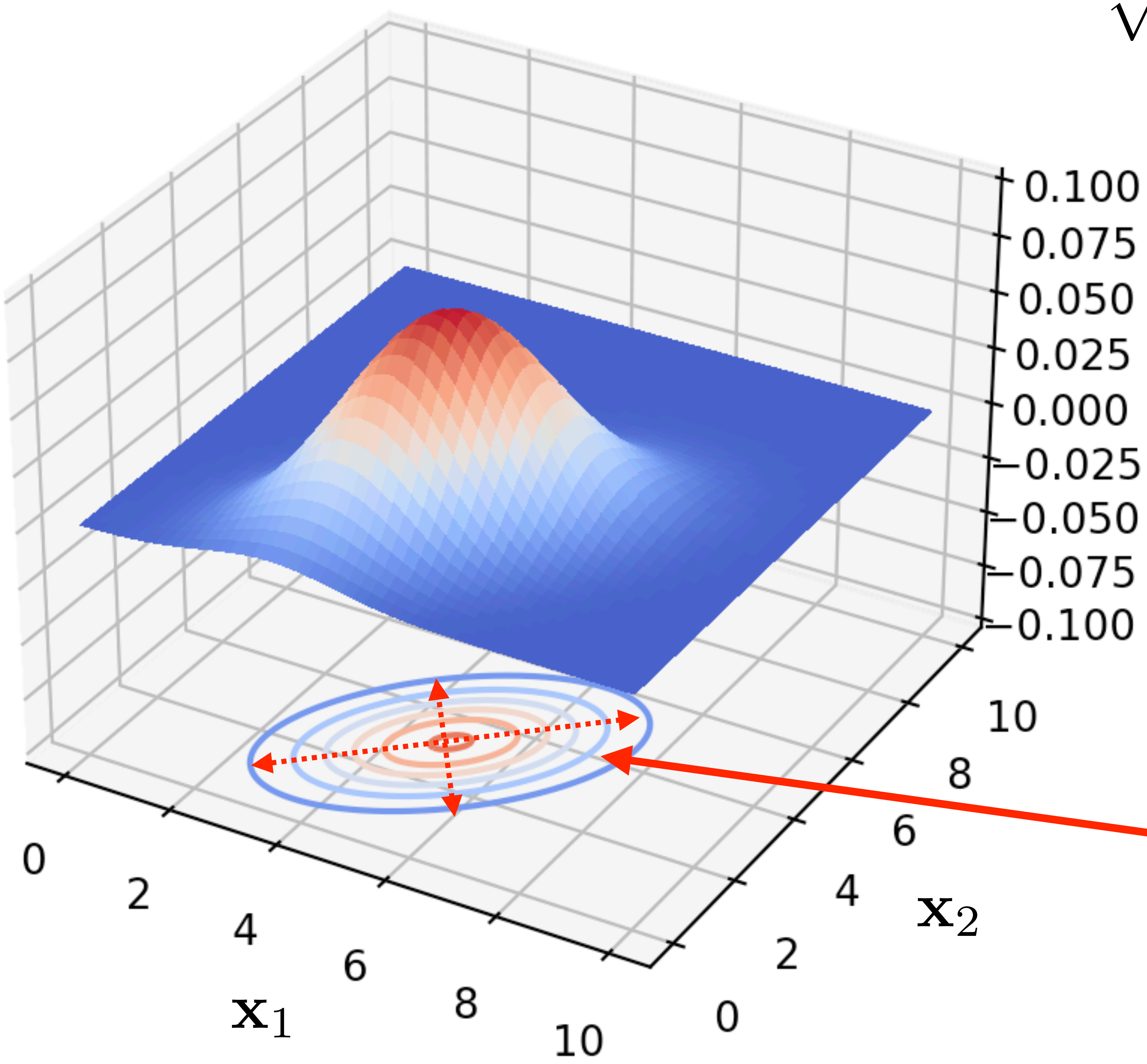
Localization - Kalman filter

Karel Zimmermann

Multivariate gaussian

$$p(\mathbf{x}) = \mathcal{N}_{\mathbf{x}}(\boldsymbol{\mu}, \boldsymbol{\Sigma}) = \frac{\exp\Big(-\frac{1}{2}(\mathbf{x}-\boldsymbol{\mu})^\top \boldsymbol{\Sigma}^{-1}(\mathbf{x}-\boldsymbol{\mu})\Big)}{\sqrt{(2\pi)^n \det(\boldsymbol{\Sigma})}}$$



$\mathbf{x} \in \mathcal{R}^n$ … real n-dimensional random column vector

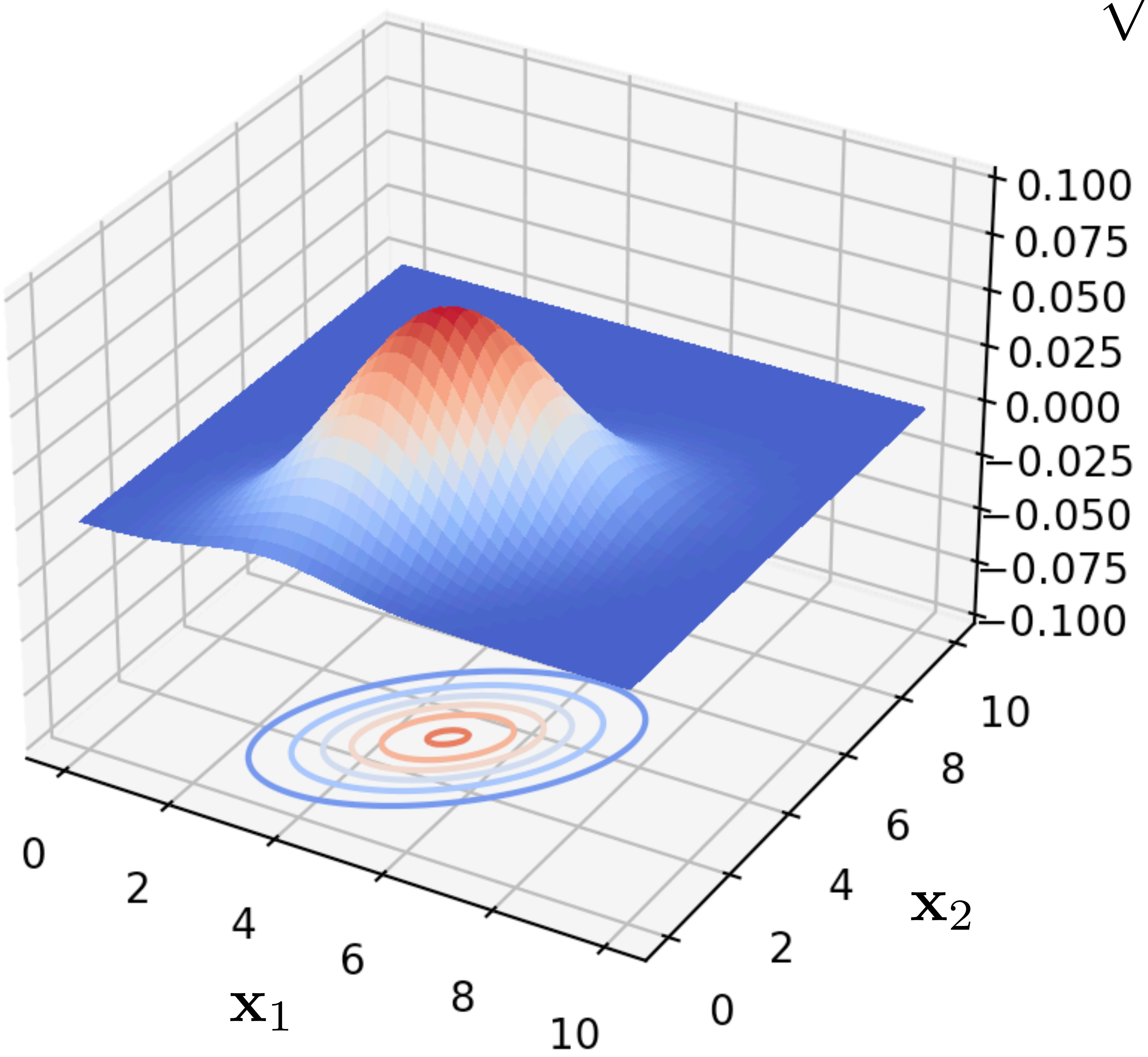
$\boldsymbol{\mu} \in \mathcal{R}^n$ … real n-dimensional mean

$\boldsymbol{\Sigma} \in \mathcal{R}^{n \times n}$ … symmetric positive definite covariance matrix

eigenvalues and eigenvectors of $\boldsymbol{\Sigma}$ determine ellipse axes

Multivariate gaussian

$$p(\mathbf{x}) = \mathcal{N}_{\mathbf{x}}(\boldsymbol{\mu}, \boldsymbol{\Sigma}) = \frac{\exp\left(-\frac{1}{2}(\mathbf{x}-\boldsymbol{\mu})^\top \boldsymbol{\Sigma}^{-1}(\mathbf{x}-\boldsymbol{\mu})\right)}{\sqrt{(2\pi)^n \det(\boldsymbol{\Sigma})}}$$



Gaussian distributions are closed under:

- Affine transformation
- Chain rule
- Marginalization
- Conditioning

System model

$$p(\mathbf{x}) = \mathcal{N}_{\mathbf{x}}(\boldsymbol{\mu}, \boldsymbol{\Sigma}) = \frac{\exp\Big(-\frac{1}{2}(\mathbf{x}-\boldsymbol{\mu})^\top \boldsymbol{\Sigma}^{-1}(\mathbf{x}-\boldsymbol{\mu})\Big)}{\sqrt{(2\pi)^n \det(\boldsymbol{\Sigma})}}$$

Linear system:

$$\mathbf{x}_t = \mathbf{A}_t \mathbf{x}_{t-1} + \mathbf{B}_t \mathbf{u}_t$$

$$\mathbf{z}_t = \mathbf{C}_t \mathbf{x}_t$$

Let’s add Gaussian noise

System model

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Linear system with Gaussian noise:

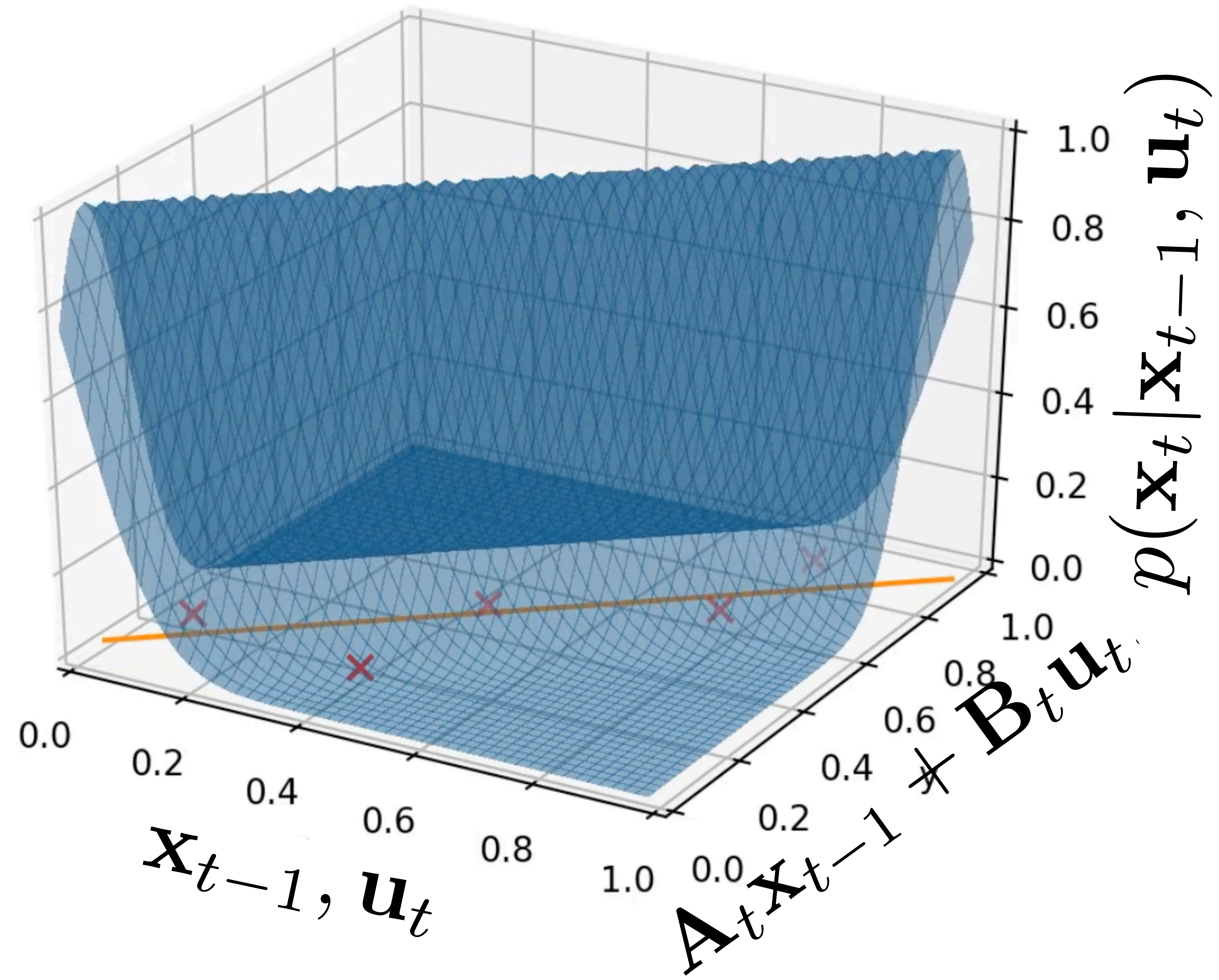
$$p(\mathbf{x}_t|\mathbf{x}_{t-1}, \mathbf{u}_t) = \mathcal{N}_{\mathbf{x}_t}(\mathbf{A}_t\mathbf{x}_{t-1} + \mathbf{B}_t\mathbf{u}_t,\ \mathbf{R}_t)$$

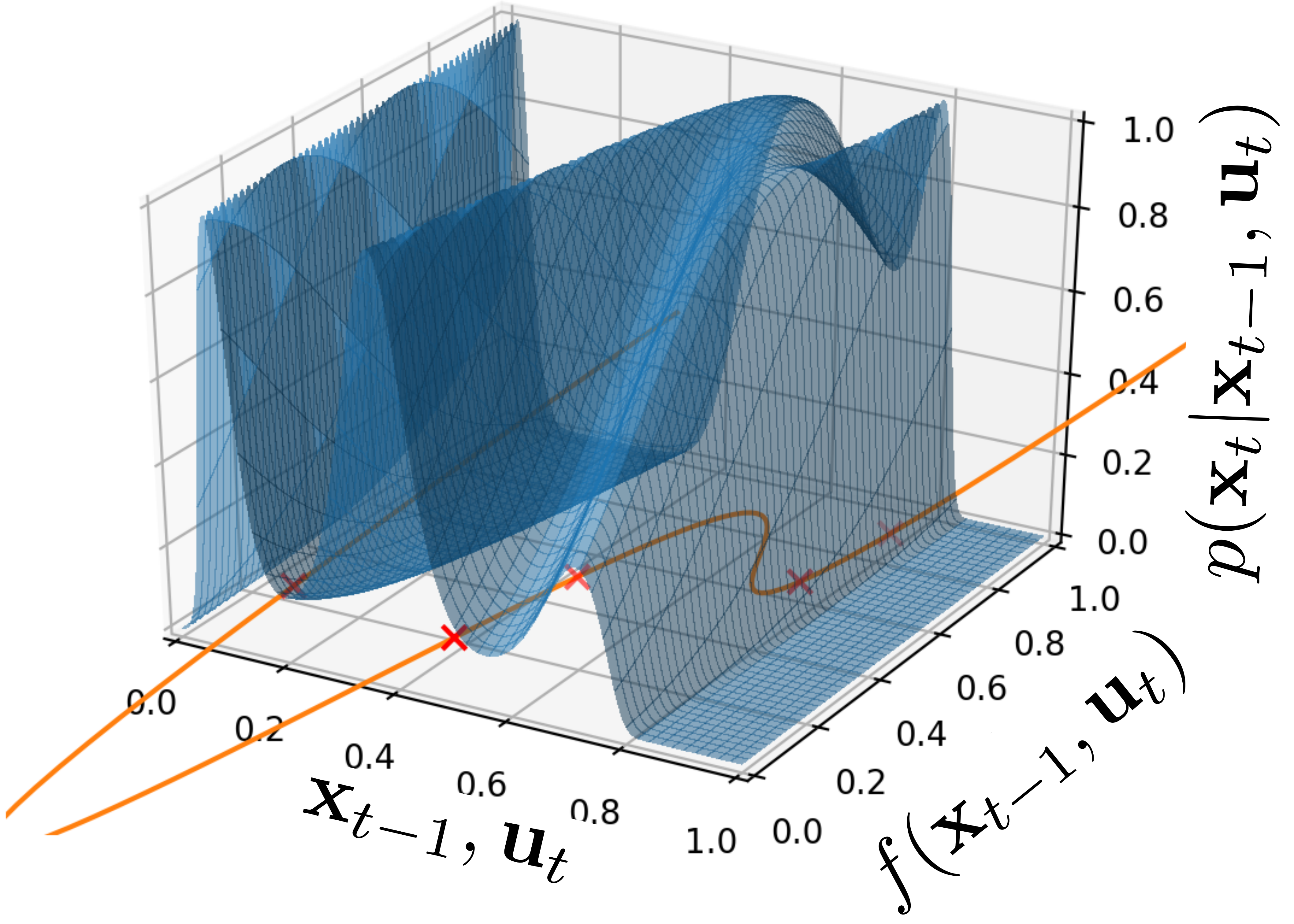
$$p(\mathbf{z}_t|\mathbf{x}_t) = \mathcal{N}_{\mathbf{z}_t}(\mathbf{C}_t\mathbf{x}_t,\ \mathbf{Q}_t)$$

Non-linear system with Gaussian noise:

$$p(\mathbf{x}_t|\mathbf{x}_{t-1}, \mathbf{u}_t) = \mathcal{N}_{\mathbf{x}_t}(f(\mathbf{x}_{t-1}, \mathbf{u}_t),\ \mathbf{R}_t)$$

$$p(\mathbf{z}_t|\mathbf{x}_t) = \mathcal{N}_{\mathbf{z}_t}(h(\mathbf{x}_t),\ \mathbf{Q}_t)$$





Kalman filter

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Bayes filter:

$$\text{bel}(\mathbf{x}_t) = \eta \cdot p(\mathbf{z}_t|\mathbf{x}_t) \cdot \underbrace{\int p(\mathbf{x}_t|\mathbf{x}_{t-1}, \mathbf{u}_t) \cdot \text{bel}(\mathbf{x}_{t-1}) d\mathbf{x}_{t-1}}_{\overline{\text{bel}}(\mathbf{x}_t)}$$

Kalman filter:

Prediction step

$$\overline{\text{bel}}(\mathbf{x}_t) = \mathcal{N}_{\mathbf{x}_t}(\overline{\boldsymbol{\mu}}_t, \overline{\boldsymbol{\Sigma}}_t)$$

$$\overline{\boldsymbol{\mu}}_t = \mathbf{A}_t\boldsymbol{\mu}_{t-1} + \mathbf{B}_t\mathbf{u}_t$$

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Measurement update step

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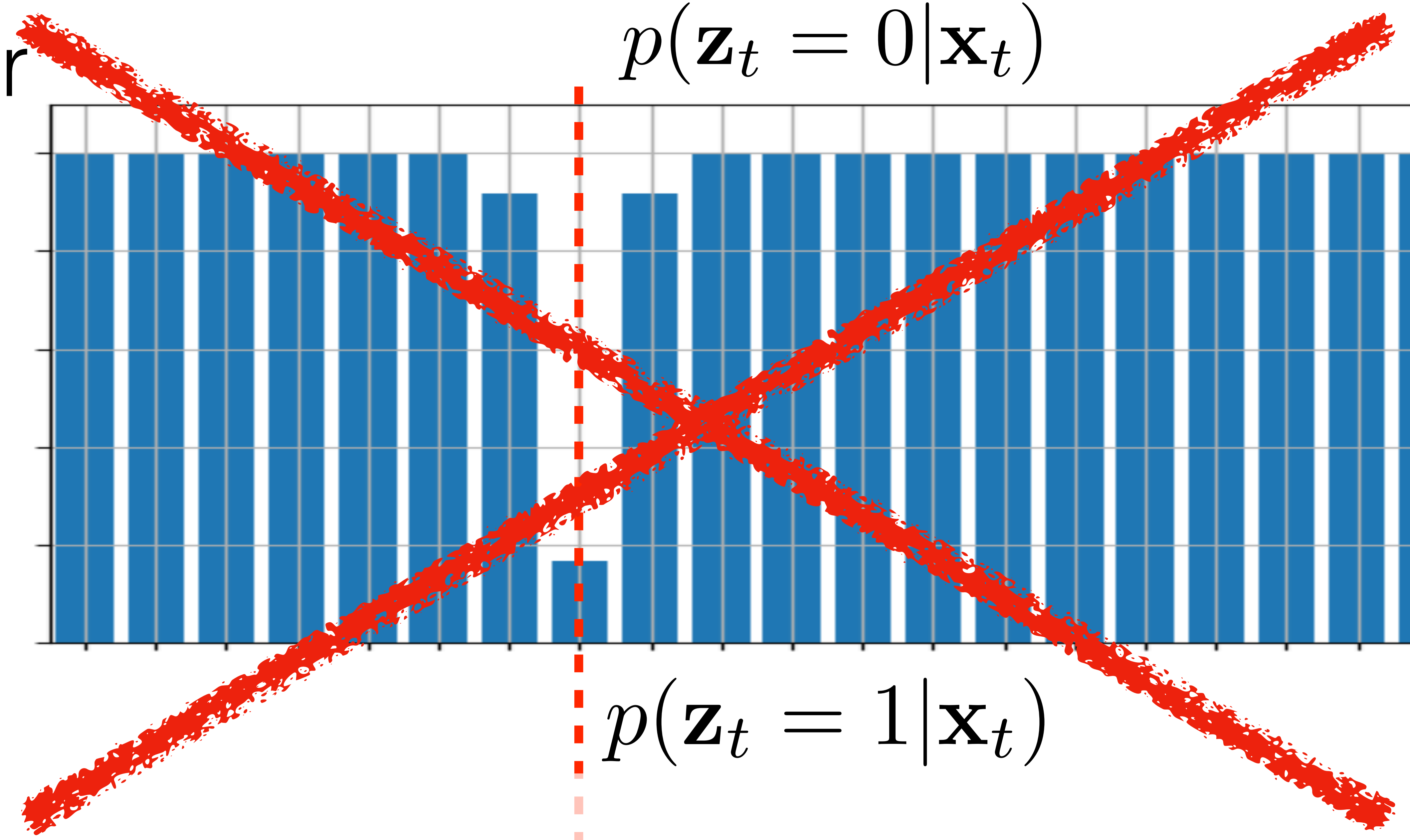

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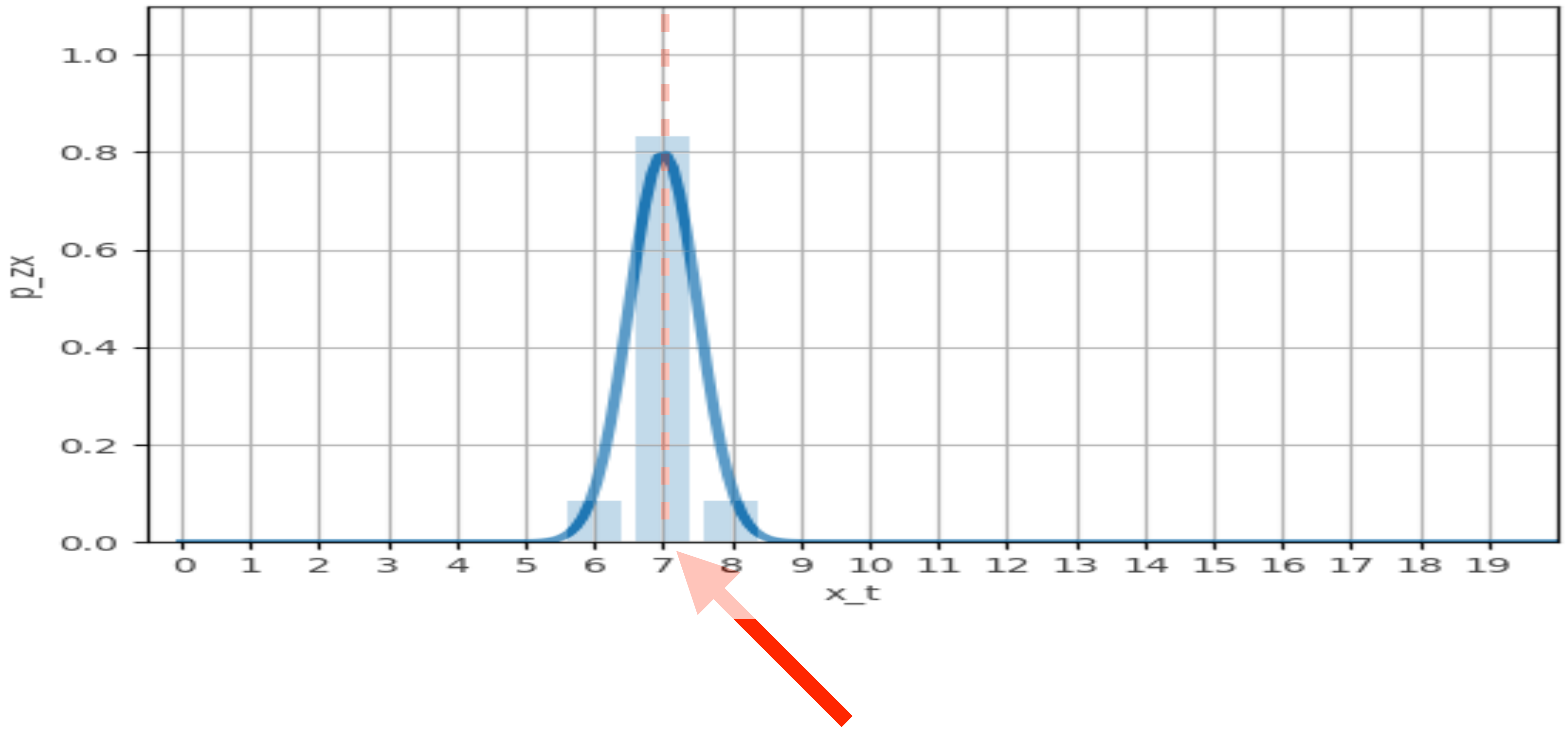

one marker at known locations
+
inaccurate sensor

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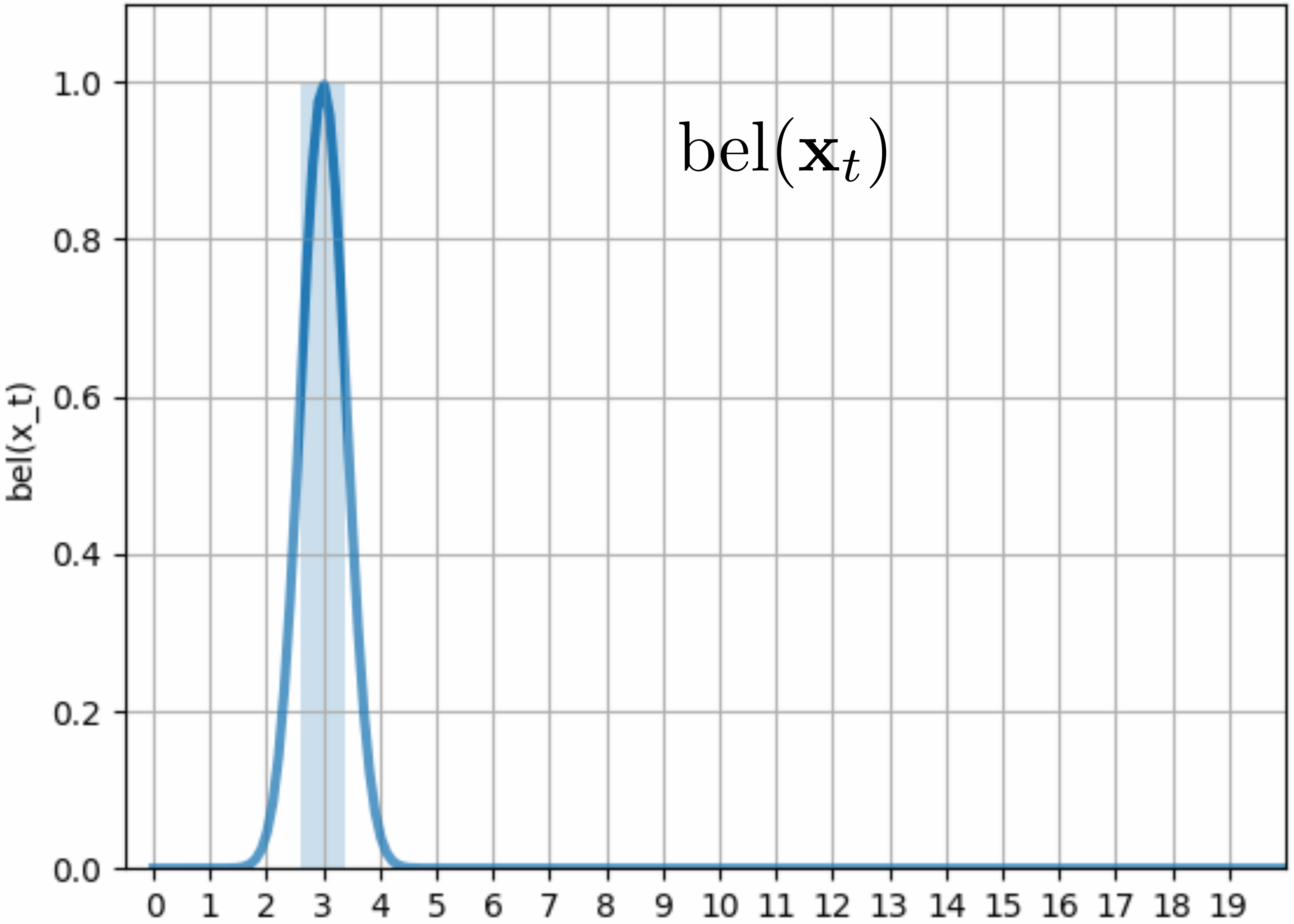

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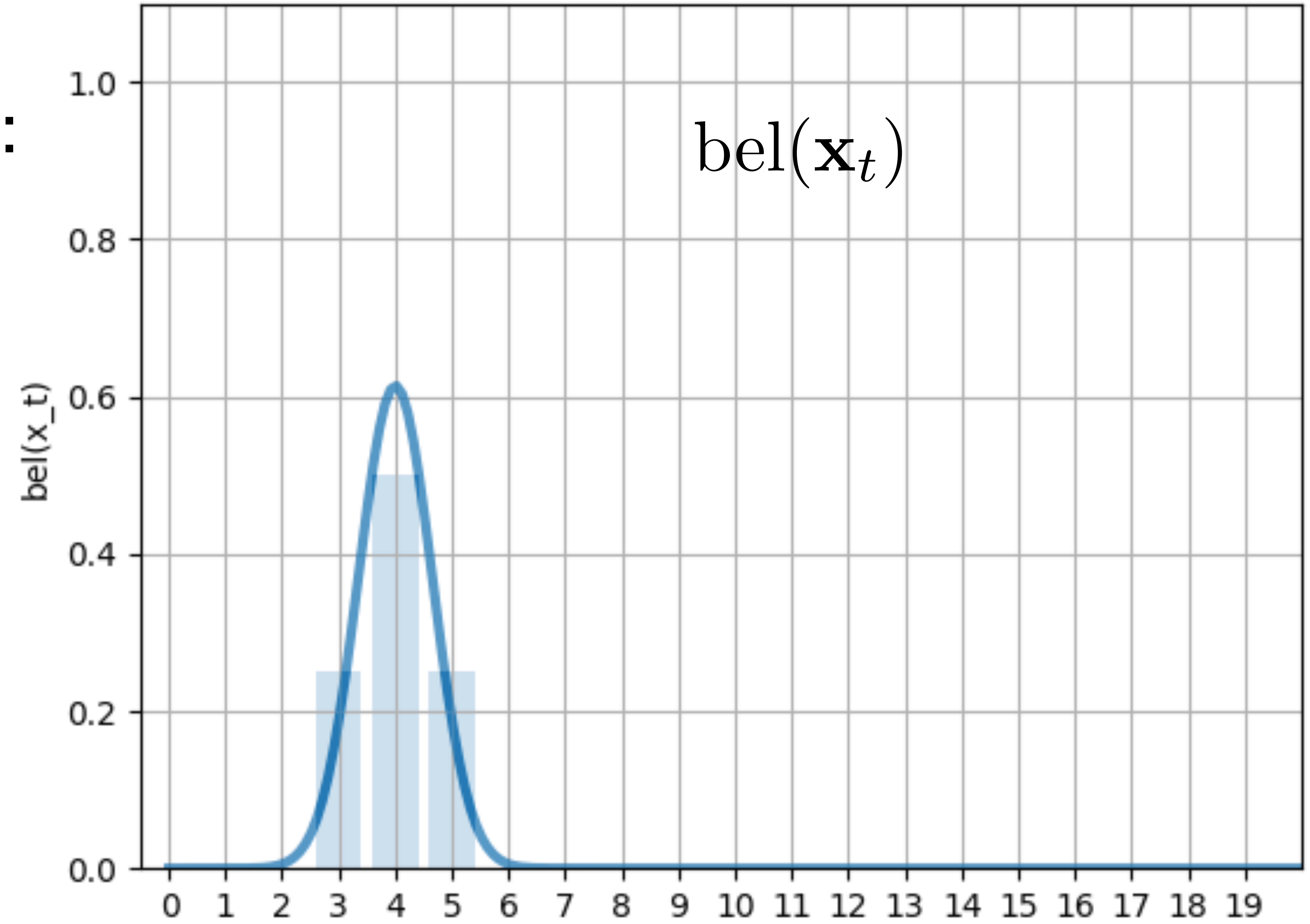

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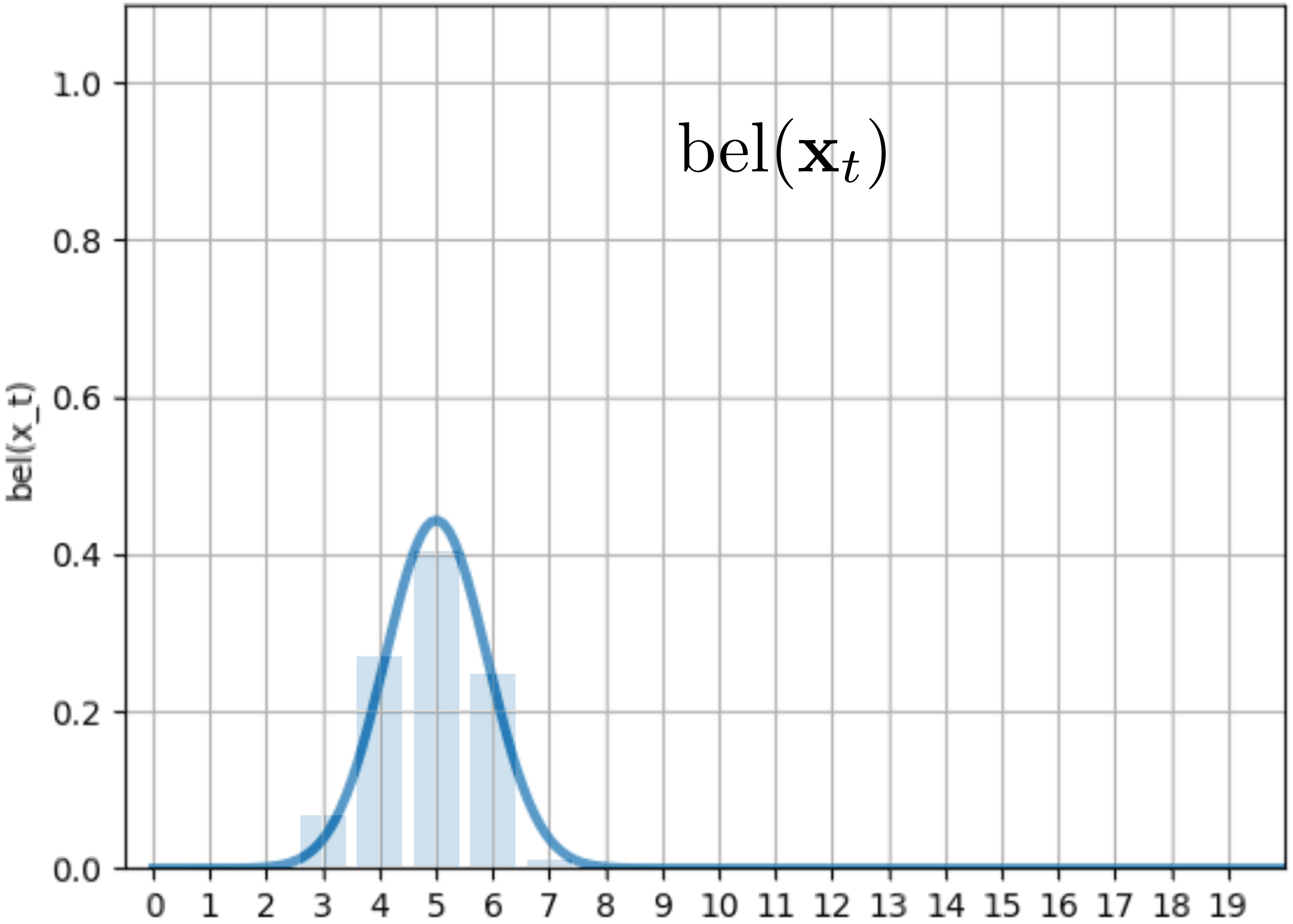

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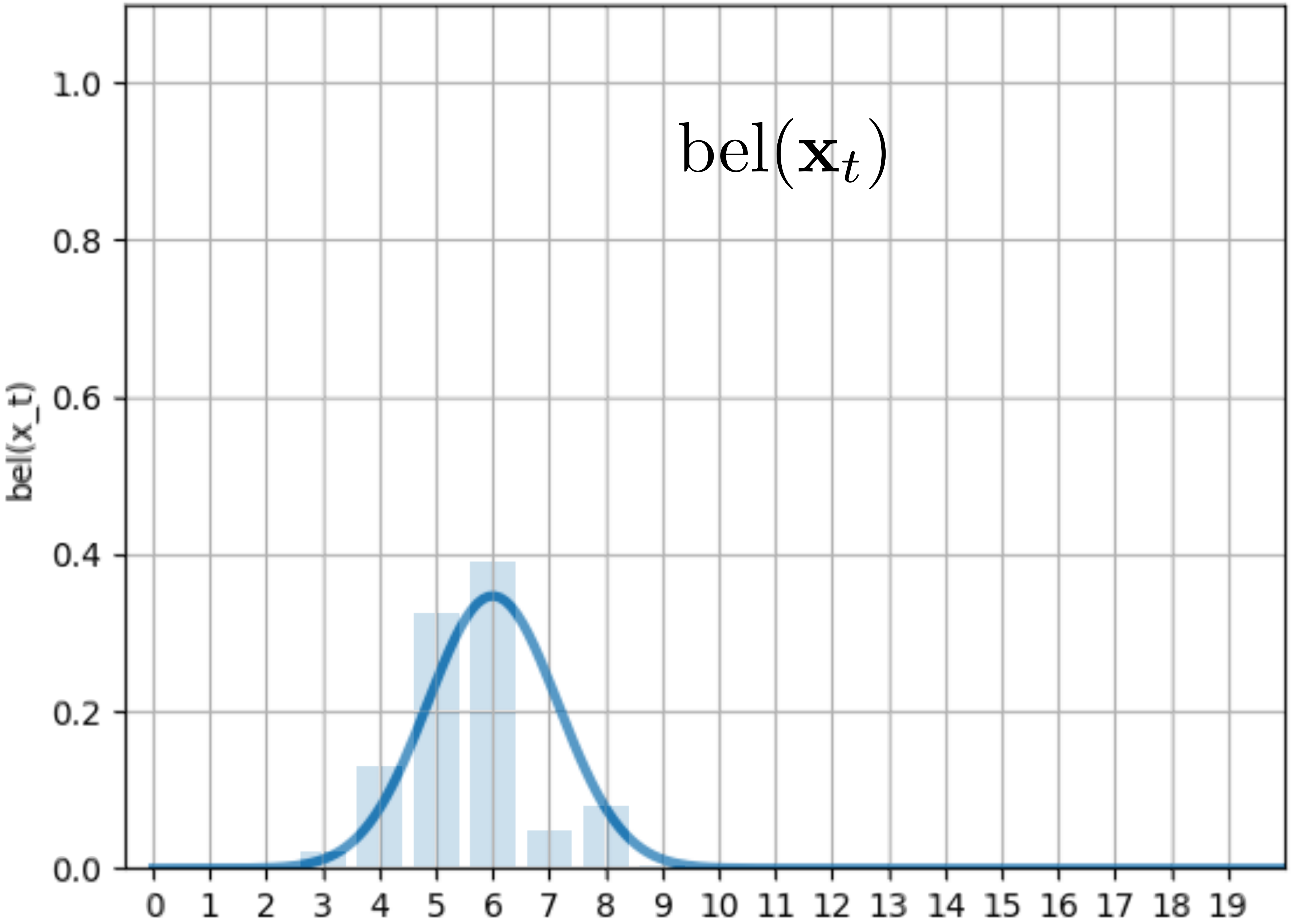

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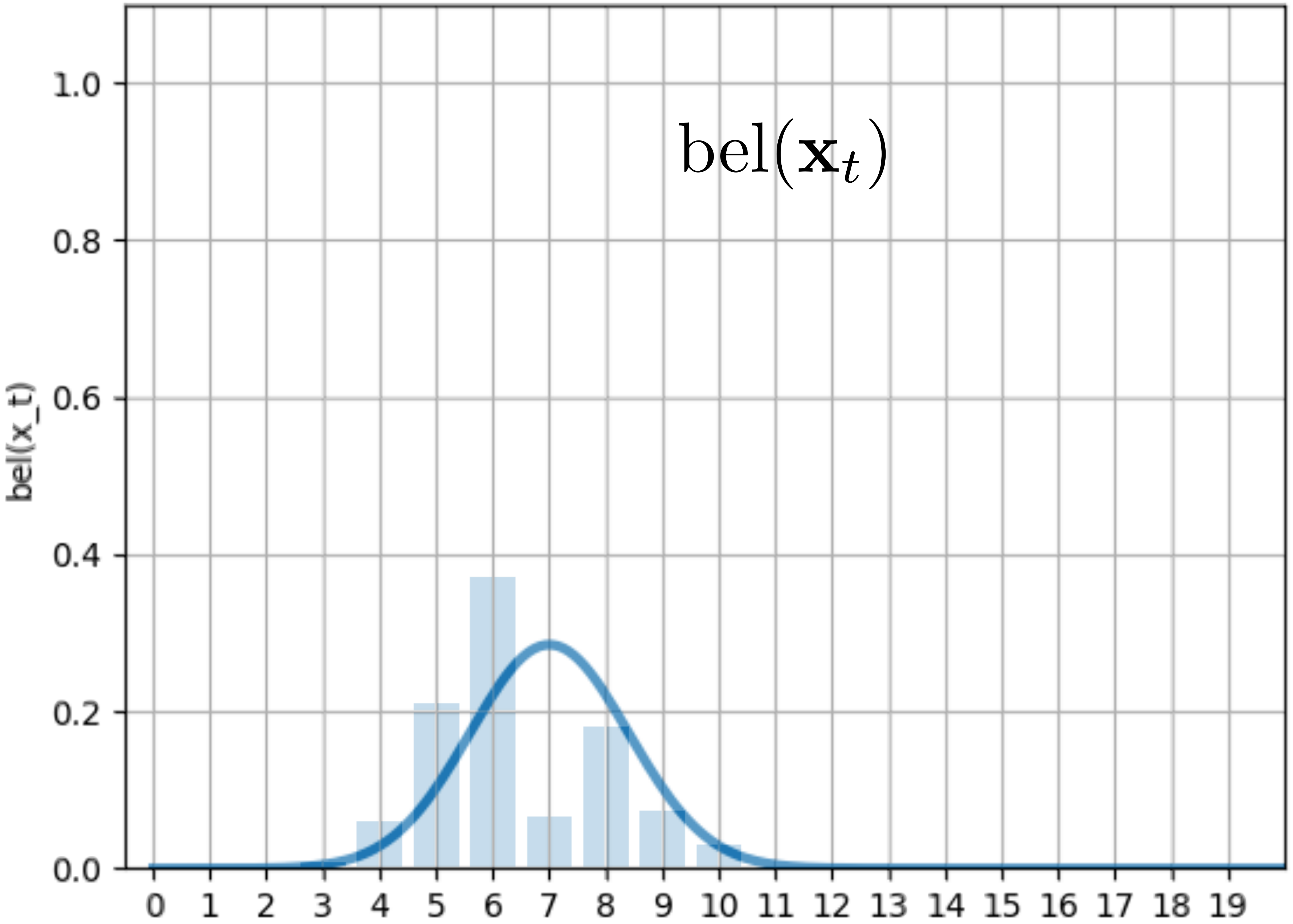

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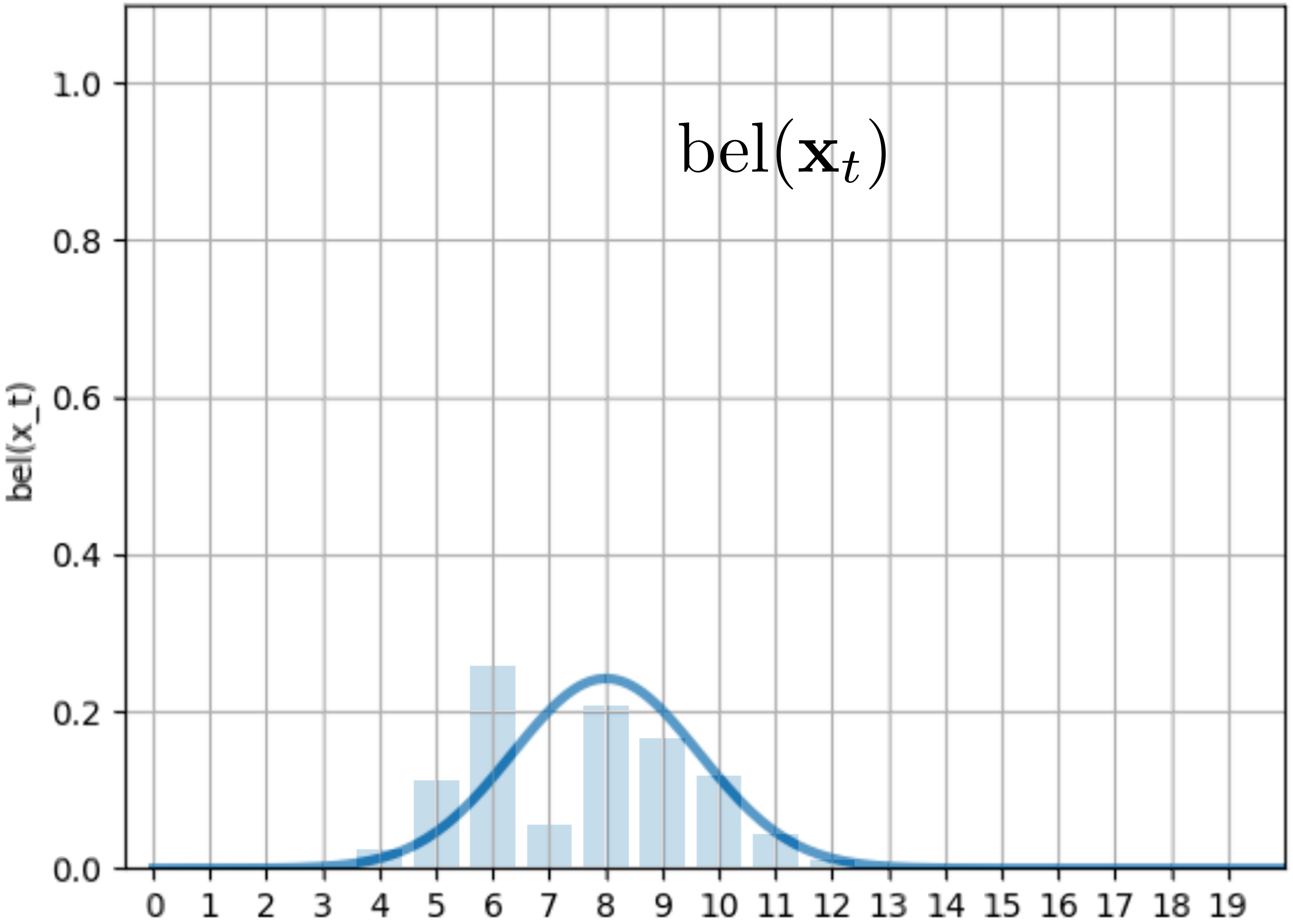

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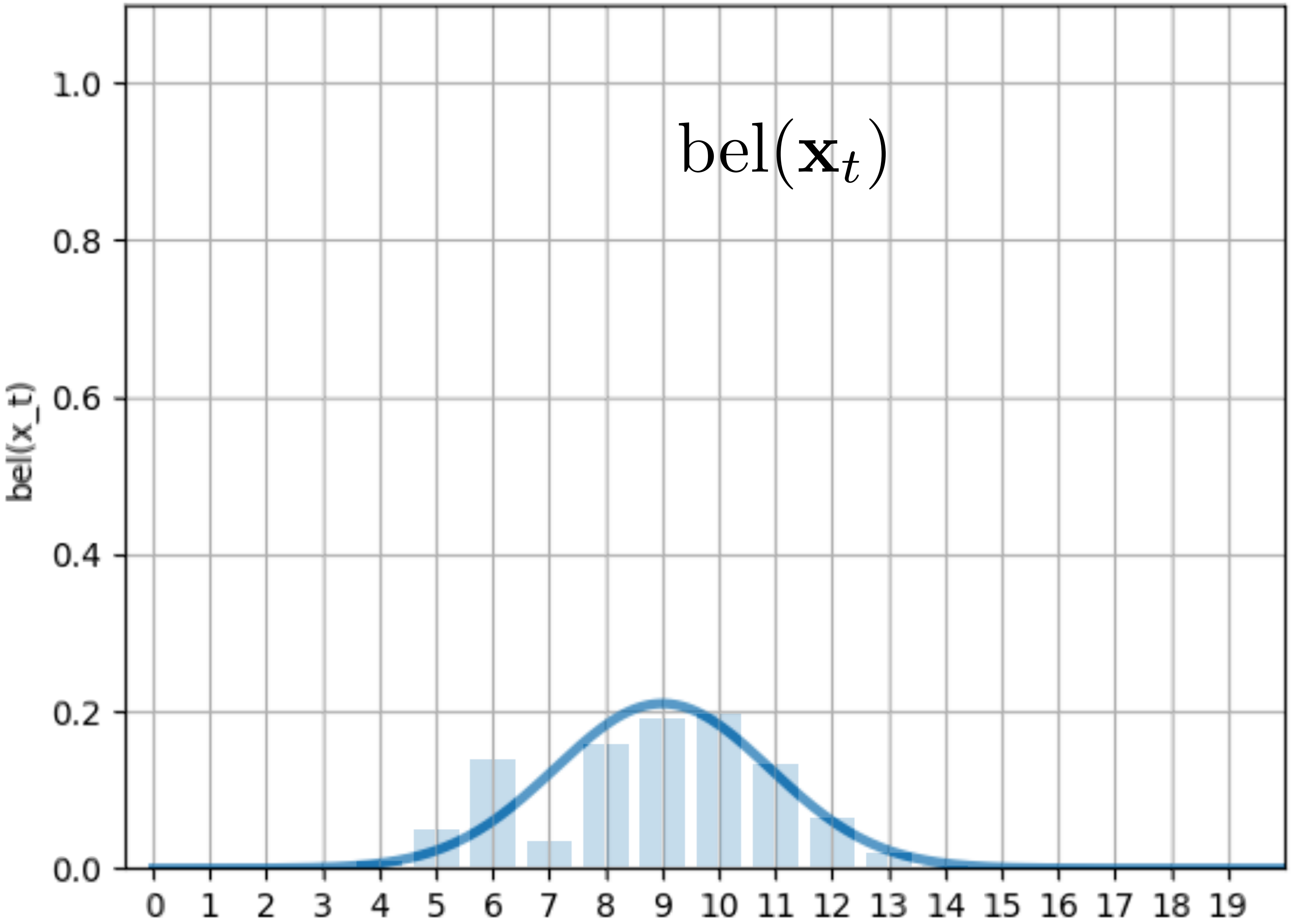

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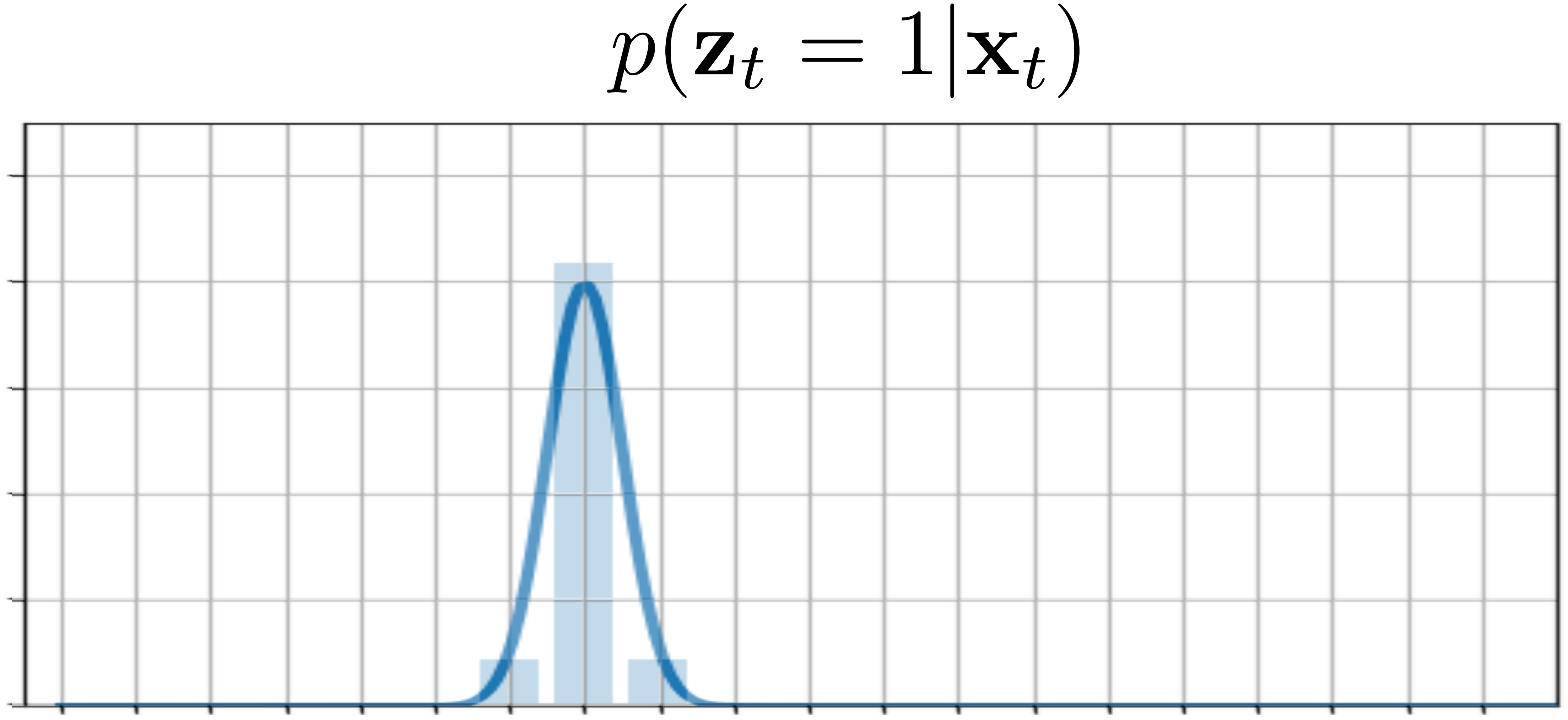

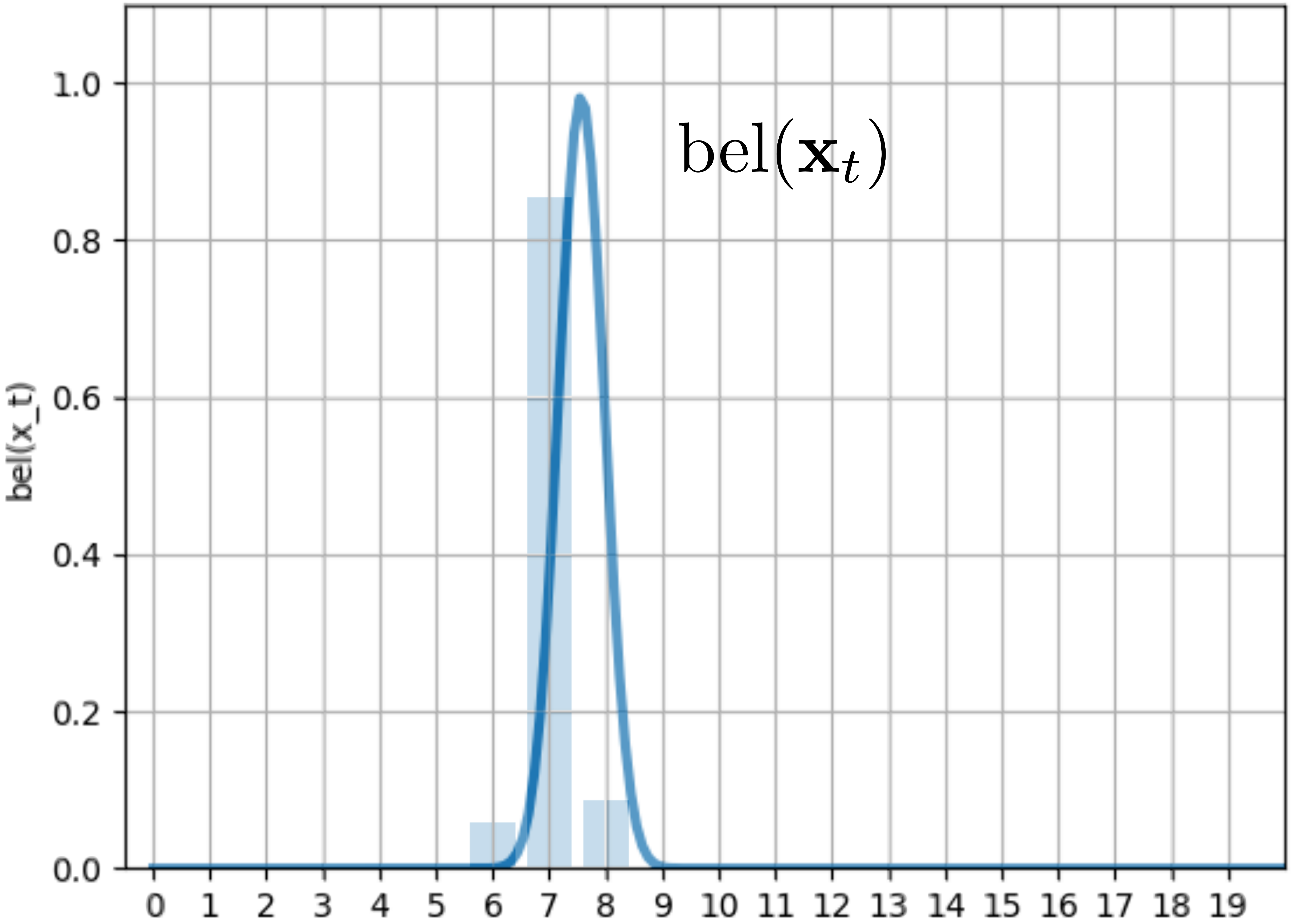

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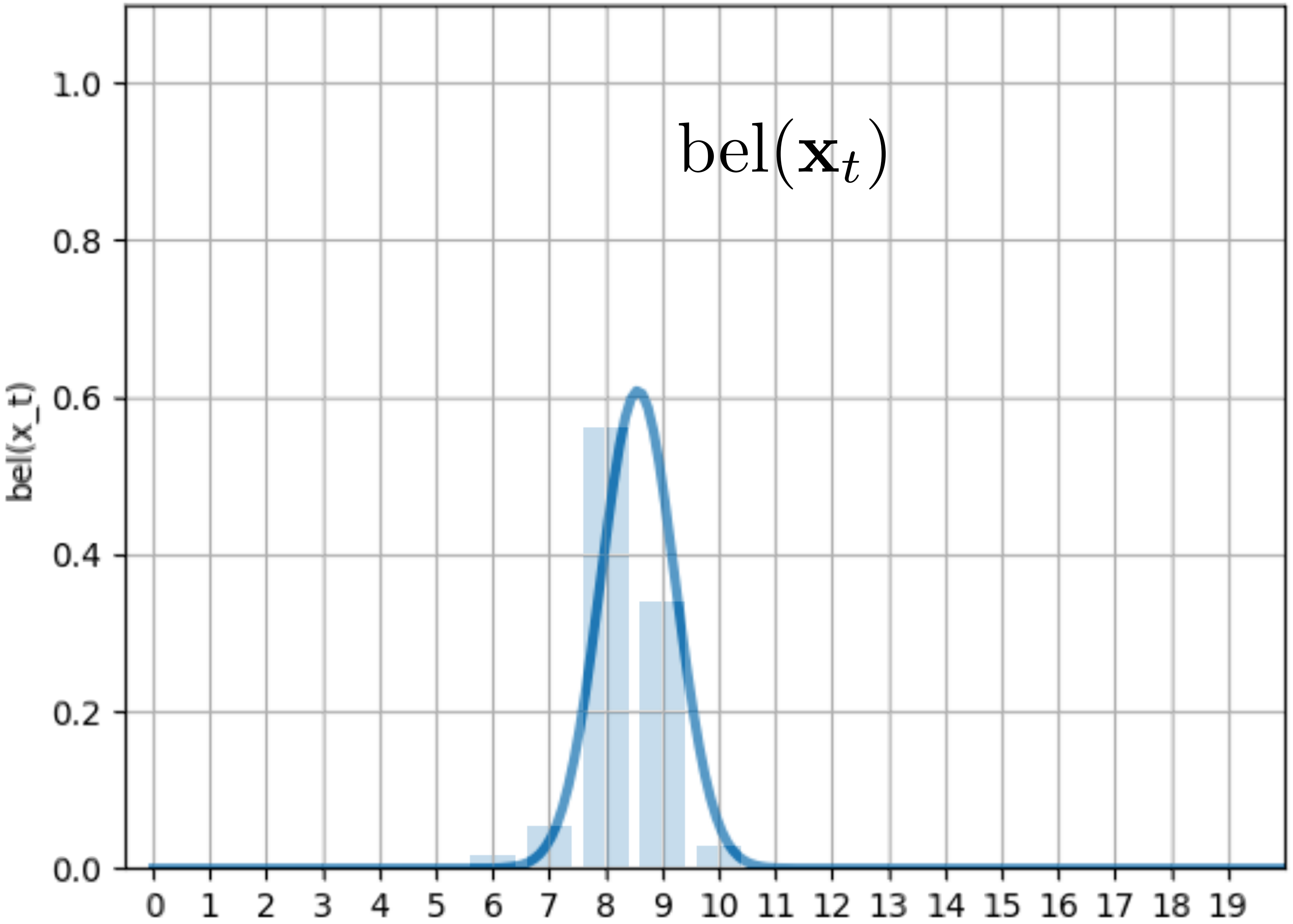

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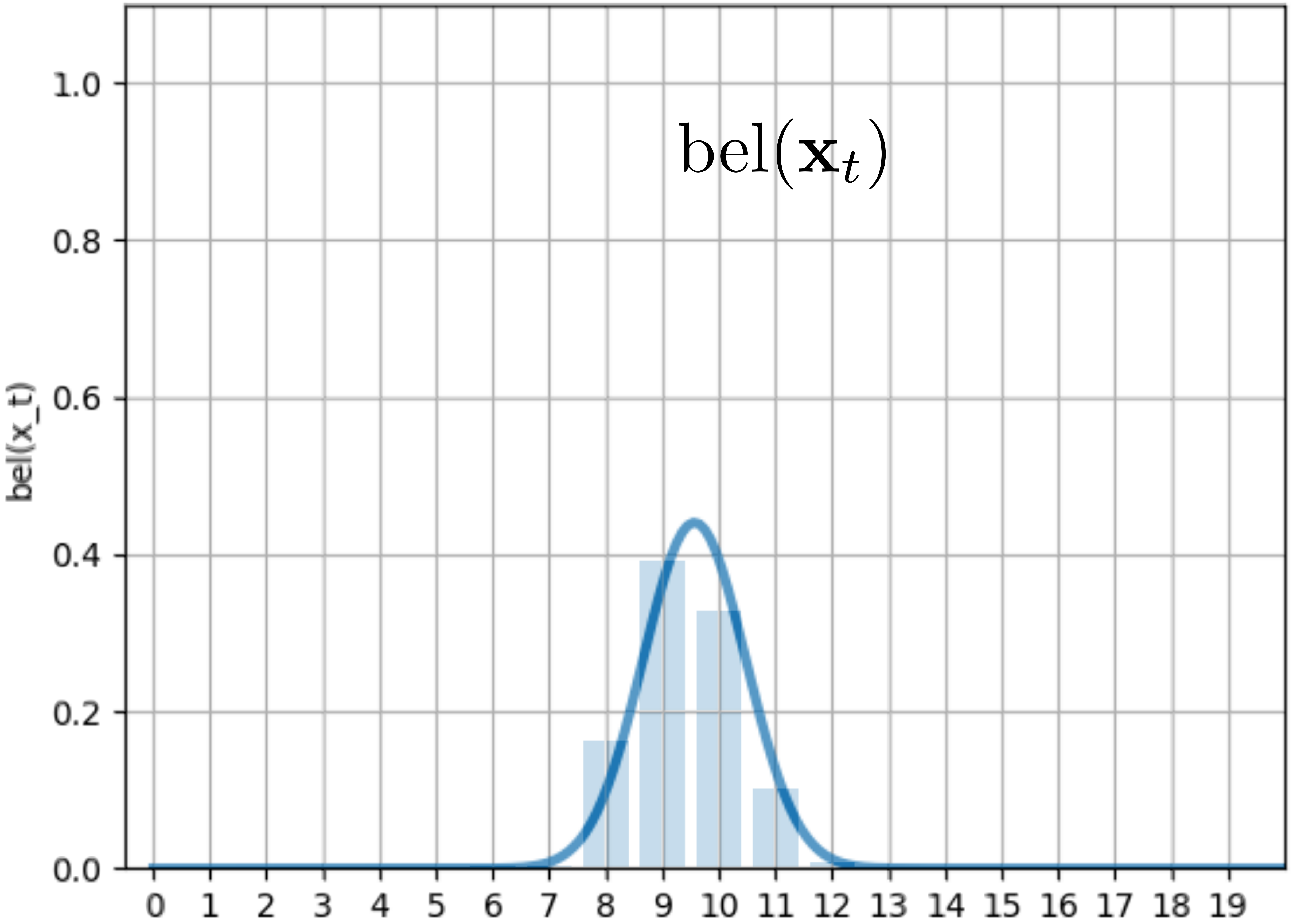

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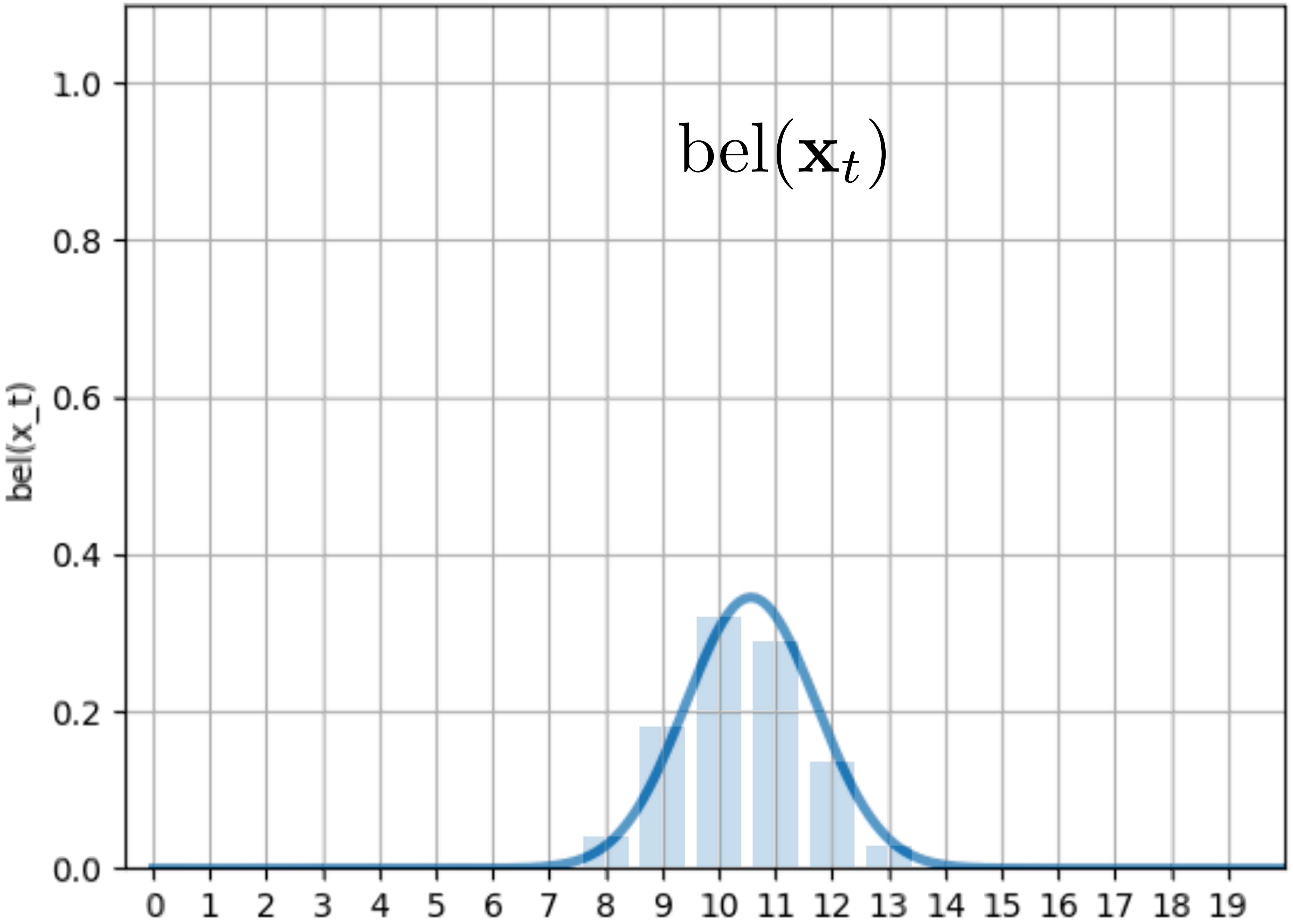

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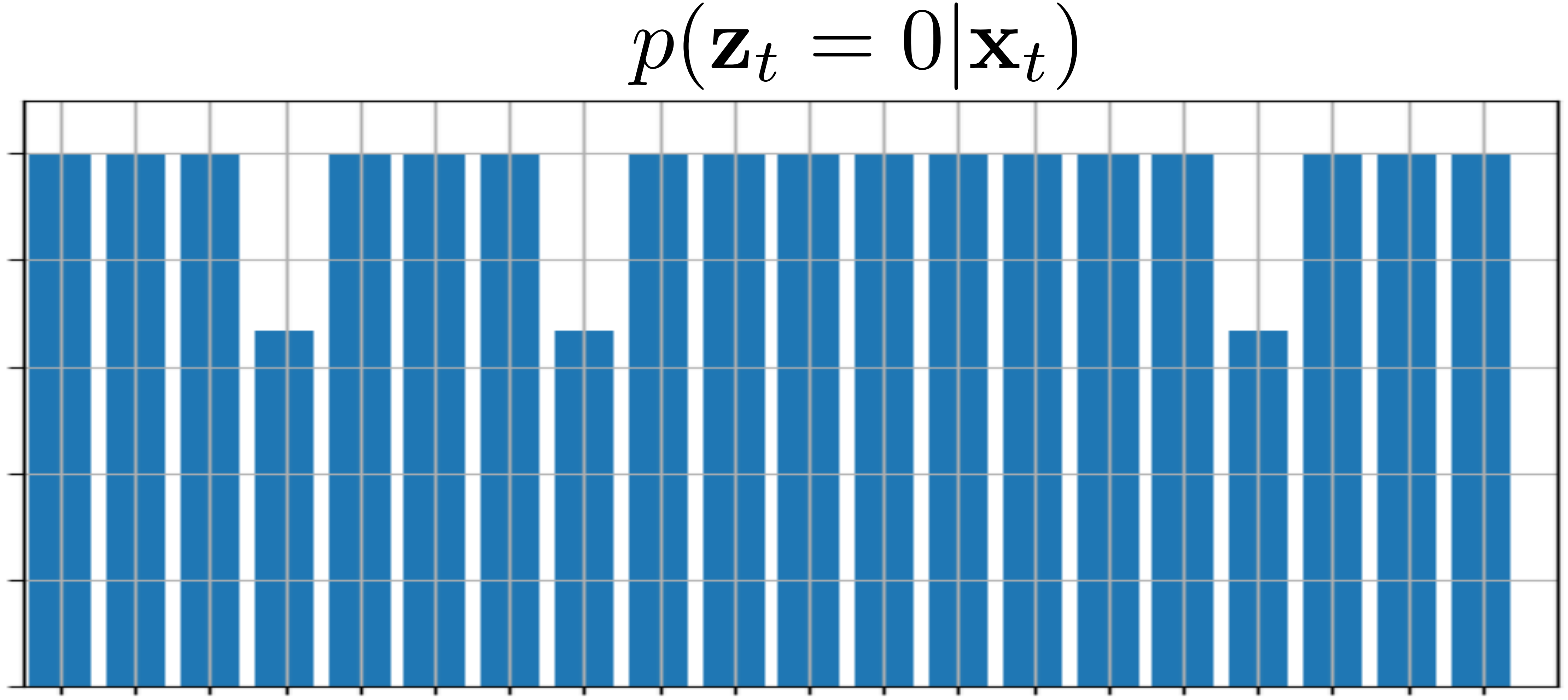
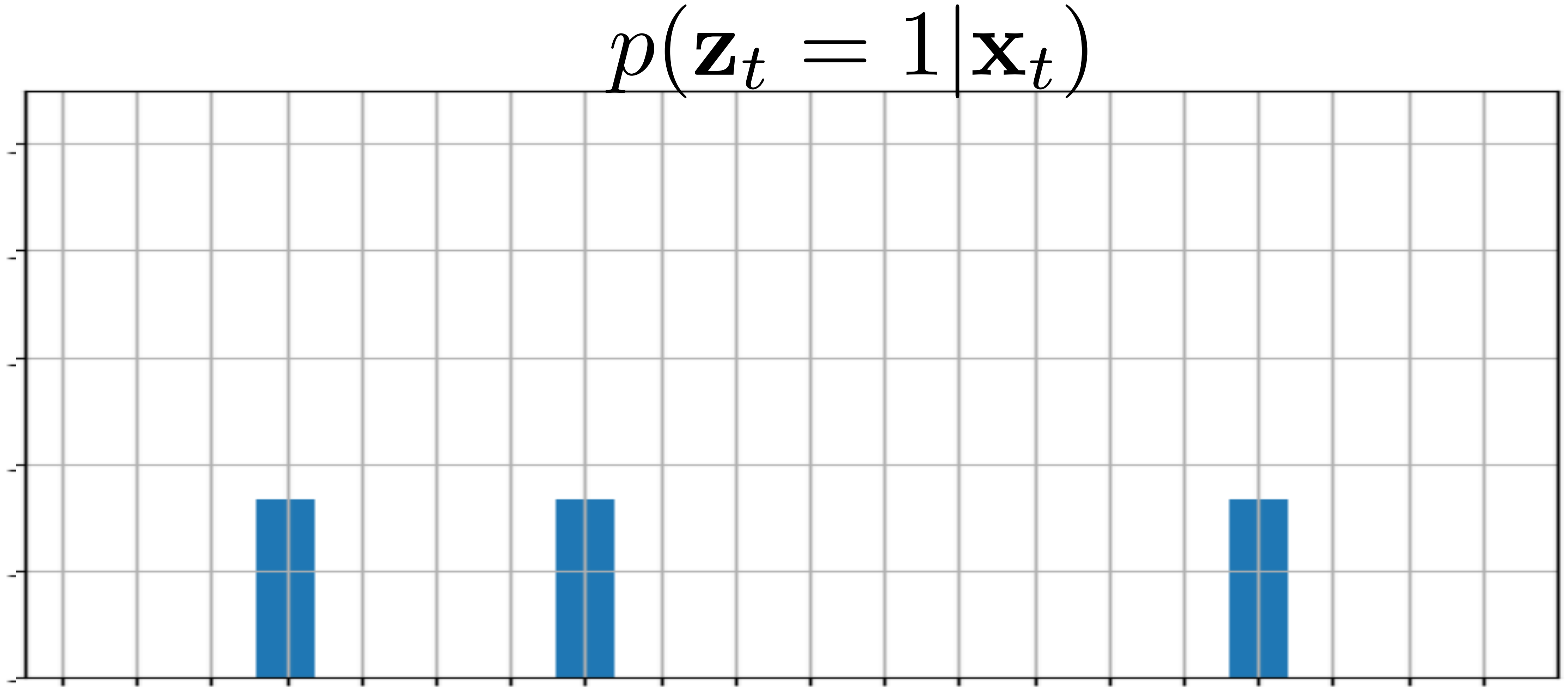
$$\mathbf{K}_t = \overline{\boldsymbol{\Sigma}}_t \mathbf{C}_t^\top (\mathbf{C}_t \overline{\boldsymbol{\Sigma}}_t \mathbf{C}_t^\top + \mathbf{Q}_t)^{-1}$$
$$\boldsymbol{\mu}_t = \overline{\boldsymbol{\mu}}_t + \mathbf{K}_t (\mathbf{z}_t - \mathbf{C}_t \overline{\boldsymbol{\mu}}_t)$$
$$\boldsymbol{\Sigma}_t = (\mathbf{I} - \mathbf{K}_t \mathbf{C}_t) \overline{\boldsymbol{\Sigma}}_t$$
$$\mathrm{bel}(\mathbf{x}_t) = \mathcal{N}_{\mathbf{x}_t}(\boldsymbol{\mu}_t, \boldsymbol{\Sigma}_t)$$

4. Repeat from 2:

$$t = t + 1$$





3 undistinguishable markers

Can we replicate the experiment with 3 markers???

2D example: state = (x … position, v …velocity)

$$p(\mathbf{x}_t|\mathbf{x}_{t-1},\mathbf{u}_t) = \mathcal{N}_{\mathbf{x}_t}\Big(\underbrace{\begin{bmatrix}1 & 1\\0 & 1\end{bmatrix}}_{\mathbf{A}_t}\mathbf{x}_{t-1} + \underbrace{\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}}_{\mathbf{B}_t}\mathbf{u}_t,\ \underbrace{\begin{bmatrix}0.01 & 0\\0 & 0.01\end{bmatrix}}_{\mathbf{R}_t}\Big)$$

$$p(\mathbf{z}_t|\mathbf{x}_t) = \mathcal{N}_{\mathbf{z}_t}\Big(\underbrace{\begin{bmatrix}1 & 0\end{bmatrix}}_{\mathbf{C}_t}\mathbf{x}_{t-1},\ \underbrace{\begin{bmatrix}0.3\end{bmatrix}}_{\mathbf{Q}_t}\Big)$$

2. Prediction step (new action $\mathbf{u}_t$ performed):

$$\overline{\boldsymbol{\mu}}_t = \mathbf{A}_t\boldsymbol{\mu}_{t-1} + \mathbf{B}_t\mathbf{u}_t$$
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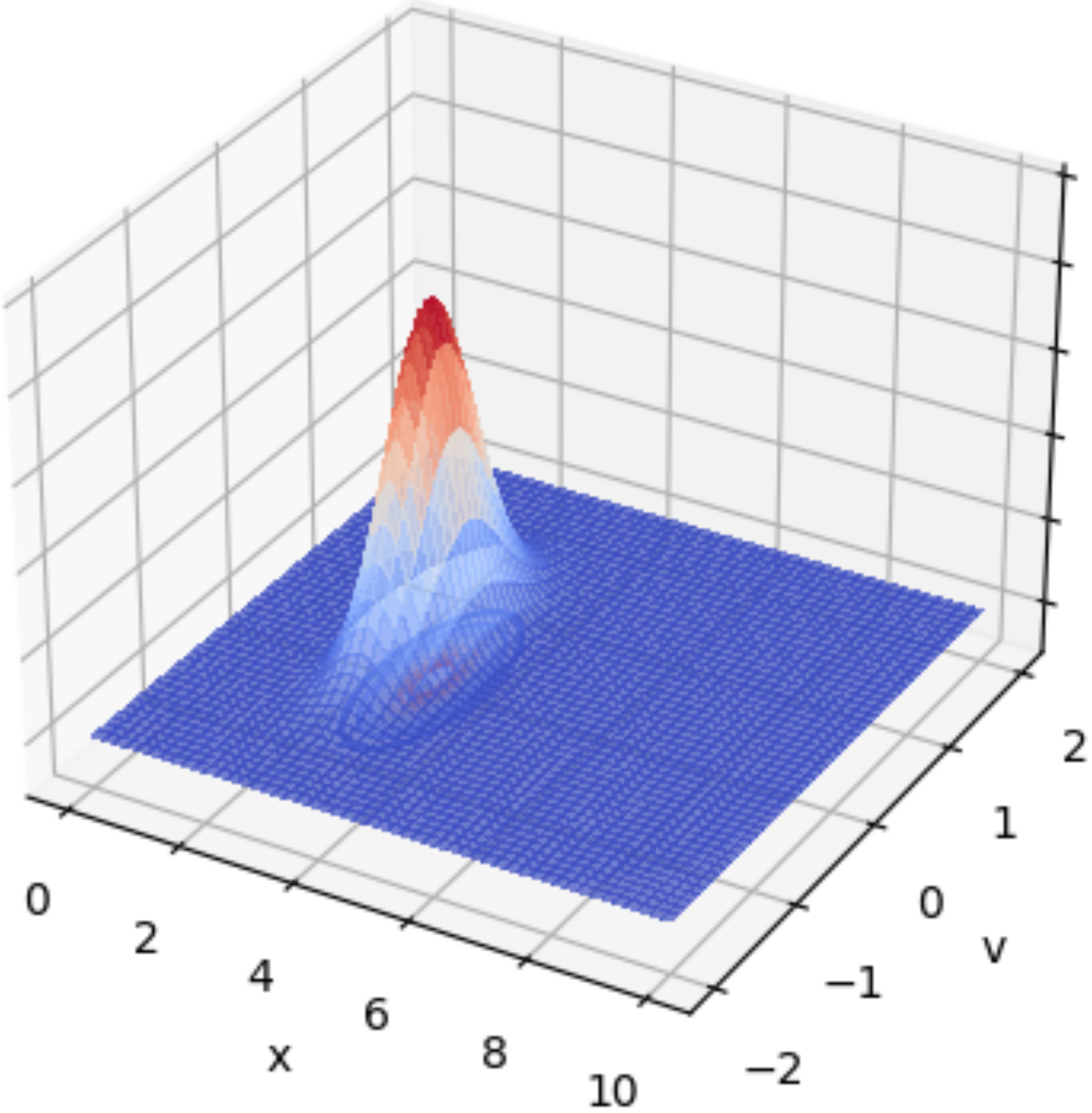

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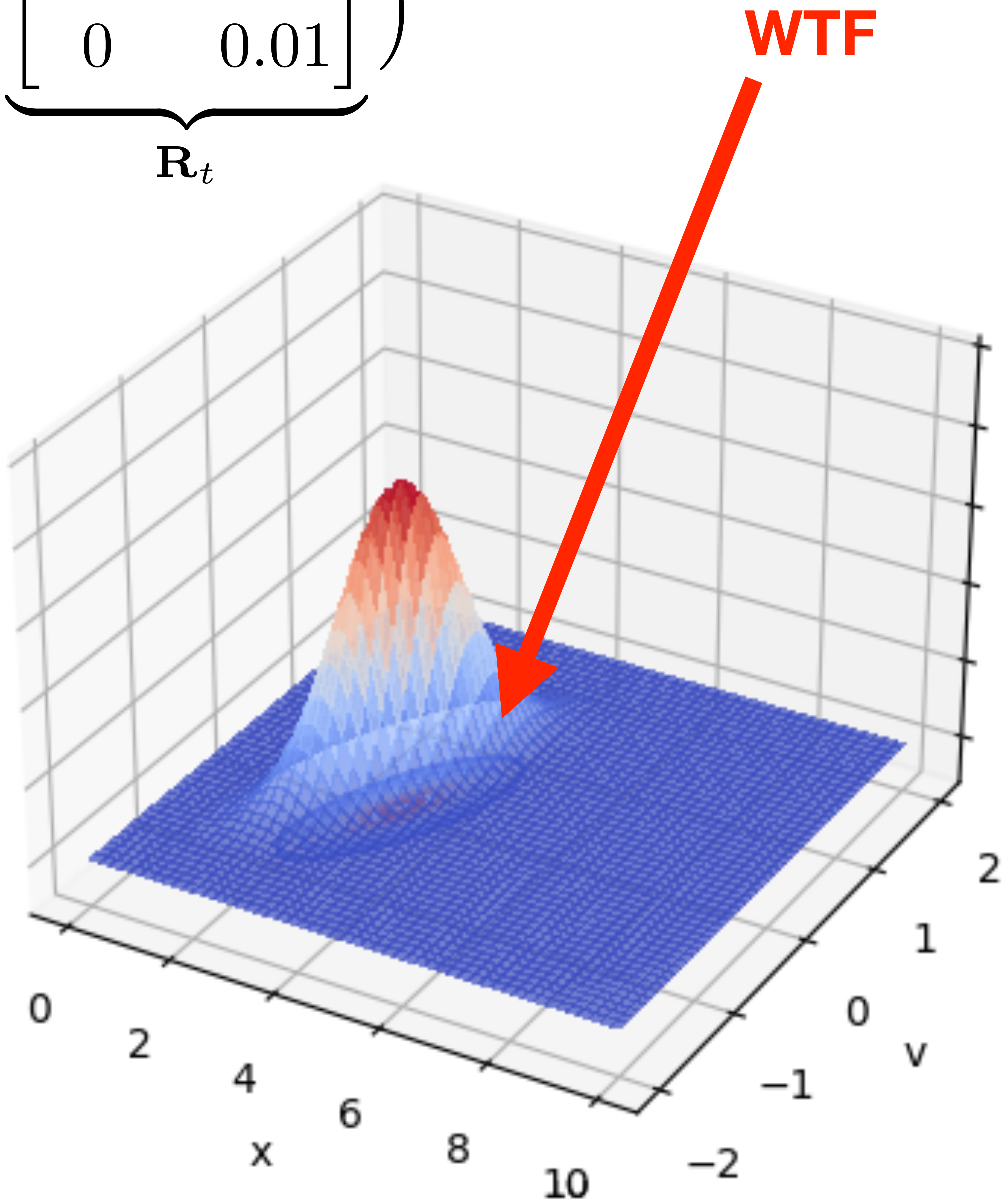

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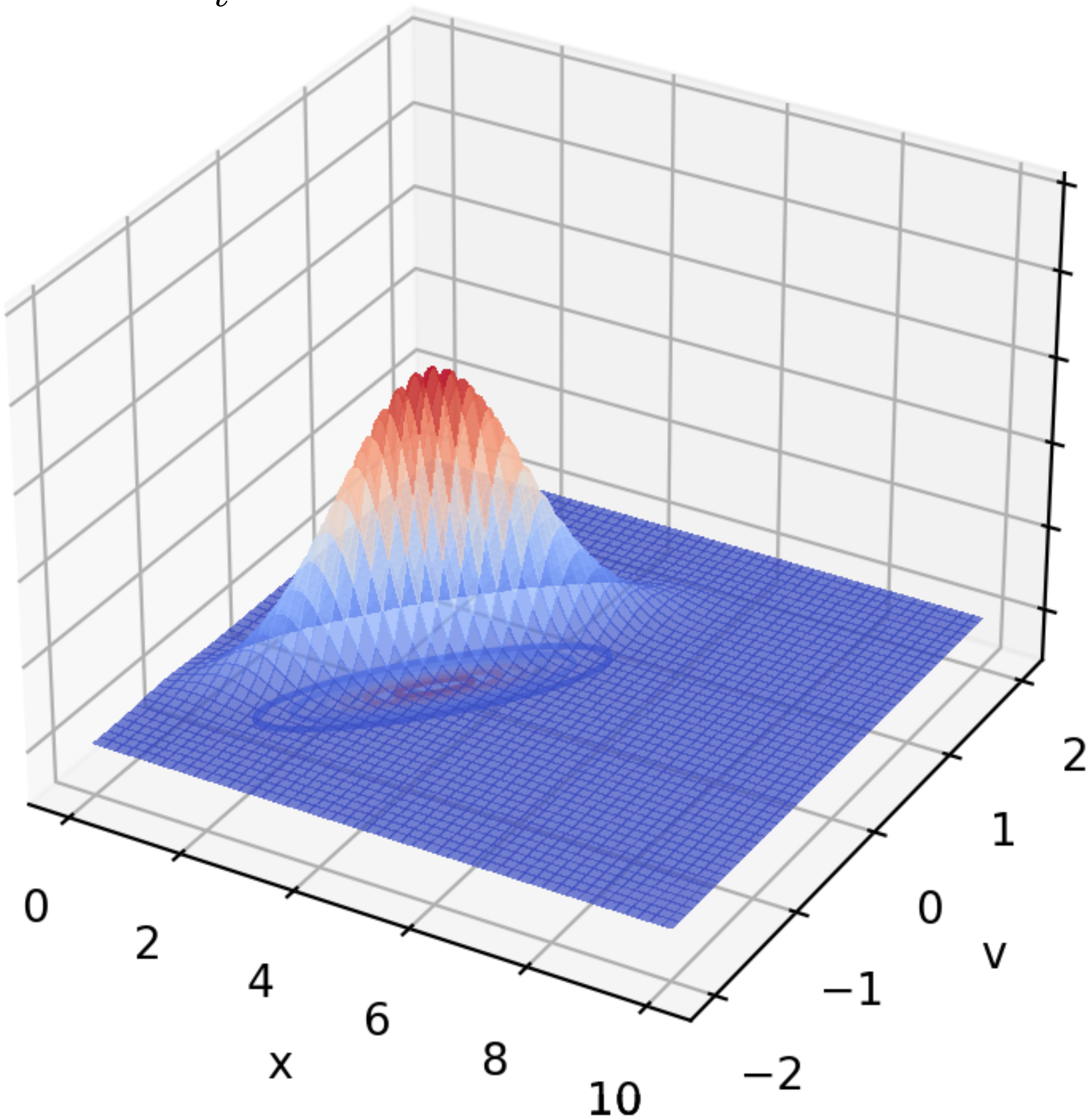

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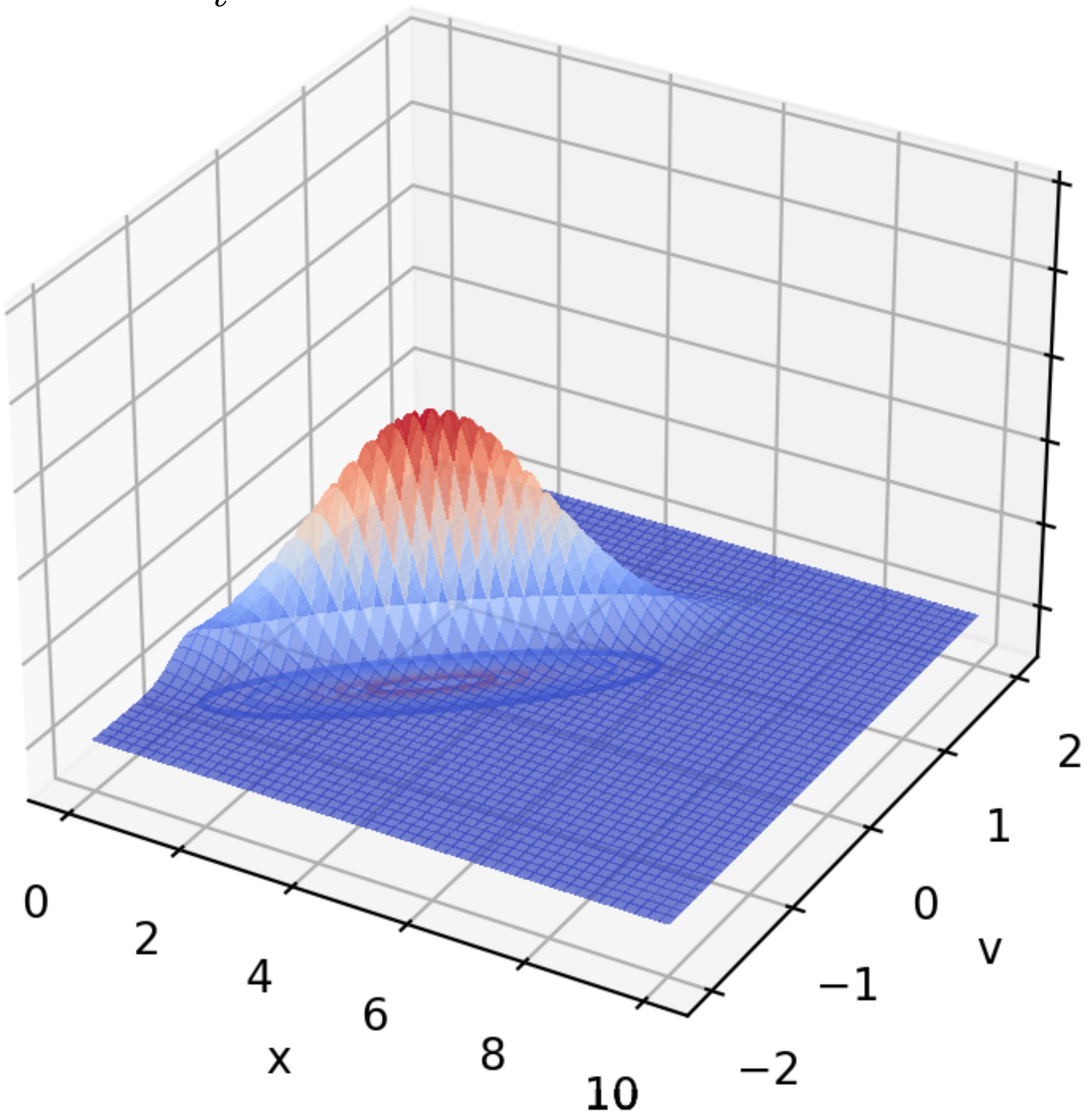

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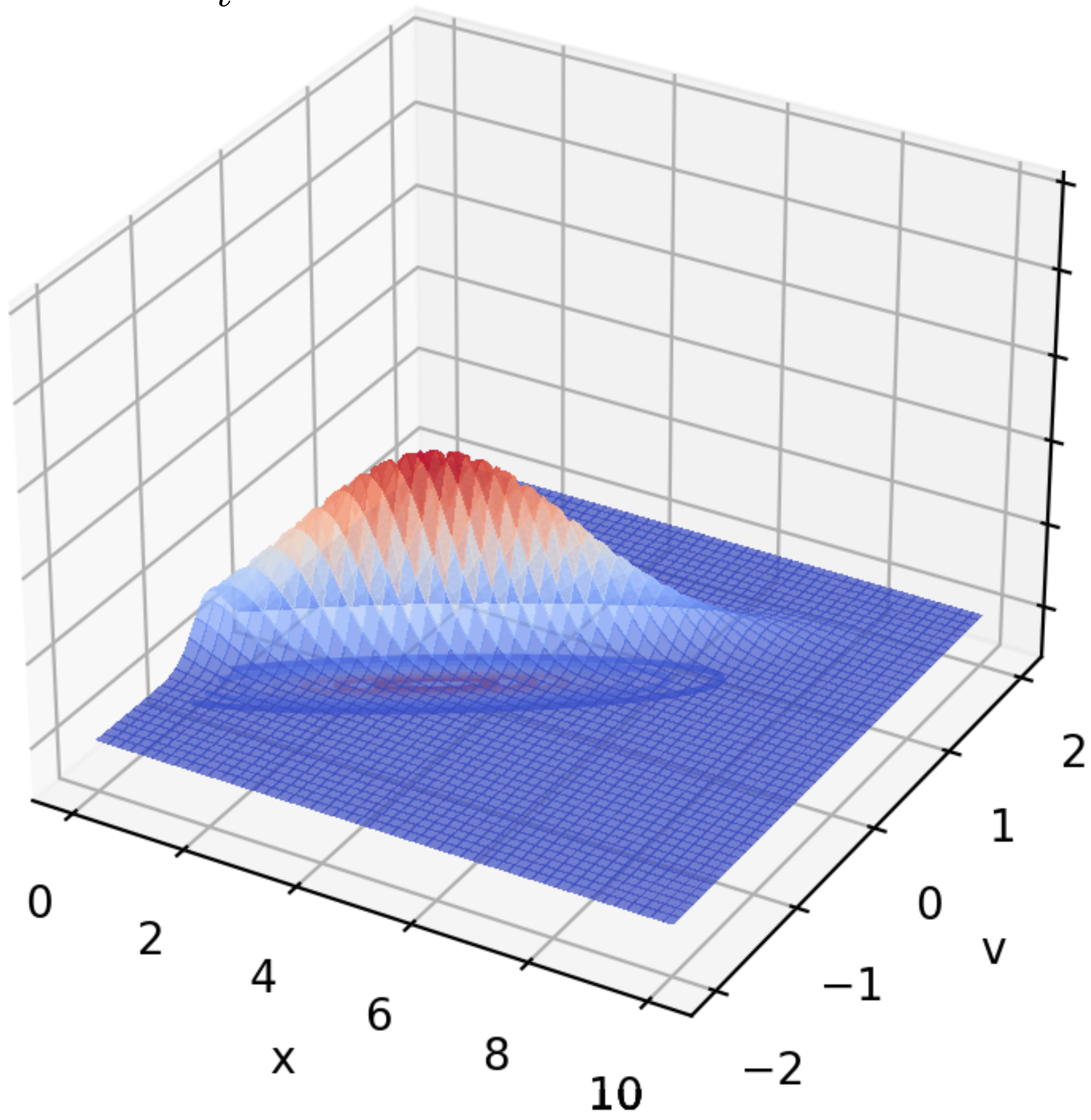

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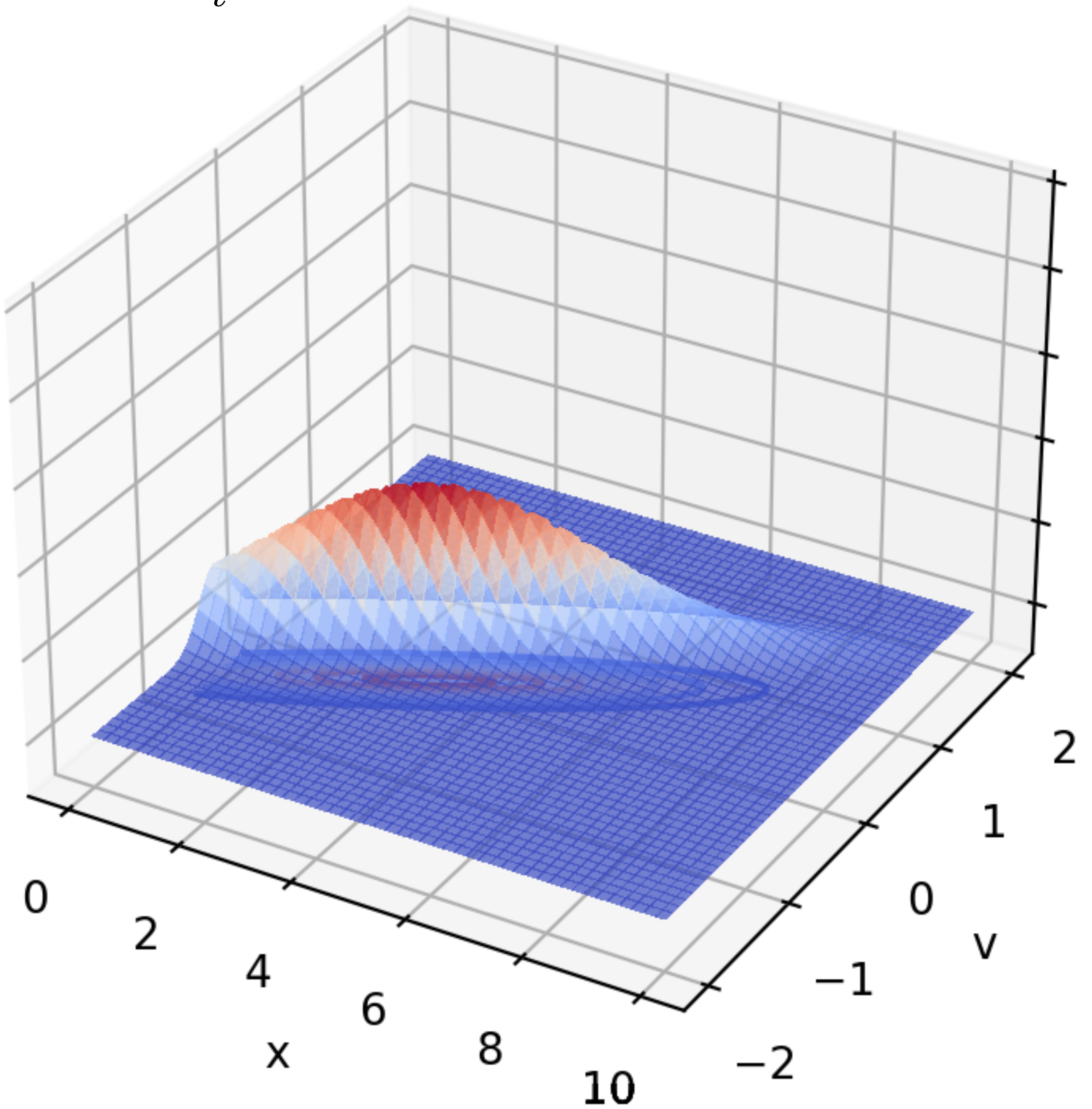

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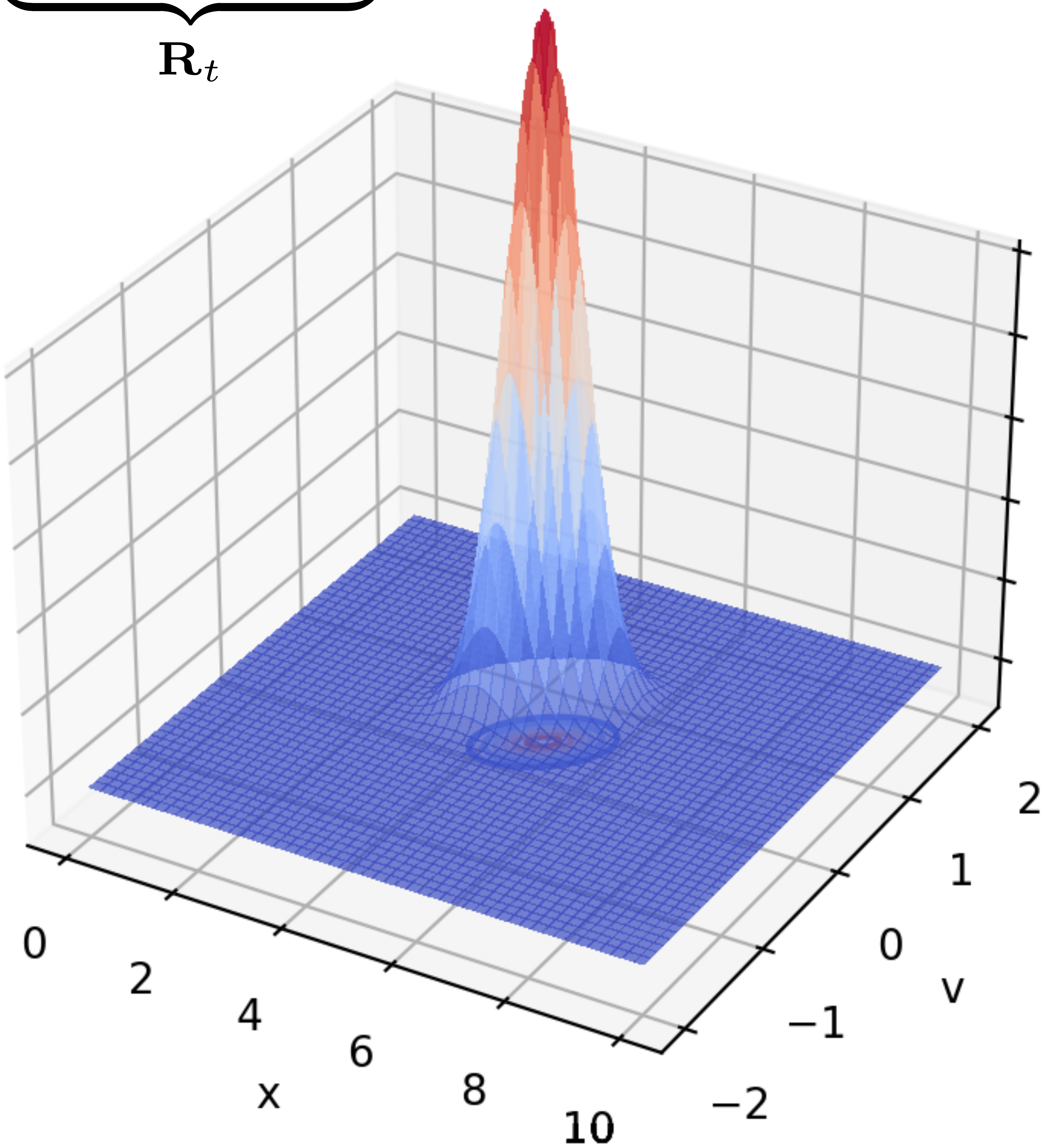

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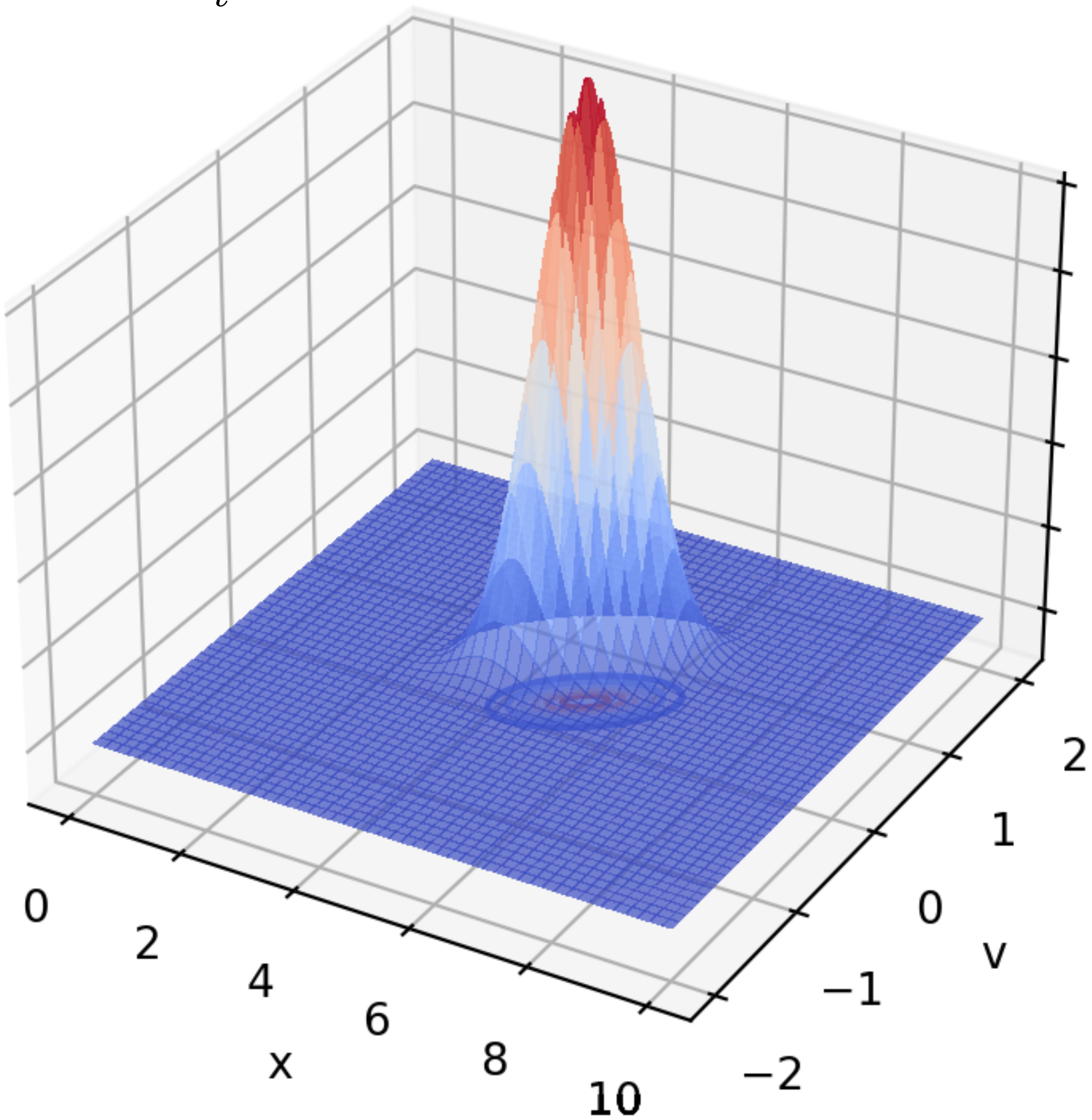

2D example: state = (x … position, v …velocity)

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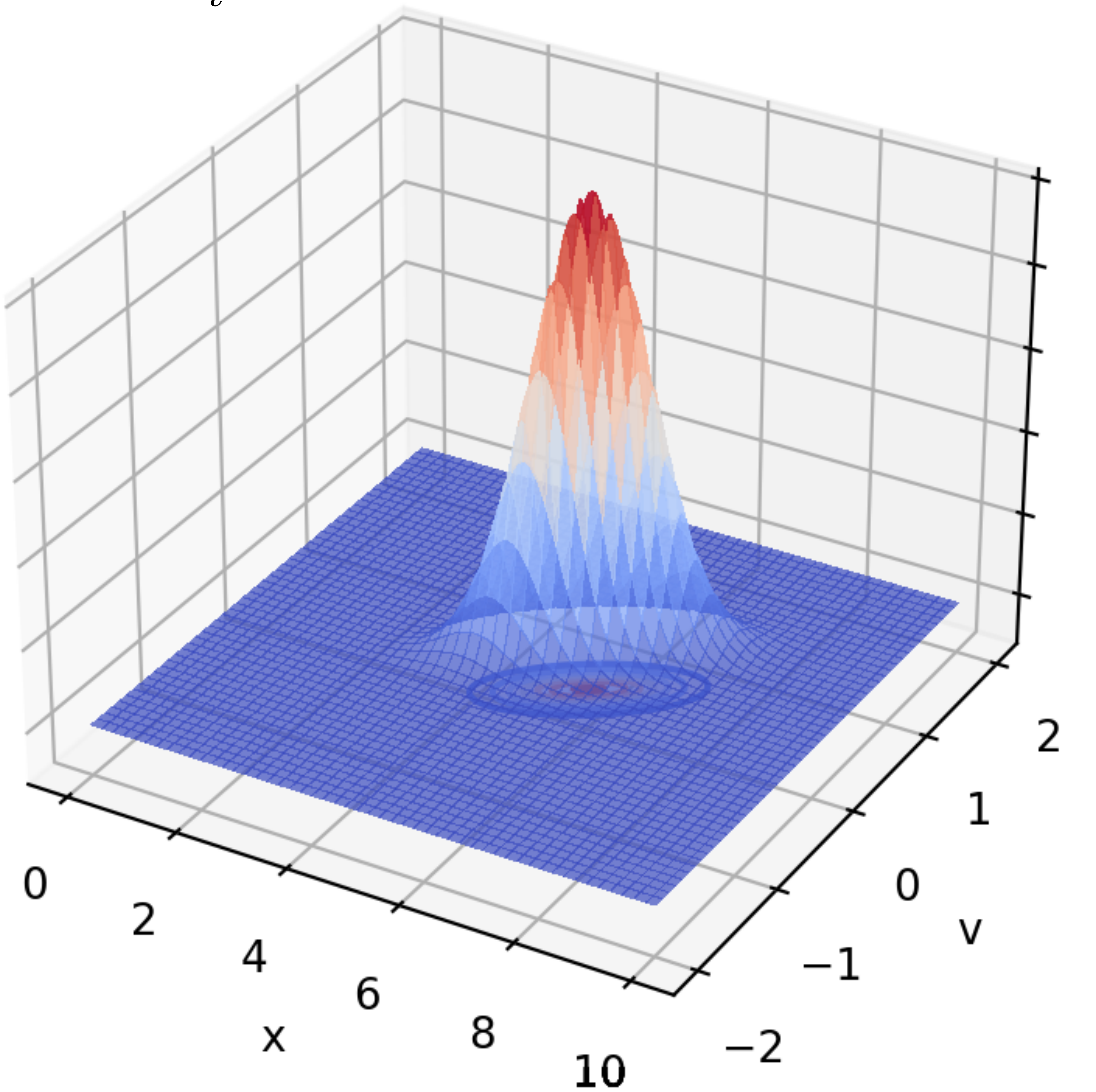

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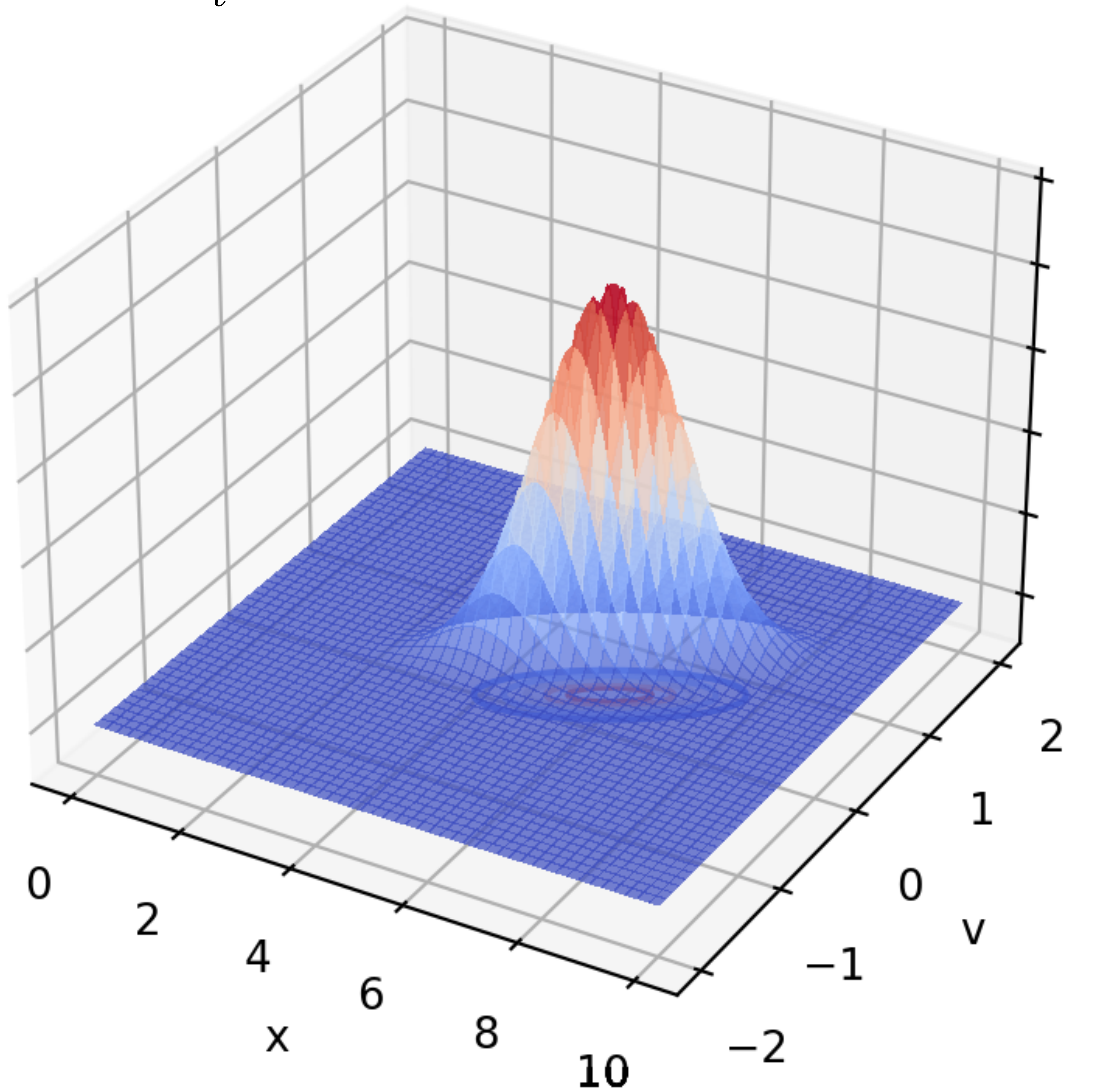

Summary Kalman Filter

- Kalman filter is optimal observer of the state for linear systems under Gaussian noise
- Kalman filter is Bayes filter where measurement and transition probabilities are linear-gaussians.
- It nicely scales to higher dimension but the linearity and gaussianity yields significant limitations
    - Example 18-dimensional state space
        - Bayes bel: Each dimension 10 discrete values => 10^18 parameters
        - KF bel: Continuous Gaussian representation ==> 18^2+18=342 params
- Extended Kalman filter removes the linearity limitation but loses the optimality