

Neuroinformatics 2021

March 9, 2022

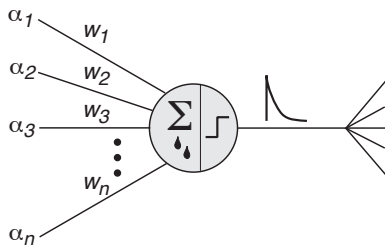
Lecture 4: Simplified neuron and population models

The leaky integrate-and-fire neuron

$$\tau_m \frac{dv(t)}{dt} = -(v(t) - E_L) + RI(t), \quad (1)$$

$$v(t^f) = \vartheta. \quad (2)$$

$$\lim_{\delta \rightarrow 0} v(t^f + \delta) = v_{\text{res}}, \quad (3)$$



The leaky integrate-and-fire neuron - analytical solution

- ▶ Very short input current

$$\tau_m \frac{dv(t)}{dt} + v(t) = 0$$

$$v(t) = \exp\left(-\frac{t}{\tau_m}\right)$$

- ▶ Constant small current $RI < \theta$

$$\frac{dv}{dt} = 0$$

$$v = RI$$

$$v(t) = RI\left(1 - \exp\left(-\frac{t}{\tau_m}\right)\right) + \frac{v(t=0)}{RI} \exp\left(-\frac{t}{\tau_m}\right)$$

(4)

The leaky integrate-and-fire neuron - CODE

```
%% Simulation of (leaky) integrate-and-fire neuron
clear; clf;

%% parameters of the model
dt=0.1;          % integration time step [ms]
tau=10;         % time constant [ms]
E_L=-65;       % resting potential [mV]
theta=-55;     % firing threshold [mV]
RI_ext=12;     % constant external input [mA/Ohm]

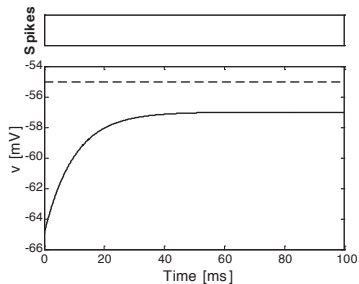
%% Integration with Euler method
t_step=0; v=E_L;
for t=0:dt:100;
    t_step=t_step+1;
    s=v>theta;
    v=s*E_L+(1-s)*(v-dt/tau*((v-E_L)-RI_ext));
    v_rec(t_step)=v;
    t_rec(t_step)=t;
    s_rec(t_step)=s;
end

%% Plotting results
subplot('position',[0.13 0.13 1-0.26 0.6])
plot(t_rec,v_rec);
hold on; plot([0 100],[-55 -55],'--');
axis([0 100 -66 -54]);
xlabel('Time [ms]'); ylabel('v [mV]')

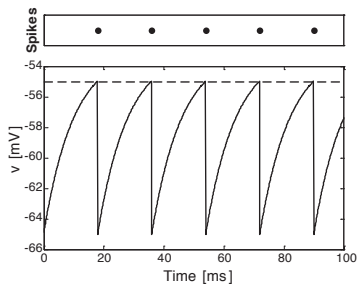
subplot('position',[0.13 0.8 1-0.26 0.1])
plot(t_rec,s_rec,'.','markersize',20);
axis([0 100 0.5 1.5]);
set(gca,'xtick',[],'ytick',[])
ylabel('Spikes')
```

The leaky integrate-and-fire neuron (cont.)

A. External input $RI_{\text{ext}} = 8 \text{ mV} < \text{threshold}$



B. External input $RI_{\text{ext}} = 12 \text{ mV} > \text{threshold}$



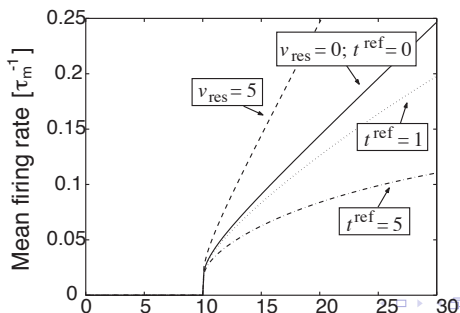
The LIF-neuron (cont.): Gain function

- ▶ Gain transfer activation
- ▶ The inverse of the first passage time t^f defines the **firing rate**
- ▶ Spikes occurs at $t = t^f = 0$, let's substitute $v(t = 0) = v_{\text{res}}$, $v(t) = \vartheta$ into (4), t_{ref} is absolute refractory period

$$t^f = -\tau_m \ln\left(\frac{\vartheta - RI}{v_{\text{res}} - RI}\right)$$

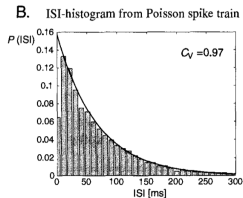
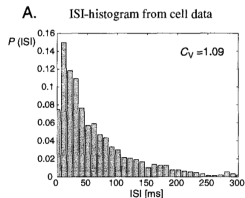
$$\bar{r} = \left(t^{\text{ref}} - \tau_m \ln \frac{\vartheta - RI}{v_{\text{res}} - RI}\right)^{-1}$$

(5)



Spike-time variability

- ▶ Inter-spike interval (ISI)
- ▶ IF neuron - constant ISI $C_V = 0$, A-cortical cell(Broadmann area), B- simulation from spike train
- ▶ regular firing in V1: $C_V = 0.5 \dots 1$



$$C_V = \frac{\sigma}{\mu}$$

$$pdf^{exp}(x, \lambda) = \lambda e^{-\lambda x}$$

$$pdf^{poisson}(x, \lambda) = \sum_{i=1}^x \lambda^i \frac{e^{-\lambda}}{i!}$$

Poisson Spike Train - CODE

```
%% Generation of Poisson spike train with refractoriness
clear; clf;
fr_mean=15/1000;      % mean firing rate
%% generating poisson spike train
lambda=1/fr_mean;     % inverse firing rate
ns=1000;              % number of spikes to be generated
isi1=-lambda.*log(rand(ns,1)); % generation of expo. distr. ISIs
%% Delete spikes that are within refractory period
isi=0;
for i=1:ns;
    if rand>exp(-isi1(i)^2/32);
        isi=isi+1;
        isi(isi)=isi1(i);
    end
end
%% Plotting histogram and calculating cv
hist(isi,50);        % Plot histogram of 50 bins
cv=std(isi)/mean(isi) % coefficient of variation
```


Sources of noise

- ▶ diffuse propagation of neurotransmitter across synaptic cleft
- ▶ propagation of the membrane potential along dendrites with varying geometry
- ▶ biochemical processes
- ▶ probabilistic nature of transmitter release by axonal spikes
- ▶ simulation of all these irregularities by INCLUDING NOISE

Noise models I

- ▶ Stochastic threshold

$$v \rightarrow v + \eta^1(t)$$

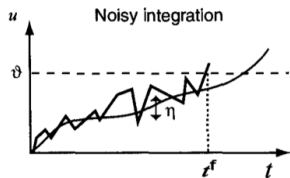
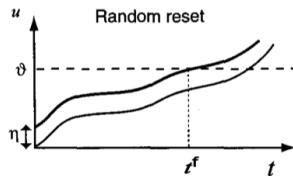
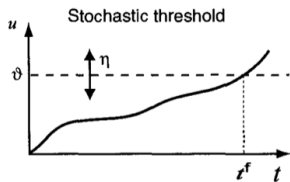
- ▶ Random reset

$$v^{\text{res}} \rightarrow u^{\text{res}} + \eta^2(t)$$

- ▶ Noisy integration

$$\tau_m \frac{dv}{dt} \rightarrow -v(t) + RI_{\text{ext}} + \eta^3(t) \quad (7)$$

Noise models II



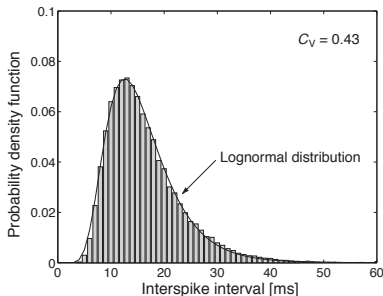
Variability of real neuron

- ▶ Model (7): normally distributed current I_{ext}

$$I_{ext} = I_{ext} + \eta, \eta \in N(0, 1)$$

- ▶ Normal pdf \rightarrow very good approximation considering independent synaptic inputs from many equally distributed neurons
- ▶ Simulation: $R\hat{I}_{ext} = 12mV$, $\vartheta = 10mV$
- ▶ log normal pdf

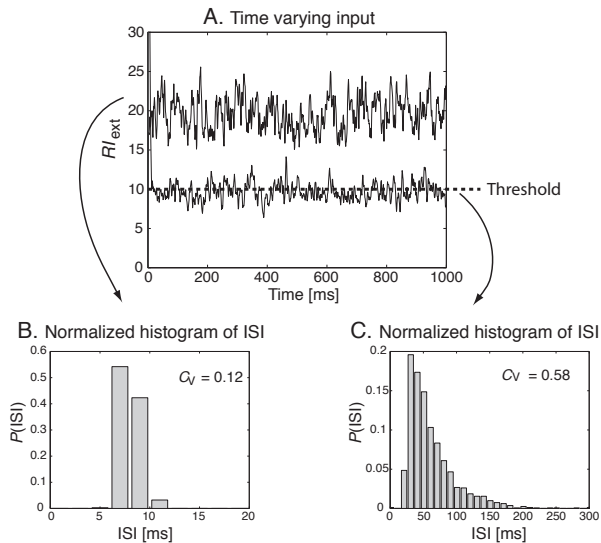
$$pdf^{lognormal}(x; \mu, \sigma) = \frac{1}{x\sigma\sqrt{2\pi}} \exp\left(\frac{-(\log(x) - \mu)^2}{2\sigma^2}\right)$$



The LIF-neuron noise simulation I

- ▶ real neuron with 5000 presynaptic neuron
- ▶ 10 % simulation \rightarrow 500 Poisson-distributed spike trains (6) with refractory corrections
- ▶ mean firing rate = 20 Hz, after correction 19.3 Hz, refractory constant 2 ms.
- ▶ each presynaptic spike \rightarrow EPSP in form of α function (??)
- ▶ $\omega = 0.5 \rightarrow$ regular firing, $C_V = 0.12$, average rate 118 Hz.
- ▶ $\omega = 0.25 \rightarrow$ irregular firing, $C_V = 0.58$, average rate 16 Hz. The $C_V >$ lower bound found in experiments

The LIF-neuron noise simulation II



Further Readings

- Wolfgang Maass and Christopher M. Bishop (eds.) (1999), **Pulsed neural networks**, MIT Press.
- Wulfram Gerstner (2000), **Population dynamics of spiking neurons: fast transients, asynchronous states, and locking**, in **Neural Computation** 12: 43–89.
- Eugene M. Izhikevich (2003), **Simple Model of Spiking Neurons**, in **IEEE Transactions on Neural Networks**, 14: 1569–1072.
- Eugene M. Izhikevich (2004), **Which model to use for cortical spiking neurons?**, in **IEEE Transactions on Neural Networks**, 15: 1063–1070.
- Warren McCulloch and Walter Pitts (1943) **A logical calculus of the ideas immanent in nervous activity**, in **Bulletin of Mathematical Biophysics** 7:115–133.
- Huge R. Wilson and Jack D. Cowan (1972), **Excitatory and inhibitory interactions in localized populations of model neurons**, in **Biophys. J.** 12:1–24.
- Nicolas Brunel and Xiao-Jing Wang, (2001), **Effects of neuromodulation in a cortical network model of working memory dominated by recurrent inhibition**, in **Journal of Computational Neuroscience** 11: 63–85.