Fifth homework assignment announced 19. 5. 2023, due 2. 6. 2023 (prior to the exam as a zip file in Brute)

In this homework assignment, you are expected to collect 20 points, by working on exercises of your own choice out of the list provided below.

Discrete Quantum Walks

1. Demonstrate that the directional bias of a Hadamard walker, as in Figure 6.1, depends on the initial coin state. (3 points)

2. Verify that Fig. 6.3 is indeed correct (make your own plot). Explain what would we expect to see if we measured after each iteration of U. (3 points)

Quantum Walk on a Complete Graph

6. The trap, as defined in Fig. 6.5 makes no sense for a quantum walk (although this is not quite the case for a Szegedy walk). Explain why. (2 points)

7. Analyze the convergence properties of iterated applications of the unitary operator $U^n |x\rangle$ that implements a quantum walk. Under what conditions does $U^n|x\rangle$ converge, and how does the unitary property of preserving distances in the Hilbert space play a role in this convergence? (4 points)

8. Derive the probability of success at step 3 and step 4 for the quantum walk on K_4. (4 points)

9. Simulate the quantum walk on K_4 for a large number of steps and for $N \sim 1000$. Show that the analogue of Fig. 6.6 shows an oscillatory behavior. (4 points)

Szegedy Walks

9. Simulate a Szegedy walk on your favorite graph. Compare the validity of your results against `QuantumWalk.jl`. (6 points)

Continuous-time Quantum Walks

10. Prove that Eq. (6.25) holds. Hint: Use a characteristic property about the columns of L. (4 points)

Quantum Walk on the Hypercube

11. Below Eq. (6.33) we read: "Note that $U(\pi/2)$ flips every bit of the state ... the opposite vertex of the hypercube." Pick your favorite $n > 2$ and demonstrate this. (3 points)

Quantum Amplitude Estimation and Monte Carlo Sampling

12. Derive the intermediate steps between Eqs. (6.57) and (6.58). (4 points)

Quantum Adiabatic Computation

13. Consider $H_{\text{clock init}}$ and assume that the initial clock state is other than $|0\rangle^{\otimes L} := |0^L\rangle^c$. Show that for $L = 4$ and for initial clock state $|0010\rangle$ we get a penalty in the energy. (4 points)

14. Prove that the state $|\gamma_0\rangle$ is an eigenstate of H_{init} with zero eigenvalue. (4 points)

Variational Quantum Algorithms

13. Showcase a small instance (e.g., $n \geq 3$, L = 1, 2), where QAOA produces the global optimum. (4 points)

14. Showcase a small instance (e.g., $n \geq 3$, L = 1, 2), where QAOA produces a particularly bad optimum. (4 points)

15. Summarize the performance guarantees of Brownian rounding for MAXCUT, based on `https://arxiv.org/abs/1812.07769`. (4 points)

16. Explain how the performance guarantees of Brownian rounding for MAXCUT can be extended to performance guarantees of warm-started QAOA of Egger et al. (`https://arxiv.org/abs/2009.10095`). (4 points)

17. The variational quantum factoring (`https://arxiv.org/abs/1808.08927`) suggests the use of QUBO therein, and explains the QUBO. Showcase an example thereof for factoring 15, incl. the optimizer (i.e., values of all of the variables including the carry bits that attain the best possible objective-function-value). (4 points)

18. The Schnorr factoring paper (`https://arxiv.org/pdf/2212.12372.pdf`) suggests the use of QUBO therein, but does not explain the QUBO instance used. Formulate the QUBO solved in Schnorr factoring (`https://arxiv.org/pdf/2212.12372.pdf`). (6 points)