Quantum Computing

Exercises 2: Qubits

1. a) Let us consider the set $\{|0\rangle, |1\rangle\}$, that forms a basis in $\mathbb{C}^2$ (the computational basis). Calculate the vectors in $\mathbb{C}^4$:

$$|0\rangle \otimes |0\rangle \ , \ |0\rangle \otimes |1\rangle \ , \ |1\rangle \otimes |0\rangle \ , \ |1\rangle \otimes |1\rangle$$

and interpret the result.
b) Consider the Pauli matrices σ_x and σ_z . Find $\sigma_x \otimes \sigma_z$ and $\sigma_z \otimes \sigma_x$ and discuss. Both σ_x and σ_z are hermitian. Are $\sigma_x \otimes \sigma_z$ and $\sigma_z \otimes \sigma_x$ hermitian? Both σ_x and σ_z are unitary. Is $\sigma_x \otimes \sigma_z$ and $\sigma_z \otimes \sigma_x$ unitary?

2. Consider again the computational basis for, $\{|0\rangle, |1\rangle\}$. The Walsh-Hadamard transform is a 1-qubit operation, denoted by H, and performs the linear transform

$$|0\rangle \rightarrow \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \ , \ |1\rangle \rightarrow \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle).$$

a) Find the unitary operator U_H which implements H with respect to the basis $\{|0\rangle, |1\rangle\}$.
b) Find the inverse of this operator.
c) Find its matrix representation in the computational (standard) basis:

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \ , \ |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix},$$

and in the Hadamard basis:

$$|0\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix} \ , \ |1\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix}.$$

3. [Nielsen & Chuang Ex. 4.1] Find the points on the Bloch sphere which correspond to the normalized eigenvectors of the different Pauli matrices.