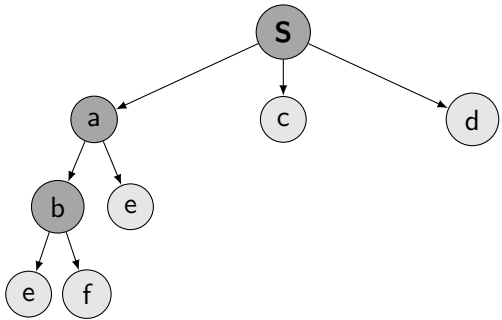

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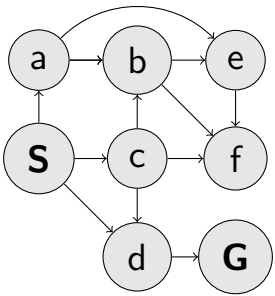

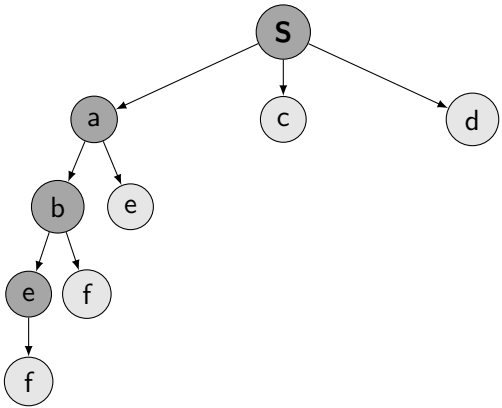

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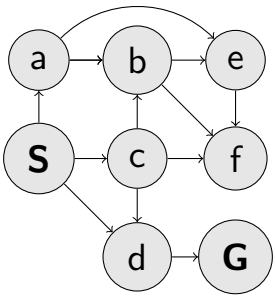

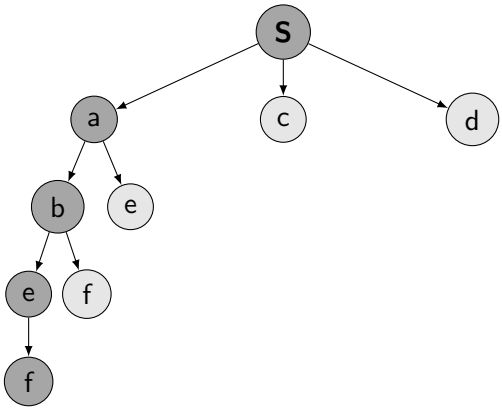

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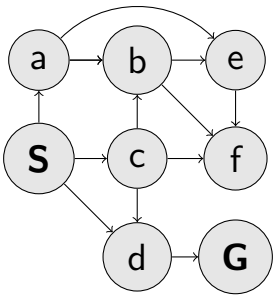

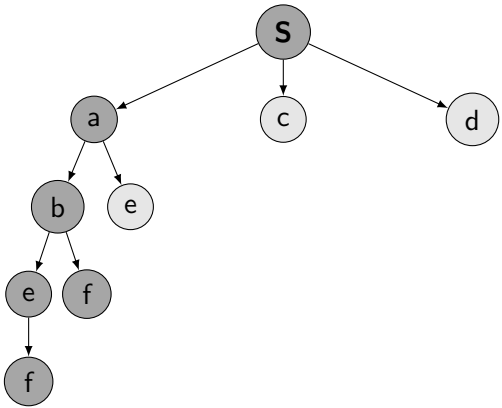

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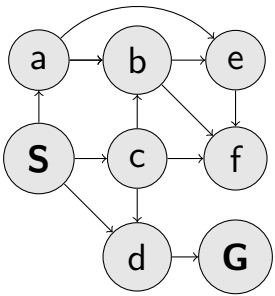

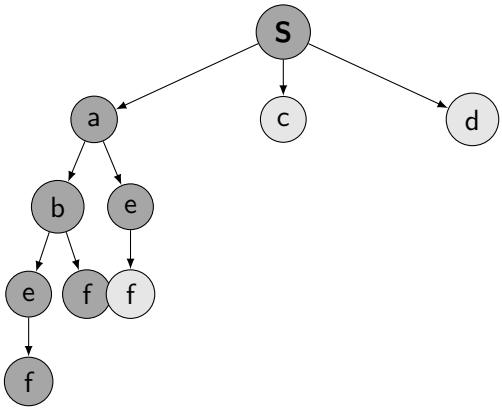

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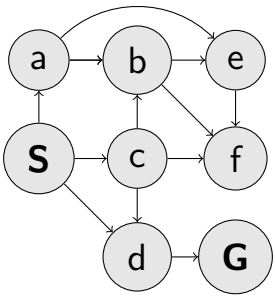

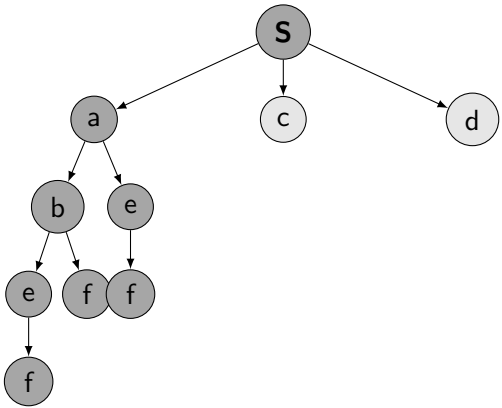

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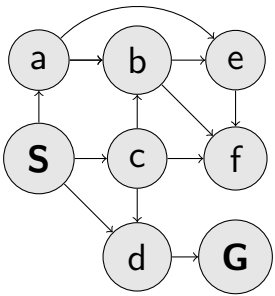

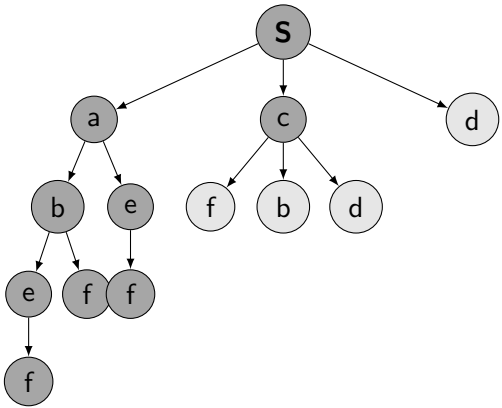

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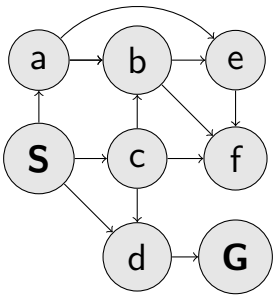

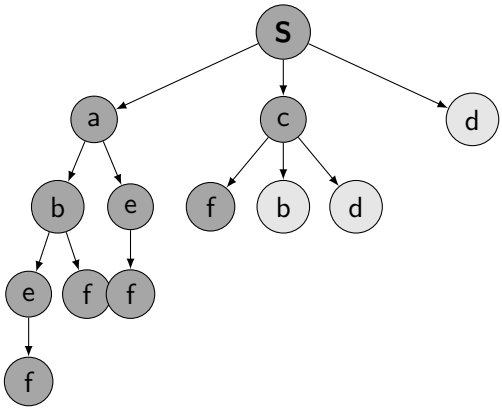

What are the DFS properties (complete, optimal, time, space)?

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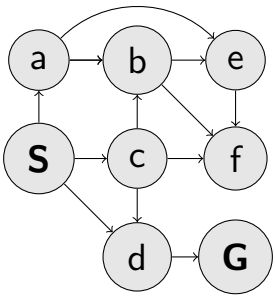

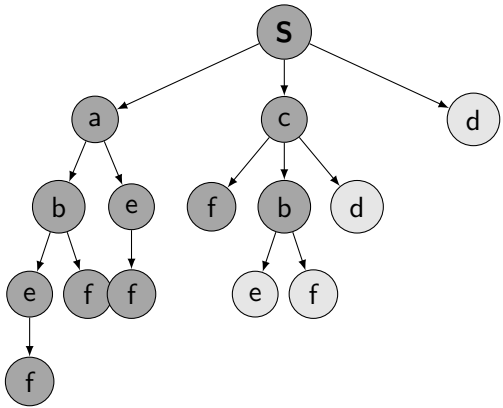

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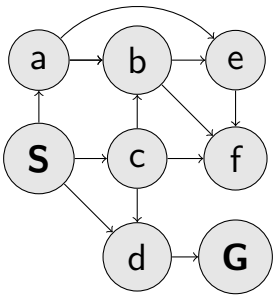

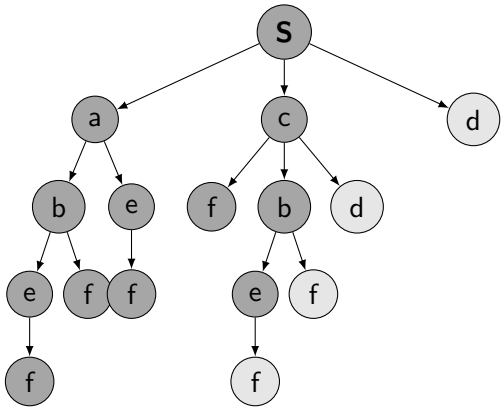

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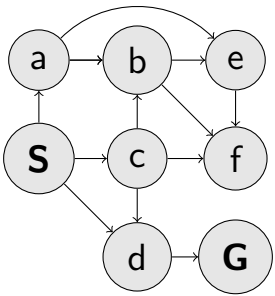

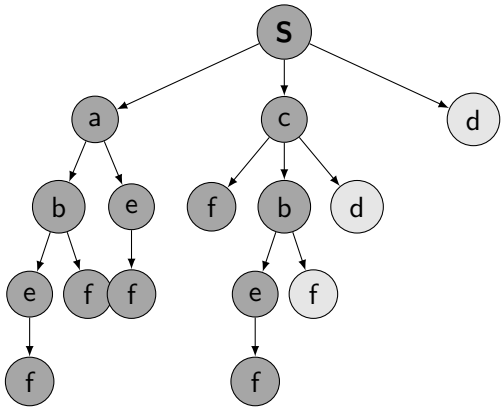

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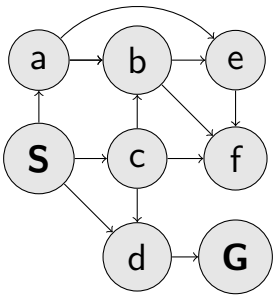

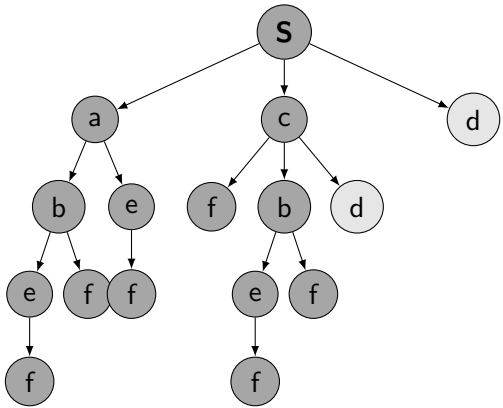

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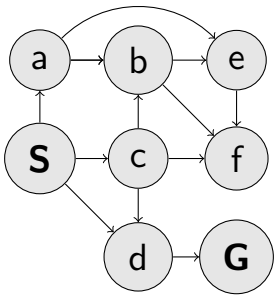

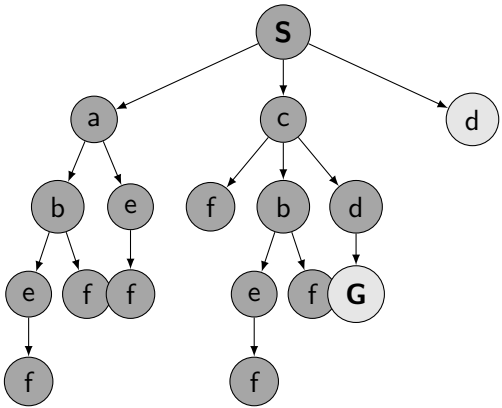

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DFS properties

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b, m, s, Time complexity?

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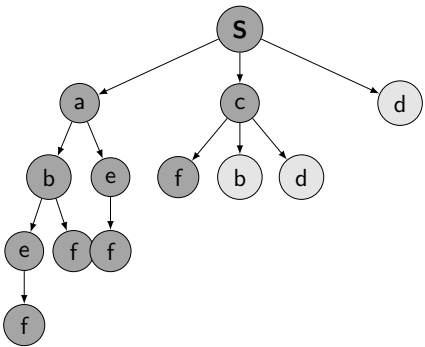
D ∞



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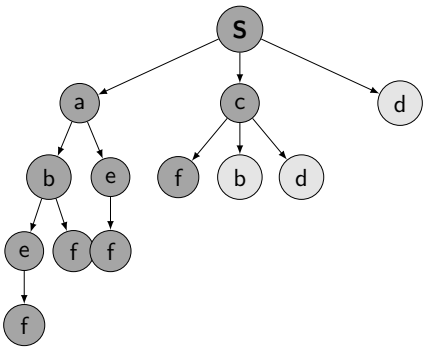
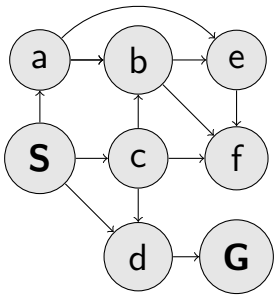


Breadth-First Search (BFS)



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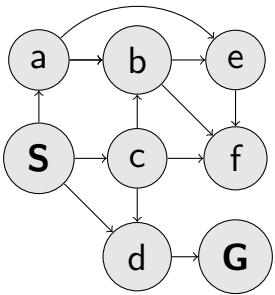

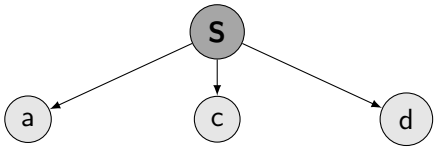

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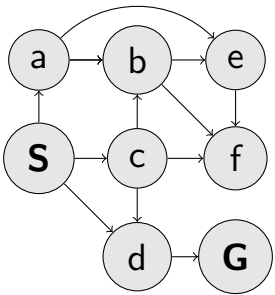

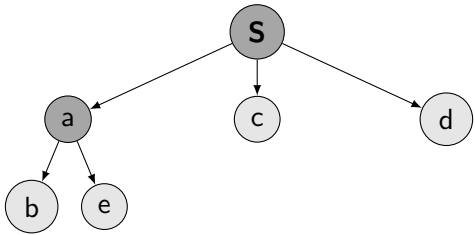

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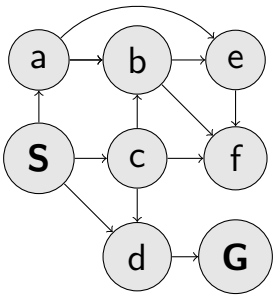

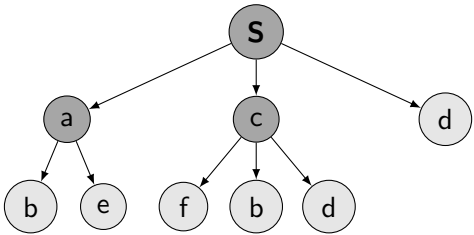

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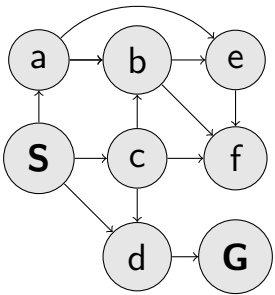

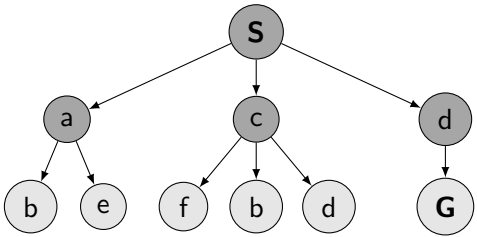

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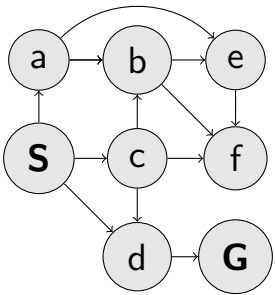

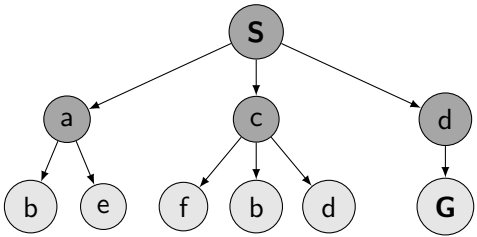

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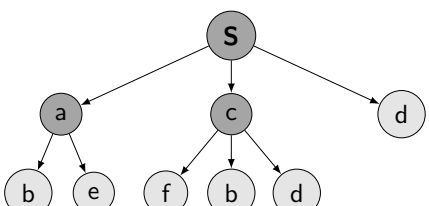

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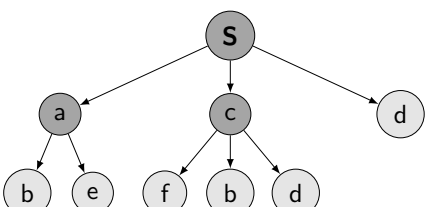

DFS vs BFS

What are (dis)advantages of the individual strategies?

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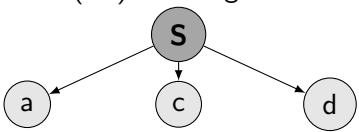

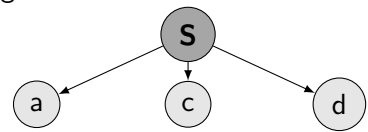

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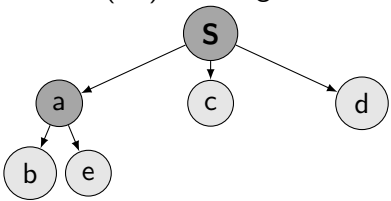

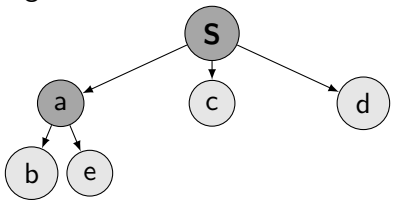

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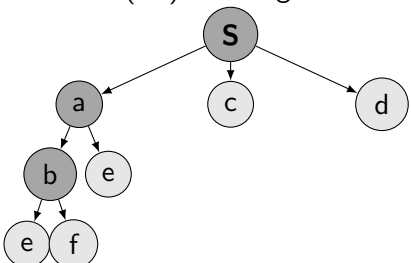

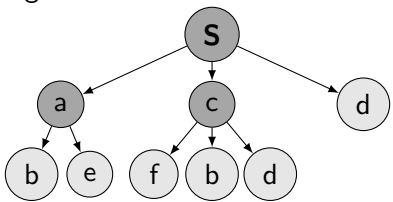

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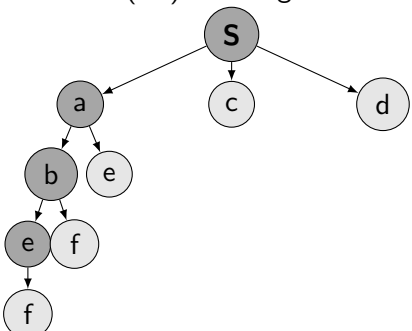

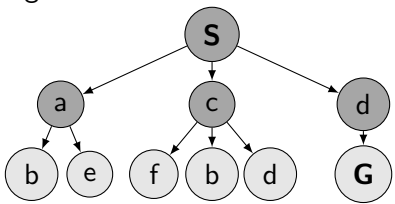

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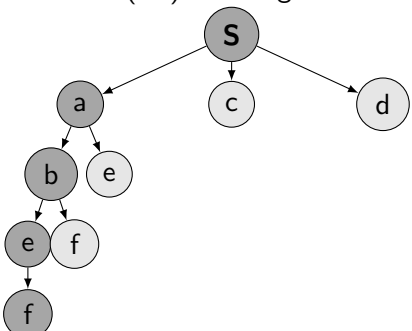

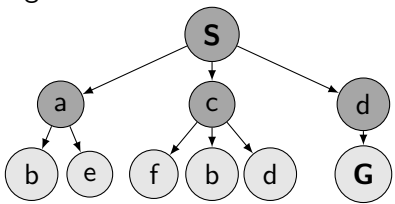

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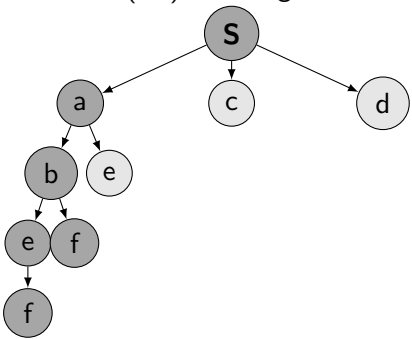

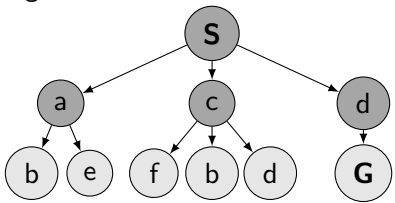

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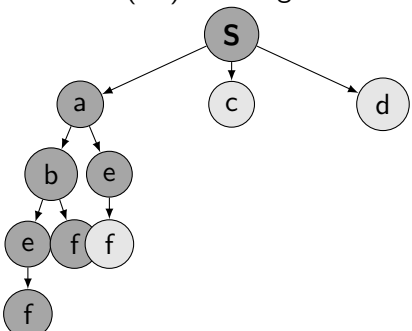

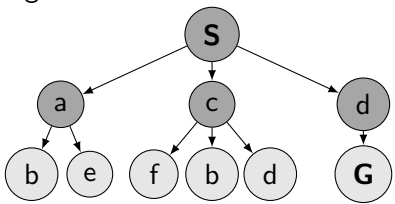

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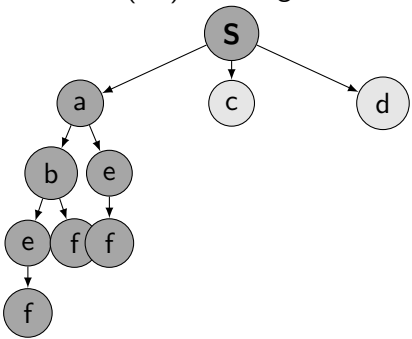

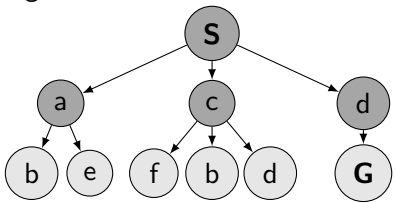

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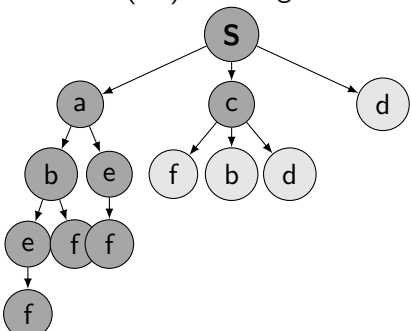

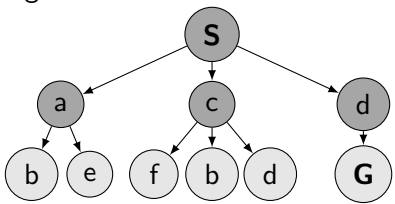

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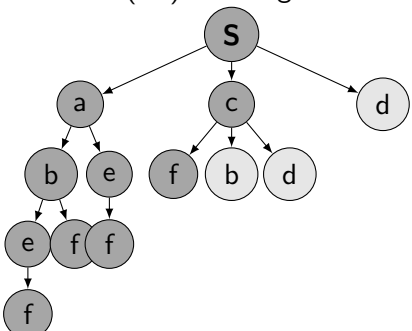

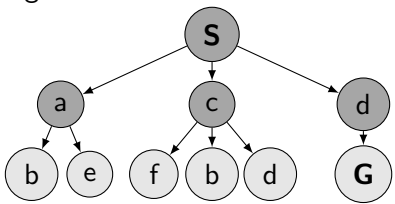

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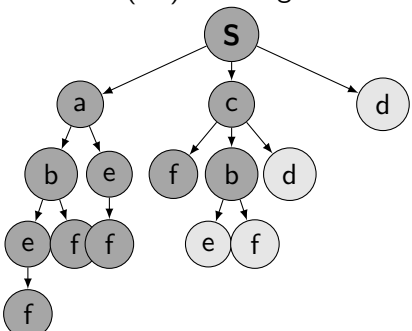

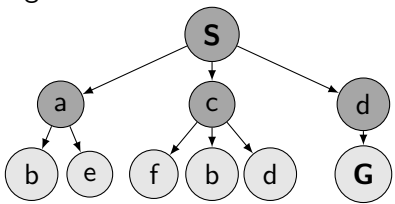

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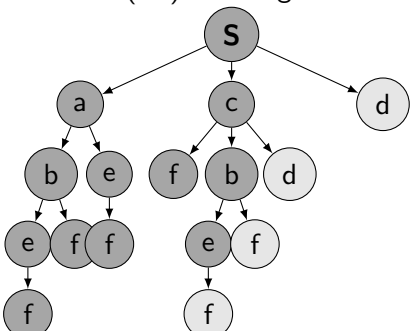

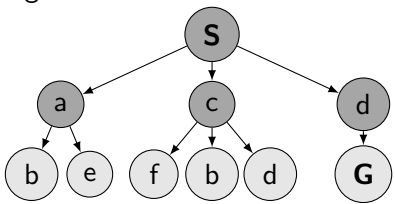

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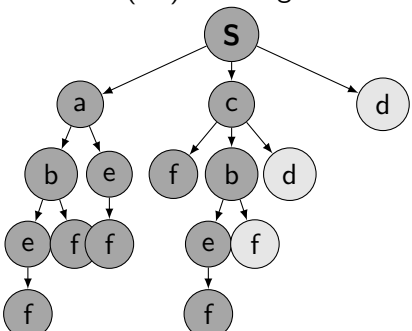

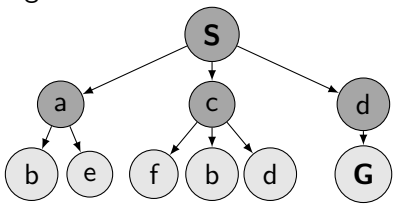

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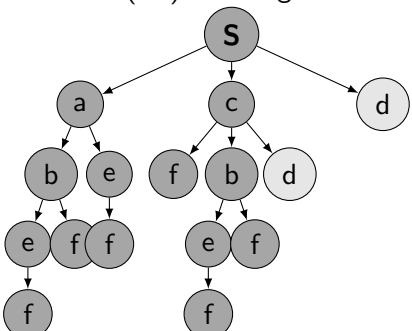

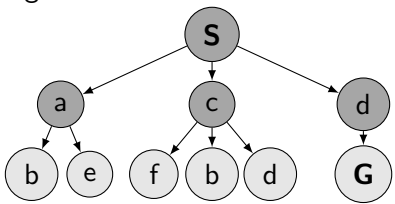

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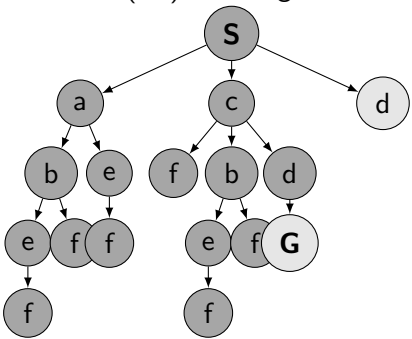

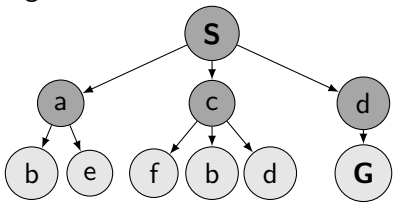

DFS with limited depth, `maxdepth=2`

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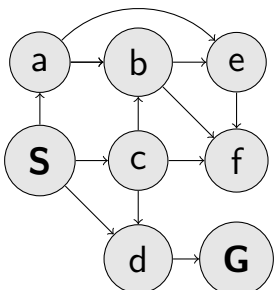

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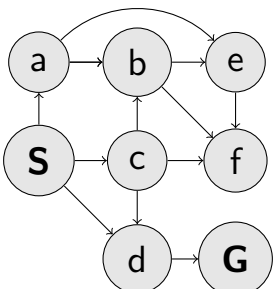

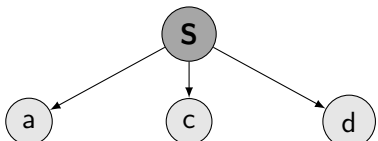

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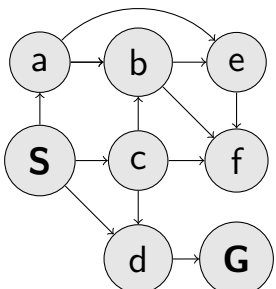

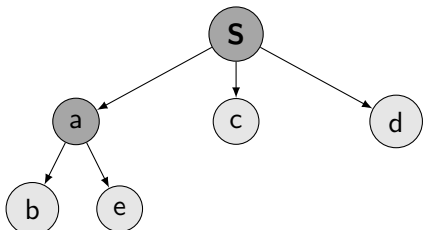

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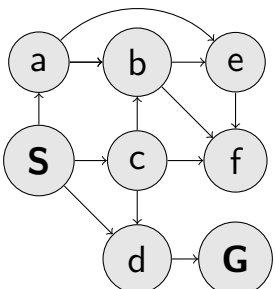

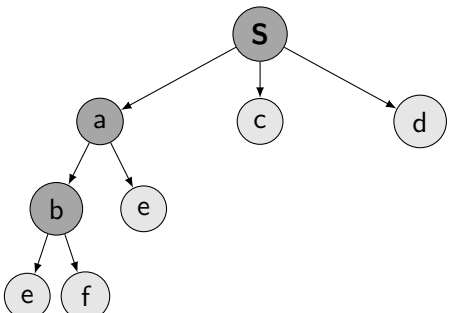

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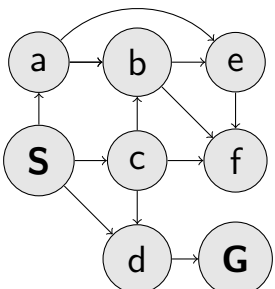

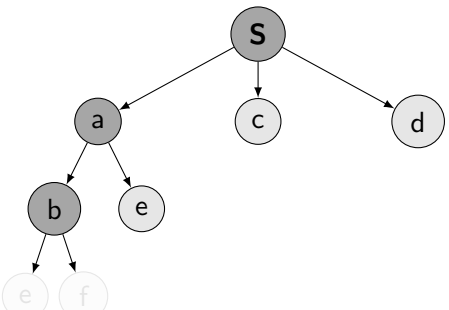

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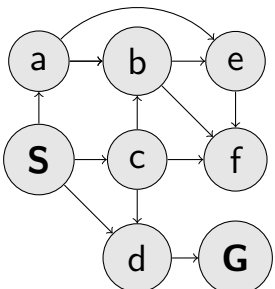

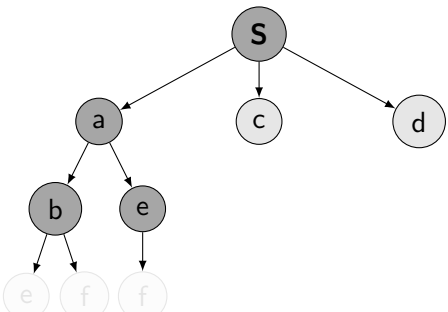

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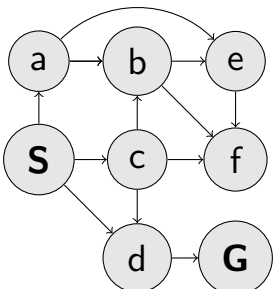

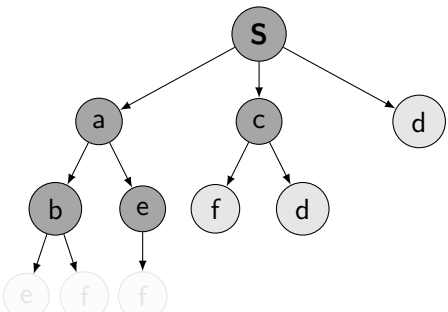

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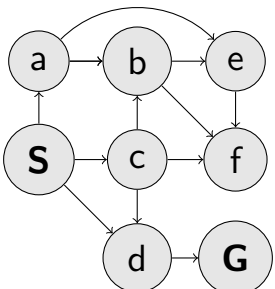

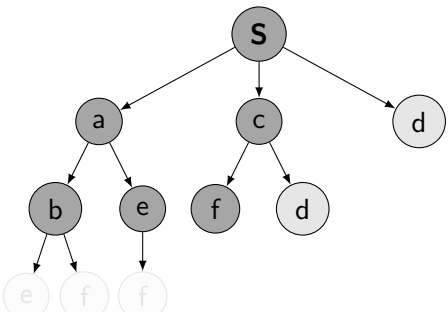

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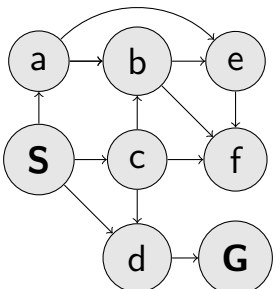

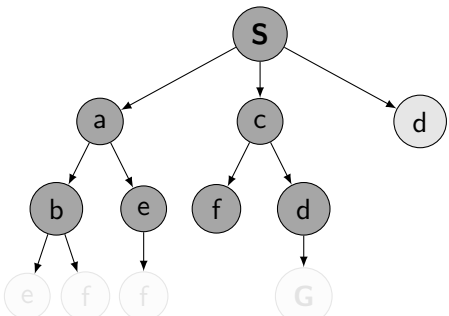

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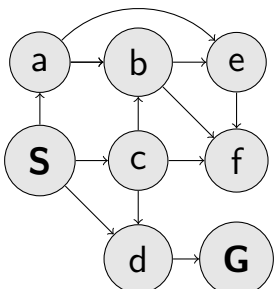

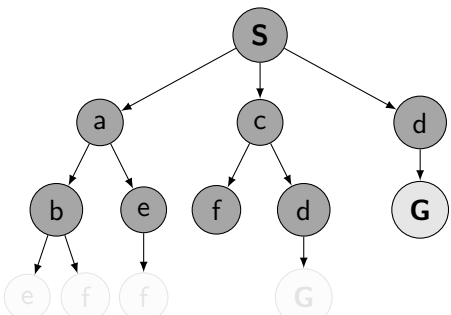

Iterative deepening DFS (ID-DFS)

- Start with `maxdepth = 1`
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- If failure, forget everything, increase `maxdepth` and repeat DFS

Is it not a terrible waste to forget everything between steps?

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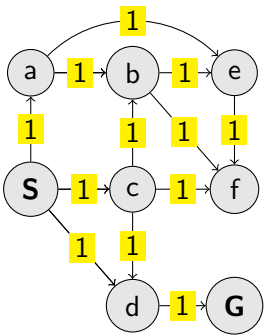

- In BFS, DFS, node ±depth was the node-value.

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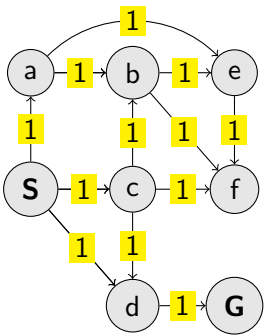

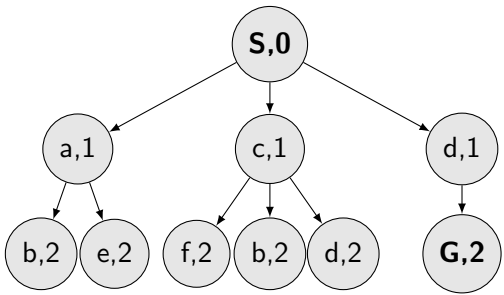

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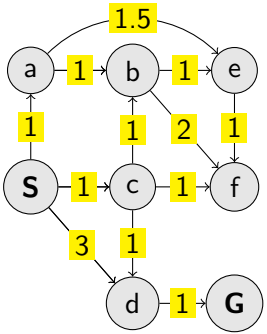

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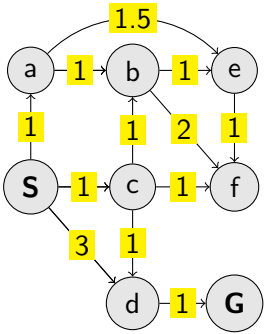

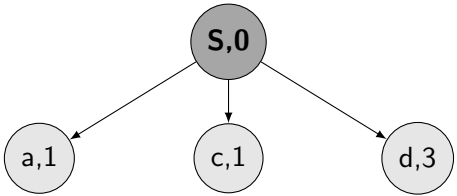

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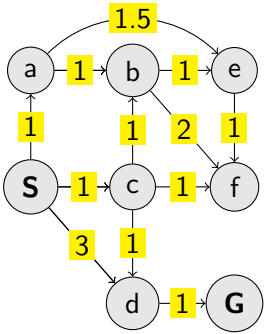

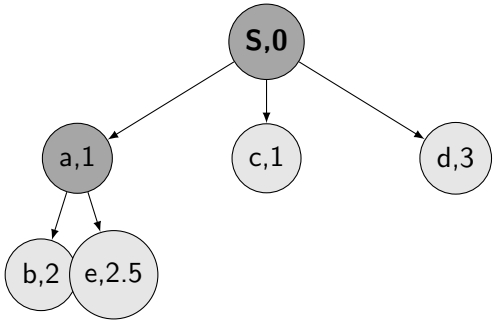

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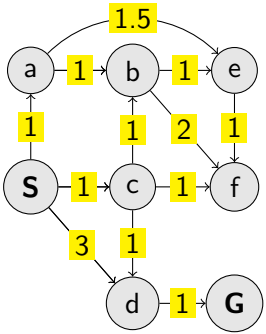

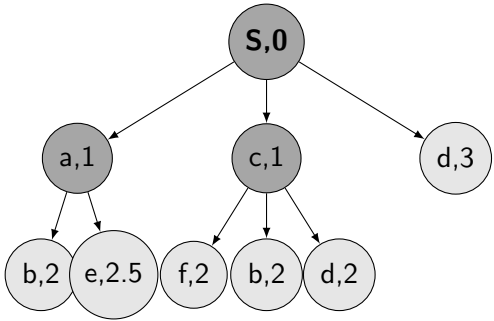

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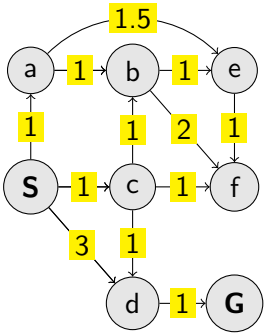

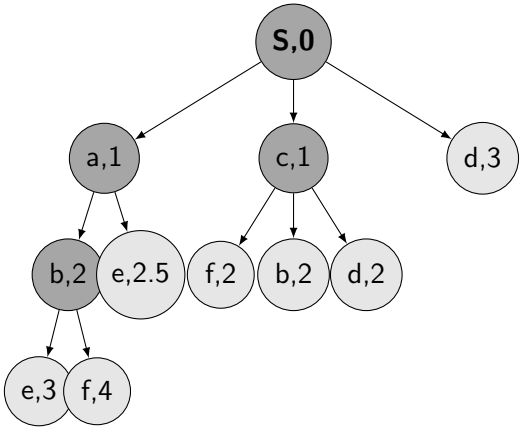

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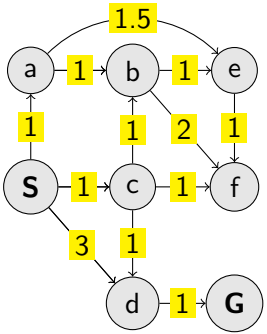

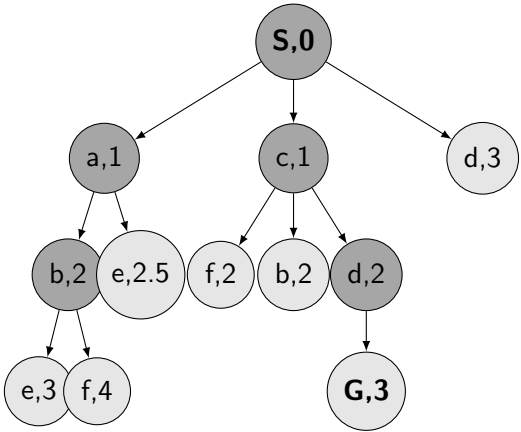

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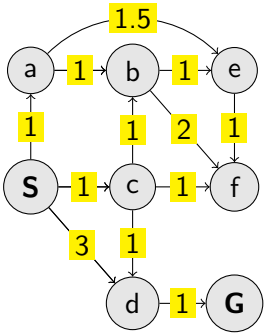

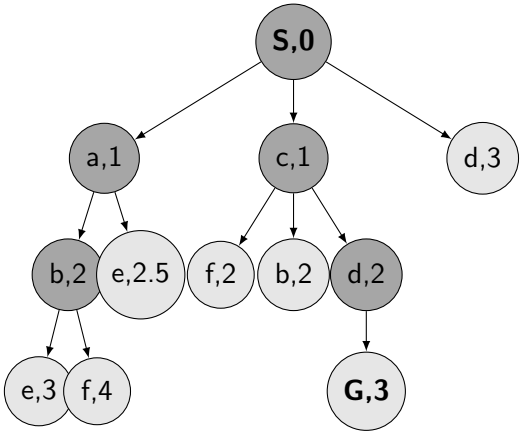

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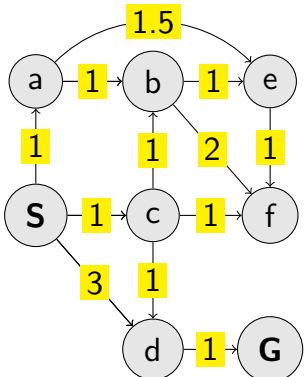

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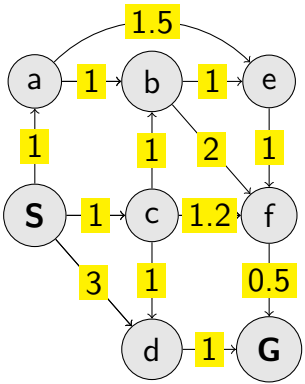

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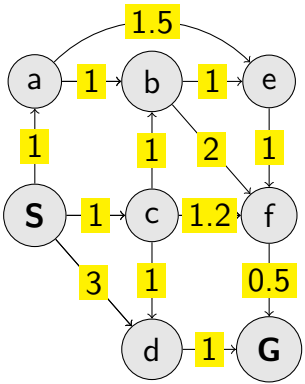

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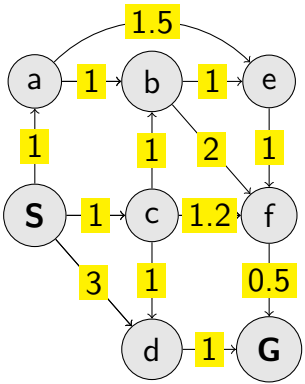

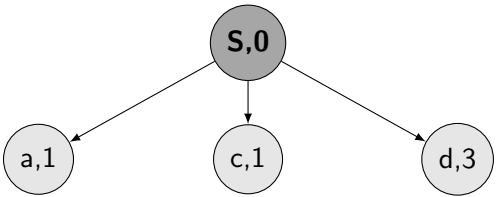

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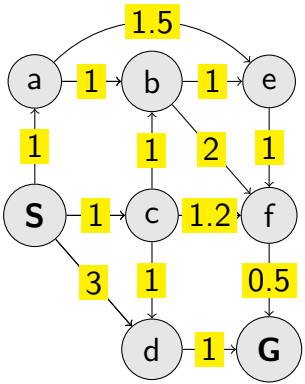

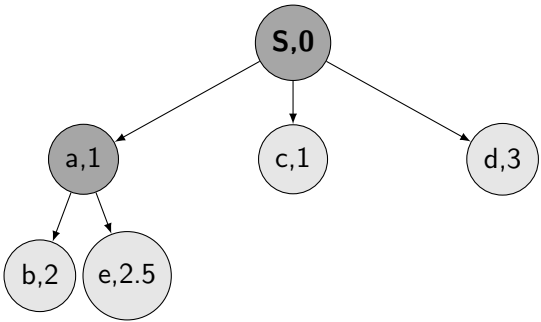

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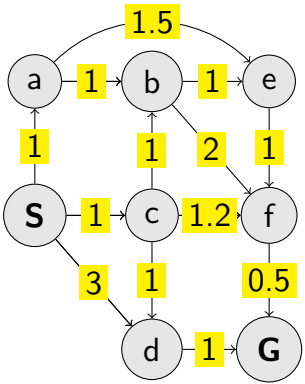

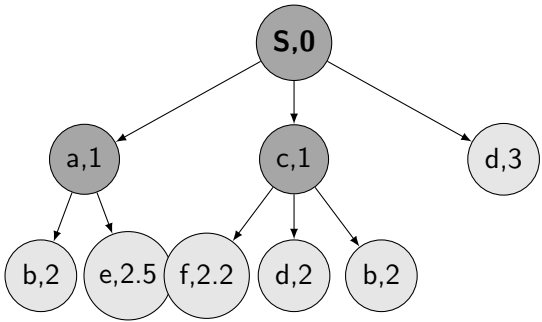

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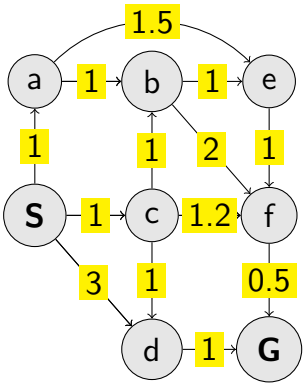

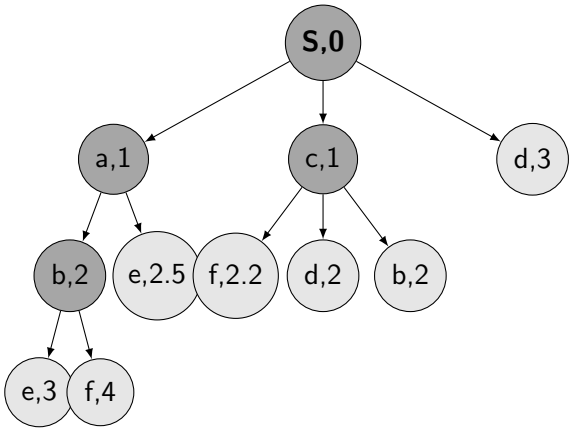

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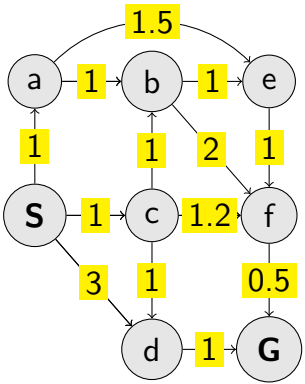

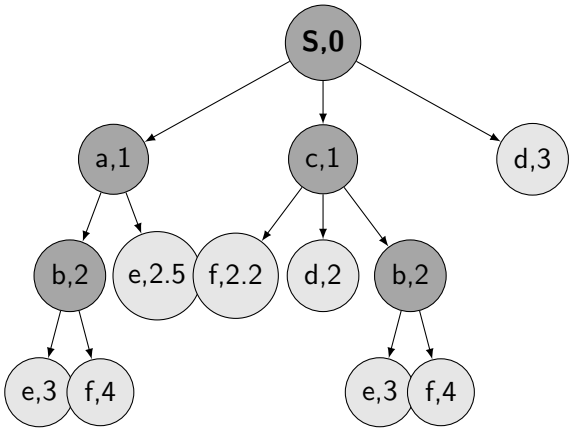

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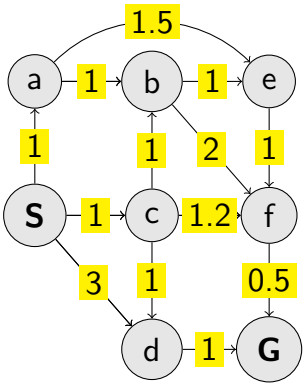

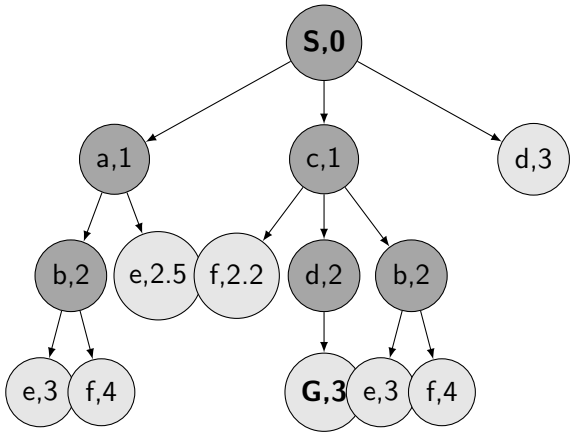

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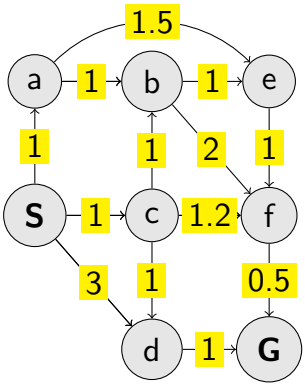

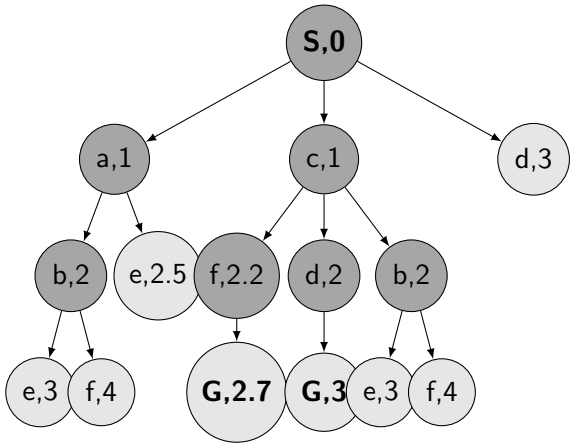

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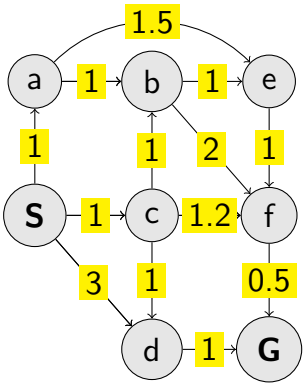

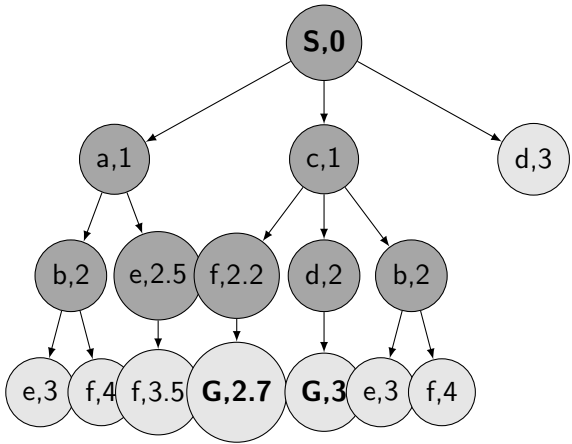

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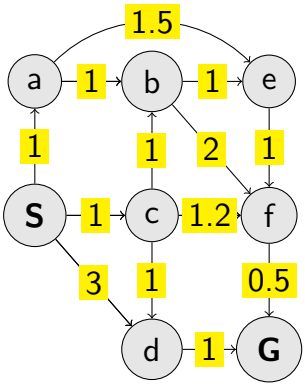

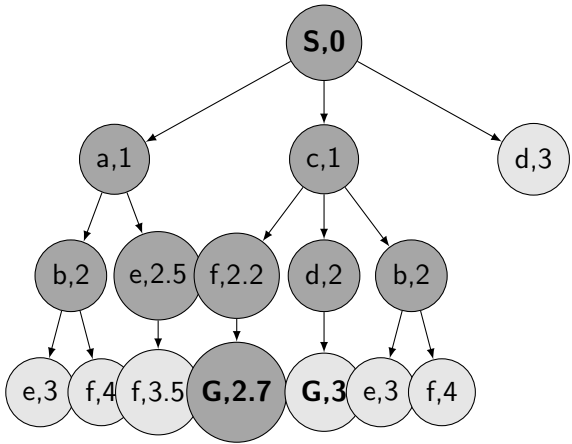

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- Space complexity?
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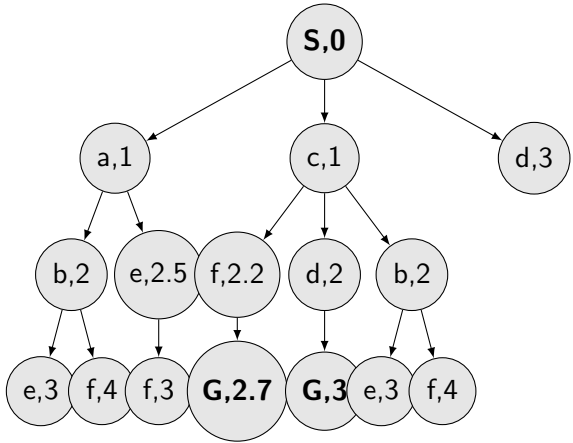
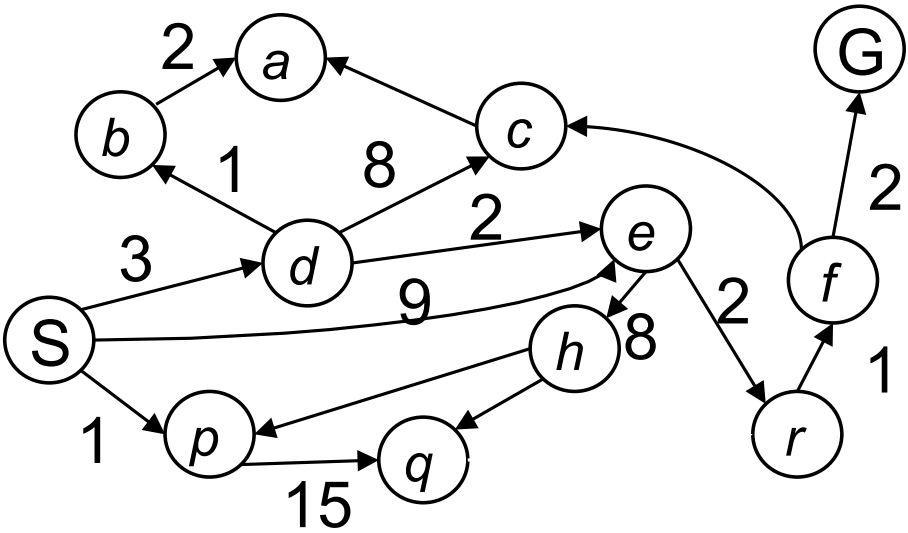


Example: Graph with costs



Infrastructure for (tree) search algorithms

What should a tree node `n` know?

- `n.state`
- `n.parent`
- `n.pathcost`

Perhaps we may add something later, if needed . . .

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How to organize nodes?

The Python examples are just suggestions, ...

- A dynamically linked structure (`list()`).
- Add a node (`list.insert(node)`).
- Take a node and remove from the structure (`node=list.pop()`).
- Check the Python modules `heapq`[1] and `queue`[2] for inspiration.

[1] https://docs.python.org/3.5/library/heapq.html
[2] https://docs.python.org/3.5/library/queue.html

What is the solution?

- We stop when Goal is reached.
- How do we construct the path?

Summary

- State space graph – an abstraction of a search problem.
- Search tree – visualization of the search algorithm run.
- Properties of search algorithms.

References, further reading

Some figures if from [2]. Chapter 2 in [1] provides a compact/dense intro into search algorithms.

[1] Steven M. LaValle.
Planning Algorithms.
Cambridge, 1st edition, 2006.
Online version available at: http://planning.cs.uiuc.edu.

[2] Stuart Russell and Peter Norvig.
Artificial Intelligence: A Modern Approach.
Prentice Hall, 3rd edition, 2010.
http://aima.cs.berkeley.edu/.