ARO 2022 **Name:** ____________________

Worksheet

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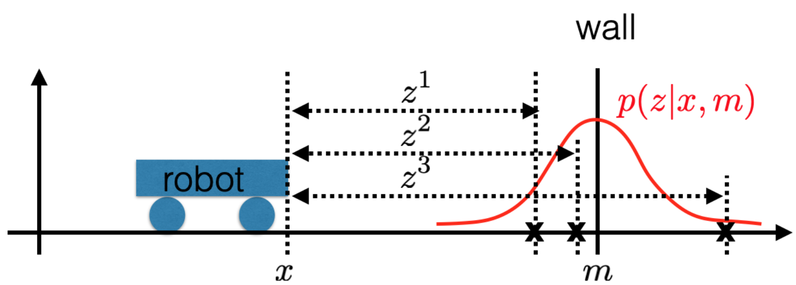
1. **MLE mapping:** Consider the robot at pose $x = 3$ measuring the distance from the wall (robot is assumed to operate only on the left side of the wall). It performs three i.i.d. measurements $z^1 = 4, z^2 = 5, z^3 = 8$ of the distance (between its pose x and wall m). Each measurement is assumed to have zero-mean Gaussian noise with the same variance σ^2.



   a) What is the probability distribution from which the measurements z has been generated (feel free to use normalizing constant K instead of $1/\sigma\sqrt{2\pi}$)?

   $p(z|x, m) =$

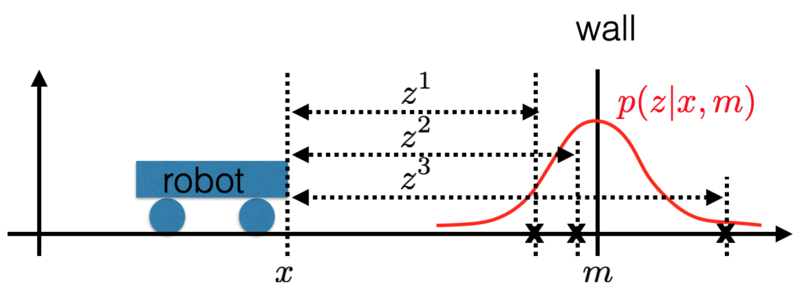
   b) Derive what is the maximum likelihood estimate of the wall position?

   $m =$

   c) What is the least squares estimate of the wall position?

   $m =$

2. **MLE localization:** Let us assume that robot changed its location (i.e. it moved wrt mapping example). The motion model as well as the performed action are completely unknown, however the pose of the wall m from the previous computations is assumed to be valid (i.e. wall has not moved in the coordinate frame of the world). The robot performs another three i.i.d. measurements $z^1 = 1, z^2 = 2, z^3 = 9$ with the same sensor at this new pose x.



a) What is the maximum likelihood estimate of its pose?

$x =$

b) Assume that we have improved quality of the sensor, that the resulting variance σ^2 is two times smaller. What is the maximum likelihood estimate of its pose, given this new sensor?

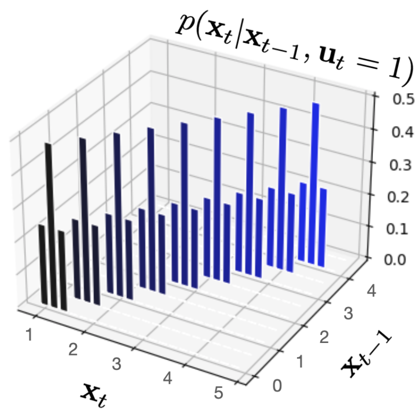
$x =$

c) Let us assume that some of the measurements can be outliers. Assume that the maximum distance between the measured pose and the true pose (i.e. inlier tolerance) is equal to 2 and perform several iterations of the RANSAC algorithm to estimate the inliers. What is the maximum likelihood estimate of the pose from the inliers?

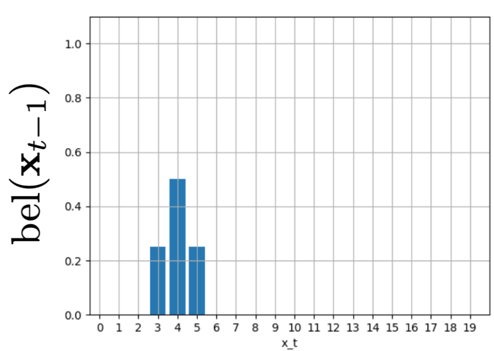
$x =$

3. **Bayes filter:** Let us assume robot operating in one-dimensional world. Its pose in time t is denoted as $\mathbf{x}_t$. It uses Bayes filter to recursively update posterior belief about its pose. Consider the state-transition probability $p(\mathbf{x}_t|\mathbf{x}_{t-1}, \mathbf{u}_t)$ depicted on the following image. The accurate values are as follows:

   $p(\mathbf{x}_t = 1|\mathbf{x}_{t-1} = 1, \mathbf{u}_t = 1) = 0.25,$
   $p(\mathbf{x}_t = 2|\mathbf{x}_{t-1} = 1, \mathbf{u}_t = 1) = 0.5,$
   $p(\mathbf{x}_t = 3|\mathbf{x}_{t-1} = 1, \mathbf{u}_t = 1) = 0.25$



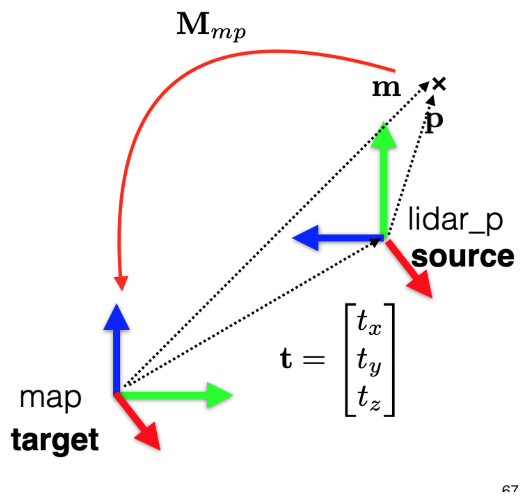
   You are given posterior belief $\text{bel}(\mathbf{x}_{t-1})$ from the previous time $t-1$. See following image for an outline.



   Robot performs action $\mathbf{u}_t = 1$ in time t. What is the belief after the prediction step?
   Hint: $\overline{\text{bel}}(\mathbf{x}_t) = \int p(\mathbf{x}_t|\mathbf{x}_{t-1}, \mathbf{u}_t)\text{bel}(\mathbf{x}_{t-1})d\mathbf{x}_{t-1}$

$\overline{\text{bel}}(\mathbf{x}_t) =$

4. **Transformations in homogeneous coordinates:** Lidar measures point $\mathbf{p}$ in its own coordinate frame. The color encodes axes as follows: x-axis = red, y-axis=green, z-axis=red. The translation of the lidar in the map coordinate frame is $\mathbf{t} = [0, 10, 10]$, rotation is $+90°$ around x-axis.



a) What is the homogeneous transformation between the lidar and the map?

$\mathbf{M}_{mp} =$

b) Given the point $\mathbf{p} = [1, 2, 3]^\top$ in the lidar coordinate frame, what are its coordinates $\mathbf{m}$ in the map coordinate frame ?

$\mathbf{m} =$

c) Given the point $\mathbf{m} = [1, 2, 3]^\top$ in the map coordinate frame, what are its coordinates $\mathbf{p}$ in the lidar coordinate frame ?

$\mathbf{p} =$

5. **3D PCL $\Rightarrow$ RGB:** Consider a perspective camera with the following intrinsic camera parameters K, camera rotation matrix R, and translation vector t:

$$\mathbf{K} = \begin{bmatrix} 500 & 0 & 500 \\ 0 & 500 & 250 \\ 0 & 0 & 1 \end{bmatrix}, \mathbf{R} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}, \mathbf{t} = \begin{bmatrix} -2 \\ 0 \\ -1 \end{bmatrix},$$

a) Construct camera projection matrix $\mathbf{P} \in \mathcal{R}^{3\times 4}$:

$\mathbf{P} =$

b) Project point

$$\mathbf{q} = \begin{bmatrix} 1 \\ 3 \\ 0 \end{bmatrix}$$

into the camera. What are pixel coordinates $\mathbf{u} \in \mathcal{R}^2$ of the projection?

$u_1 =$

$u_2 =$

6. **RGBD $\Rightarrow$ 3D PCL:** Consider the RGBD perspective camera with the previously defined parameters:

$$\mathbf{K} = \begin{bmatrix} 500 & 0 & 500 \\ 0 & 500 & 250 \\ 0 & 0 & 1 \end{bmatrix}, \mathbf{R} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}, \mathbf{t} = \begin{bmatrix} -2 \\ 0 \\ -1 \end{bmatrix},$$

You are given pixel coordinates $\mathbf{u} = [500; -50]$. Depth estimated at this coordinates by the RGBD sensor is $D(\mathbf{u}) = 3$. What is corresponding 3D point $\mathbf{q} \in \mathcal{R}^3$?
Hint: Start working in camera coordinate frame (i.e. ignore provided $\mathbf{R}$ and $\mathbf{t}$) and find point $\mathbf{p}$ such that $p_z = D(\mathbf{u})$ and its projection is $\mathbf{u}$. Then transform point $\mathbf{p}$ to world c.f. $\mathbf{q}$ such that $\mathbf{p} = \mathbf{R}\mathbf{q} + \mathbf{t}$. You can easily verify that projection of $\mathbf{q}$ to the given yields provided pixel $\mathbf{u}$.

7. What is true for the configuration space?

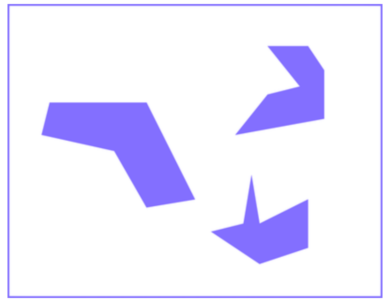
- Configuration space is a set of all possible configurations of the robot
- It has as many dimensions as the dimension of the workspace
- It has the same number of dimensions as the number of degrees of freedom of the robot
- It has the same number of dimensions as the number of degrees of freedom of the workspace
- The number of dimensions of the configuration space is the Minkowski sum of degrees of freedom of the robot and the degrees of freedom of the robot

8. Let's assume a planar robotic manipulator with n revolute joints.

- The corresponding configurations space is n-dimensional
- The corresponding configurations space is (n+1)-dimensional
- The corresponding configurations space is (n-1)-dimensional
- The corresponding configurations space is 2(n-1)-dimensional
- The corresponding configurations space is (2n)-dimensional

9. Consider 2D polygonal workspace (example is depicted in the figure) and a circular robot. Let C_1 is the configurations space of a circular robot with radius r_1 moving in the workspace. Let C_2 is the configurations space of another circular robot with radius $r_2, r_2 > r_1$ moving in the same workspace. What you can say about the two configuration spaces?

- Both have the same number of dimensions
- The number of dimensions of C_2 is bigger, because the 2nd robot is bigger
- Volume of collision-free region $C^1_{free} \subseteq C_1$ is the same as volume of $C^2_{free} \subseteq C_2$
- Volume of collision-free region $C^1_{free} \subseteq C_1$ is smaller than volume of collision-free region $C^2_{free} \subseteq C_2$



- Volume of collision-free region $C^1_{free} \subseteq C_1$ is bigger than volume of collision-free region $C^2_{free} \subseteq C_2$

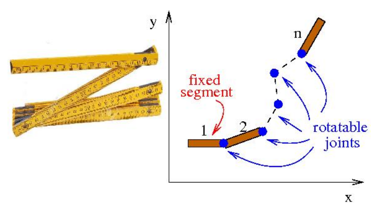
10. What is probabilistic completness of path/motion planning algorithms?

11. Describe how RRT works (few sentences, or use drawing with explanation).

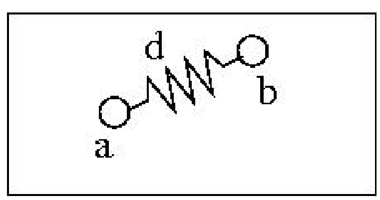
12. When is RRT terminated?

13. What is goal-bias in Rapidly-exploring Random Tree (RRT)?

14. Let assume a folding wooden meter with n segments. Let assume that the meter is fixed at its 1st segment (i.e., the first segment cannot move at all). How many degrees of freedom the meter has?



15. Consider path planning for two disc robots moving in 2D space with obstacles (heading of the robots is not important). The distance between the particles d is variable and it must be in the range $d \in (0, 1)$ meters. How many degrees of freedom has the corresponding configuration space?



16. Consider paths planned by RRT, PRM and Voronoi diagram method. Which path has the maximum clearance?