Problem solving by search II

Tomáš Svoboda, Matěj Hoffmann, and Petr Pošík

Vision for Robots and Autonomous Systems, Center for Machine Perception
Department of Cybernetics
Faculty of Electrical Engineering, Czech Technical University in Prague

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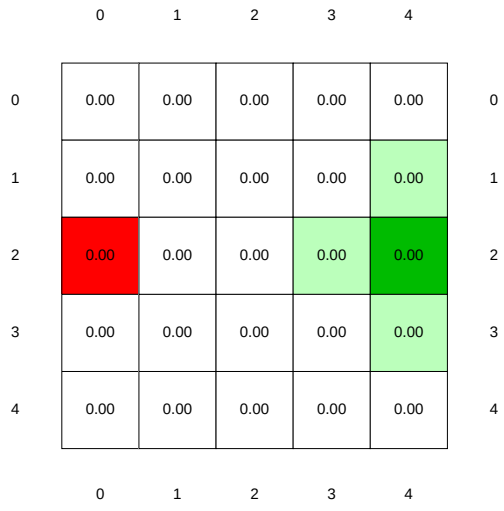
Notes

Outline

- Graph search
- Heuristics (how to search faster)
- Greedy
- A*. A-star search.

Notes

A Maze, what could possibly go wrong?



https://youtu.be/WKSoedfRZQ4

Notes

Analyze the demo run (BFS). What happened? Why did it take that long?
Because it is TREE_SEARCH...
Many loops are created and all nodes with depth < 7 need to be expanded first. Goal is at depth 8.
Notes for teacher:
Working note for demo:
`python3 easy_search_agents.py`
`'n' for next`
`'s' for skip`
code settings:
`MAP = 'maps/easy/easy2.bmp'`
`TREE_SEARCH = True`
`node_type = 'BFS'`
How to decode printout on command line:

- Every iteration ends with: `print('End of while loop: length of the frontier:',len(frontier), 'length of the expanded:', len(expanded_states), frontier, frontier.is_empty())`
- But note that the algo is written in a general way (like UCS), stopping after expanding the goal node – that is why you see also depth 9 in the frontier notes at the end.
- Size of the visualiation can be altered in `./kuimaze/maze.py`, look for `MAX_CELL_SIZE`

Tree search the maze

function TREE_SEARCH(env) **return** a solution or failure
    initialize the frontier
    while frontier **do**
        node ← frontier.pop()
        if goal in node **then return** node
        end if
        child_nodes ← env.expand(node.state)
        Add child_nodes to frontier
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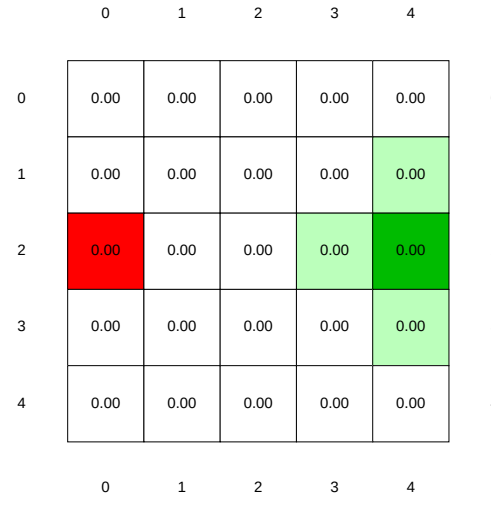
end function



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Make a `frontier` and `expand` columns on a paper and follow the algorithm by putting and removing (scratching out) nodes from the list.

Note that there are many more nodes than states (*search tree* vs. *state space*).

Tree search seems hugely ineffective. Note that this is (also) because of the state space. It's a maze with undirected egdes. If we had directed edges, there would be much much fewer cycles.

A graph search

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        child_nodes ← env.expand(node.state)
        for all child_nodes **do**
            if *child_node.state not in explored* **then**
                add nodes to frontier
            end if
        end for
    end while
end function

Do not forget: node is not the same as state!

Notes

Think about what is node and what state. What is main difference? How are they connected? Where do they appear? What is node/state in the maze problem?
The main idea: Do not expand a state twice.
What would be a good data structure to implement the *explored* set? Yes, it would be a *set* ;) – where every element is present only once. Unlike *list*.
"What about frontier?" - if you can ensure that the first time you add a node to frontier, the state will be reached by an optimal path from start, you can also check frontier here (e.g., BFS). If you can't guarantee that, you have to be more careful.

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The BFS graph search

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function BFS_GRAPH_SEARCH(env) return a solution or failure
    node ← env.observe()
    frontier ← FIFOqueue(node)
    explored ← set()
    while frontier not empty do
        node ← frontier.pop()
        explored.add(node.state)                          ▷ Add state, not node!
        child_nodes ← env.expand(node.state)
        for all child_nodes do
            if child_node.state not in explored and not in frontier then
                if child_node contains Goal then return child_node
                end if
                frontier.insert(child_node)
            end if
        end for
    end while
end function
```


Notes

Why adding/checking `state` and not `node` in `explored` data structure? Can I do the simple presence check for all kind of graph search algorithms?
Run demo again with BFS graph search.
Notes for teacher:
TREE_SEARCH = False
Working note for demo:

```
 python3 easy_search_agents.py
'n' for next
's' for skip
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code settings:

```
 MAP = 'maps/easy/easy2.bmp'
TREE_SEARCH = False
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Result can be also seen at: https://youtu.be/4yu_nsWZ2ck

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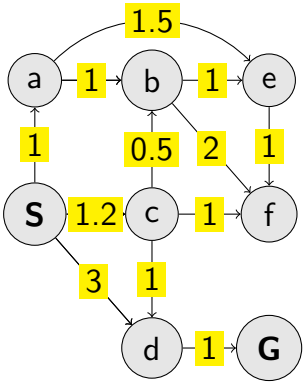
Checking `frontier` in uniform costs graph search?



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When following the algorithm (animation) use the paper list of `frontier` and `explored`
Note the extra features of UCS vs. BFS in action:

1. Update of cost:
   - "b,2" disappears as "b,1.7" appears – update with lower cost.
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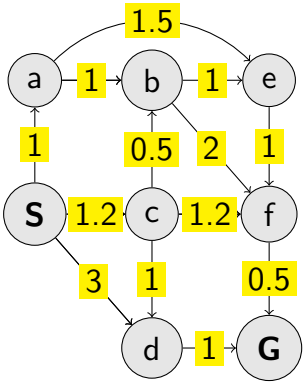

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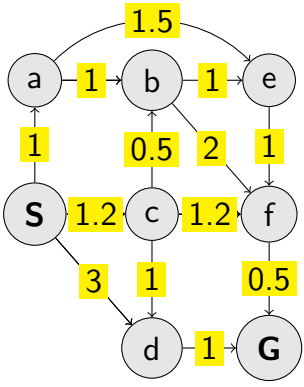

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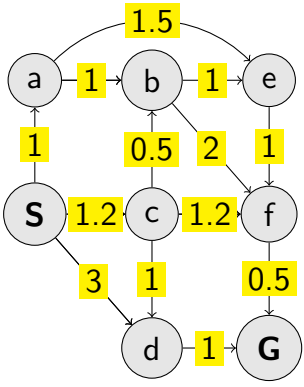

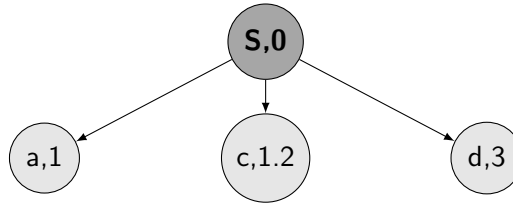

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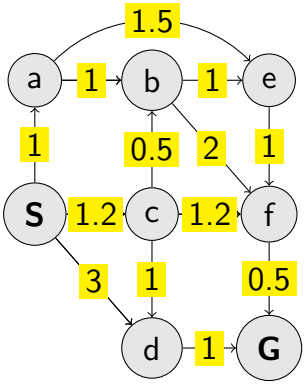

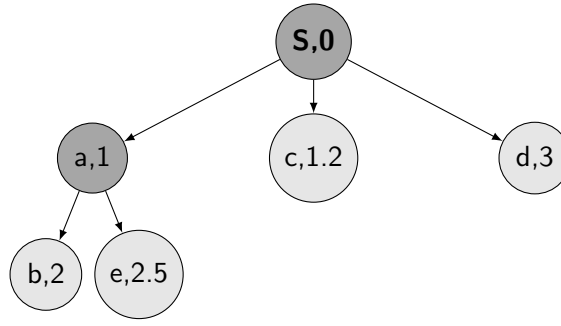

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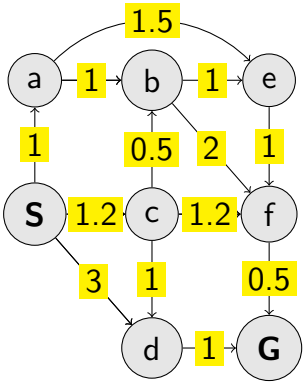

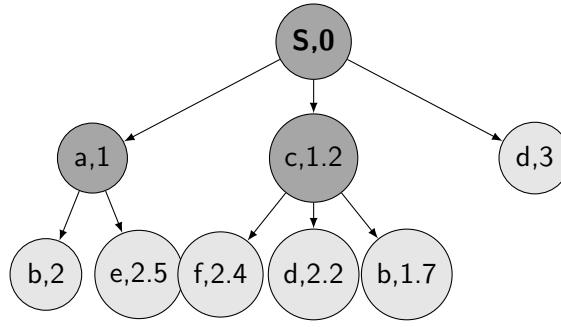

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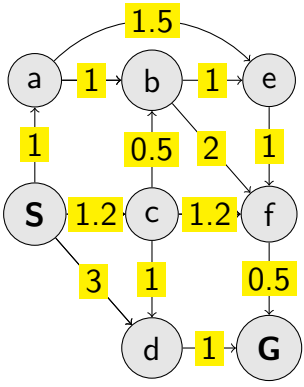

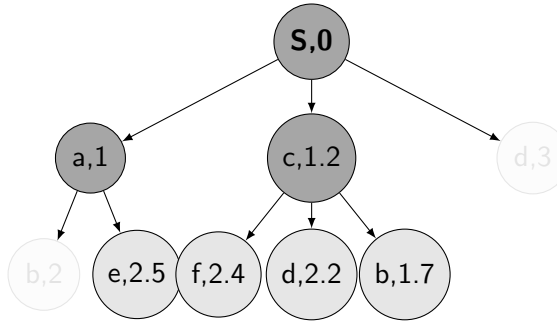

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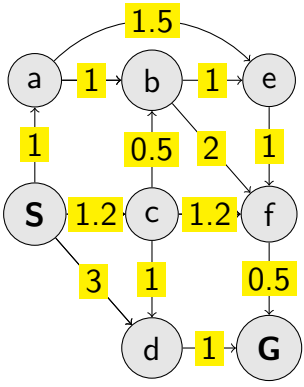

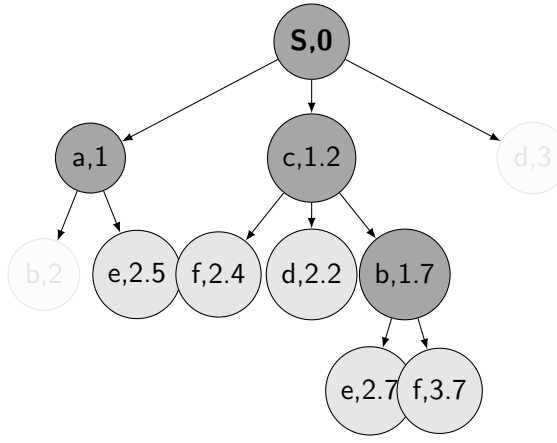

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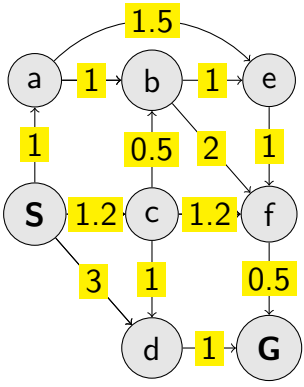

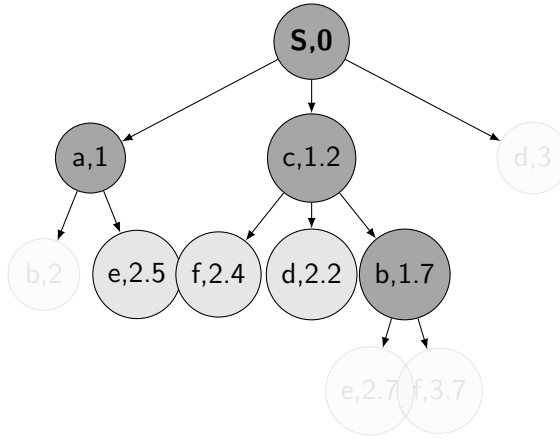

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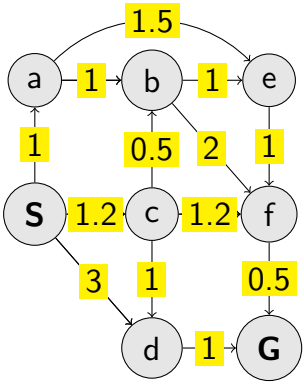

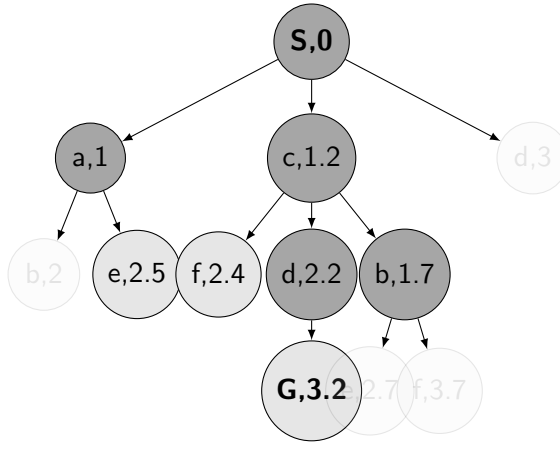

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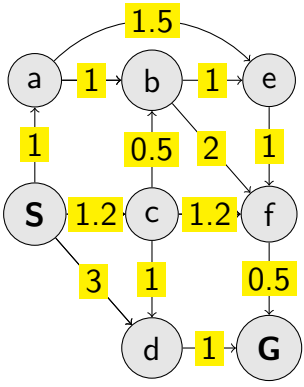

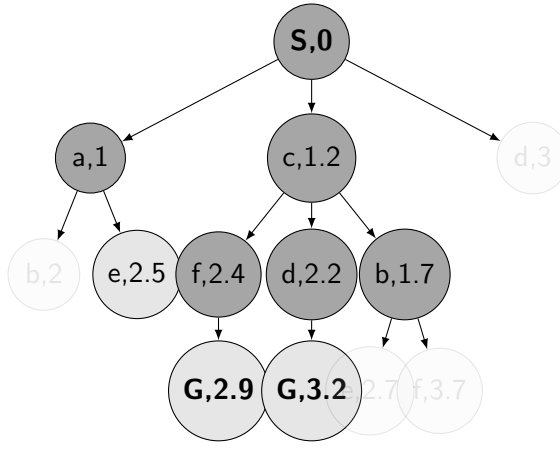

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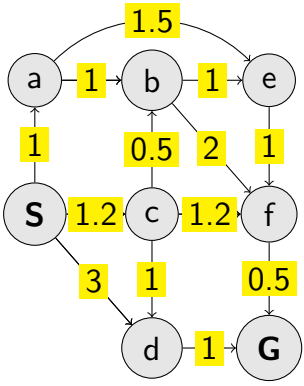

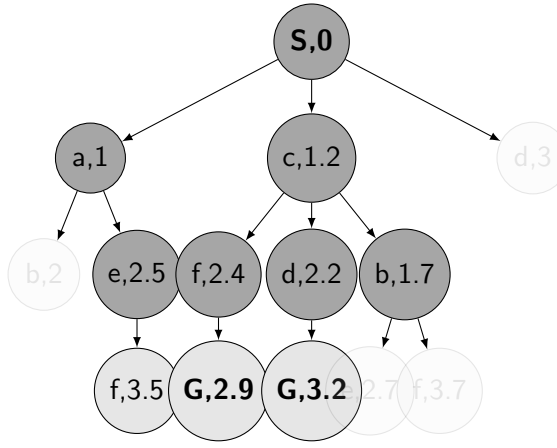

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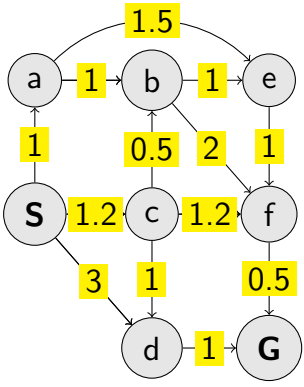

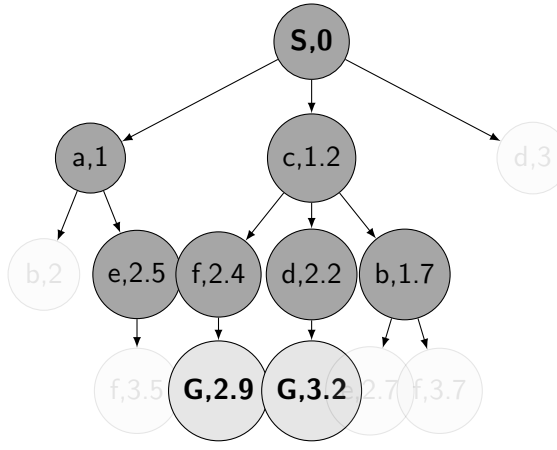

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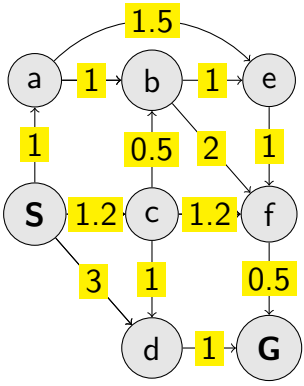

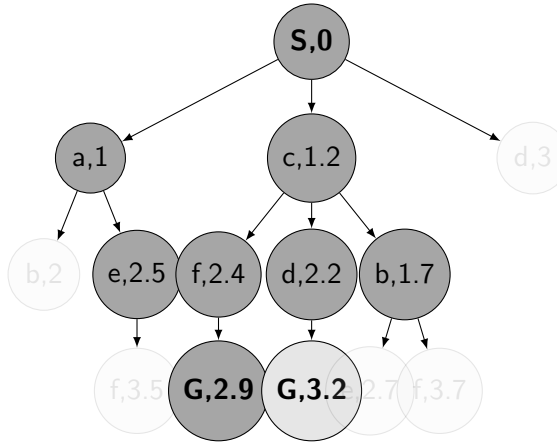

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function UCS_GRAPH_SEARCH(env) return a solution or failure
    node ← env.observe()
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    explored ← set()
    while frontier not empty do
        node ← frontier.pop()
        if node contains Goal then return node               ▷ check here!
        end if
        explored.add(node.state)
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        for all child_nodes do
            if child_node.state not in explored and not in frontier then
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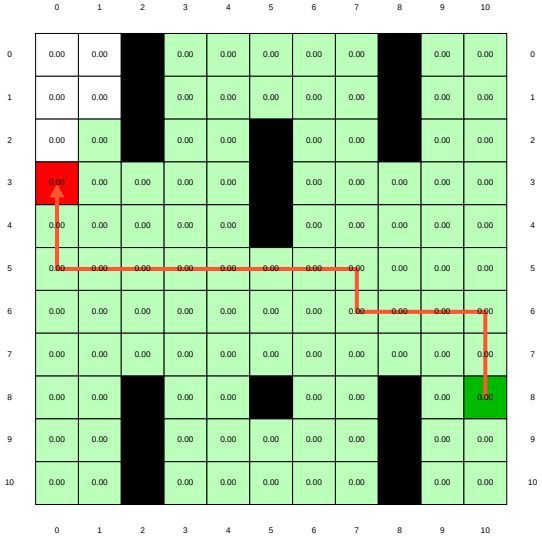
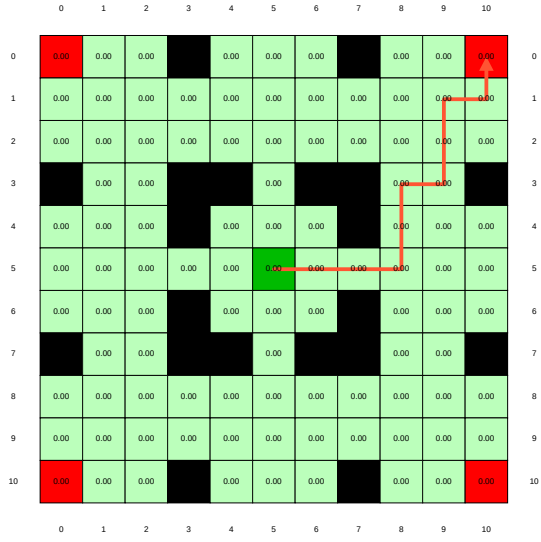
Few examples of search strategies so far



Run the demos.

Notes

What is wrong with UCS and other strategies?



Run the demo, or see https://youtu.be/TT5MY8xCgAg

Notes

Node selection, take argmin $f(n)$

Selecting next node to expand/visit (pop operation):

$$\texttt{node} \leftarrow \underset{n \in \texttt{frontier}}{\operatorname{argmin}} \; f(n)$$

What is $f(n)$ for DFS, BFS, and UCS?

- DFS:
- BFS:
- UCS:

- $f(n) = n.path\_cost$
- $f(n) = n.depth$
- $f(n) = -n.depth$

The good: (one) frontier as a priority queue
(I.e., priority queue will work universally. Still, stack (LIFO) and queue (FIFO) are (conceptually) the perfect data structures for DFS and BFS, respectively.)
The bad: All the $f(n)$ correspond to the cost from n to the start - only backward cost; cost-to-come (to n).

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Do humans look back when planing path? Is looking back important at all? If yes, when?

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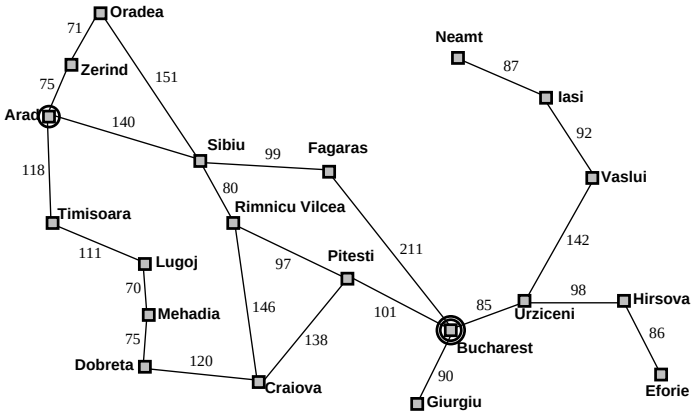
How far are we from the goal cost-to-go ? – Heuristics

- A function that estimates how close a *state* is to the goal.
- Designed for a particular problem.
- $h(n.state)$ – it is function of the state (attribute of node)
- It is often shortened as $h(n)$ – heuristic value of node n.

Notes

What happens if $h(n) =$ true cost?

Example of heuristics



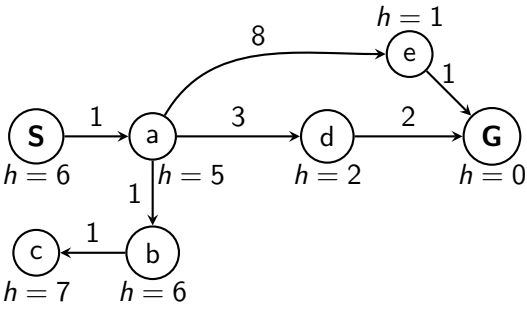
Arad	366	Mehadia	241
Bucharest	0	Neamt	234
Craiova	160	Oradea	380
Drobeta	242	Pitesti	100
Eforie	161	Rimnicu Vilcea	193
Fagaras	176	Sibiu	253
Giurgiu	77	Timisoara	329
Hirsova	151	Urziceni	80
Iasi	226	Vaslui	199
Lugoj	244	Zerind	374

Notes

Straight-line distance to Bucharest.

Illustration of *greedy* failing: Imagine going from Iasi to Fagaras. Neamt will be chosen for expansion. This will add Iasi back. Iasi is closer to Fagaras than Vaslui is and will be expanded again. Infinite loop... (3.5.1. in [2])

Greedy, take the $n^* = \operatorname{argmin}_{n \in \texttt{frontier}} h(n)$



What is wrong (and nice) with the Greedy?

Notes

Also called "Greedy best-first search" [2].
What will happen in this example:

1. Expand "S". Add "a" to frontier.
2. Expand "a". Add "b", "d", "e".
3. Expand "e" ($h = 1$). Get "G".

Wrong:

- not optimal
- not complete (tree search version) (Can be shown on the Romania example – go back.)
- (graph search version is complete only in finite state spaces)

Nice: it is simple.

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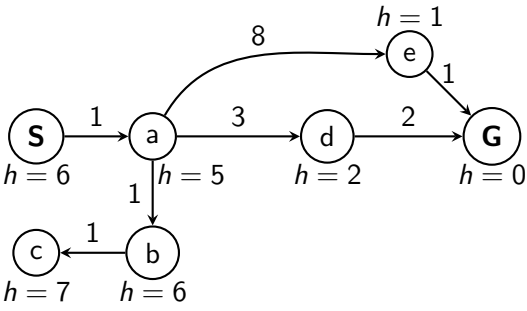

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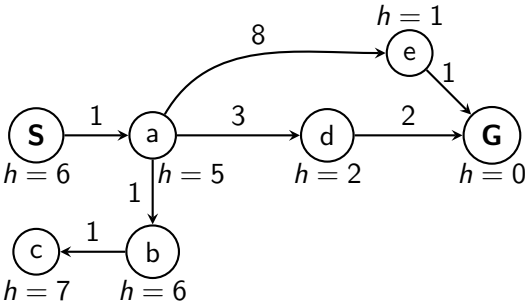

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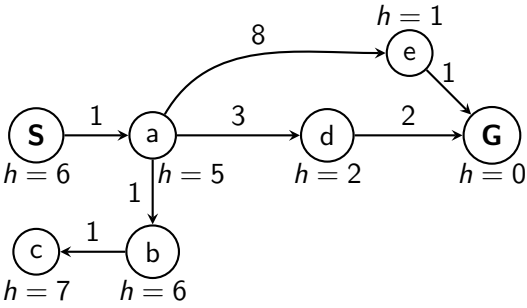

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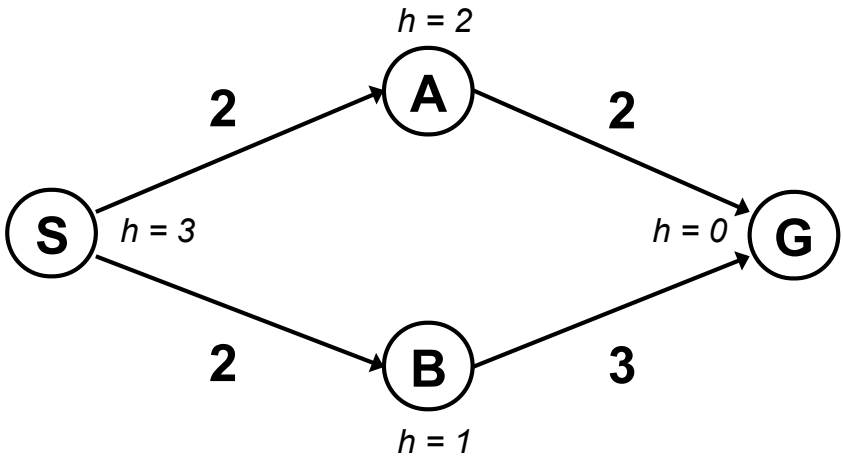

1

When poping G from `frontier`.

[1] Graph example: Dan Klein and Pieter Abbeel

Notes

1. S
   - $f(S) = g(S) + h(S) = 0 + 3 = 3$
   - expanding/poping this one and crossing out (removing from frontier)
2. $S \to A$
   - $f(A) = g(A) + h(A) = 2 + 2 = 4$
3. $S \to B$
   - $f(B) = g(B) + h(B) = 2 + 1 = 3$
   - expanding this one and crossing out
4. $S \to B \to G$
   - $f(G) = g(G) + h(G) = 5 + 0 = 5$
   - Should I stop now? No. Pop $S \to A$ with $f = 4$.
5. $S \to A \to G$
   - $f(G) = g(G) + h(G) = 4 + 0 = 4$
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Note: h is a function of the state. g is a function of a node (the path matters).

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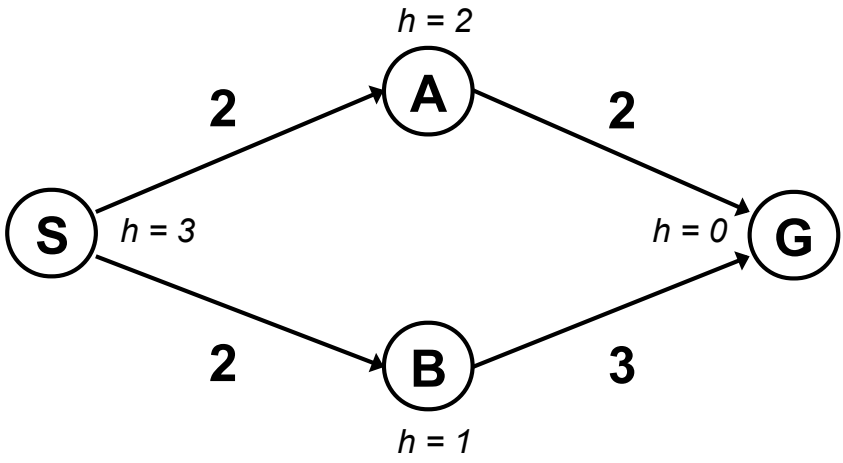

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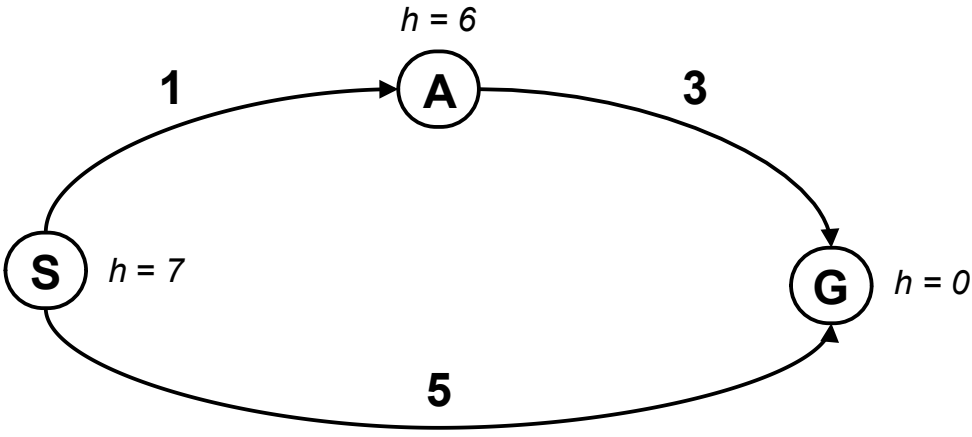
Is A* optimal?



2

What is the problem?

[2] Graph example: Dan Klein and Pieter Abbeel

Notes

Try to answer the question before going to the next slide.

1. S
   - $f(S) = g(S) + h(S) = 0 + 7 = 7$
   - expanding/poping this one and crossing out (removing from frontier)
2. $S \to A$
   - $f(A) = g(A) + h(A) = 1 + 6 = 7$
3. $S \to G$
   - $f(G) = g(G) + h(G) = 5 + 0 = 5$
   - This is now cheapest on the frontier. I pop/expand and I'm done.

Ooops! That's not cheapest! What went wrong?
What follows – keep for next slide. Problem with $h(A) = 6$. Overestimating the expense. (Same problem for $h(S)$.)
Estimates need to be $\leq$ actual costs.

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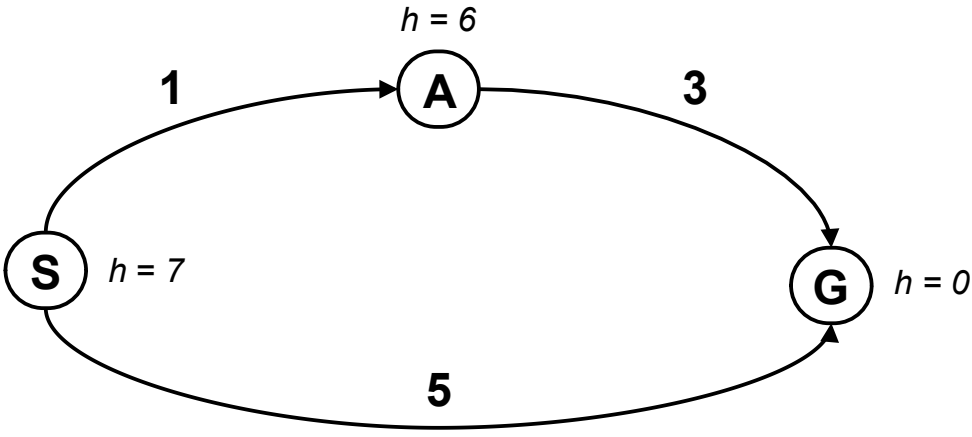

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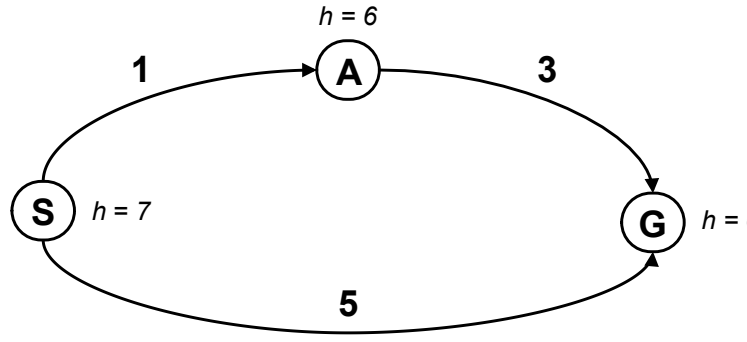
What follows – keep for next slide. Problem with $h(A) = 6$. Overestimating the expense. (Same problem for $h(S)$.)
Estimates need to be $\leq$ actual costs.

What is the right $h(A)$?

A: $0 \leq h(A) \leq 4$
B: $h(A) \leq 3$
C: $0 \leq h(A) \leq 3$
D: $0 \leq h(A)$



Notes

$h(A) \leq 3$ it means less than the actual cost of going from A to goal. Heuristic must not be overly pesimistic.
B is correct.
Negative $h(n)$ does not break the admissibility property but $h(\text{Goal}) = 0$ must be kept, always. Optimality of *tree-search* A^* is also kept if the termination waits till taking the Goal node from frontier out. We will discuss the *graph-search* version later.
For a discussion, see, e.g.

https://stackoverflow.com/questions/30067813/are-heuristic-functions-that-produce-negative-values-inadmissible

Admissible heuristics

A heuristic function h is admissible if:

$$\begin{aligned} h(n) &\leq \text{cost}(n.state, \text{Goal}_{\text{nearest}}) \\ h(\text{Goal}) &= 0 \end{aligned}$$

Notes

Even if negative heuristic value is allowed on the way to goal, does it make sense? How would you intepret $h(n) = 0$? Is it a meaningful minimum? Why?

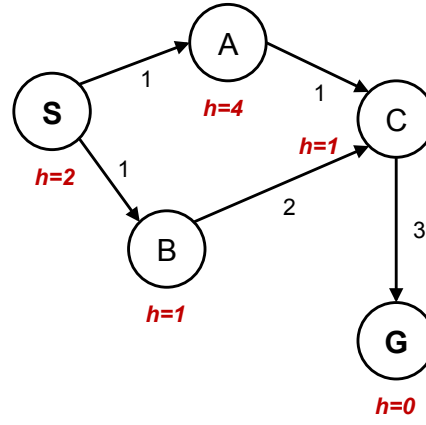
Optimality of A* tree search

A* is optimal if $h(n)$ is admissible.

Notes

A* graph search

```
function GRAPH_SEARCH(env)
    frontier.insert(startnode)
    explored ← set()
    while frontier do
        node ← frontier.pop()
        if goal in node then return node
        end if
        child_nodes ← env.expand(node.state)
        explored.add(node.state)
        for all child_nodes do
            if child_node.state not in explored then
                frontier.insert(child_node)
            end if
        end for
    end while
end function
```



What went wrong?

Graph example: Dan Klein and Pieter Abbeel

Notes

1. – $f(S) = g(S) + h(S) = 0 + 2 = 2$
   – expanding/poping this one and crossing out (removing from frontier); *explored set*: S
2. $S \to A$; $f(A) = g(A) + h(A) = 1 + 4 = 5$
3. $S \to B$; $f(B) = g(B) + h(B) = 1 + 1 = 2$
4. B is cheapest on the frontier. Expanding and removing from frontier; *explored set*: S, B
5. $B \to C$; $f(C) = g(C) + h(C) = 3 + 1 = 4$
6. C is cheapest on the frontier. Expanding and removing from frontier; *explored set*: S, B, C
7. $C \to G$; $f(G) = f(G) + h(G) = 6 + 0 = 6$
8. A is cheapest on the frontier. Expanding and removing from frontier; *explored set*: S, A, B, C
9. $A \to C$; $f(C) = f(C) + h(C) = 2 + 1 = 3$
10. C is cheapest on the frontier. But, it's on *explored set*! Can't be expanded.
11. Moving on to G, expanding and finishing.

Ooops! That's not cheapest! $cost(S \to B \to C \to G) = 6$; $cost(S \to A \to C \to G) = 5$ What went wrong?

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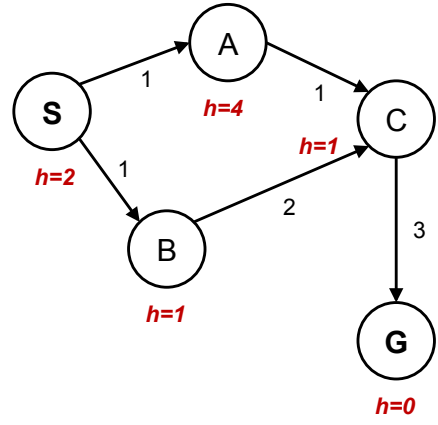

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11. Moving on to G, expanding and finishing.

Ooops! That's not cheapest! $cost(S \to B \to C \to G) = 6$; $cost(S \to A \to C \to G) = 5$ What went wrong?

What would be the proper $h(A)$?

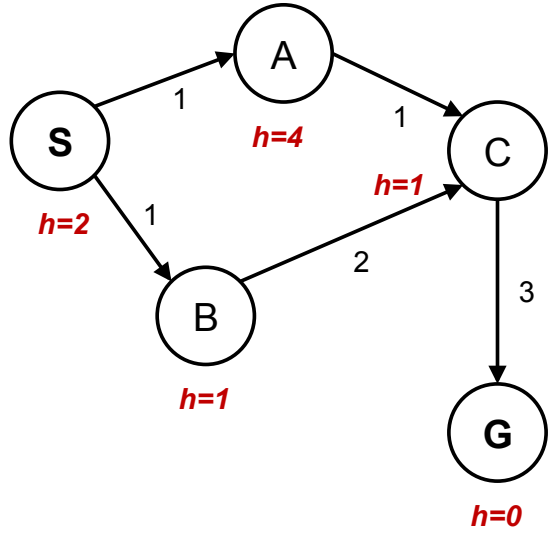
Consider other $h(s)$ fixed.

A: $h(A) = 1$

B: $h(A) = 2$

C: $1 \leq h(A) \leq 2$

D: $0 \leq h(A) \leq 1$

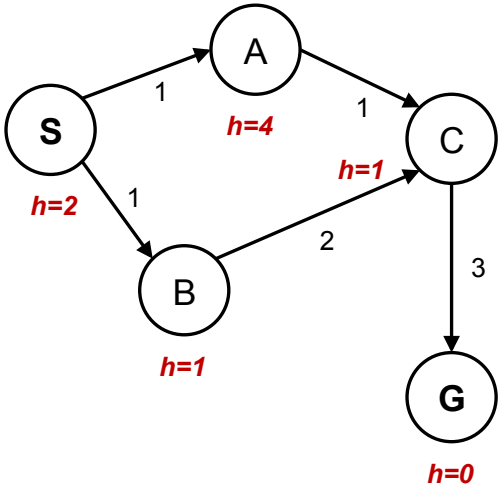


Notes

As it will be explained in the next slides: $h(A) \leq c(A, C) + h(C) = 2$

$h(S) \leq c(S, A) + h(A)$ it means $h(A) \geq h(S) - c(A, S) = 1$

Consistent heuristics



Admissible h:
$h(A) \leq$ true cost $A \to G$

Consistent h:
$h(A) - h(C) \leq$ true cost $A \to C$
in general:
$h(n) - h(s) \leq$ true cost $n \to s$ for any pair: node n and its successor s

$f(n) = g(n) + h(n)$ along a path never decreases!

Notes

Our heuristic was admissible.
With *tree search* it would have worked. It would have expanded C and found the alternative, cheaper path.
For graph search, the problem is the $A \to C \to G$ subgraph where the *consistent* heuristic condition is violated.
The general condition means we have two constraints for (A) for this particuar graph:
$h(S) - h(A) \leq c(S, A)$
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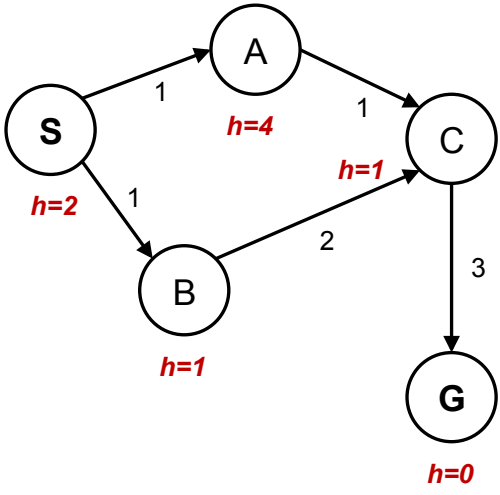
Consistent heuristics



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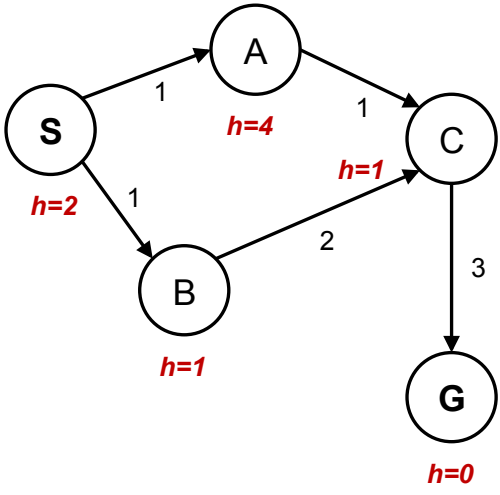

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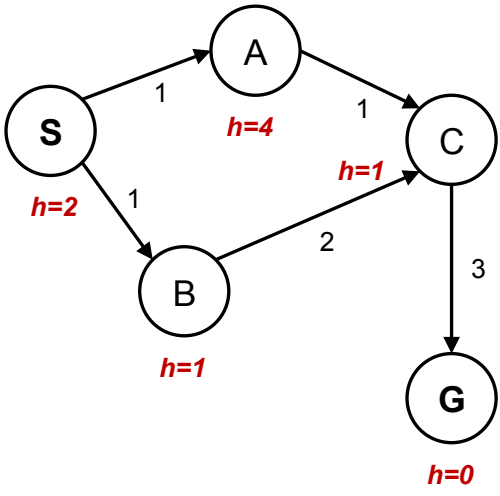

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Optimality of A*

- admissible h for tree search
- consistent h for graph search
- Are all consistent heuristics also admissible?

$h(A) - h(C) \leq \text{cost}(A \to C)$

Notes

Yes, all consistent heuristics are also admissible. Btw., it is not easy to invent a heuristics that is admissible but not consistent.

Optimality of A*

- admissible h for tree search
- consistent h for graph search
- Are all consistent heuristics also admissible?
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Yes, all consistent heuristics are also admissible. Btw., it is not easy to invent a heuristics that is admissible but not consistent.

Summary

- Graph vs Tree search – avoid repeating expansions
- Effectivness – adding heuristic estimates of cost-to-go
- Not all heuristics are equally good (admissibility, consistence, informativeness)

Notes

References, further reading

Some figures from [2]. Chapter 2 in [1] provides a compact/dense intro into search algorithms. (State space) Search algoritmhs are ubiquitous, explanations in many (text)books about Algorithms.
Nice online course from UC Berkeley (CS 188 Into to AI):
http://ai.berkeley.edu/lecture_videos.html Lecture: Informed Search.

[1] Steven M. LaValle.
Planning Algorithms.
Cambridge, 1st edition, 2006.
Online version available at: http://planning.cs.uiuc.edu.

[2] Stuart Russell and Peter Norvig.
Artificial Intelligence: A Modern Approach.
Prentice Hall, 3rd edition, 2010.
http://aima.cs.berkeley.edu/.

Notes