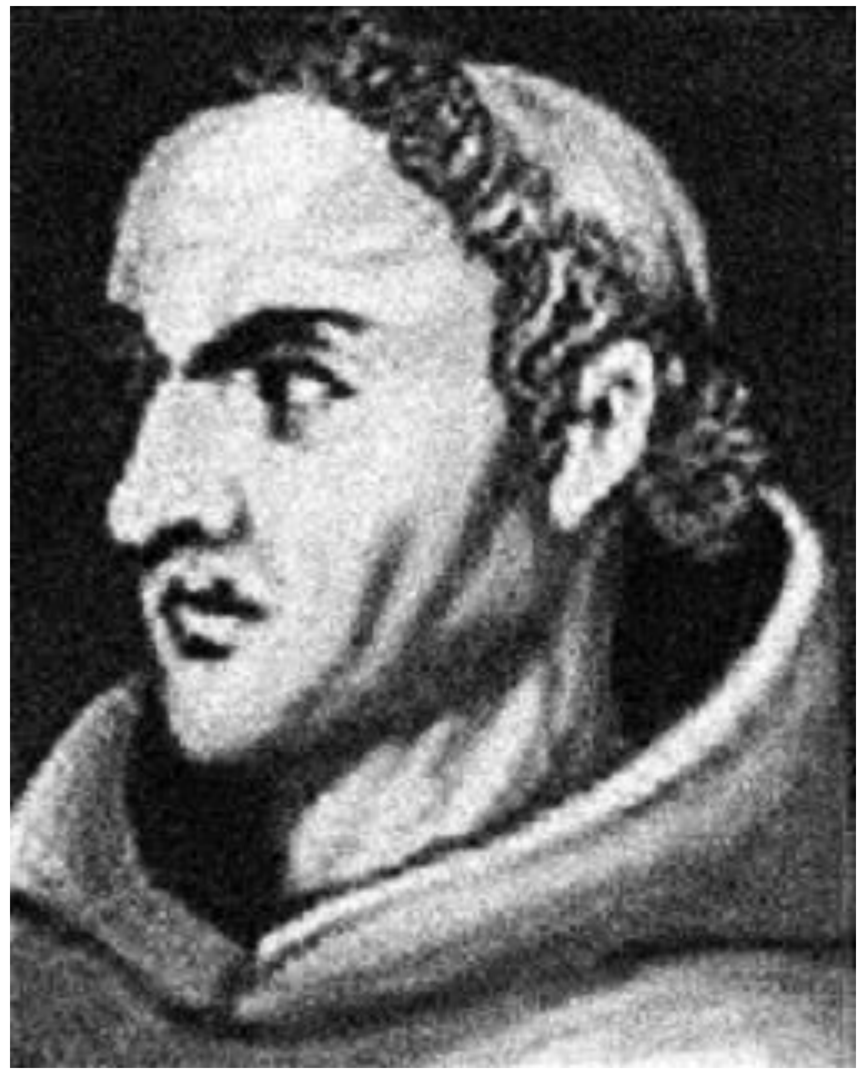
Localization: Bayes filter

Karel Zimmermann

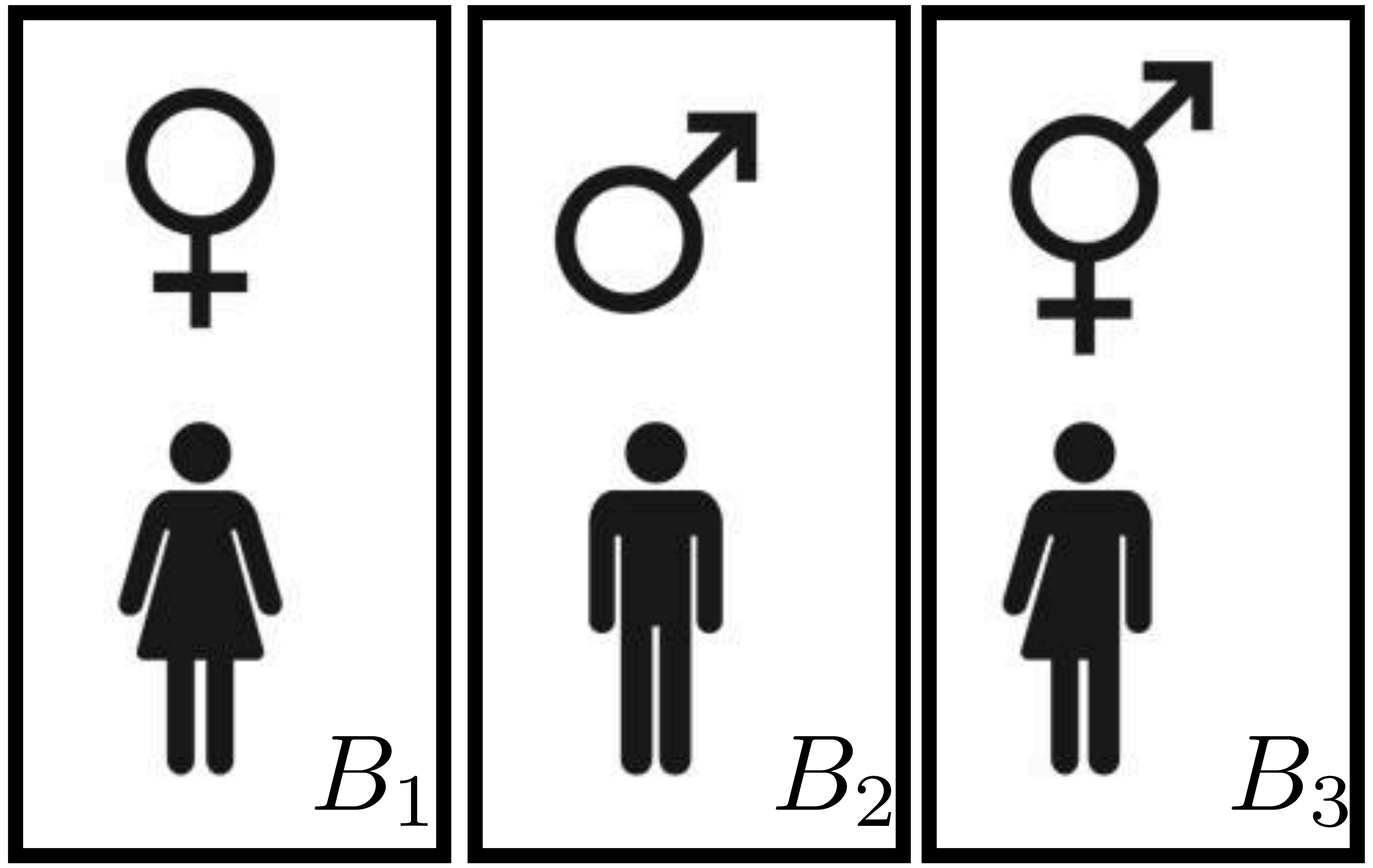
   

Prerequisites

- Bayes Rule (BR)

$$p(A|B) = \frac{p(B|A)p(A)}{p(B)}$$

- Law of total probability (LTP)



$$p(A) = \sum_B p(A|B)p(B)$$

- Conditional independence (CI)

Complete states

Complete states: $\mathbf{x}_0, \mathbf{x}_1, \ldots, \mathbf{x}_t \in \mathcal{R}^n$

Def: A is *conditionally independent* on C given B iff $p(A|B,C) = p(A|B)$

Def: State $\mathbf{x}_{t-1}$ is *complete* iff future $\mathbf{x}_t$ is conditionally independent on past given $\mathbf{x}_{t-1}$

Consequences:

state-transition probability: $p(\mathbf{x}_t|\mathbf{x}_{t-1}, \mathbf{u}_t) = p(\mathbf{x}_t|\mathbf{x}_{t-1}, \mathbf{u}_t, \mathbf{x}_{0:t-2}, \mathbf{z}_{1:t-1}, \mathbf{u}_{1:t-1})$

Problem definition

States: $\mathbf{x}_0, \mathbf{x}_1, \ldots, \mathbf{x}_t \in \mathcal{R}^n$

<u>Def:</u> A and B are <u>*conditionally independent*</u> given C iff $p(A|B,C) = p(A|C)$

<u>Def:</u> State $\mathbf{x}_t$ is <u>*complete*</u> iff future is conditionally independent on past given $\mathbf{x}_t$

<u>Consequences:</u>

measurement probability: $p(\mathbf{z}_t|\mathbf{x}_t) = p(\mathbf{z}_t|\mathbf{x}_t, \mathbf{x}_{0:t-1}, \mathbf{z}_{1:t-1}, \mathbf{u}_{1:t})$

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measurement probability: $p(\mathbf{z}_t|\mathbf{x}_t) = p(\mathbf{z}_t|\mathbf{x}_t, \mathbf{x}_{0:t-1}, \mathbf{z}_{1:t-1}, \mathbf{u}_{1:t})$

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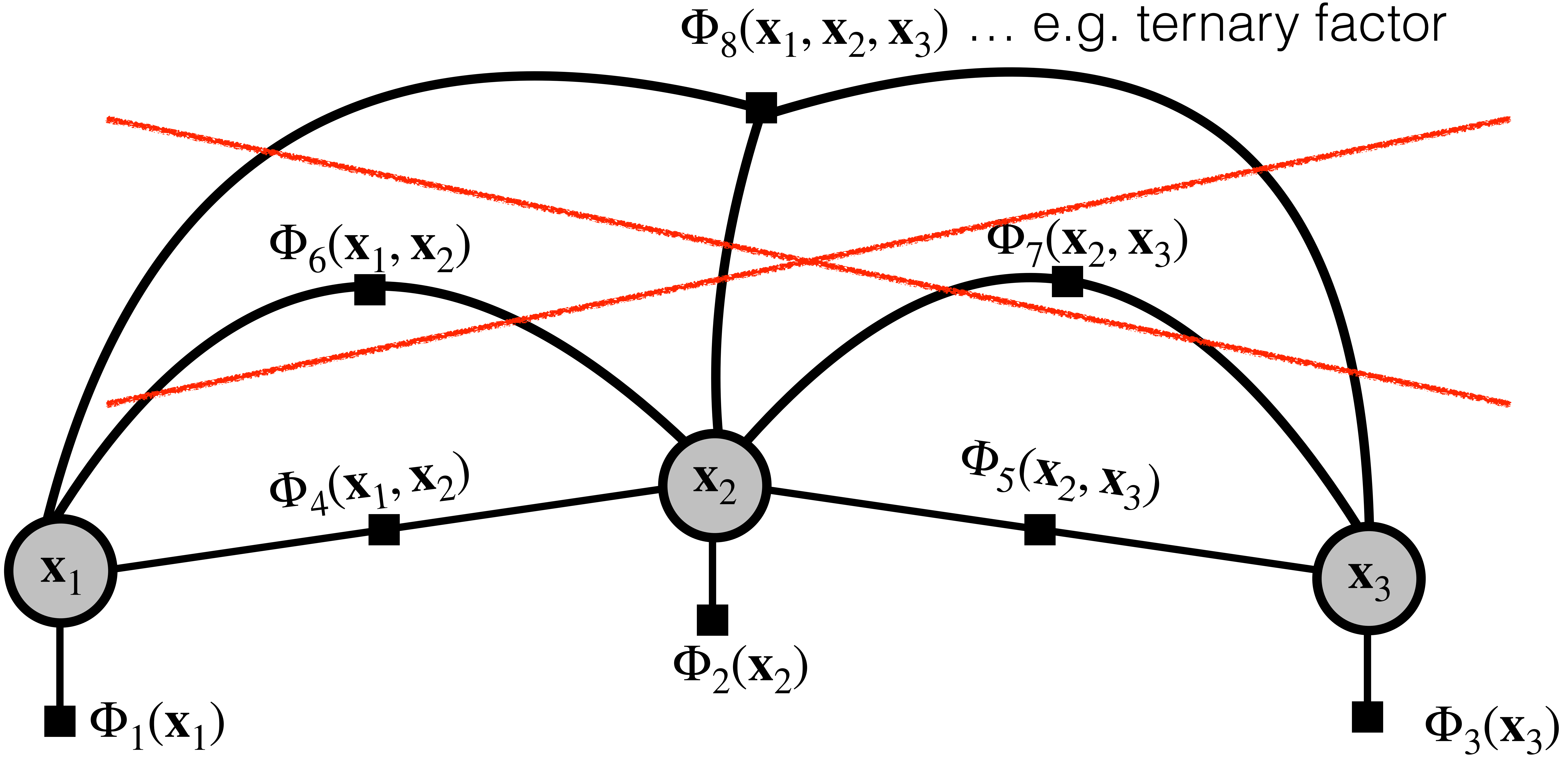
Factor graph

Def: Factor graph is bipartite graph $\mathcal{G} = \{\mathcal{U}, \mathcal{V}, \mathcal{E}\}$ with

- Two types of nodes: factors ■ $\boldsymbol{\Phi}_i \in \mathcal{U}$ and ● variables $\mathbf{x}_j \in \mathcal{V}$.
- Edges $\mathbf{e}_{ij} \in \mathcal{E}$ are always between factor nodes and variable nodes.



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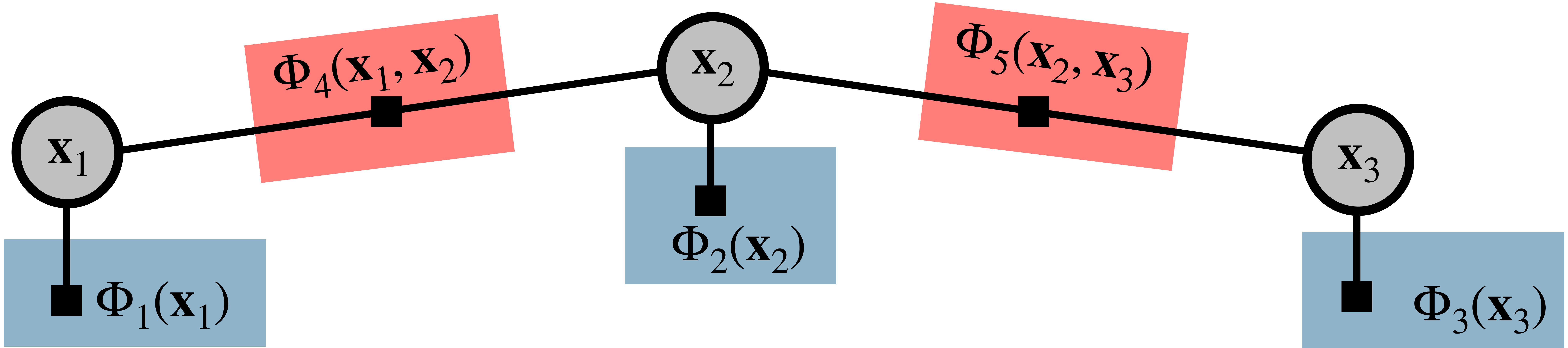

Bayes filter

Consequences:

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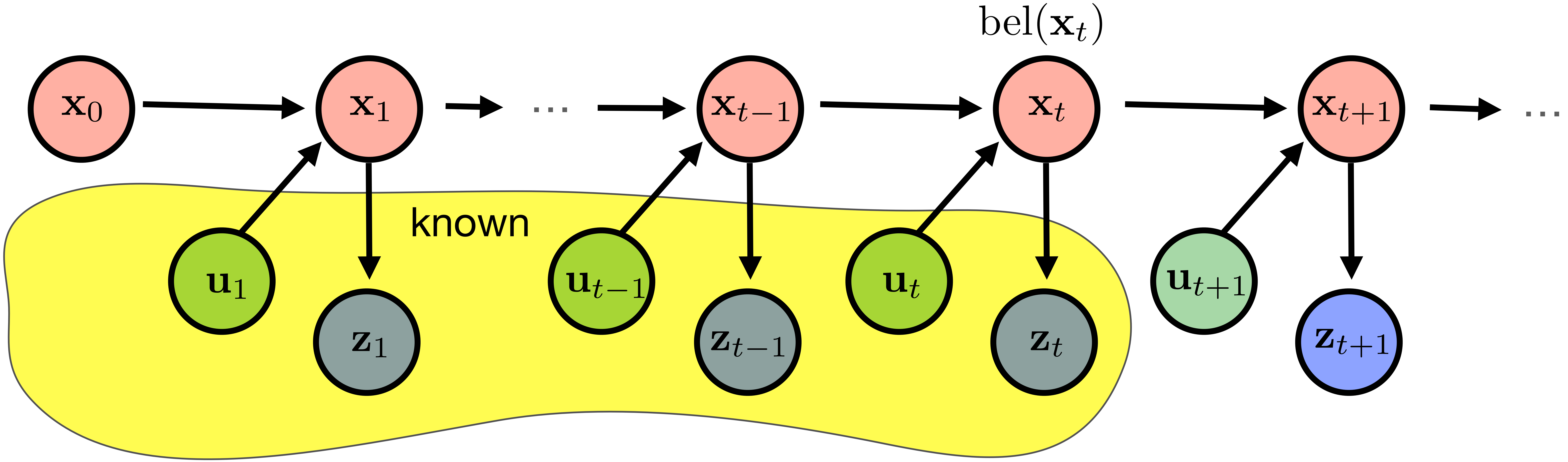
We don't have access to states - only measurements and actions are known $\mathbf{u}_1, \mathbf{z}_1, \ldots$

$\mathrm{bel}(\mathbf{x}_t) = p(\mathbf{x}_t|\mathbf{z}_{1:t}, \mathbf{u}_{1:t})$ Posterior belief (prob. distr. of current state only)



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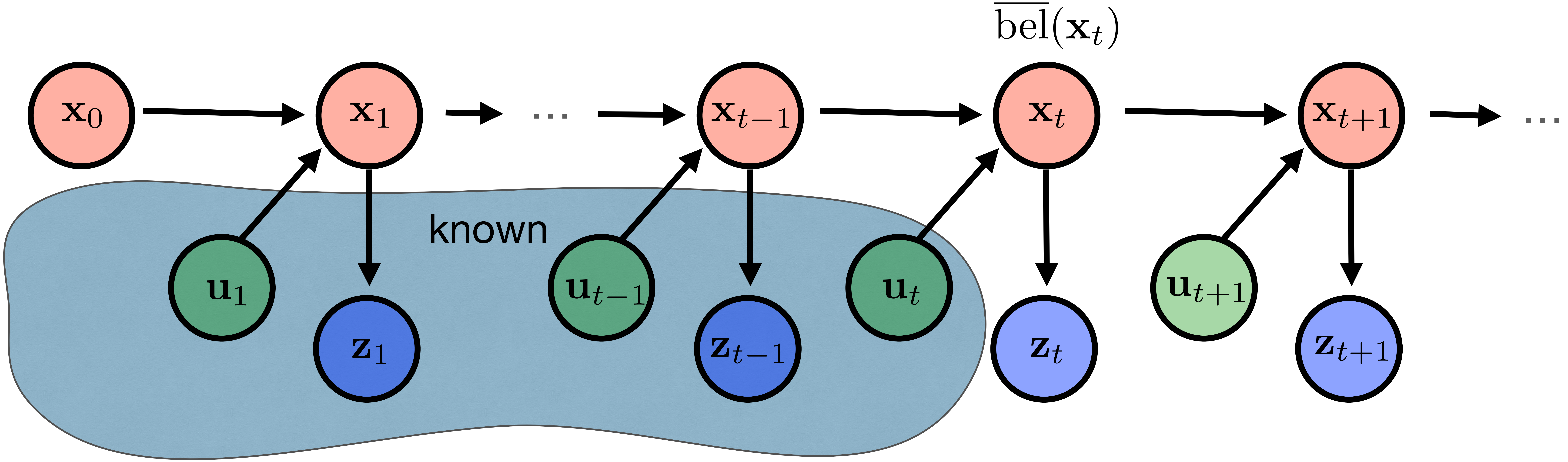

Bayes filter - derivation

Consequences:

measurement probability: $p(\mathbf{z}_t|\mathbf{x}_t) = p(\mathbf{z}_t|\mathbf{x}_t, \mathbf{x}_{0:t-1}, \mathbf{z}_{1:t-1}, \mathbf{u}_{1:t})$

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$$\overset{\text{LTP}}{=} \eta \cdot p(\mathbf{z}_t|\mathbf{x}_t) \cdot \int p(\mathbf{x}_t|\mathbf{x}_{t-1}, \mathbf{z}_{1:t-1}, \mathbf{u}_{1:t}) \cdot p(\mathbf{x}_{t-1}|\mathbf{z}_{1:t-1}, \mathbf{u}_{1:t}) d\mathbf{x}_{t-1}$$

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1. Initialization: $\text{bel}(\mathbf{x}_0)$, $t = 1$
2. Prediction step (new action $\mathbf{u}_t$ performed):

   For all $\mathbf{x}_t$

$$\overline{\text{bel}}(\mathbf{x}_t) = \int p(\mathbf{x}_t|\mathbf{x}_{t-1}, \mathbf{u}_t) \text{bel}(\mathbf{x}_{t-1}) d\mathbf{x}_{t-1}$$

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3. Measurement update (new $\mathbf{z}_t$ received):

   For all $\mathbf{x}_t$

   $$\mathrm{bel}(\mathbf{x}_t) = p(\mathbf{z}_t|\mathbf{x}_t) \overline{\mathrm{bel}}(\mathbf{x}_t)$$
   $$s = s + \mathrm{bel}(\mathbf{x}_t)$$

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   For all $\mathbf{x}_t$

   $$\mathrm{bel}(\mathbf{x}_t) = \frac{\mathrm{bel}(\mathbf{x}_t)}{s}$$

$$\text{bel}(\mathbf{x}_t) = \eta \cdot p(\mathbf{z}_t | \mathbf{x}_t) \cdot \underbrace{\int p(\mathbf{x}_t | \mathbf{x}_{t-1}, \mathbf{u}_t) \cdot \text{bel}(\mathbf{x}_{t-1}) d\mathbf{x}_{t-1}}_{\overline{\text{bel}}(\mathbf{x}_t)}$$

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4. Repeat from 2:

   $t = t + 1$

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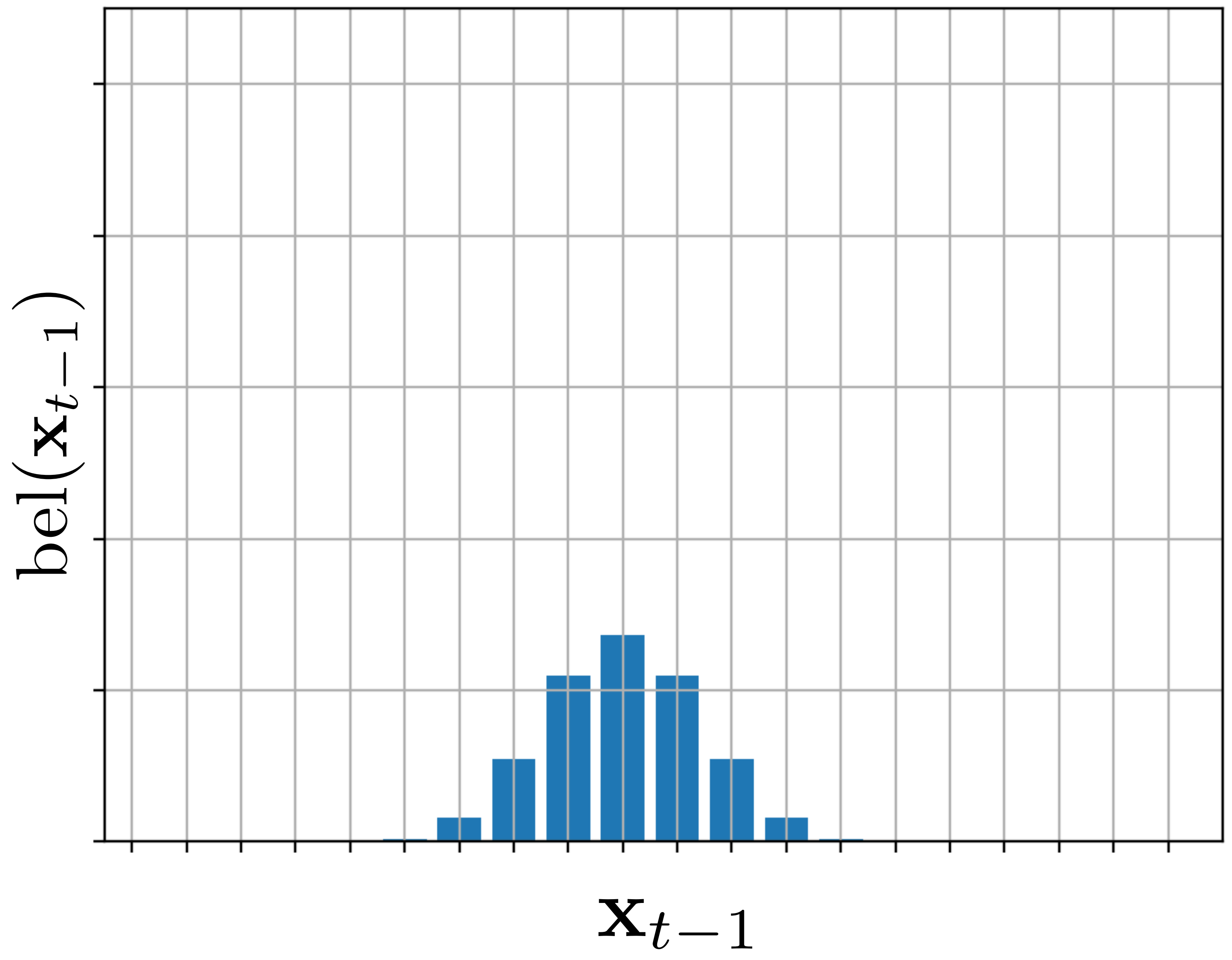

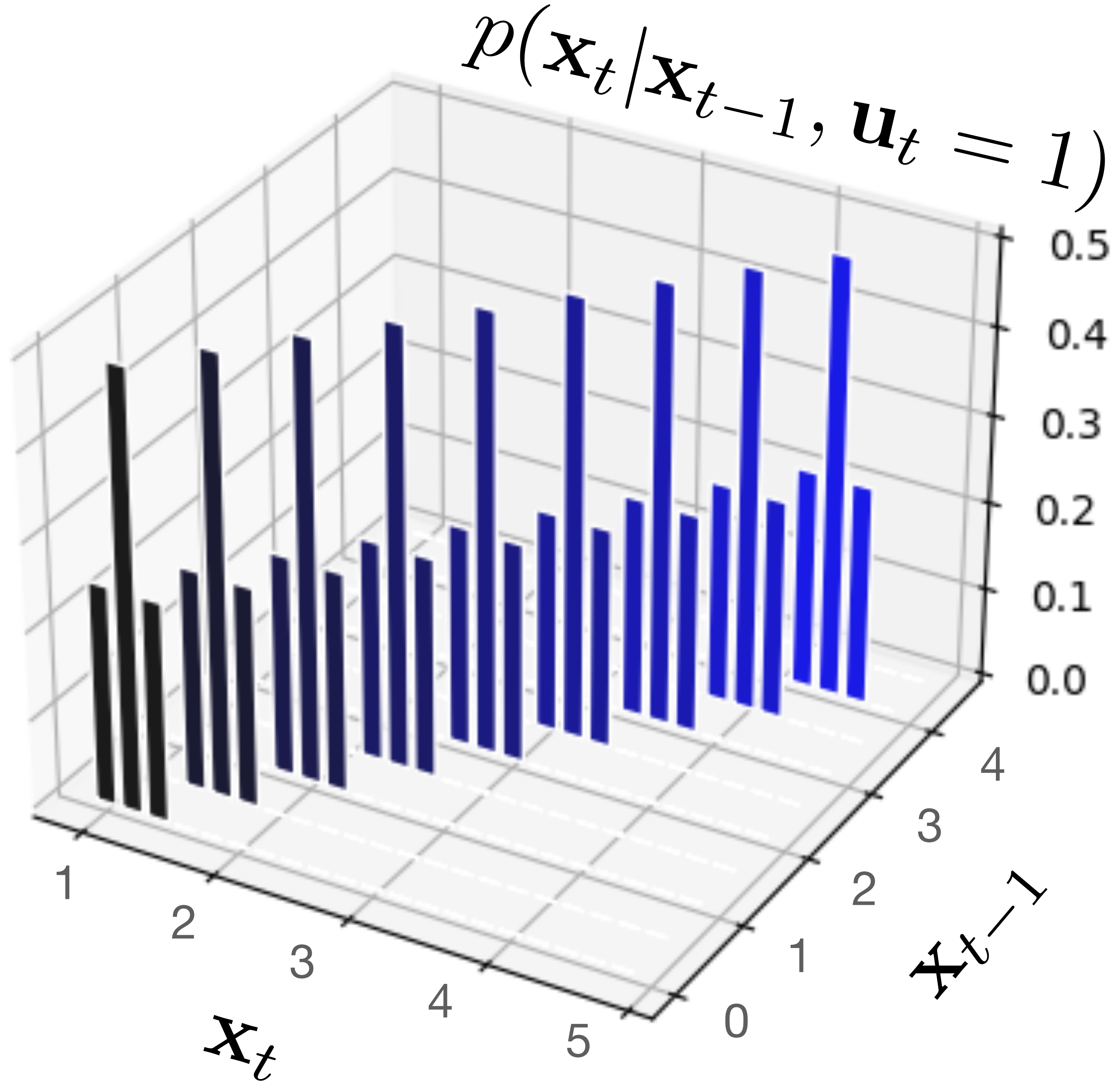

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1. Initialization: $\mathrm{bel}(\mathbf{x}_0)$, $t = 1$
2. Prediction step (new action $\mathbf{u}_t$ performed):

   For all $\mathbf{x}_t$

$$\overline{\mathrm{bel}}(\mathbf{x}_t) = \int p(\mathbf{x}_t|\mathbf{x}_{t-1}, \mathbf{u}_t)\mathrm{bel}(\mathbf{x}_{t-1})d\mathbf{x}_{t-1}$$

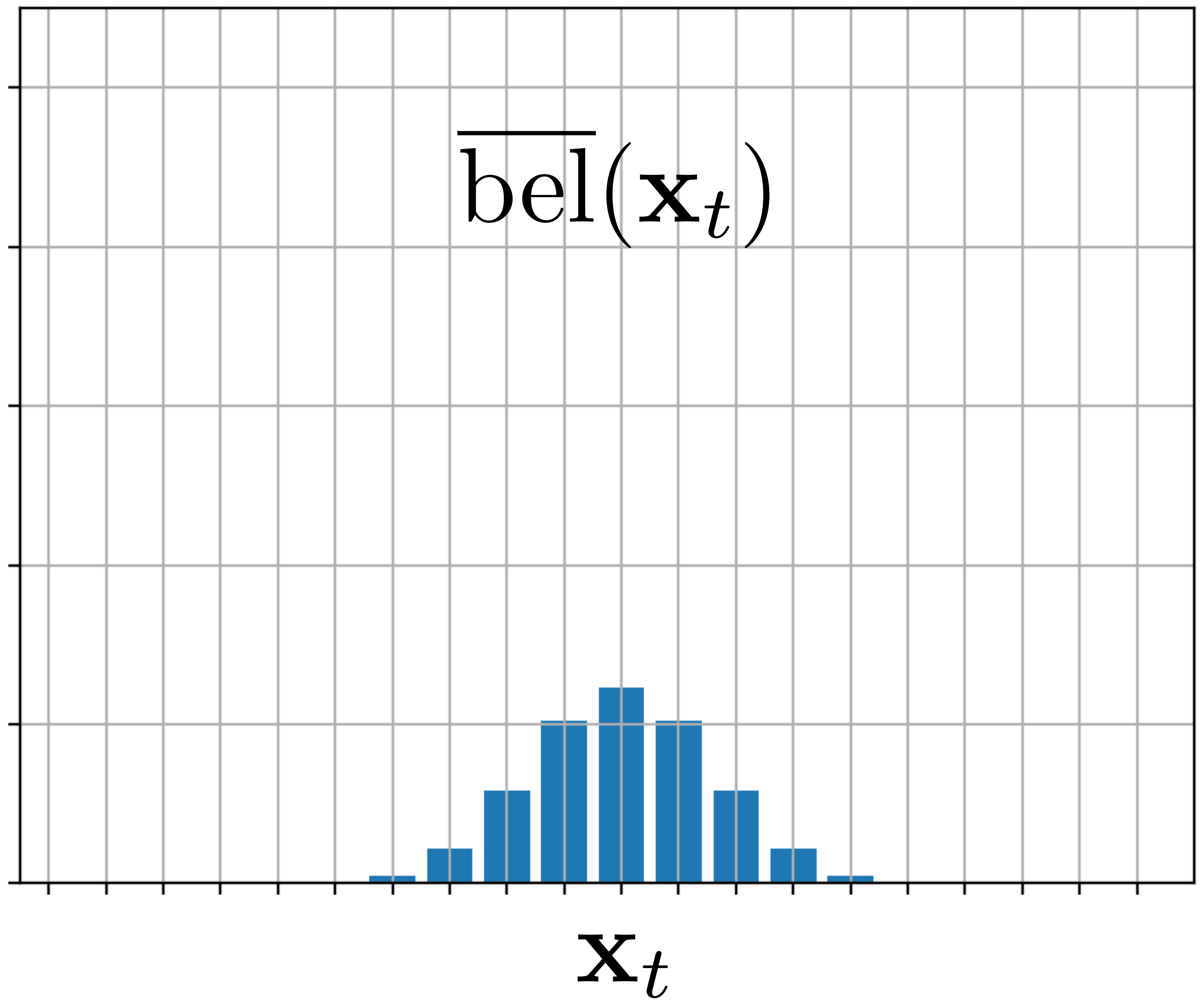
3. Measurement update (new $\mathbf{z}_t$ received):

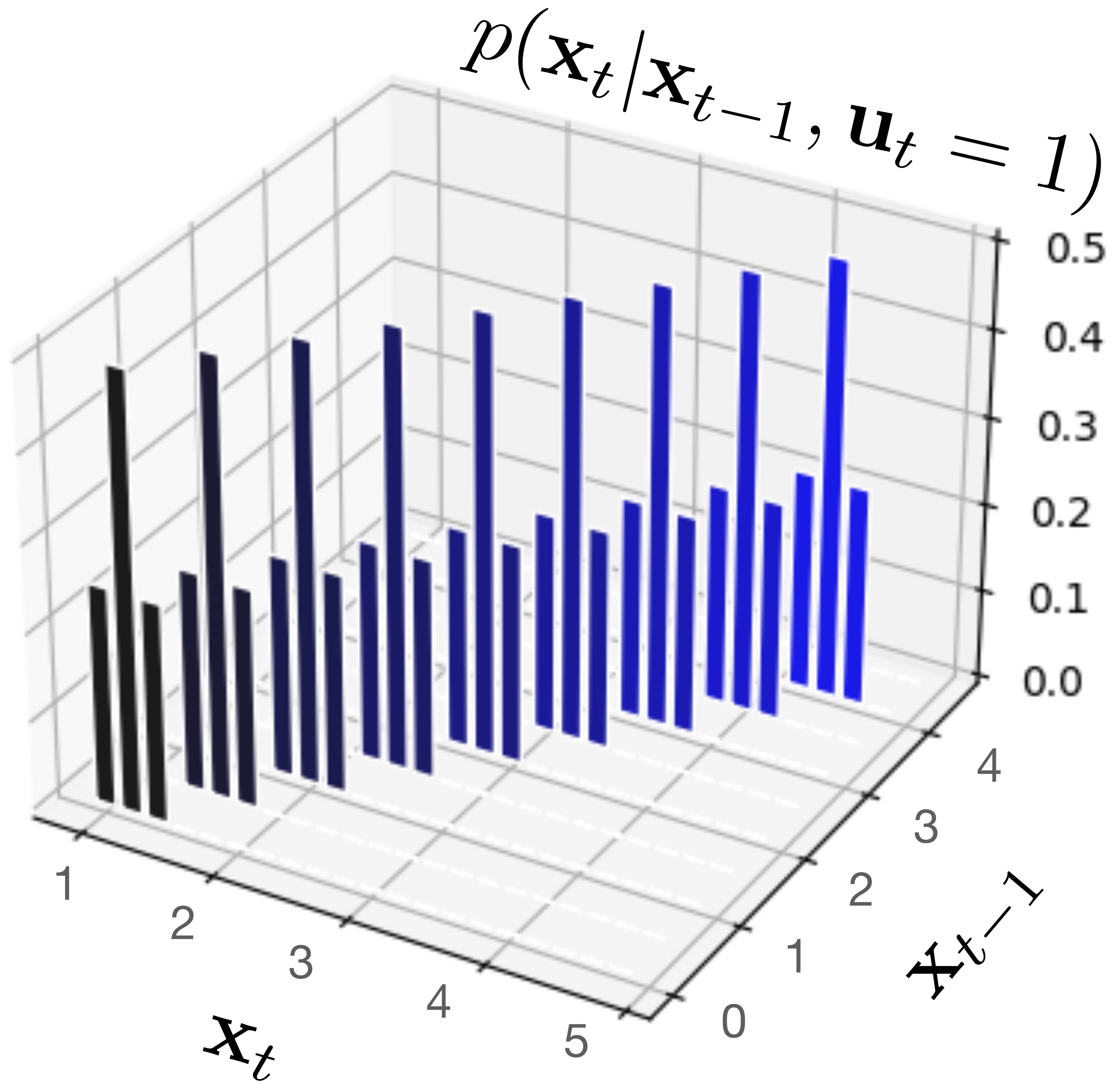
   For all $\mathbf{x}_t$

$$\mathrm{bel}(\mathbf{x}_t) = \overline{\mathrm{bel}}(\mathbf{x}_t)$$

4. Repeat from 2:

   $t = t + 1$





1. Initialization: $\mathrm{bel}(\mathbf{x}_0)$, $t = 1$
2. Prediction step (new action $\mathbf{u}_t$ performed):

   For all $\mathbf{x}_t$

$$\overline{\mathrm{bel}}(\mathbf{x}_t) = \int p(\mathbf{x}_t | \mathbf{x}_{t-1}, \mathbf{u}_t) \mathrm{bel}(\mathbf{x}_{t-1}) d\mathbf{x}_{t-1}$$

3. Measurement update (new $\mathbf{z}_t$ received):

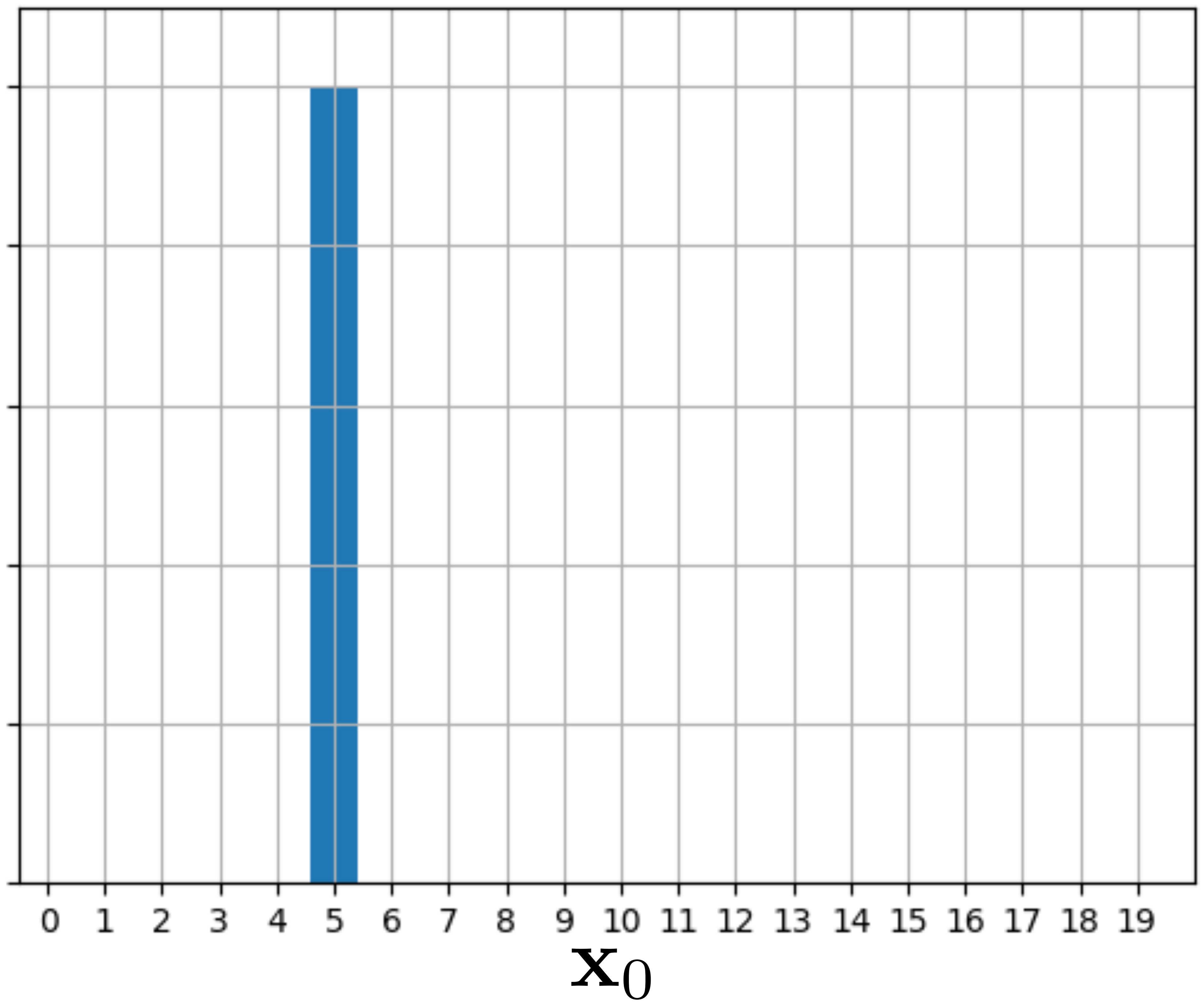
   For all $\mathbf{x}_t$

$$\mathrm{bel}(\mathbf{x}_t) = \overline{\mathrm{bel}}(\mathbf{x}_t)$$

4. Repeat from 2:

   $t = t + 1$

$\mathrm{bel}(\mathbf{x}_0)$



1. Initialization: $\text{bel}(\mathbf{x}_0)$, $t = 1$
2. Prediction step (new action $\mathbf{u}_t$ performed): $\mathbf{u}_t = +1$

   For all $\mathbf{x}_t$

$$\overline{\text{bel}}(\mathbf{x}_t) = \int p(\mathbf{x}_t | \mathbf{x}_{t-1}, \mathbf{u}_t)\text{bel}(\mathbf{x}_{t-1})d\mathbf{x}_{t-1}$$

3. Measurement update (new $\mathbf{z}_t$ received):

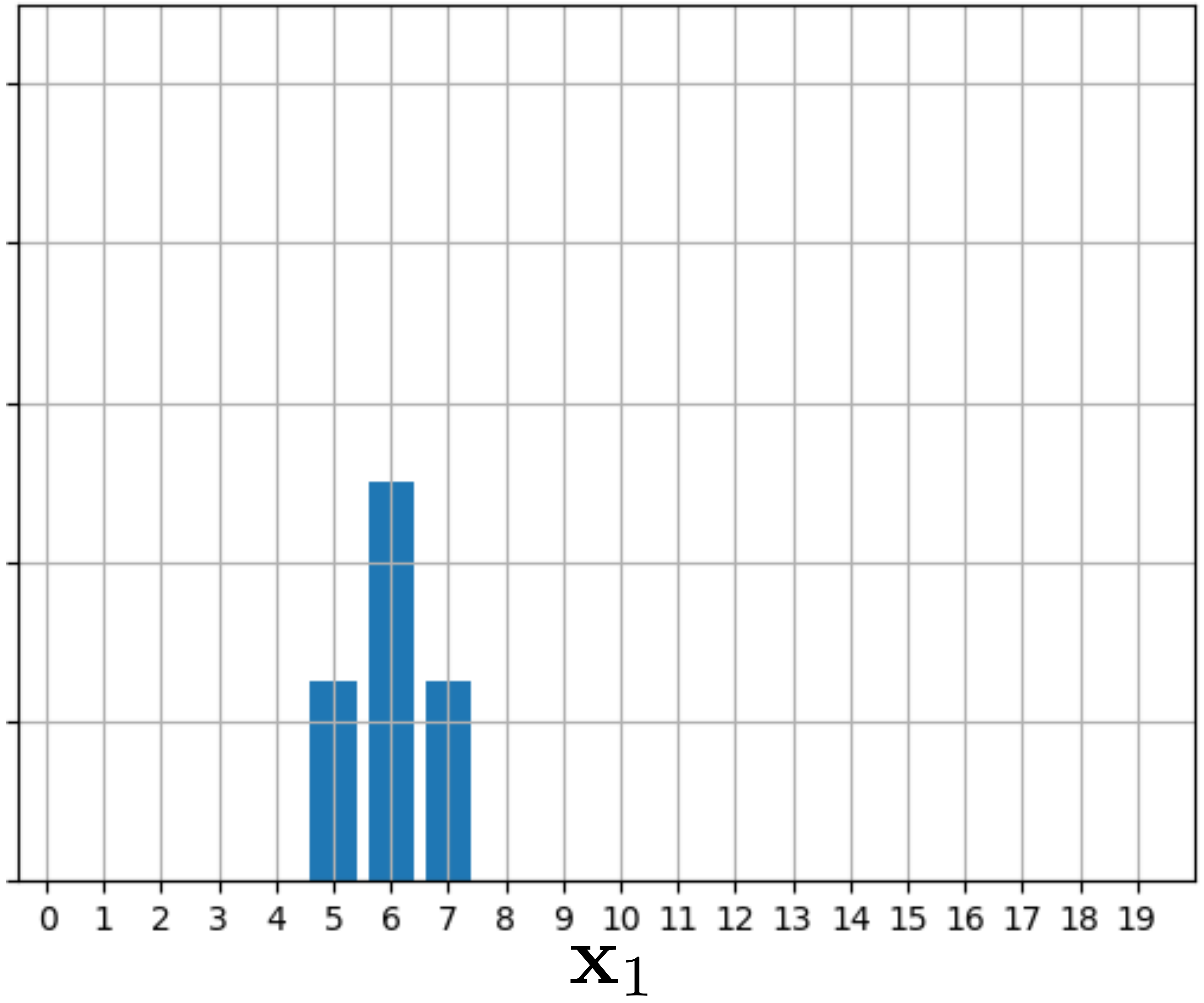
   For all $\mathbf{x}_t$

$$\text{bel}(\mathbf{x}_t) = \overline{\text{bel}}(\mathbf{x}_t)$$

4. Repeat from 2:

   $t = t + 1$

$\text{bel}(\mathbf{x}_1)$



1. Initialization: $\text{bel}(\mathbf{x}_0)$, $t = 1$
2. Prediction step (new action $\mathbf{u}_t$ performed): $\mathbf{u}_t = +1$

   For all $\mathbf{x}_t$

$$\overline{\text{bel}}(\mathbf{x}_t) = \int p(\mathbf{x}_t | \mathbf{x}_{t-1}, \mathbf{u}_t) \text{bel}(\mathbf{x}_{t-1}) d\mathbf{x}_{t-1}$$

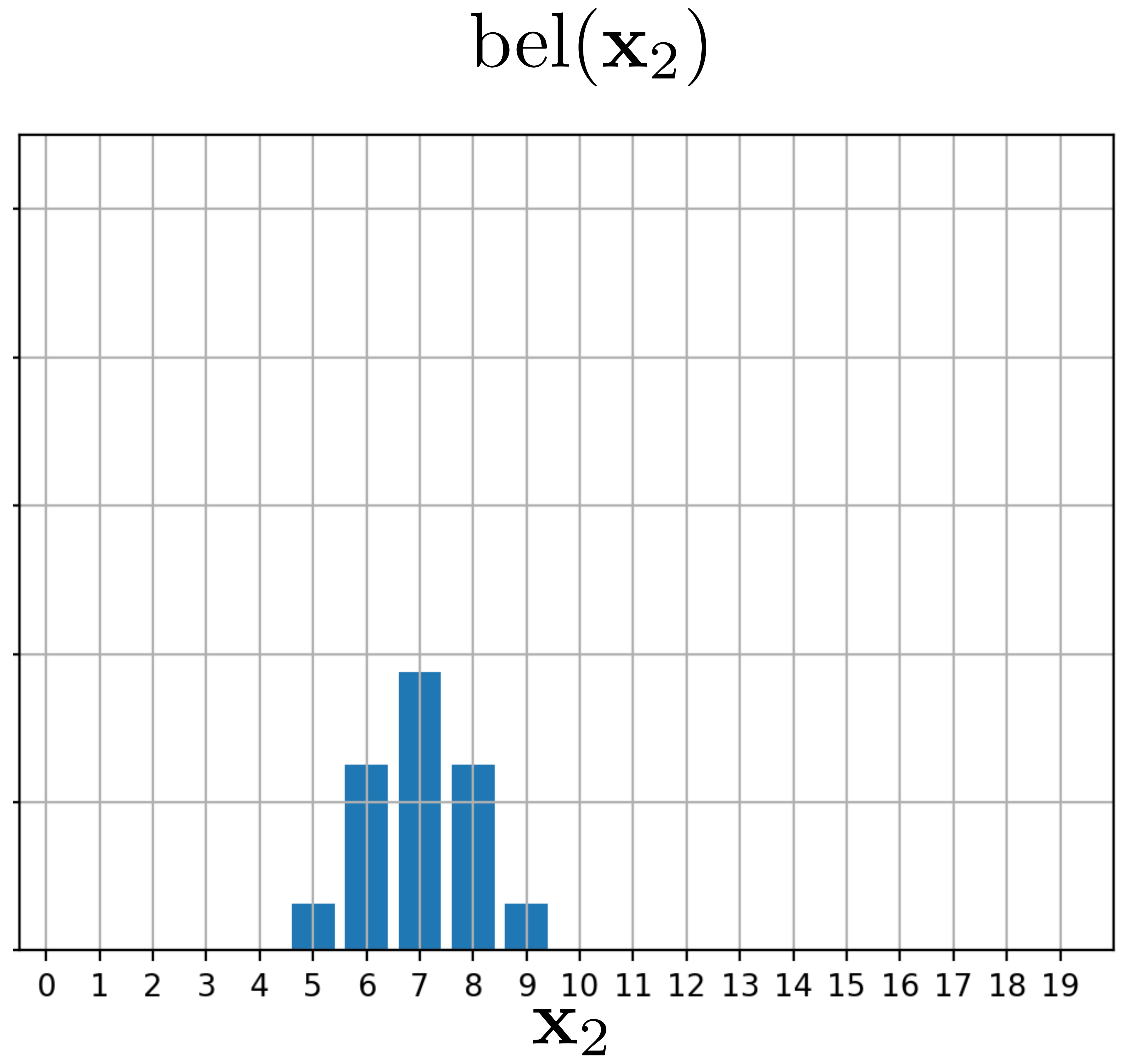
3. Measurement update (new $\mathbf{z}_t$ received):

   For all $\mathbf{x}_t$

$$\text{bel}(\mathbf{x}_t) = \overline{\text{bel}}(\mathbf{x}_t)$$

4. Repeat from 2:

   $t = t + 1$



1. Initialization: $\mathrm{bel}(\mathbf{x}_0)$, $t = 1$
2. Prediction step (new action $\mathbf{u}_t$ performed): $\mathbf{u}_t = +1$

   For all $\mathbf{x}_t$

$$\overline{\mathrm{bel}}(\mathbf{x}_t) = \int p(\mathbf{x}_t|\mathbf{x}_{t-1}, \mathbf{u}_t)\mathrm{bel}(\mathbf{x}_{t-1})d\mathbf{x}_{t-1}$$

3. Measurement update (new $\mathbf{z}_t$ received):

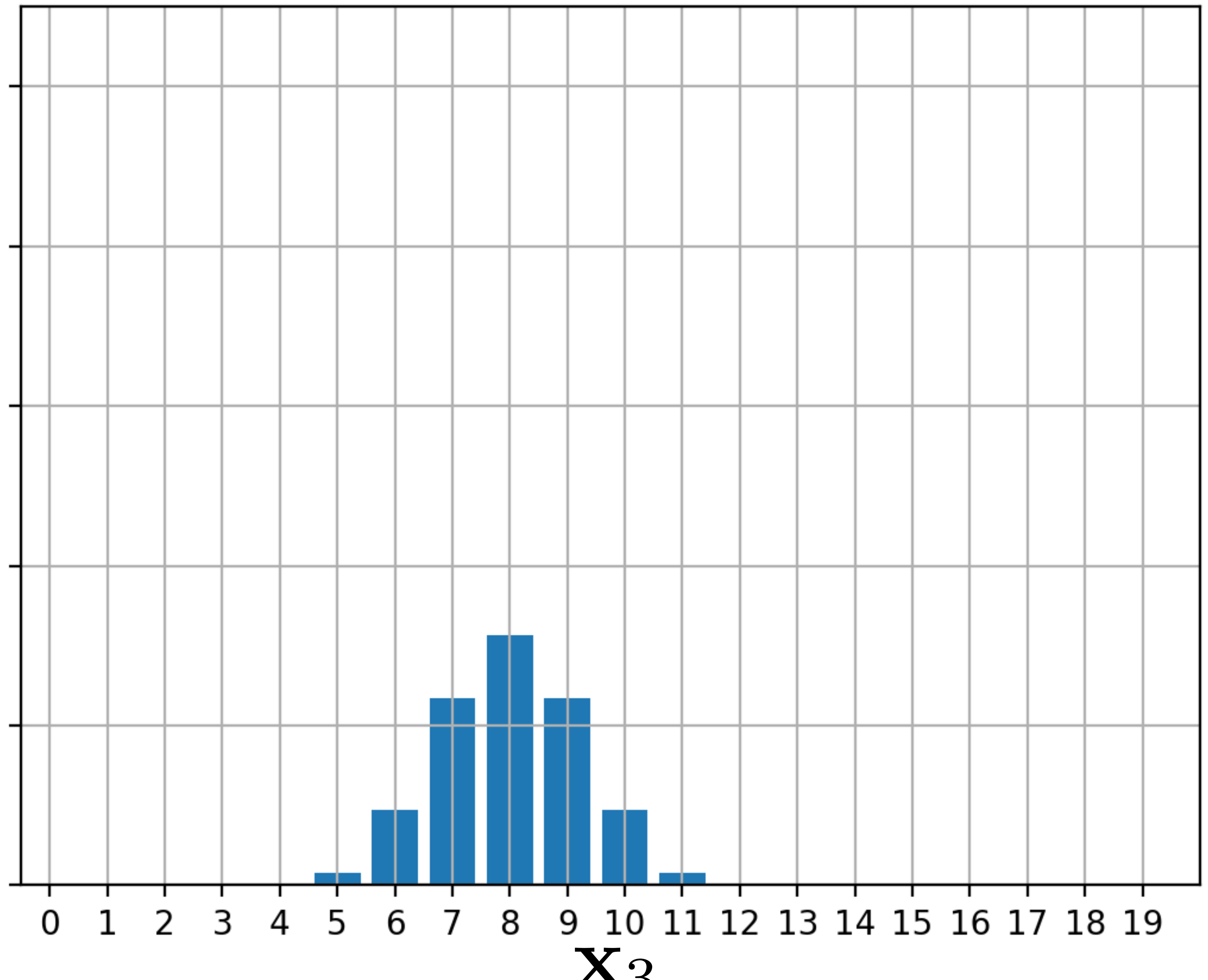
   For all $\mathbf{x}_t$

$$\mathrm{bel}(\mathbf{x}_t) = \overline{\mathrm{bel}}(\mathbf{x}_t)$$

4. Repeat from 2:

   $t = t + 1$

$\mathrm{bel}(\mathbf{x}_3)$



1. Initialization: $\mathrm{bel}(\mathbf{x}_0)$, $t = 1$
2. Prediction step (new action $\mathbf{u}_t$ performed): $\mathbf{u}_t = +1$

   For all $\mathbf{x}_t$

$$\overline{\mathrm{bel}}(\mathbf{x}_t) = \int p(\mathbf{x}_t | \mathbf{x}_{t-1}, \mathbf{u}_t) \mathrm{bel}(\mathbf{x}_{t-1}) d\mathbf{x}_{t-1}$$

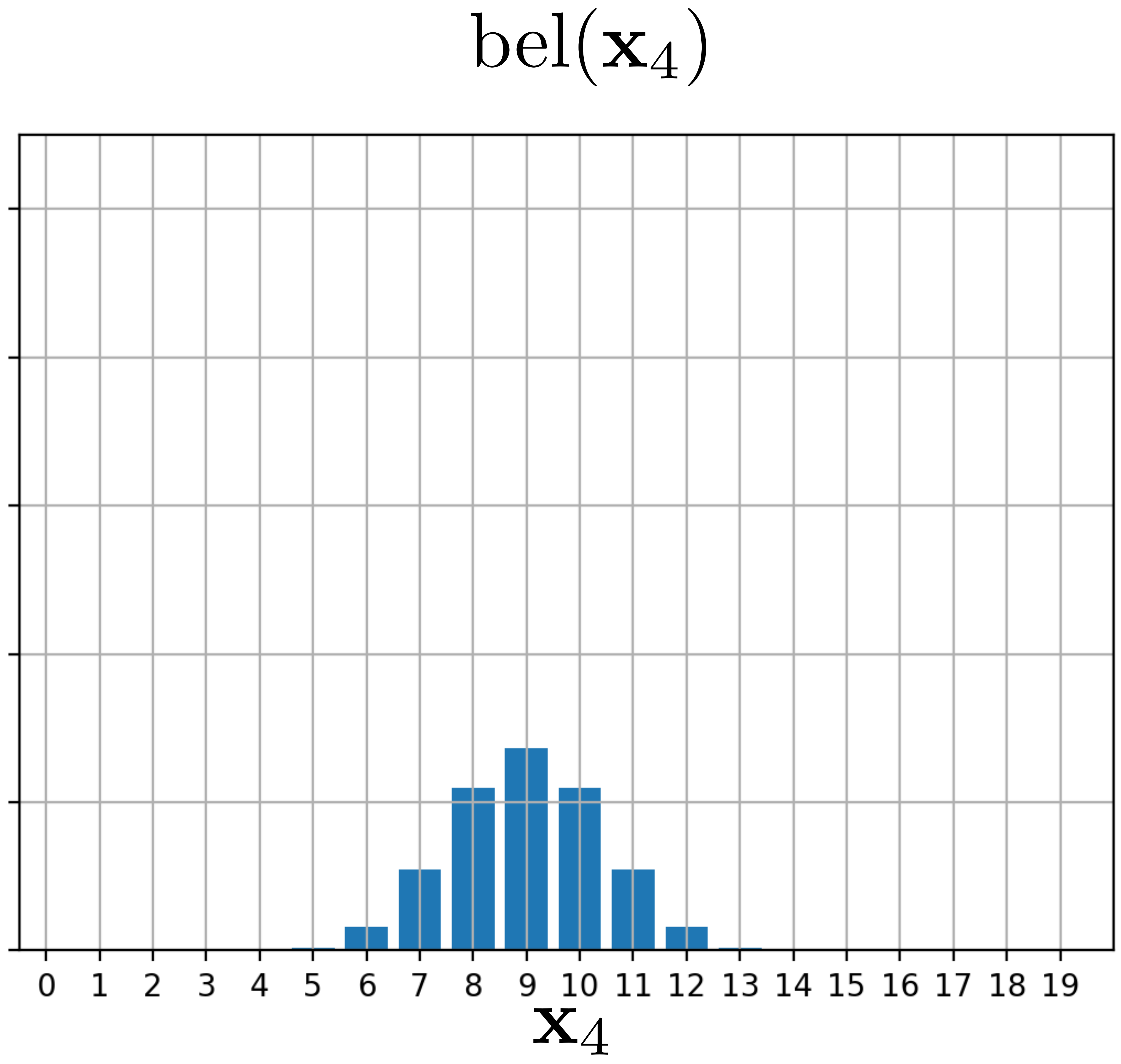
3. Measurement update (new $\mathbf{z}_t$ received):

   For all $\mathbf{x}_t$

$$\mathrm{bel}(\mathbf{x}_t) = \overline{\mathrm{bel}}(\mathbf{x}_t)$$

4. Repeat from 2:

   $t = t + 1$



Bayes filter

1. Initialization: $\text{bel}(\mathbf{x}_0)$, $t = 1$
2. Prediction step (new action $\mathbf{u}_t$ performed): $\mathbf{u}_t = +1$

   For all $\mathbf{x}_t$

$$\overline{\text{bel}}(\mathbf{x}_t) = \int p(\mathbf{x}_t | \mathbf{x}_{t-1}, \mathbf{u}_t) \text{bel}(\mathbf{x}_{t-1}) d\mathbf{x}_{t-1}$$

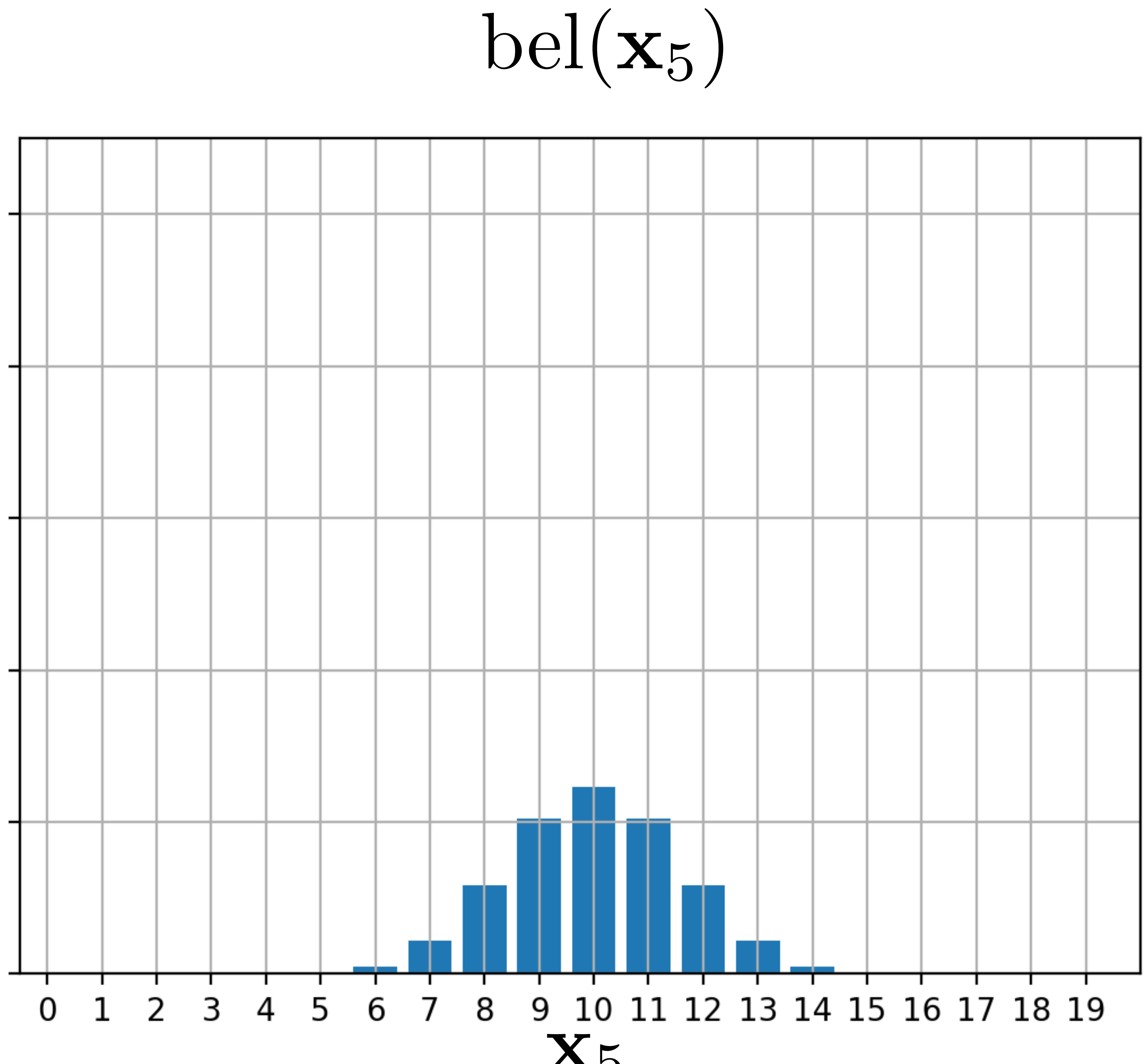
3. Measurement update (new $\mathbf{z}_t$ received):

   For all $\mathbf{x}_t$

$$\text{bel}(\mathbf{x}_t) = \overline{\text{bel}}(\mathbf{x}_t)$$

4. Repeat from 2:

   $t = t + 1$



Bayes filter

1. Initialization: $\mathrm{bel}(\mathbf{x}_0)$, $t = 1$
2. Prediction step (new action $\mathbf{u}_t$ performed):

   For all $\mathbf{x}_t$

$$\overline{\mathrm{bel}}(\mathbf{x}_t) = \int p(\mathbf{x}_t|\mathbf{x}_{t-1}, \mathbf{u}_t)\mathrm{bel}(\mathbf{x}_{t-1})d\mathbf{x}_{t-1}$$

3. Measurement update (new $\mathbf{z}_t$ received):

   For all $\mathbf{x}_t$

$$\mathrm{bel}(\mathbf{x}_t) = p(\mathbf{z}_t|\mathbf{x}_t)\overline{\mathrm{bel}}(\mathbf{x}_t)$$

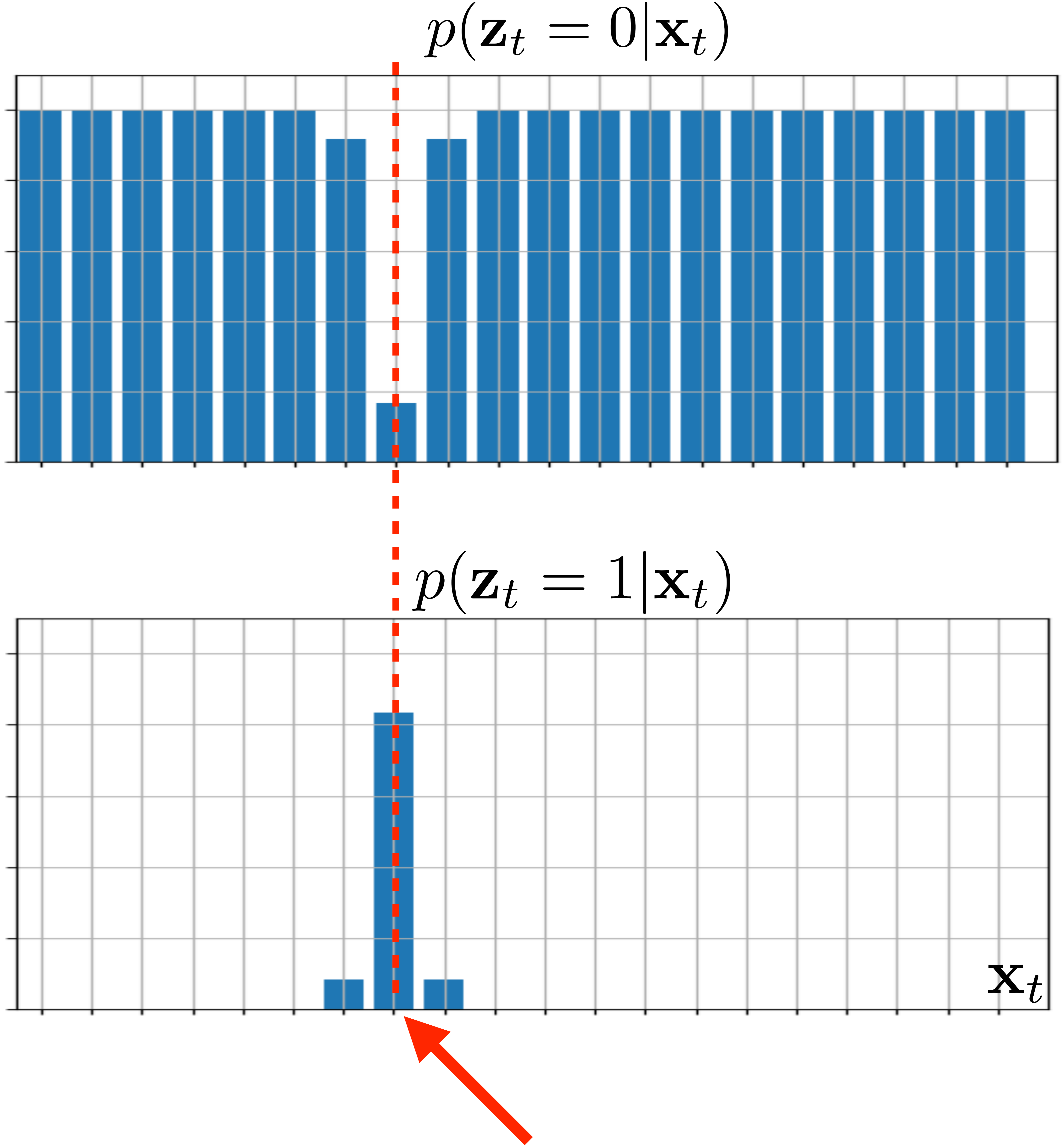
$$s = s + \mathrm{bel}(\mathbf{x}_t)$$

   For all $\mathbf{x}_t$

$$\mathrm{bel}(\mathbf{x}_t) = \frac{\mathrm{bel}(\mathbf{x}_t)}{s}$$

4. Repeat from 2:

$$t = t + 1$$



one marker at known locations
+
inaccurate sensor

Bayes filter

1. Initialization: $\mathrm{bel}(\mathbf{x}_0)$, $t = 1$
2. Prediction step (new action $\mathbf{u}_t$ performed):

   For all $\mathbf{x}_t$

   $$\overline{\mathrm{bel}}(\mathbf{x}_t) = \int p(\mathbf{x}_t|\mathbf{x}_{t-1}, \mathbf{u}_t)\mathrm{bel}(\mathbf{x}_{t-1})d\mathbf{x}_{t-1}$$

3. Measurement update (new $\mathbf{z}_t$ received):

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   $$s = s + \mathrm{bel}(\mathbf{x}_t)$$

   For all $\mathbf{x}_t$

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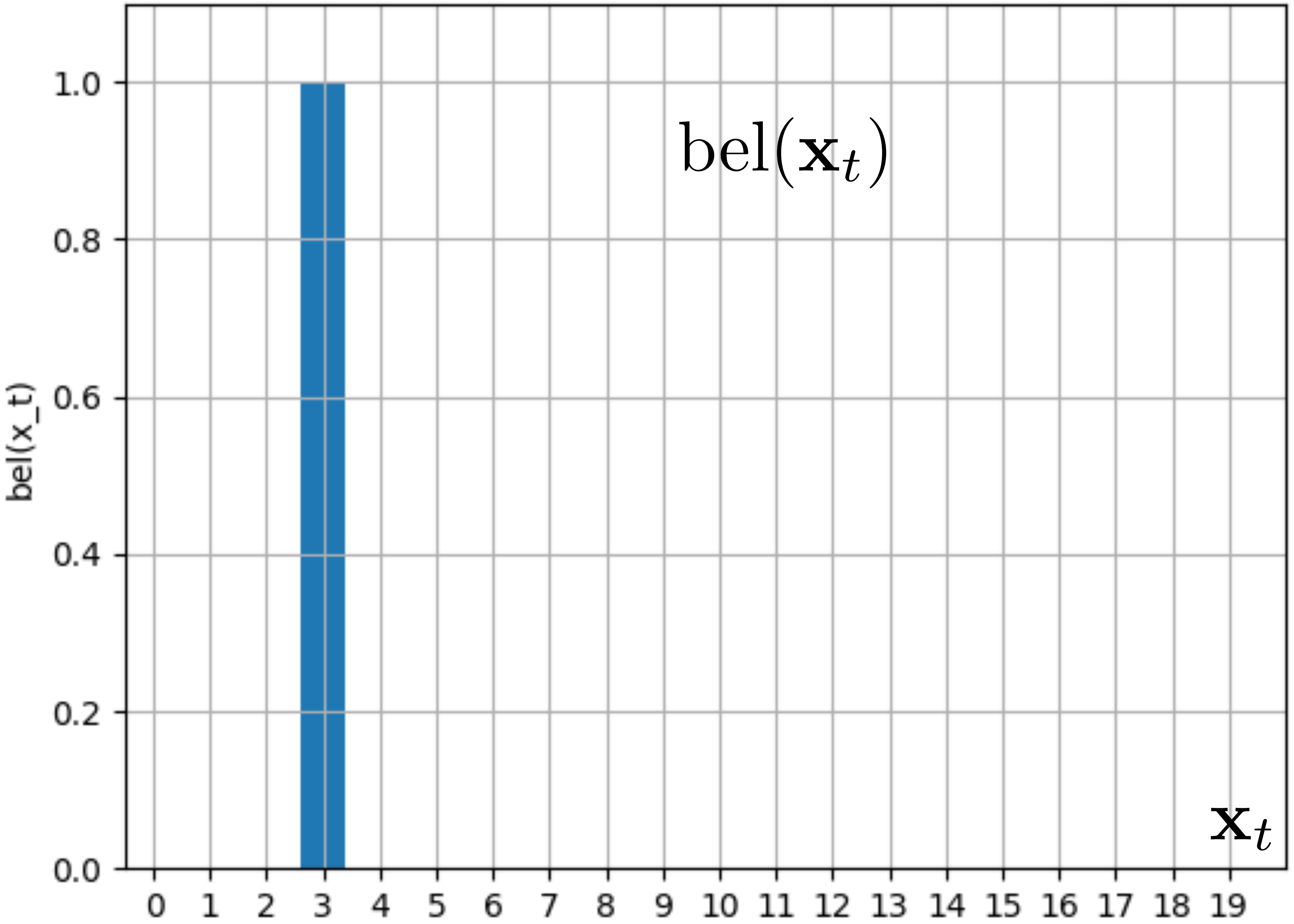

Bayes filter

1. Initialization: $\mathrm{bel}(\mathbf{x}_0)$, $t = 1$
2. Prediction step (new action $\mathbf{u}_t$ performed):

   For all $\mathbf{x}_t$

   $$\overline{\mathrm{bel}}(\mathbf{x}_t) = \int p(\mathbf{x}_t|\mathbf{x}_{t-1}, \mathbf{u}_t)\mathrm{bel}(\mathbf{x}_{t-1})d\mathbf{x}_{t-1}$$

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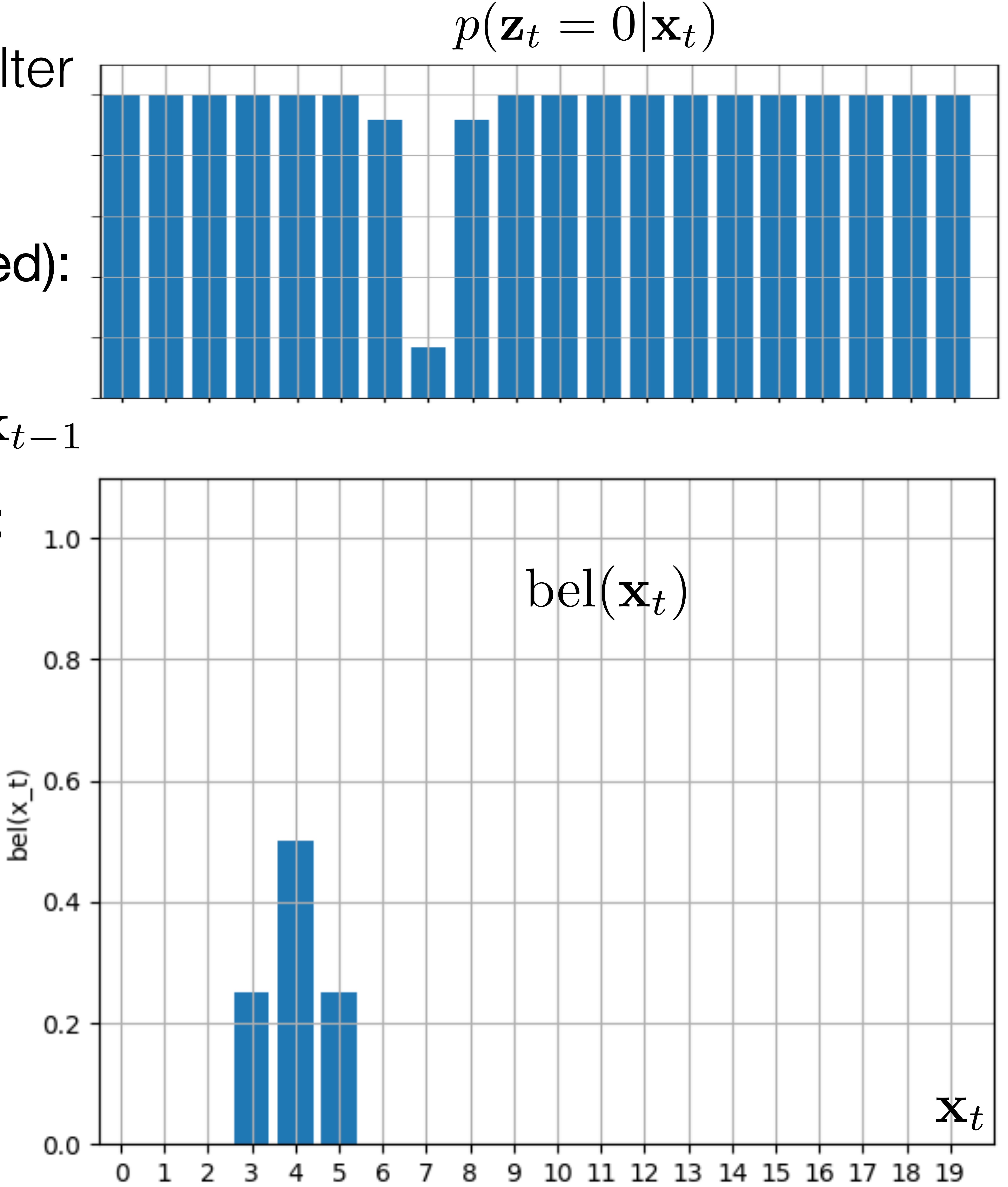

Bayes filter

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2. Prediction step (new action $\mathbf{u}_t$ performed):

   For all $\mathbf{x}_t$

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   For all $\mathbf{x}_t$

   $$\mathrm{bel}(\mathbf{x}_t) = p(\mathbf{z}_t | \mathbf{x}_t)\overline{\mathrm{bel}}(\mathbf{x}_t)$$
   $$s = s + \mathrm{bel}(\mathbf{x}_t)$$

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   $$\mathrm{bel}(\mathbf{x}_t) = \frac{\mathrm{bel}(\mathbf{x}_t)}{s}$$

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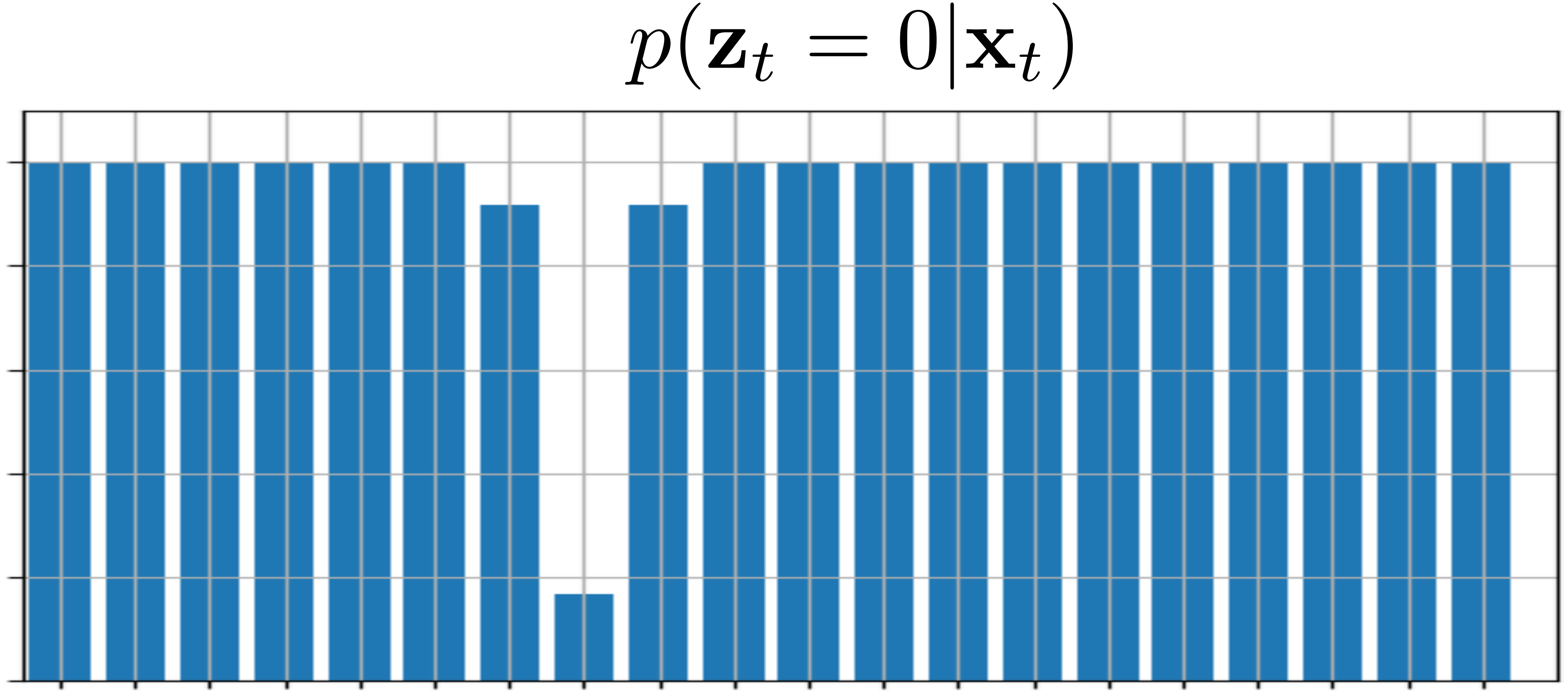

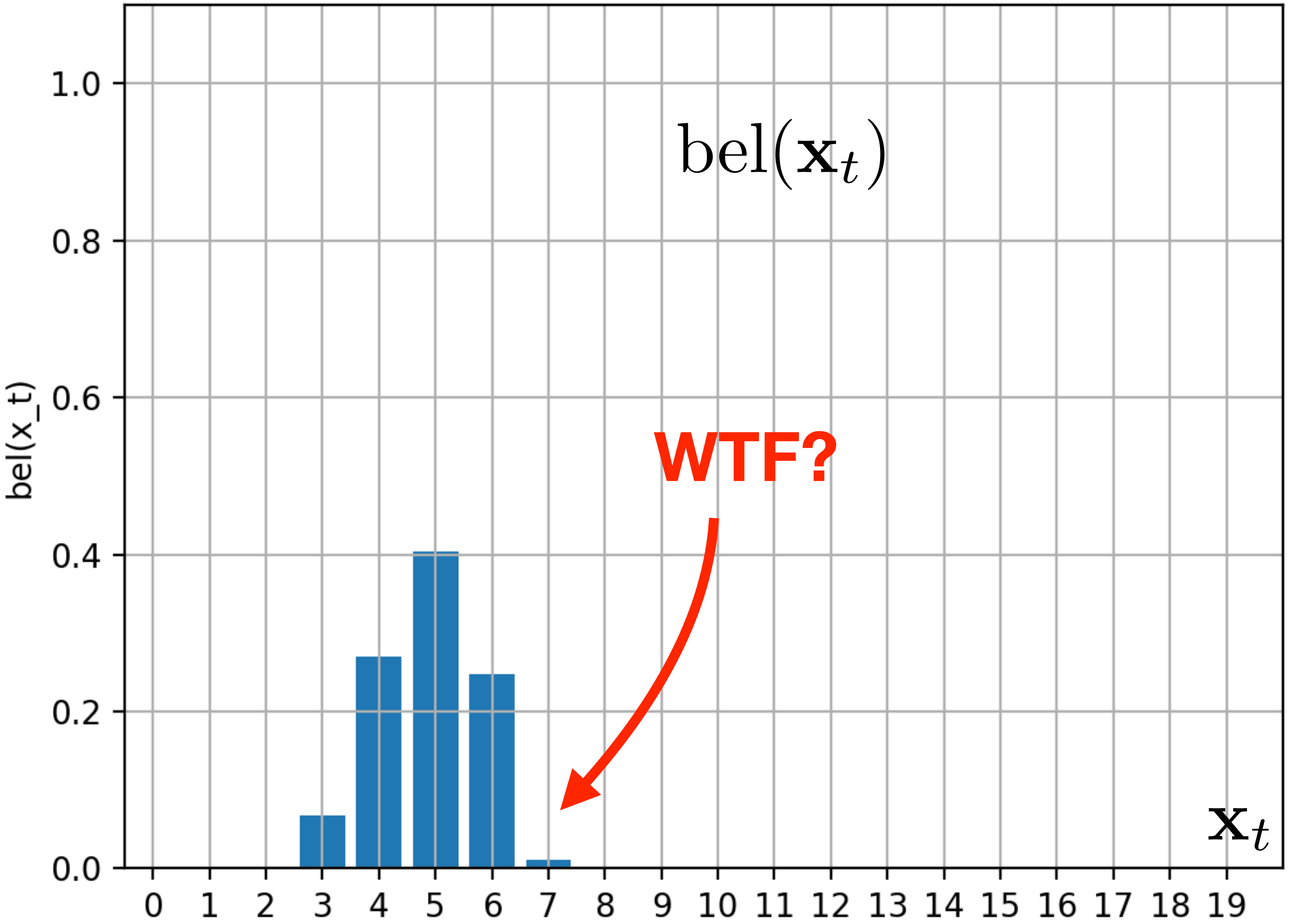

Bayes filter

1. Initialization: $\text{bel}(\mathbf{x}_0)$, $t = 1$
2. Prediction step (new action $\mathbf{u}_t$ performed):

   For all $\mathbf{x}_t$

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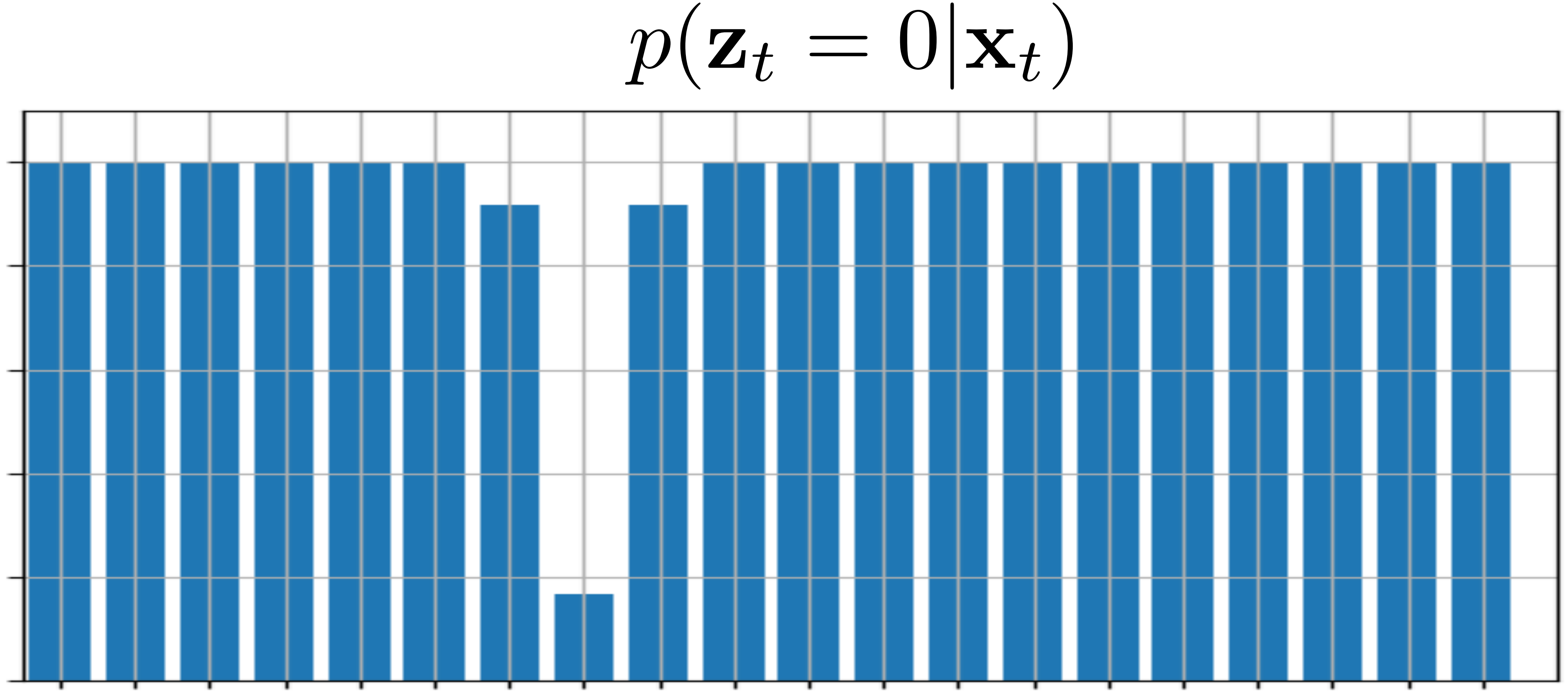

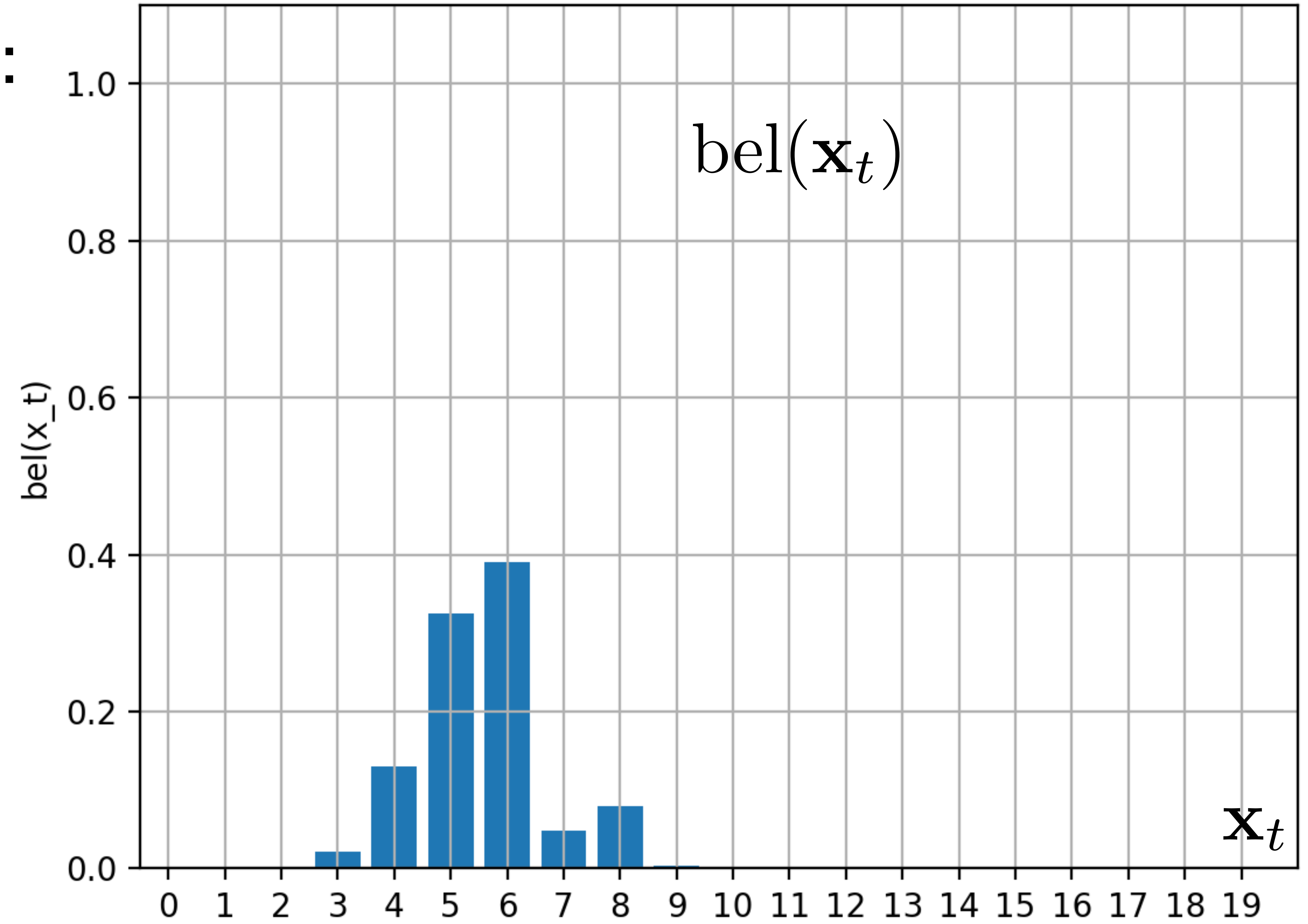

Bayes filter

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2. Prediction step (new action $\mathbf{u}_t$ performed):

   For all $\mathbf{x}_t$

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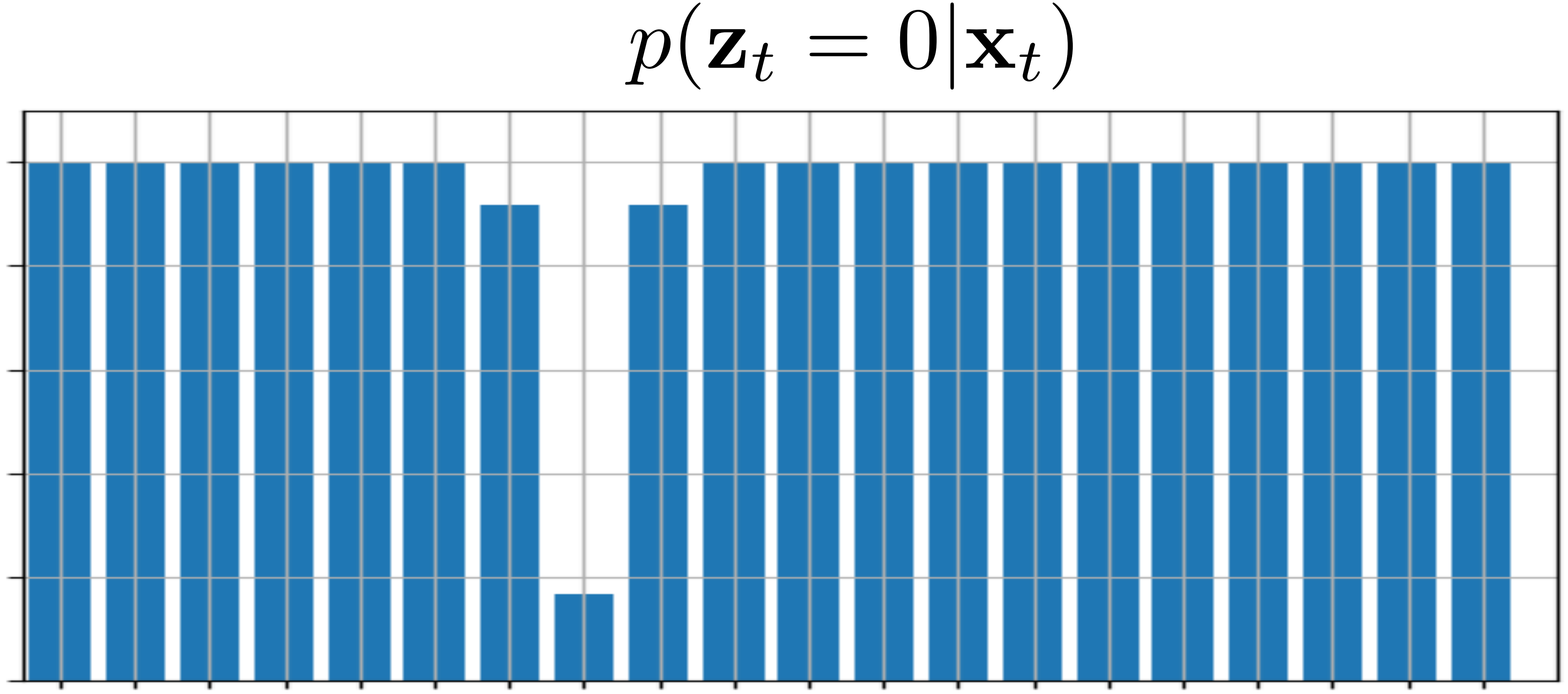

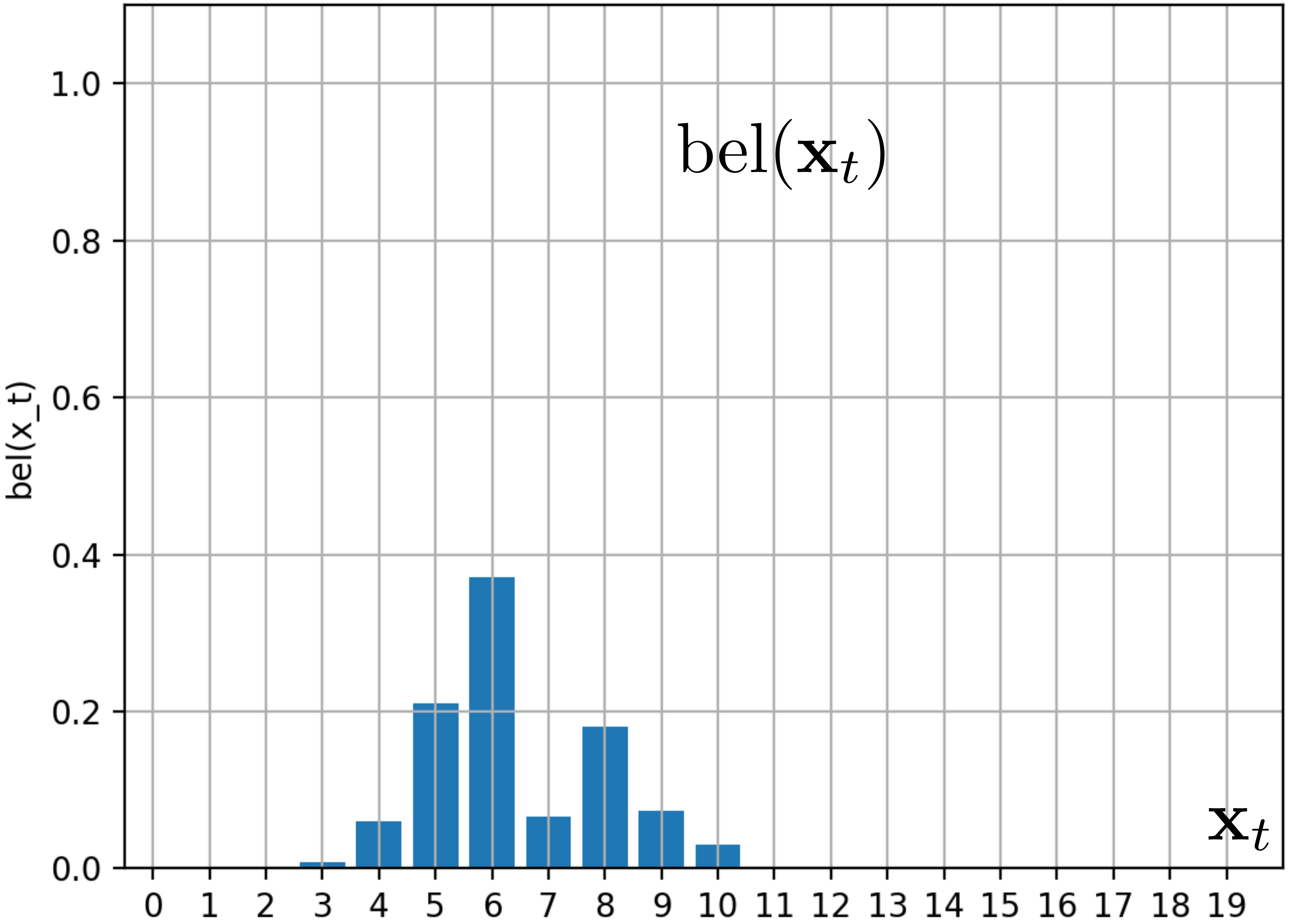

Bayes filter

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2. Prediction step (new action $\mathbf{u}_t$ performed):

   For all $\mathbf{x}_t$

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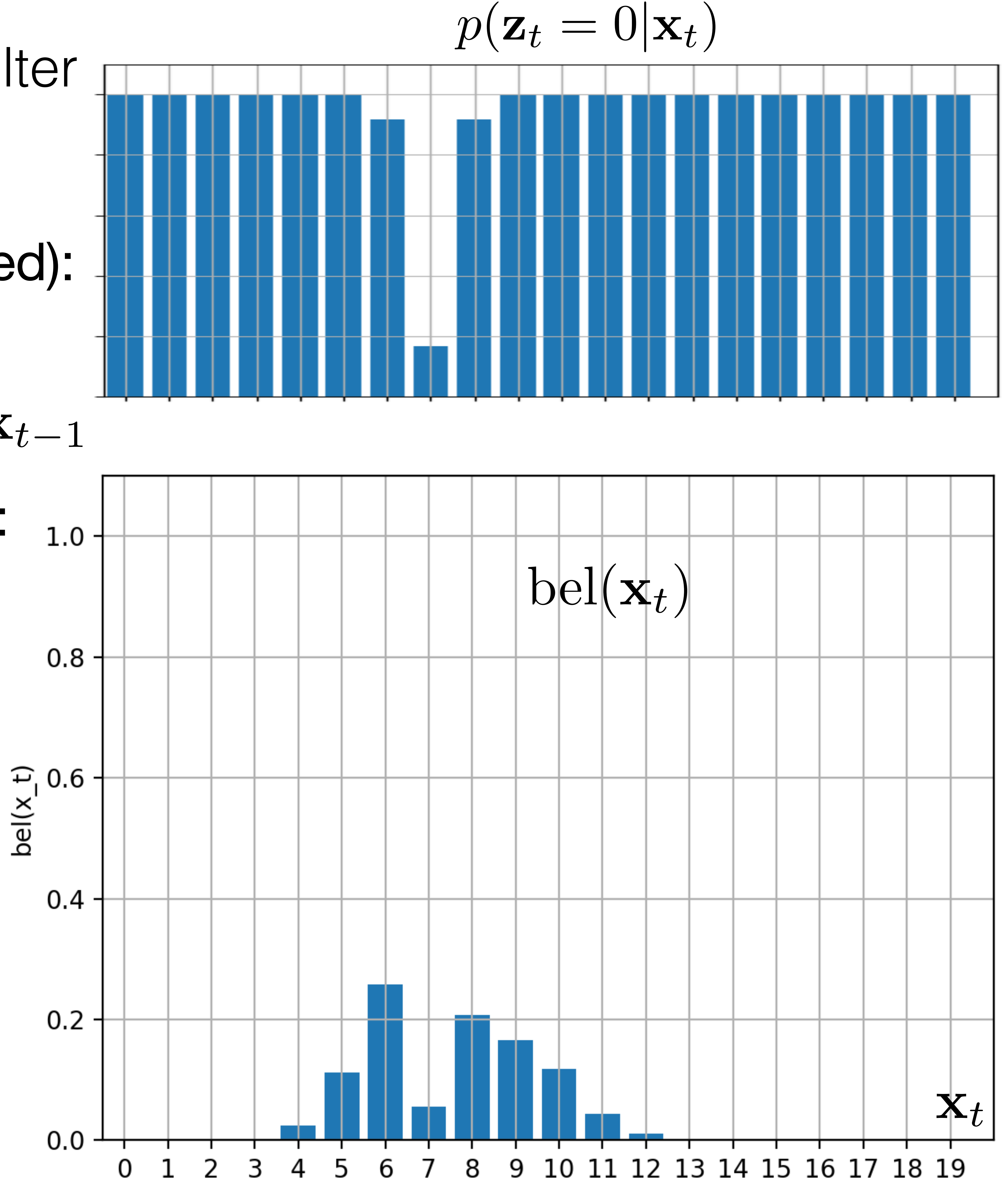
   For all $\mathbf{x}_t$

   $$\text{bel}(\mathbf{x}_t) = \frac{\text{bel}(\mathbf{x}_t)}{s}$$

4. Repeat from 2:

   $t = t + 1$



Bayes filter

1. Initialization: $\text{bel}(\mathbf{x}_0)$, $t = 1$
2. Prediction step (new action $\mathbf{u}_t$ performed):

   For all $\mathbf{x}_t$

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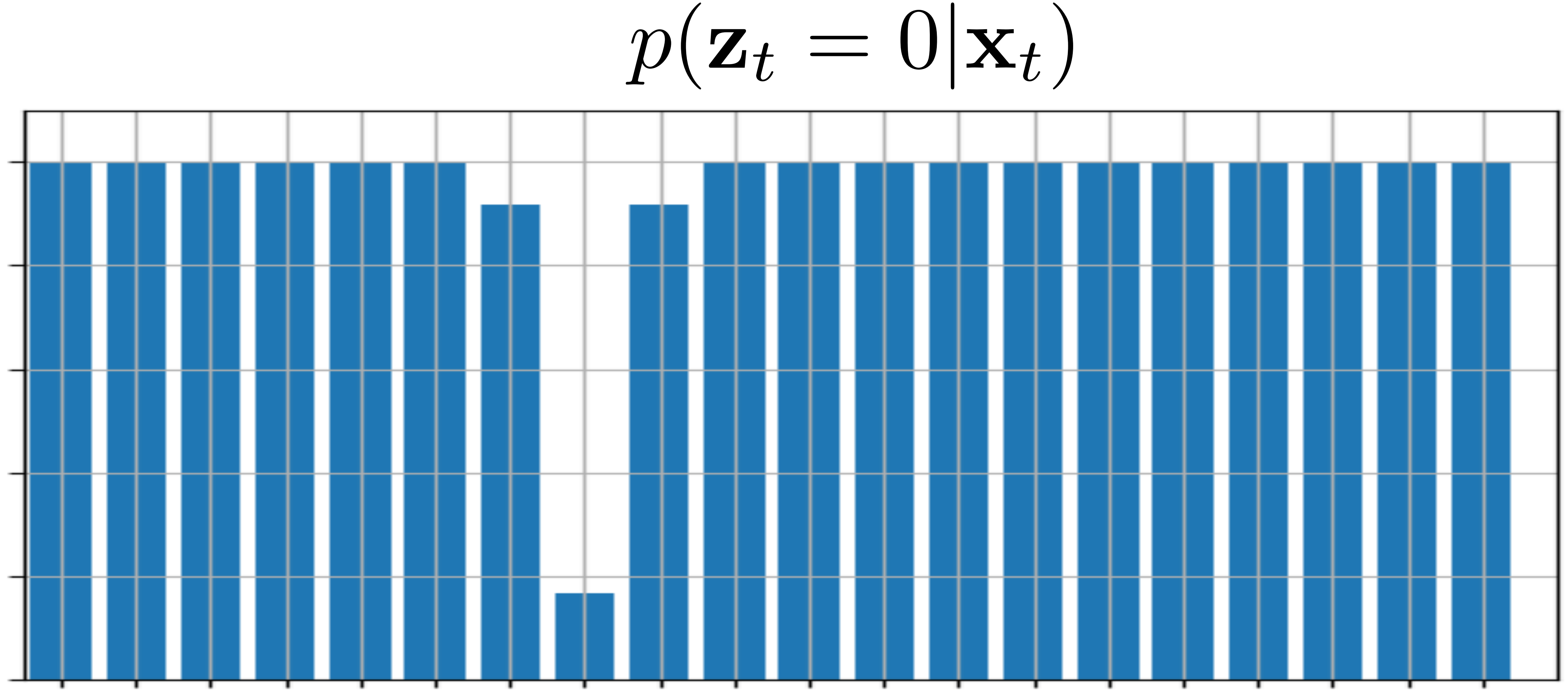

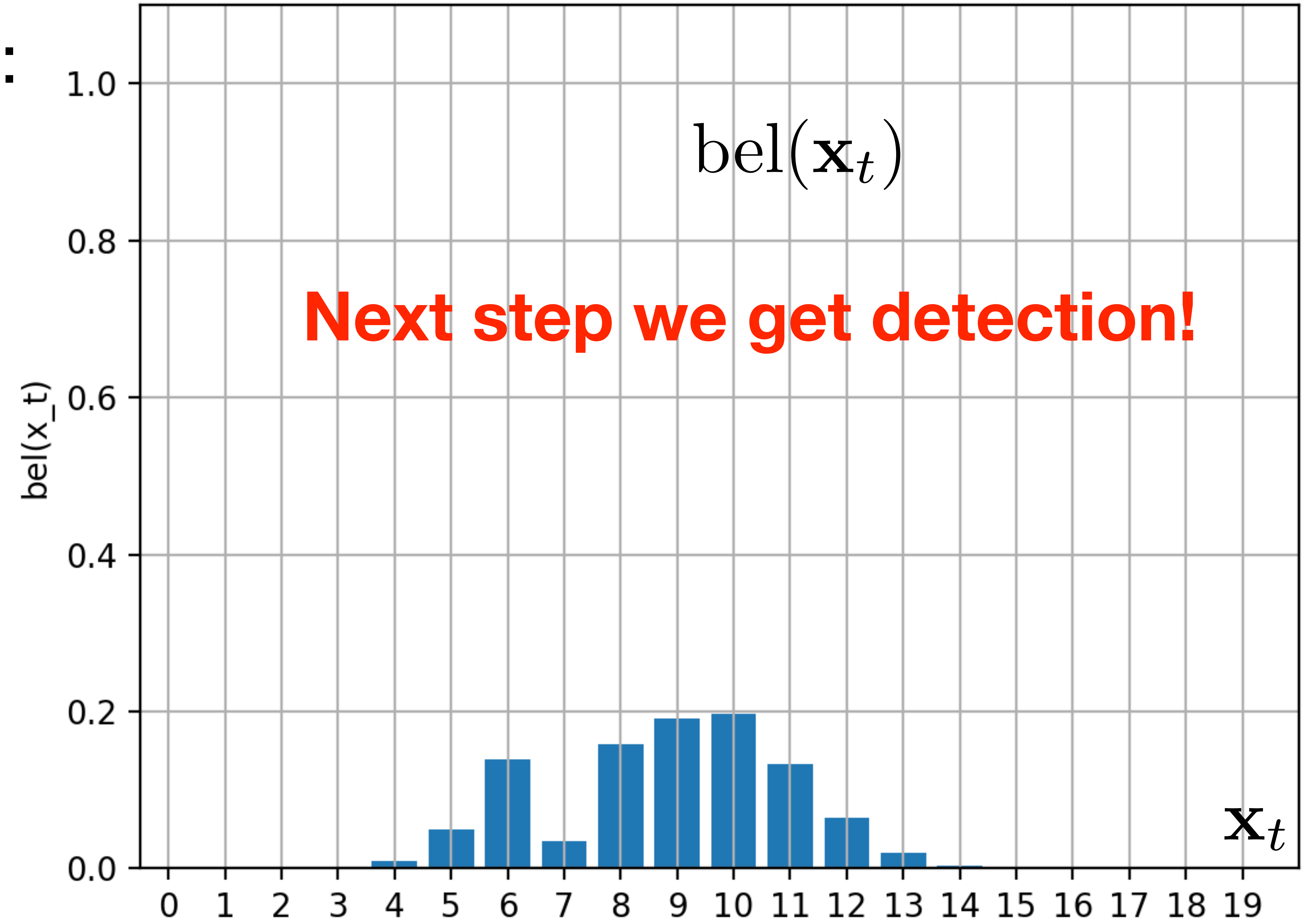

Bayes filter

1. Initialization: $\text{bel}(\mathbf{x}_0)$, $t = 1$
2. Prediction step (new action $\mathbf{u}_t$ performed):

   For all $\mathbf{x}_t$

   $$\overline{\text{bel}}(\mathbf{x}_t) = \int p(\mathbf{x}_t | \mathbf{x}_{t-1}, \mathbf{u}_t) \text{bel}(\mathbf{x}_{t-1}) d\mathbf{x}_{t-1}$$

3. Measurement update (new $\mathbf{z}_t$ received):

   For all $\mathbf{x}_t$

   $$\text{bel}(\mathbf{x}_t) = p(\mathbf{z}_t | \mathbf{x}_t) \overline{\text{bel}}(\mathbf{x}_t)$$

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   For all $\mathbf{x}_t$

   $$\text{bel}(\mathbf{x}_t) = \frac{\text{bel}(\mathbf{x}_t)}{s}$$

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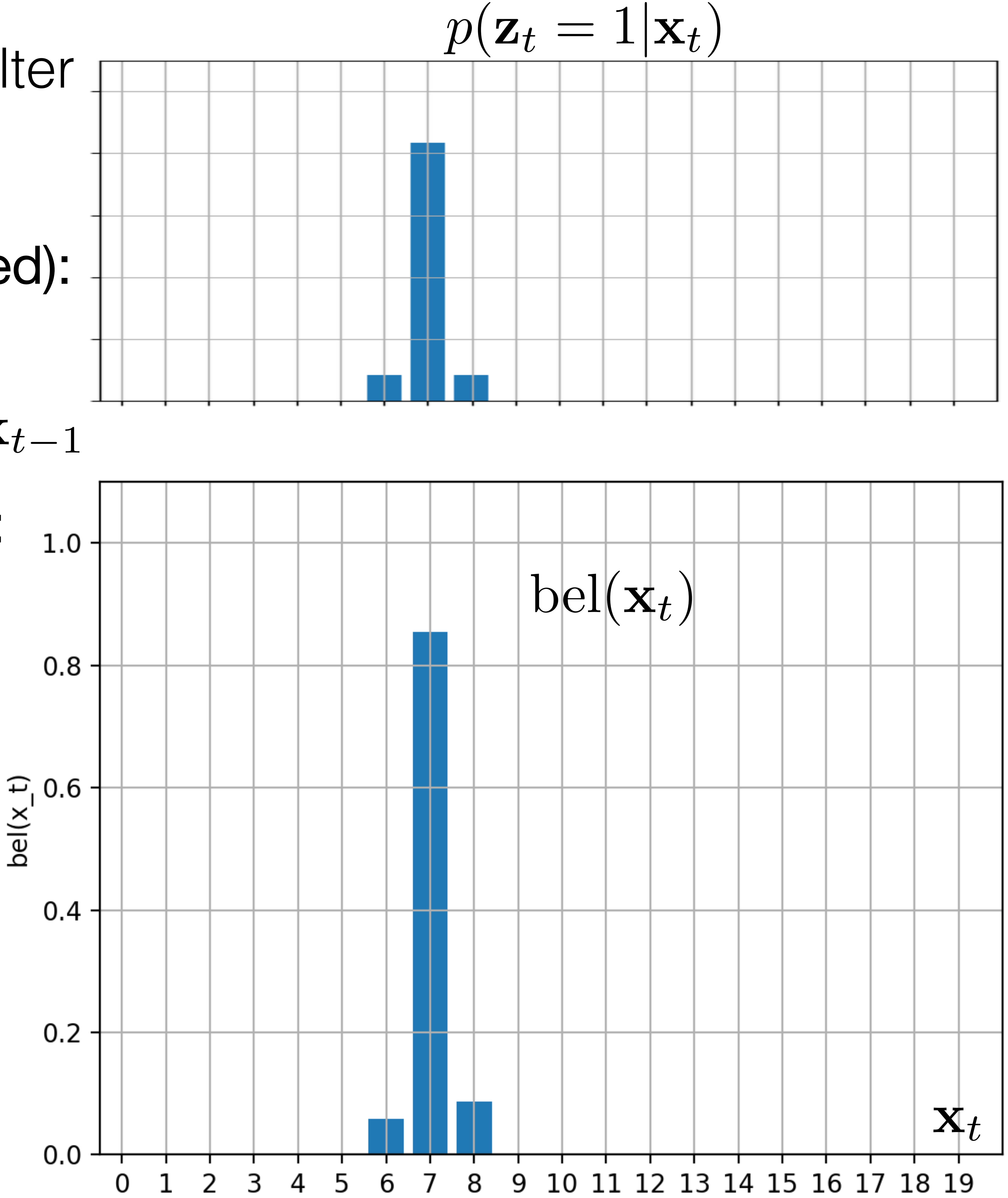

Bayes filter

1. Initialization: $\mathrm{bel}(\mathbf{x}_0)$, $t = 1$
2. Prediction step (new action $\mathbf{u}_t$ performed):

   For all $\mathbf{x}_t$

   $$\overline{\mathrm{bel}}(\mathbf{x}_t) = \int p(\mathbf{x}_t|\mathbf{x}_{t-1}, \mathbf{u}_t)\mathrm{bel}(\mathbf{x}_{t-1})d\mathbf{x}_{t-1}$$

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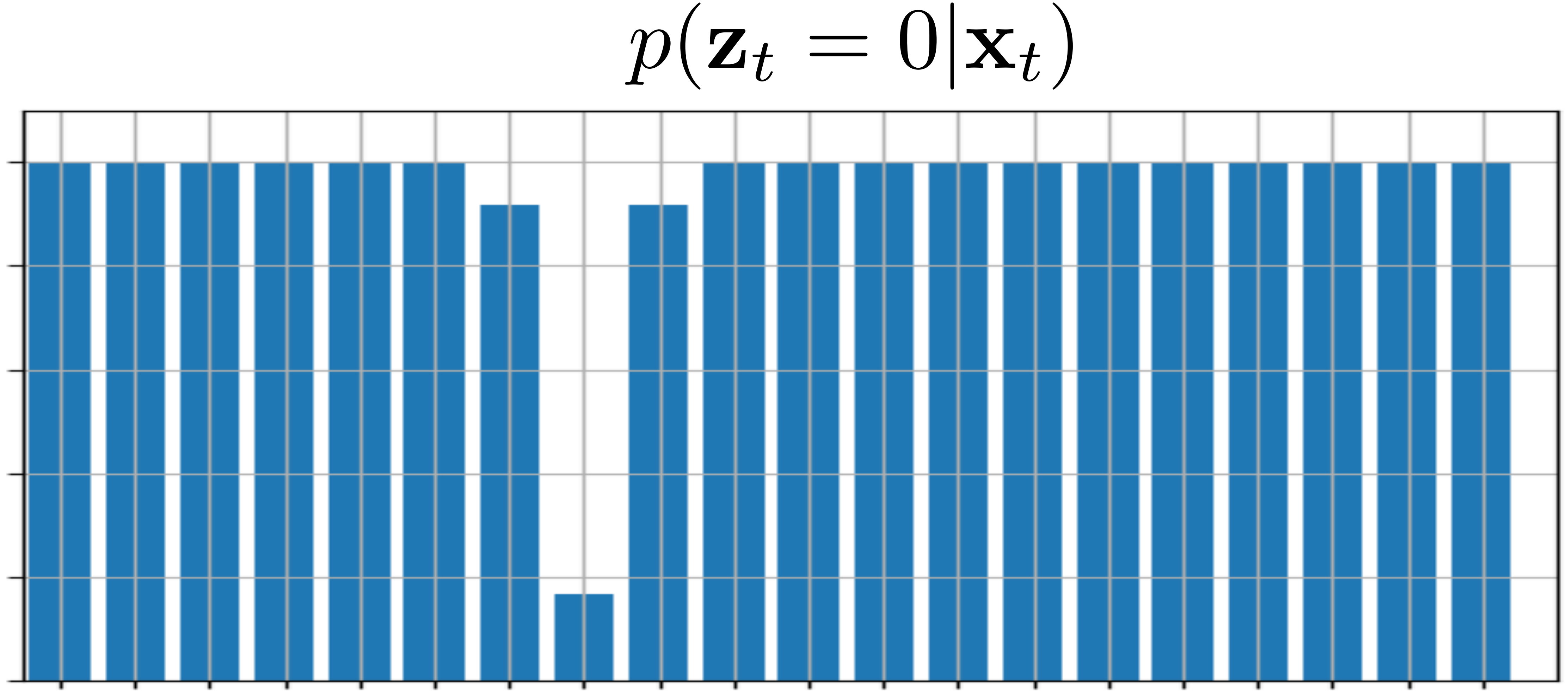
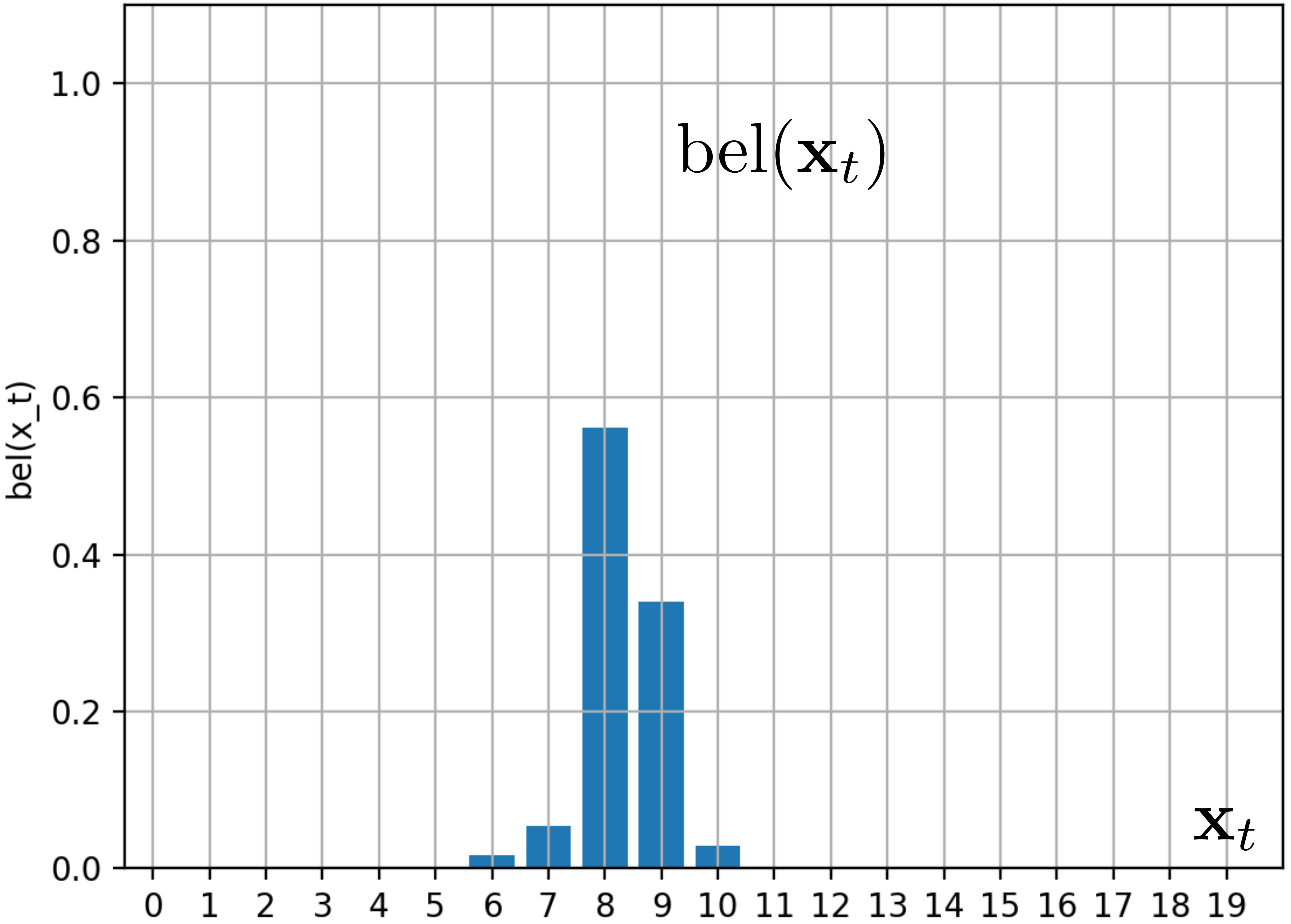
   $$\mathrm{bel}(\mathbf{x}_t) = \frac{\mathrm{bel}(\mathbf{x}_t)}{s}$$

4. Repeat from 2:

   $t = t + 1$





Bayes filter

1. Initialization: $\text{bel}(\mathbf{x}_0)$, $t = 1$
2. Prediction step (new action $\mathbf{u}_t$ performed):

   For all $\mathbf{x}_t$

   $$\overline{\text{bel}}(\mathbf{x}_t) = \int p(\mathbf{x}_t | \mathbf{x}_{t-1}, \mathbf{u}_t)\text{bel}(\mathbf{x}_{t-1})d\mathbf{x}_{t-1}$$

3. Measurement update (new $\mathbf{z}_t$ received):

   For all $\mathbf{x}_t$

   $$\text{bel}(\mathbf{x}_t) = p(\mathbf{z}_t | \mathbf{x}_t)\overline{\text{bel}}(\mathbf{x}_t)$$
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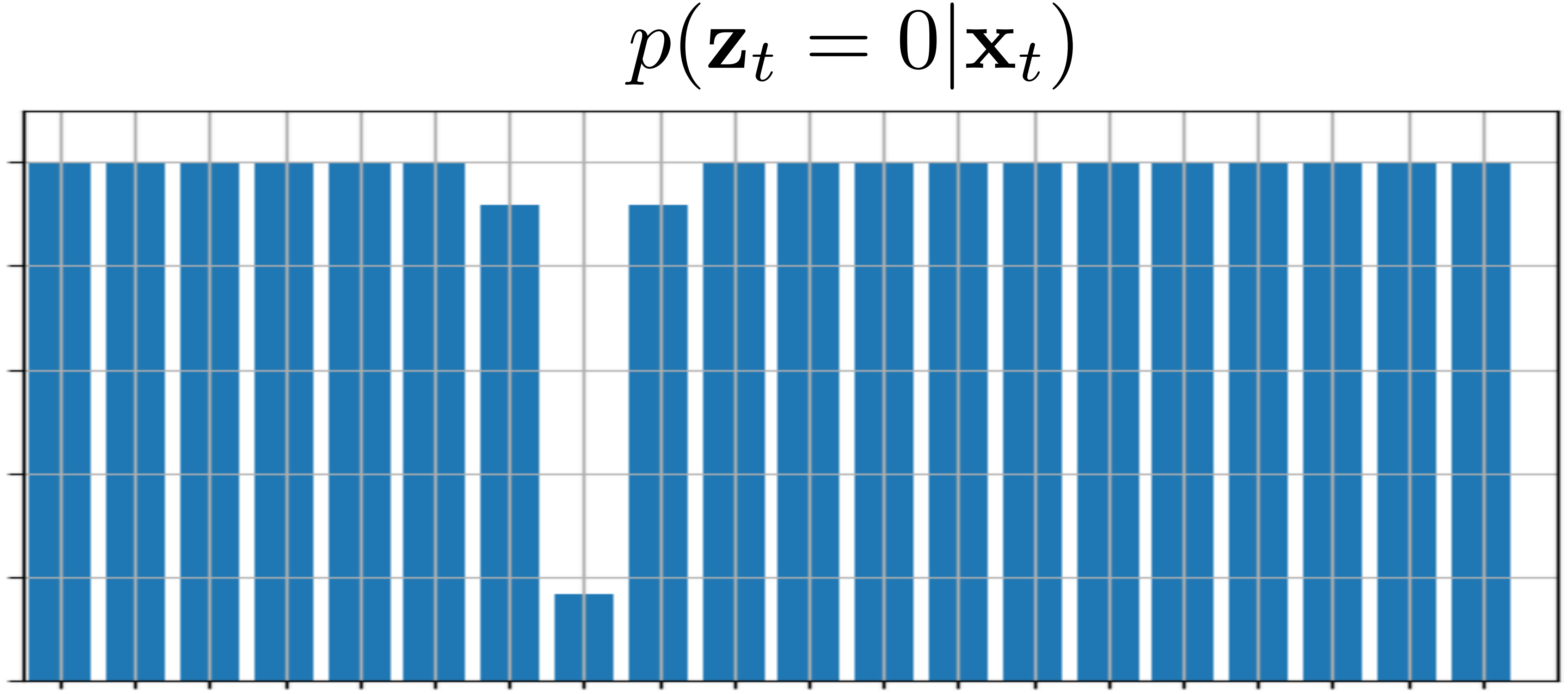
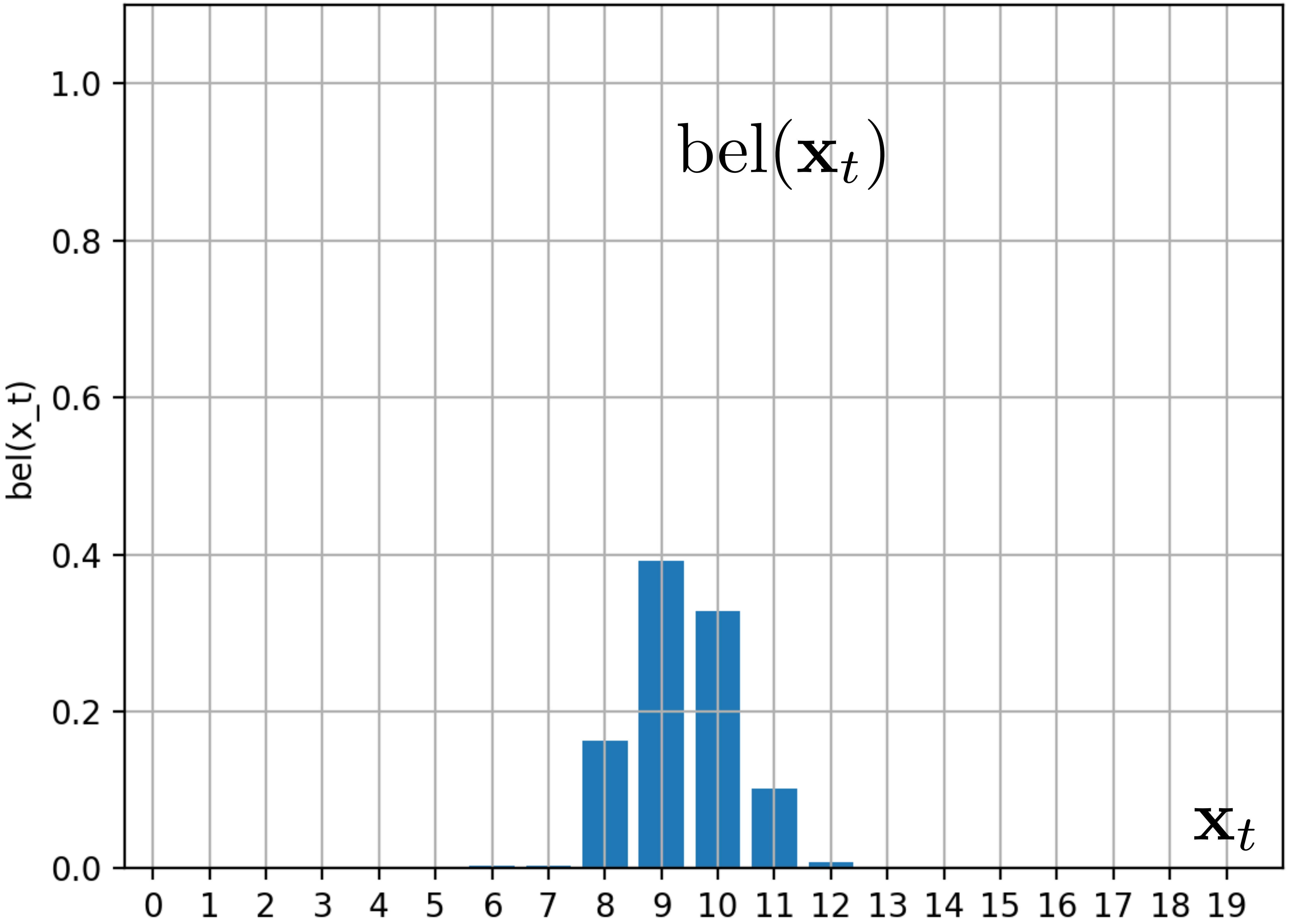
   For all $\mathbf{x}_t$

   $$\text{bel}(\mathbf{x}_t) = \frac{\text{bel}(\mathbf{x}_t)}{s}$$

4. Repeat from 2:

   $t = t + 1$





Bayes filter

1. Initialization: $\mathrm{bel}(\mathbf{x}_0)$, $t = 1$
2. Prediction step (new action $\mathbf{u}_t$ performed):

   For all $\mathbf{x}_t$

$$\overline{\mathrm{bel}}(\mathbf{x}_t) = \int p(\mathbf{x}_t|\mathbf{x}_{t-1}, \mathbf{u}_t)\mathrm{bel}(\mathbf{x}_{t-1})d\mathbf{x}_{t-1}$$

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   For all $\mathbf{x}_t$

$$\mathrm{bel}(\mathbf{x}_t) = \frac{\mathrm{bel}(\mathbf{x}_t)}{s}$$

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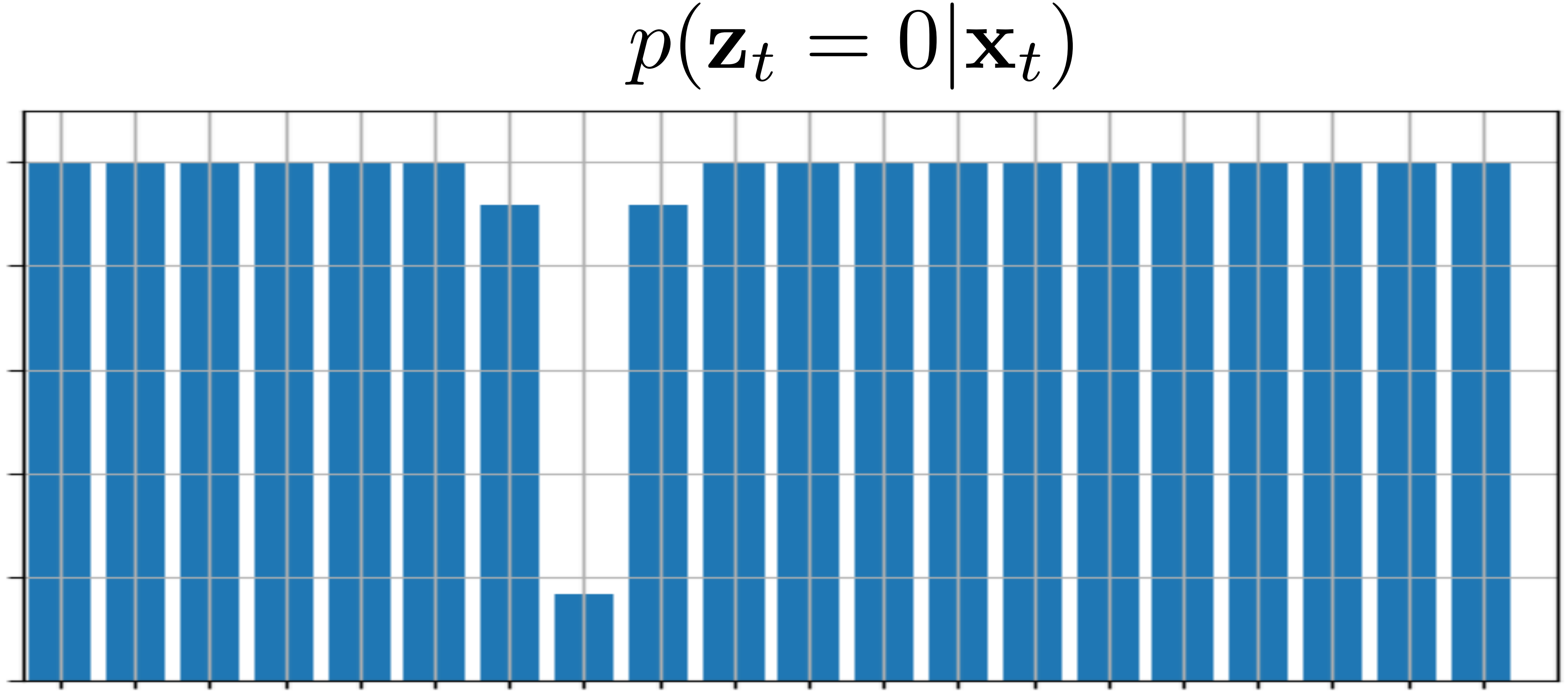
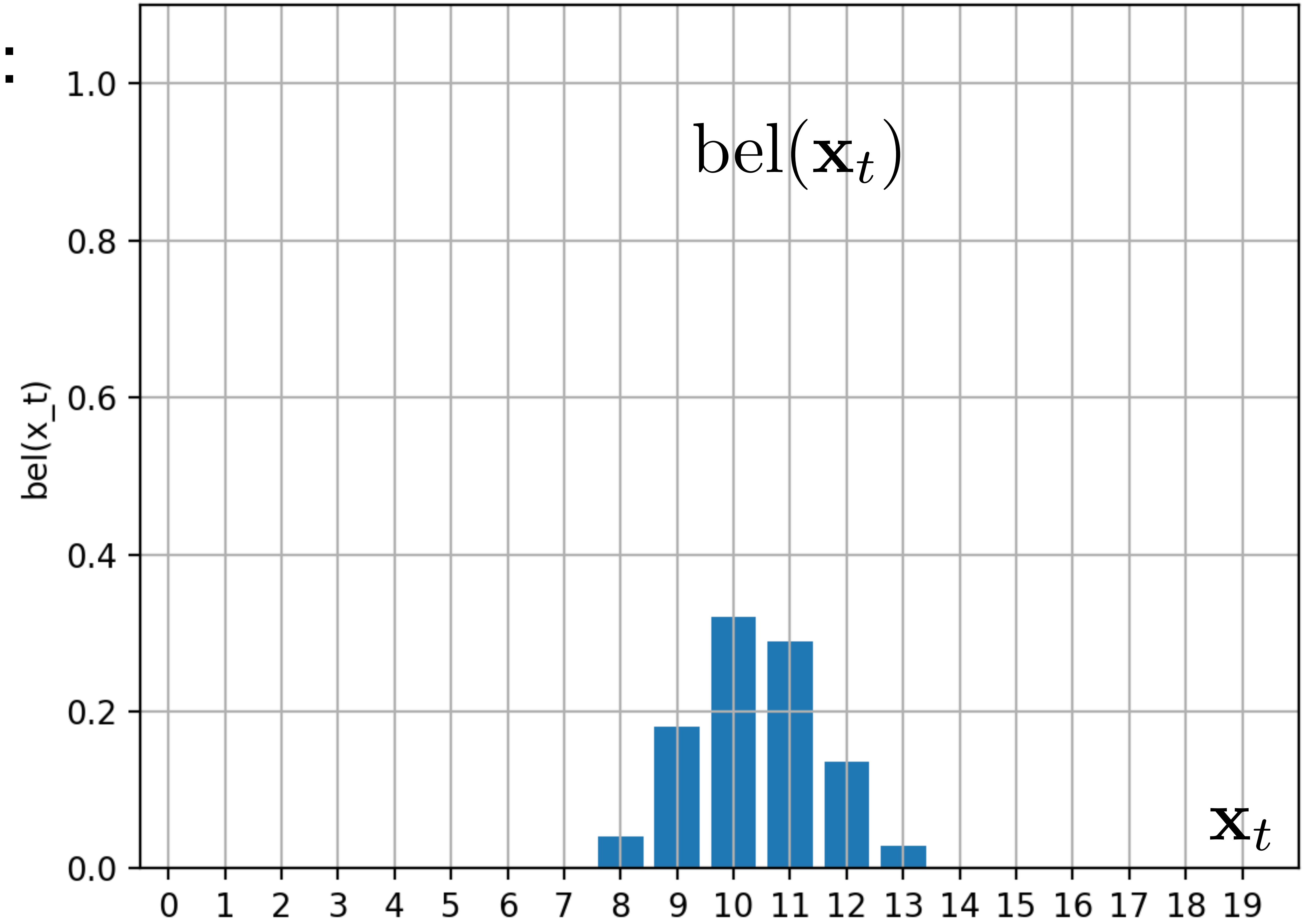
   $t = t + 1$





Bayes filter

1. Initialization: $\mathrm{bel}(\mathbf{x}_0)$, $t = 1$
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For all $\mathbf{x}_t$

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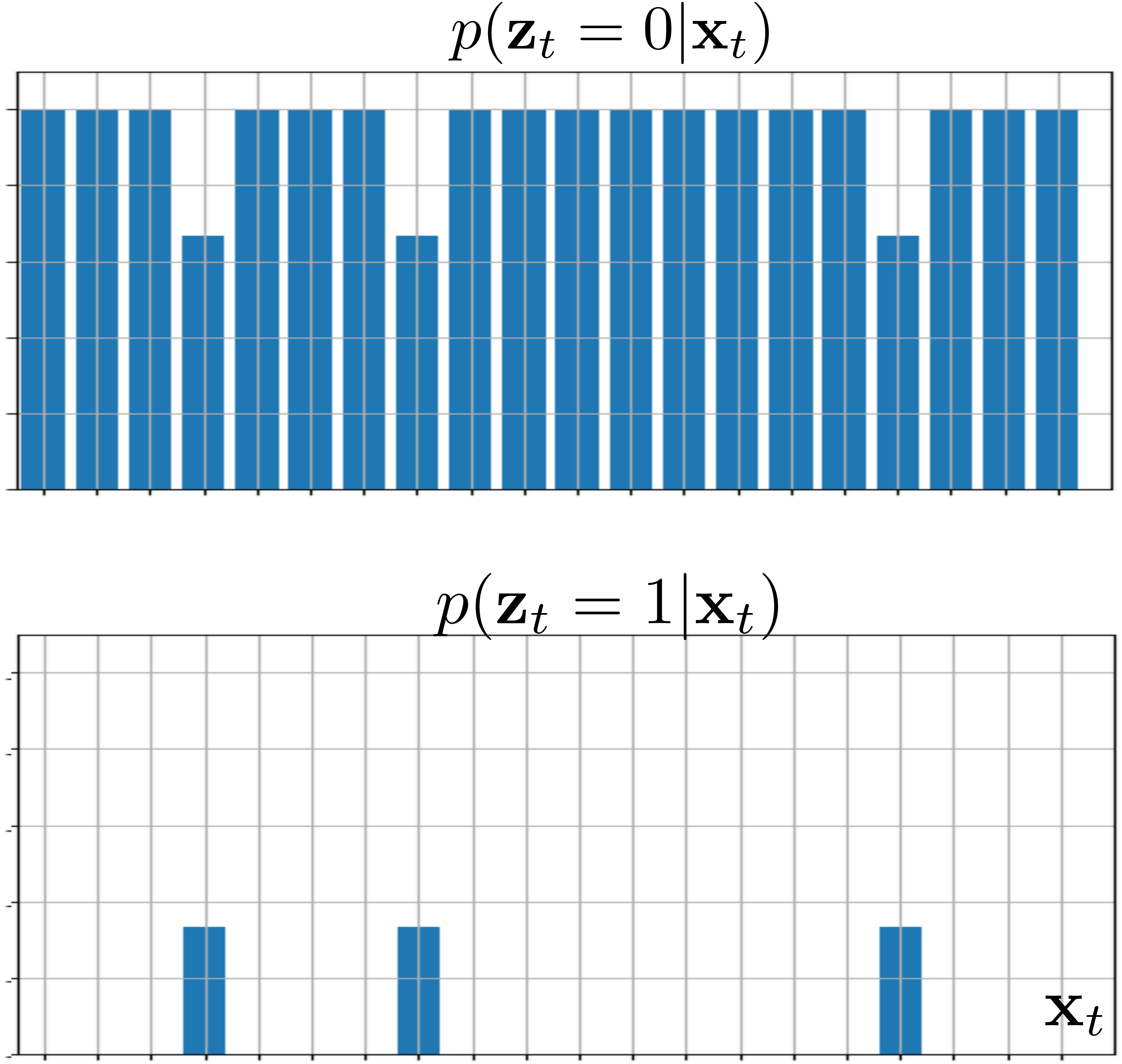

3 undistinguishable markers

Bayes filter

1. Initialization: $\mathrm{bel}(\mathbf{x}_0)$, $t = 1$
2. Prediction step (new action $\mathbf{u}_t$ performed):

   For all $\mathbf{x}_t$

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   For all $\mathbf{x}_t$

   $$\mathrm{bel}(\mathbf{x}_t) = \frac{\mathrm{bel}(\mathbf{x}_t)}{s}$$

4. Repeat from 2:

   $$t = t + 1$$

Bayes filter

1. Initialization: $\text{bel}(\mathbf{x}_0)$, $t = 1$
2. Prediction step (new action $\mathbf{u}_t$ performed):

For all $\mathbf{x}_t$

$$\overline{\text{bel}}(\mathbf{x}_t) = \int p(\mathbf{x}_t|\mathbf{x}_{t-1}, \mathbf{u}_t)\text{bel}(\mathbf{x}_{t-1})d\mathbf{x}_{t-1}$$

3. Measurement update (new $\mathbf{z}_t$ received):

For all $\mathbf{x}_t$

$$\text{bel}(\mathbf{x}_t) = p(\mathbf{z}_t|\mathbf{x}_t)\overline{\text{bel}}(\mathbf{x}_t)$$

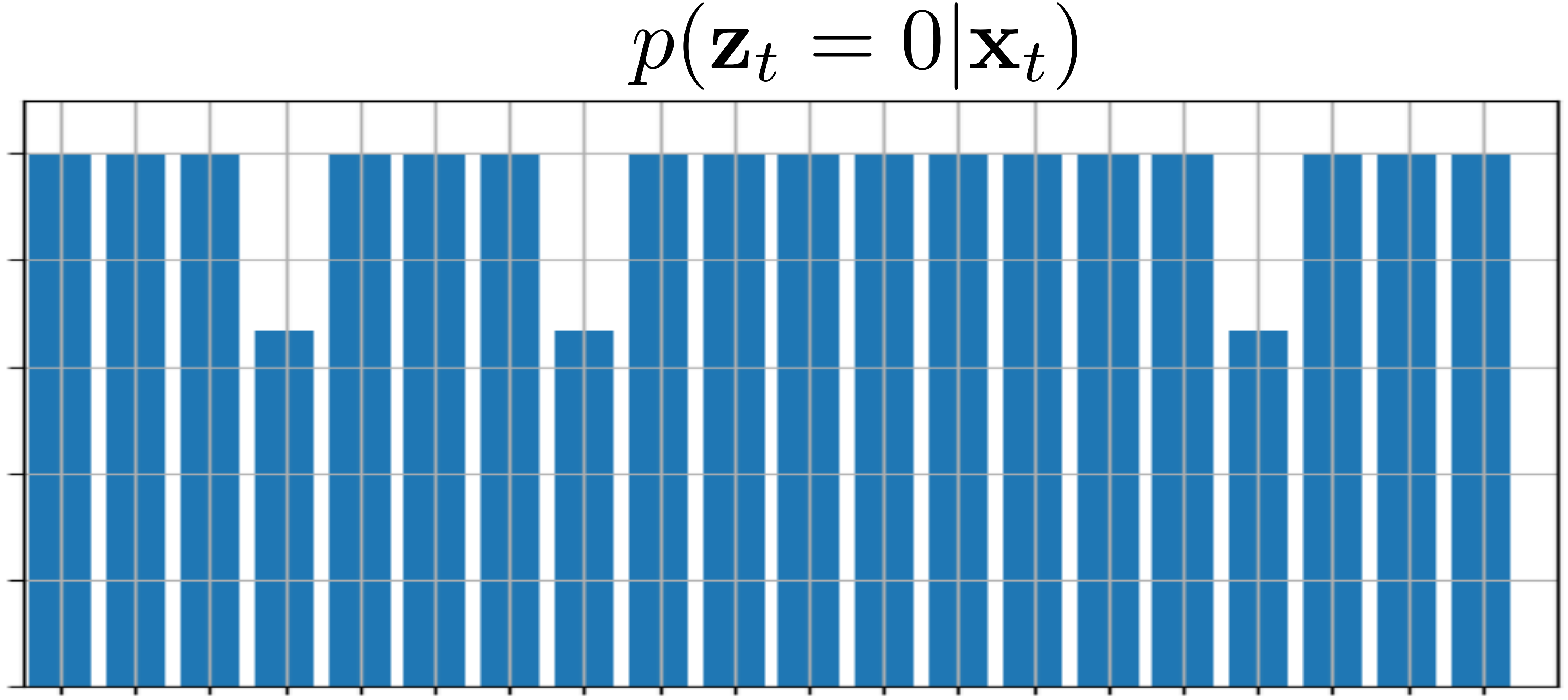
$$s = s + \text{bel}(\mathbf{x}_t)$$

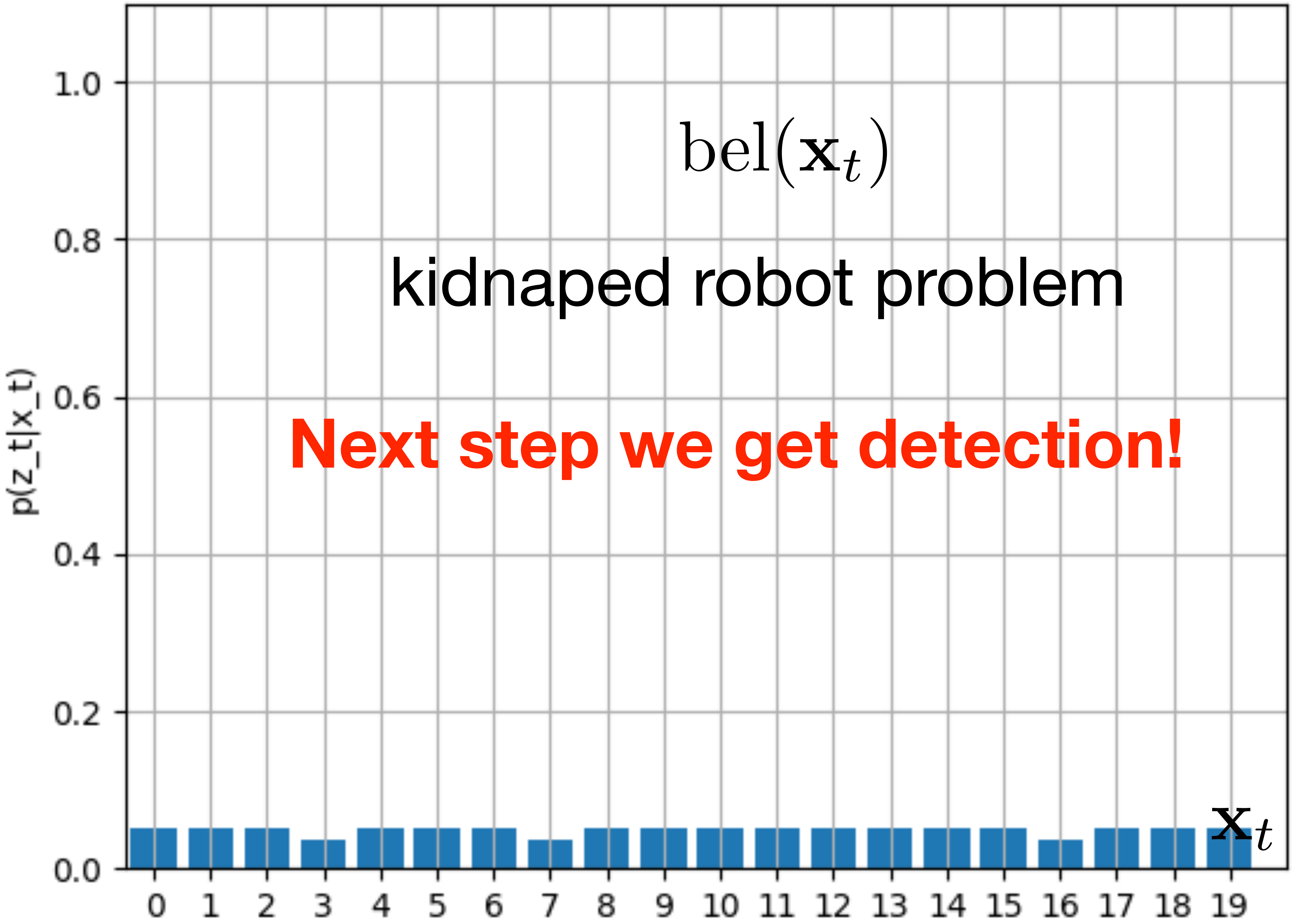
For all $\mathbf{x}_t$

$$\text{bel}(\mathbf{x}_t) = \frac{\text{bel}(\mathbf{x}_t)}{s}$$

4. Repeat from 2:

$t = t + 1$





Bayes filter

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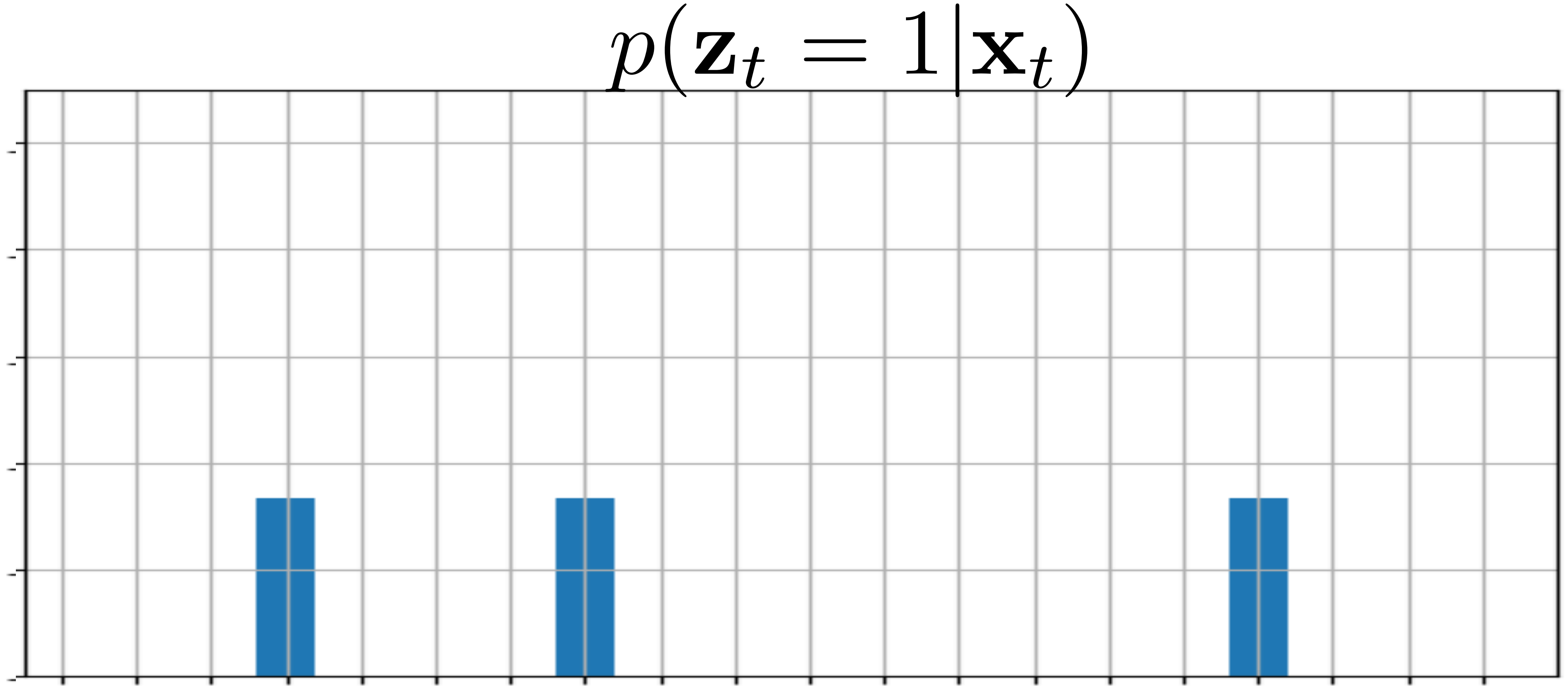

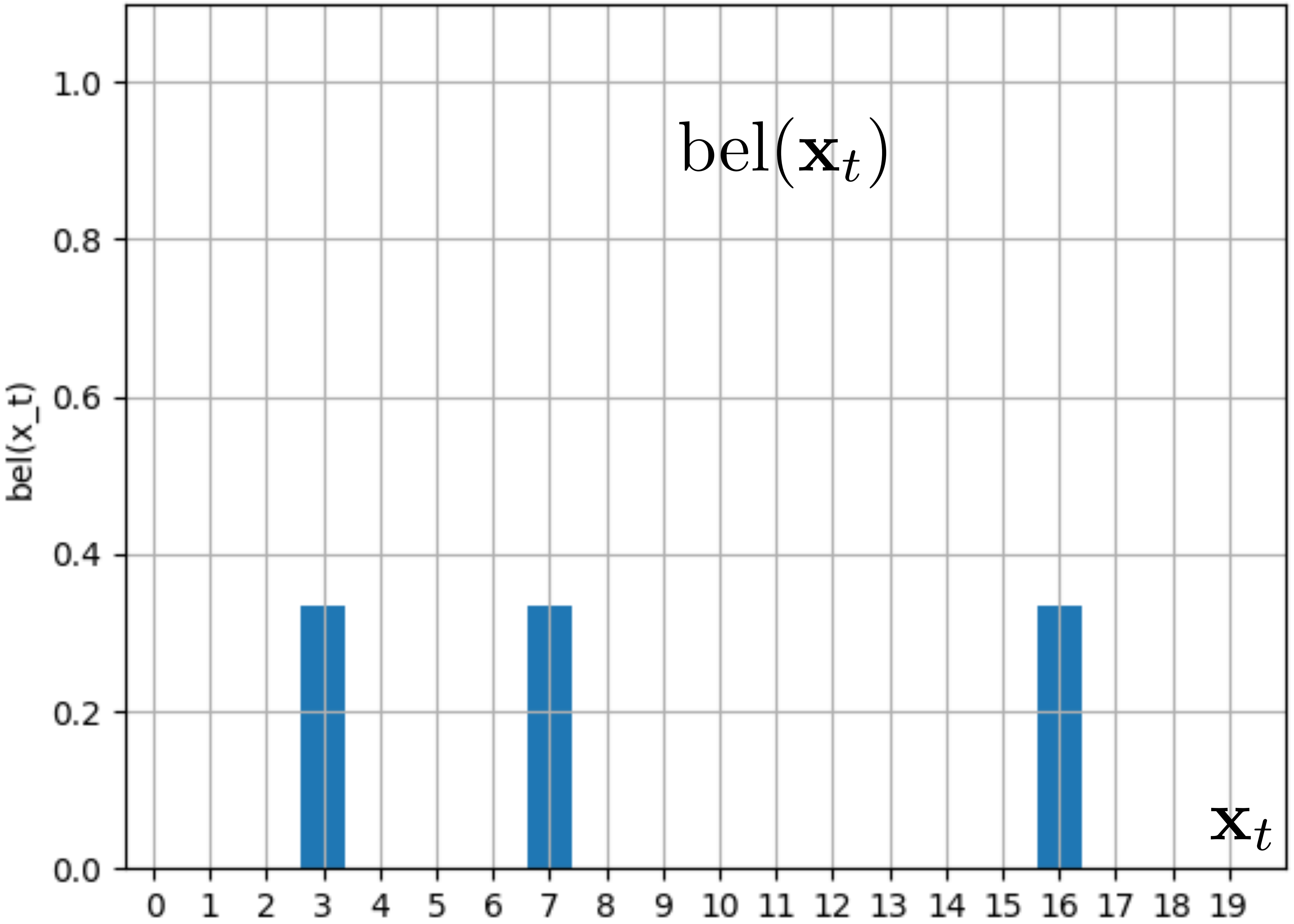

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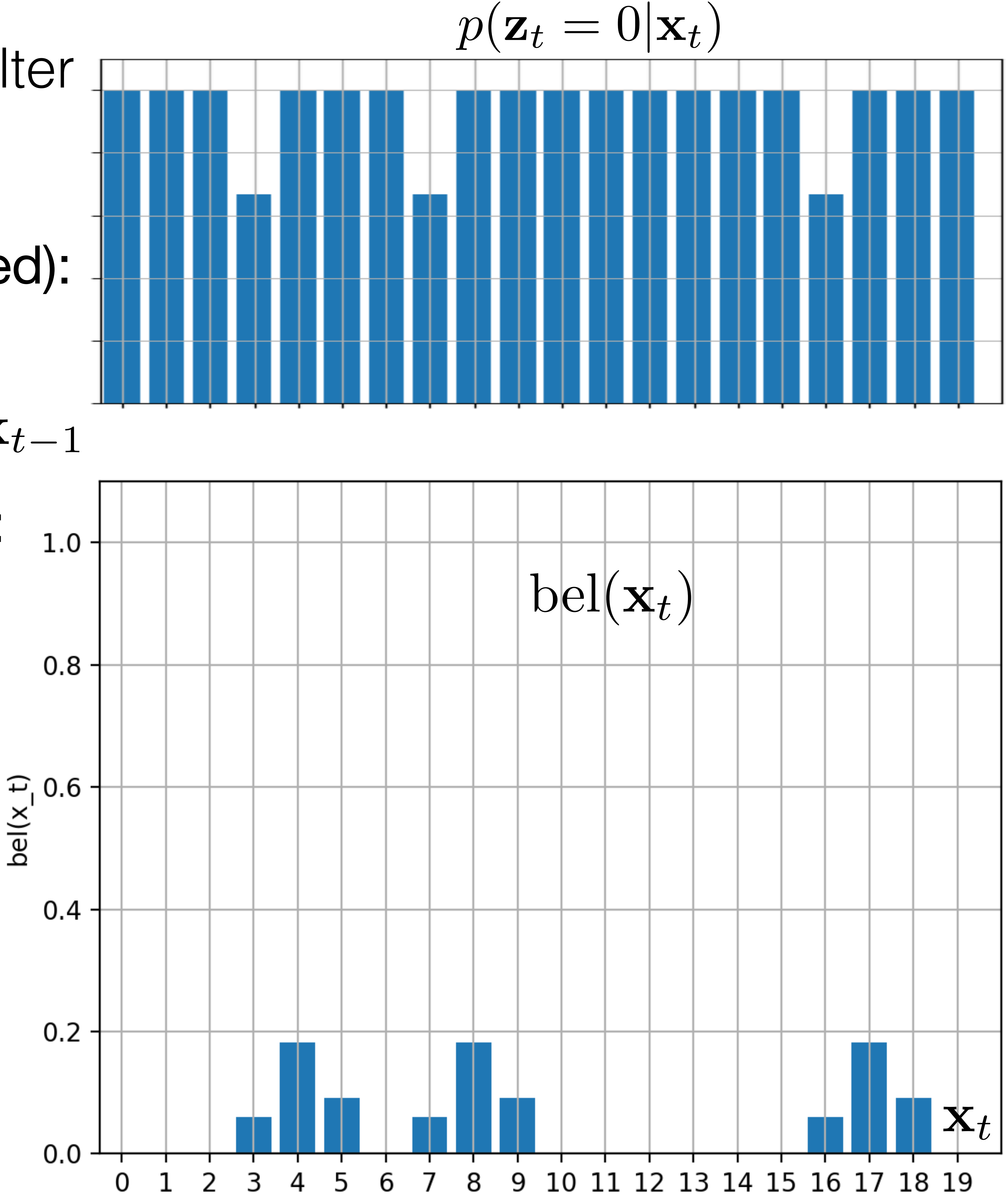

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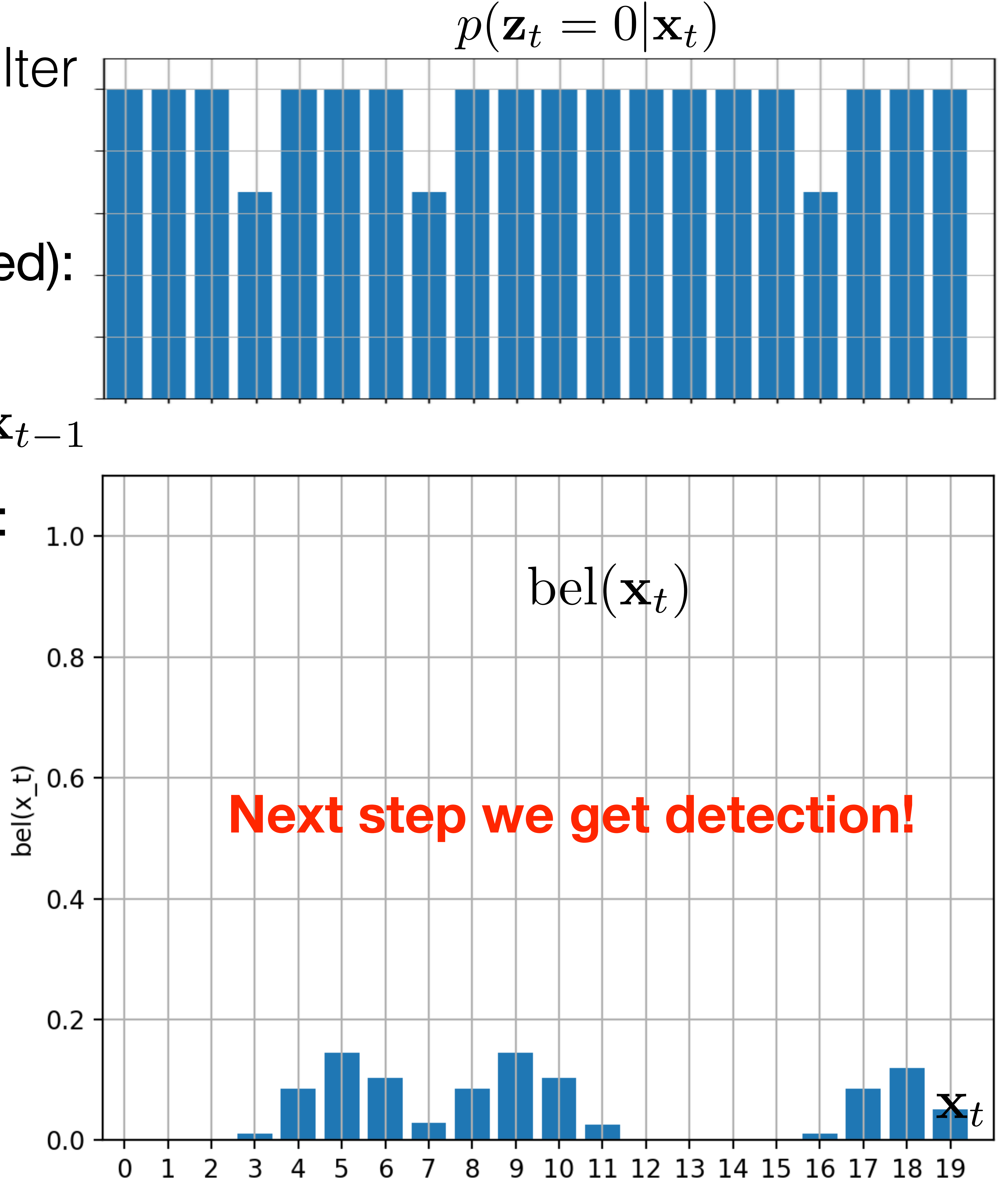

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breakpoint

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For all $\mathbf{x}_t$

$$\text{bel}(\mathbf{x}_t) = \frac{\text{bel}(\mathbf{x}_t)}{s}$$ **= ???**

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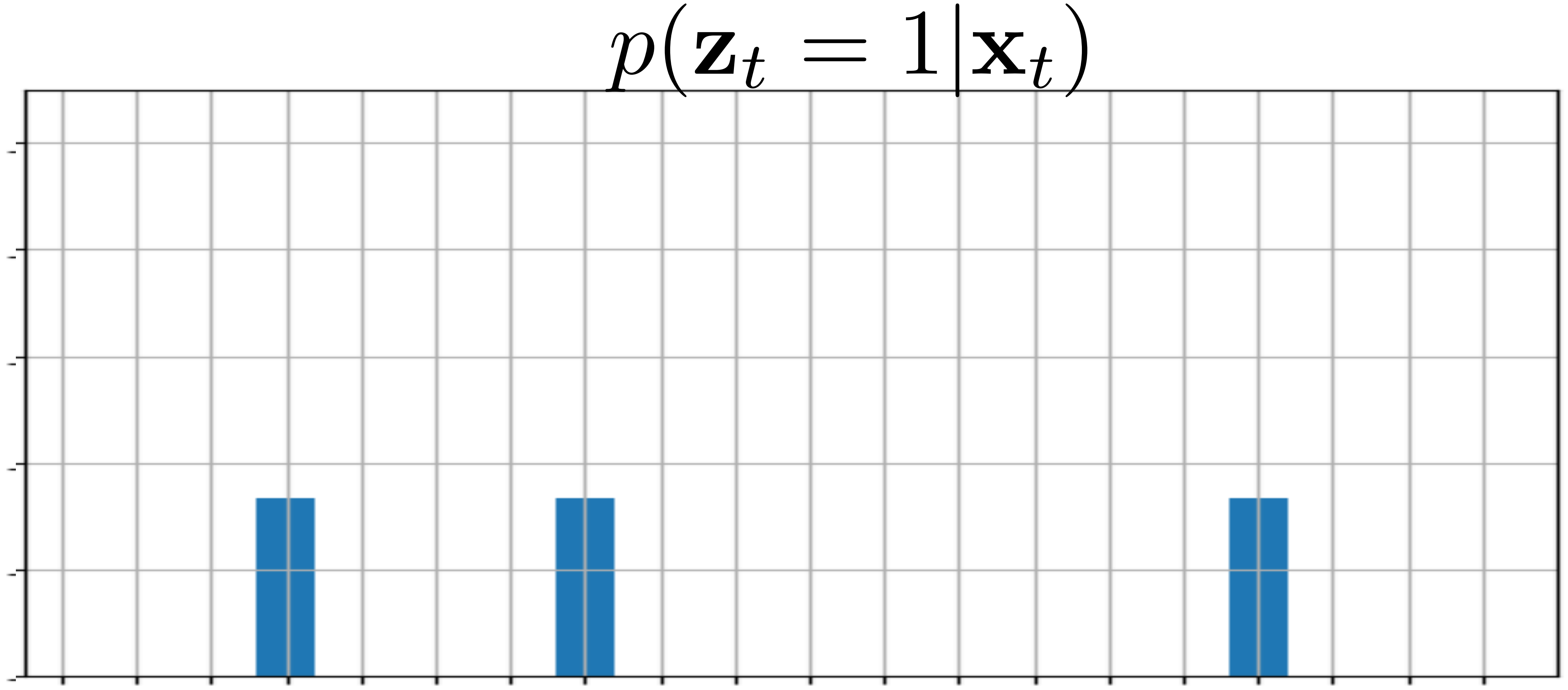

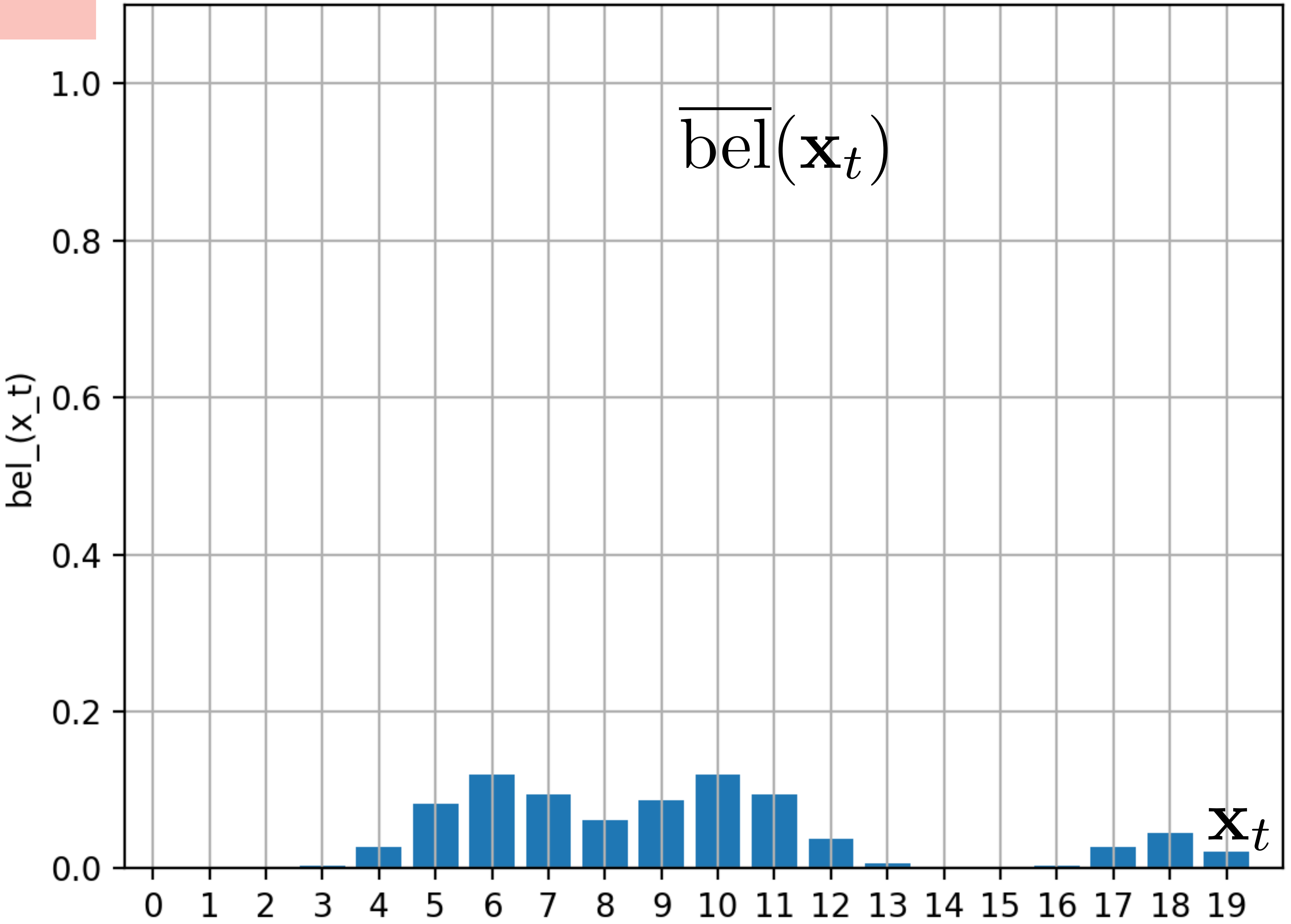

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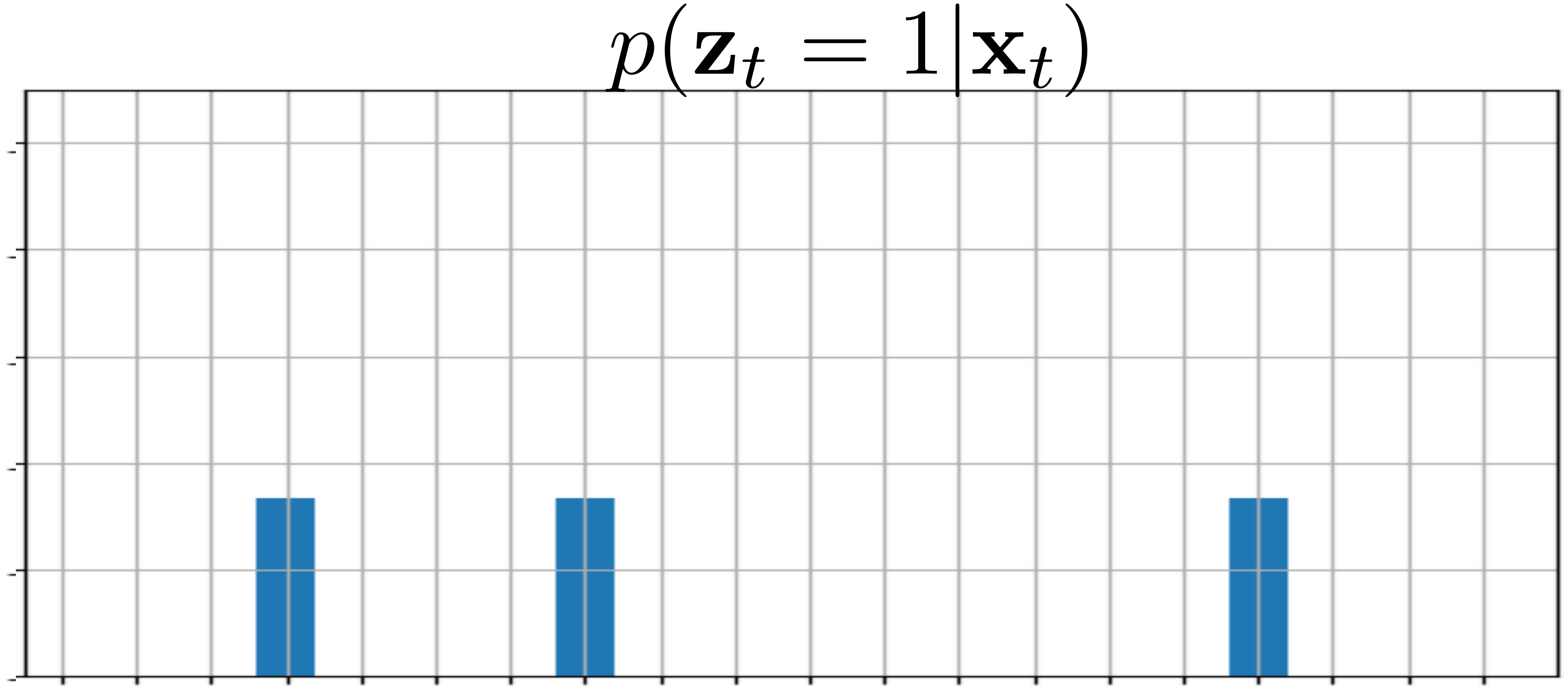

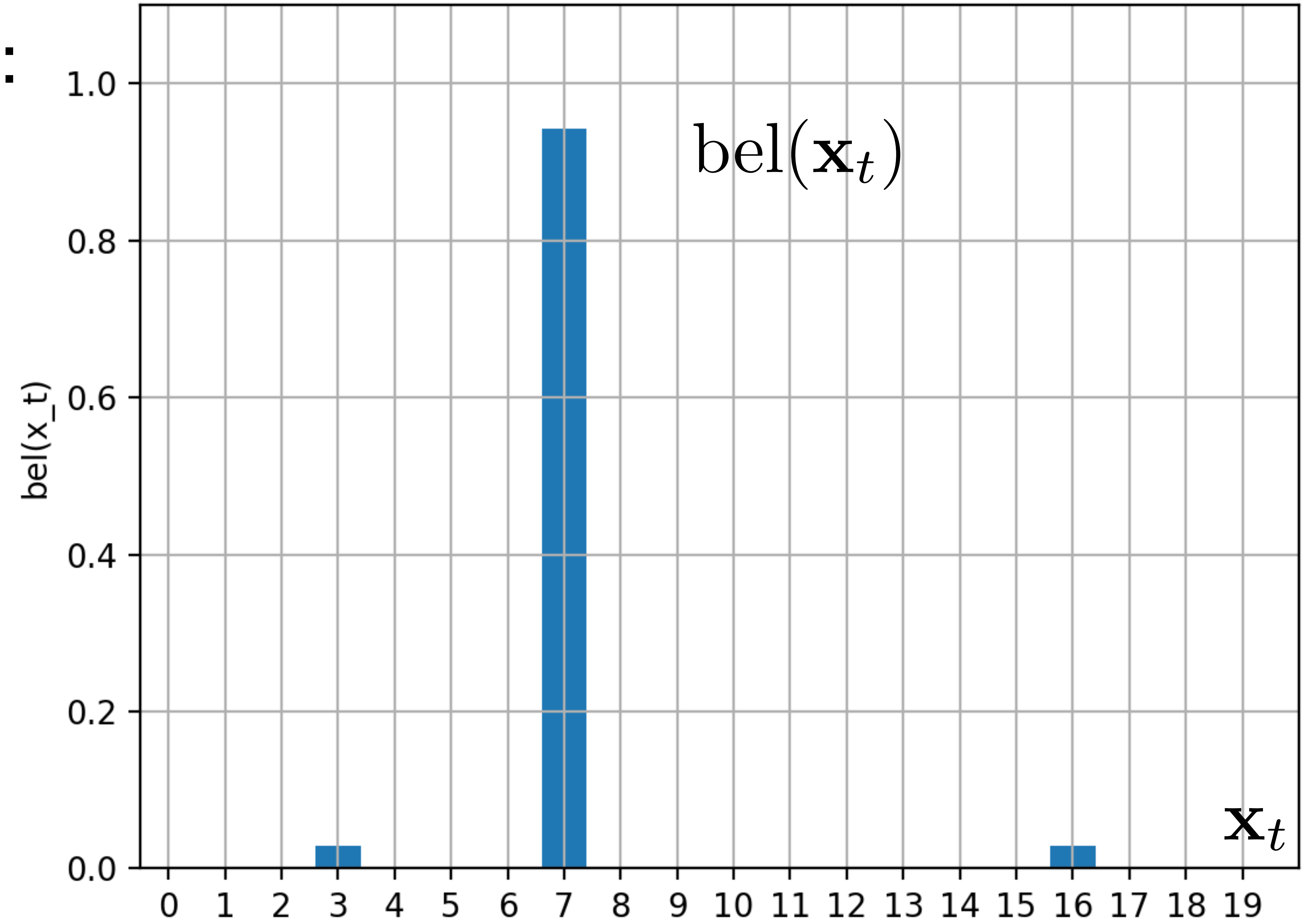

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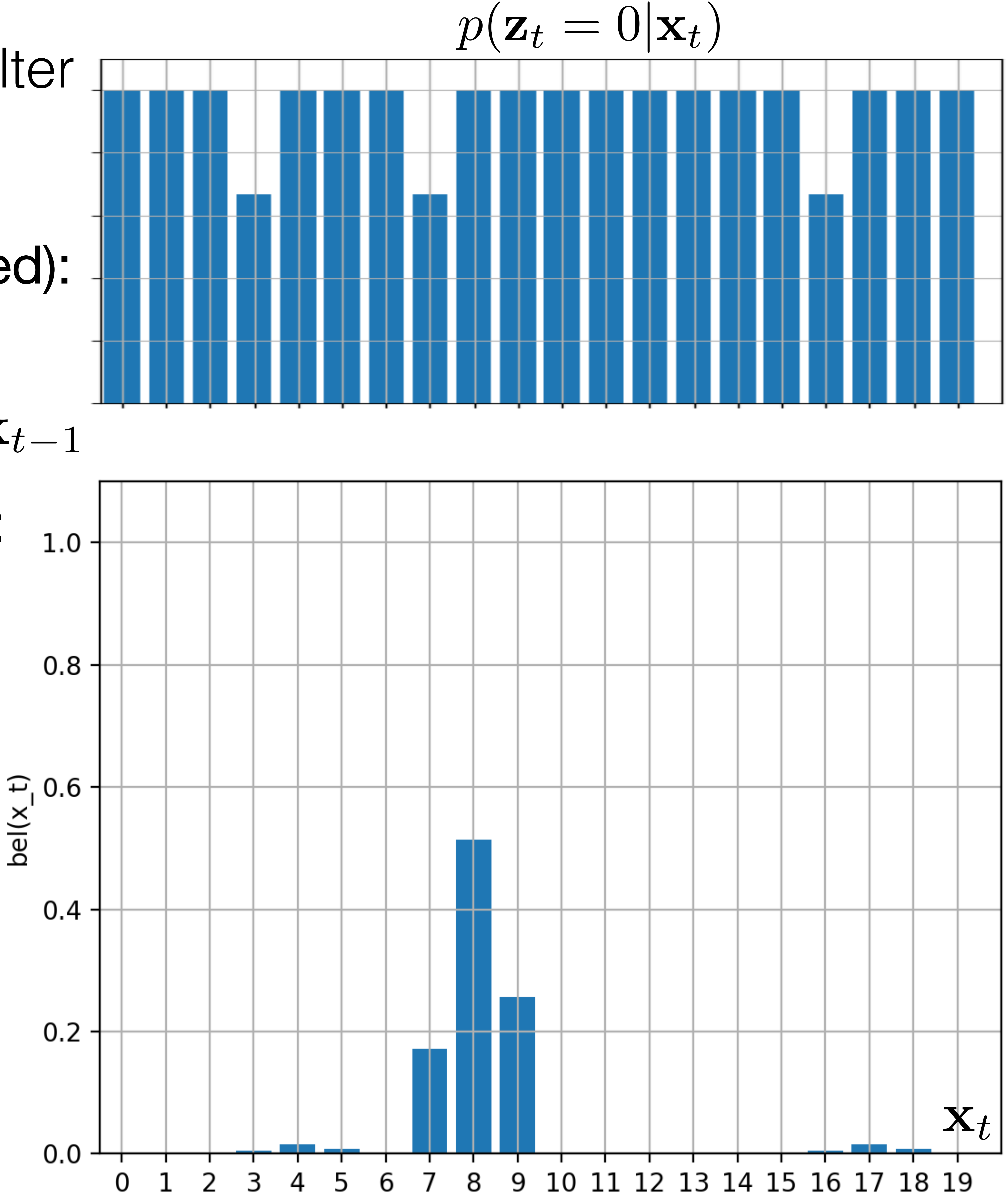

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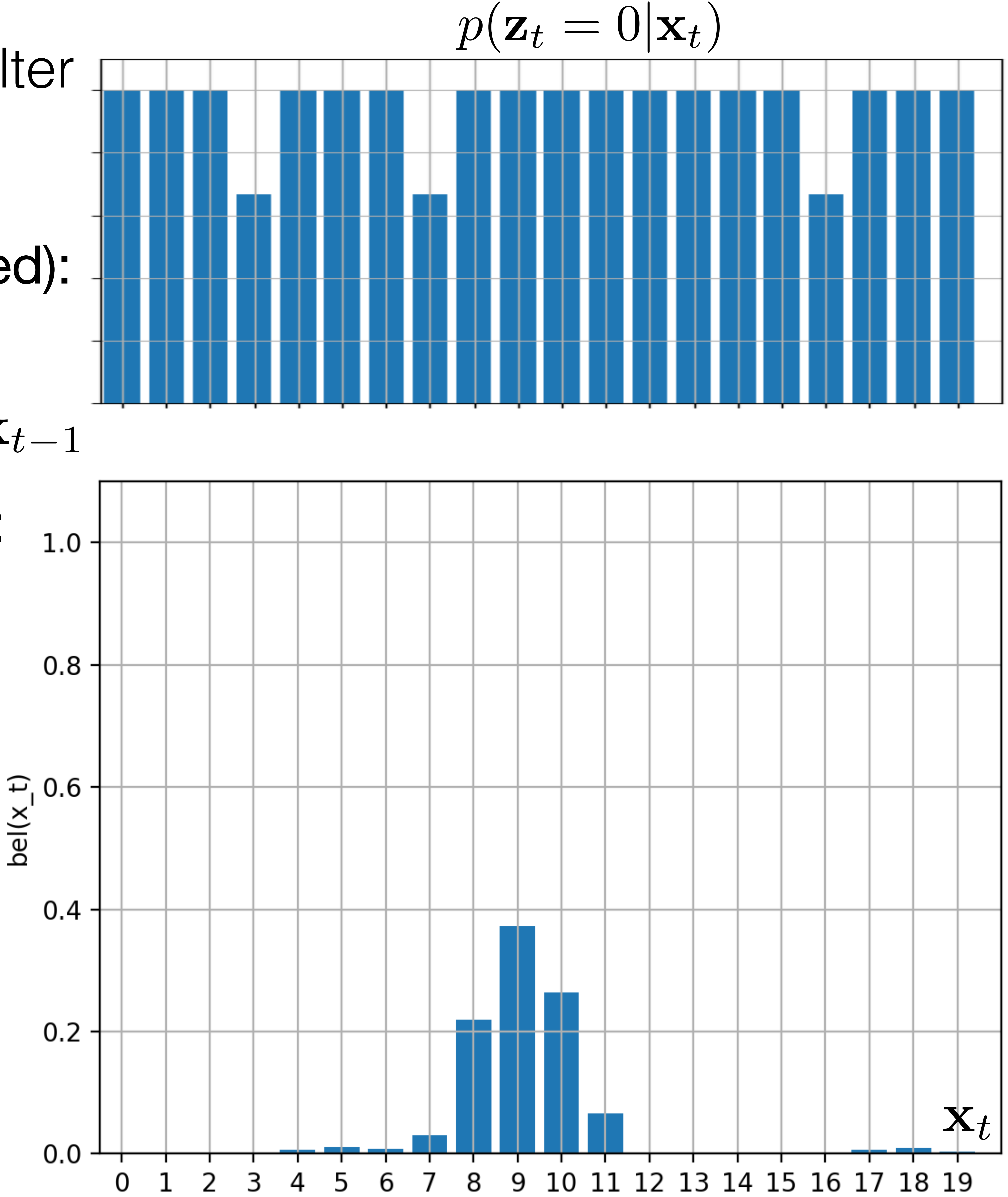

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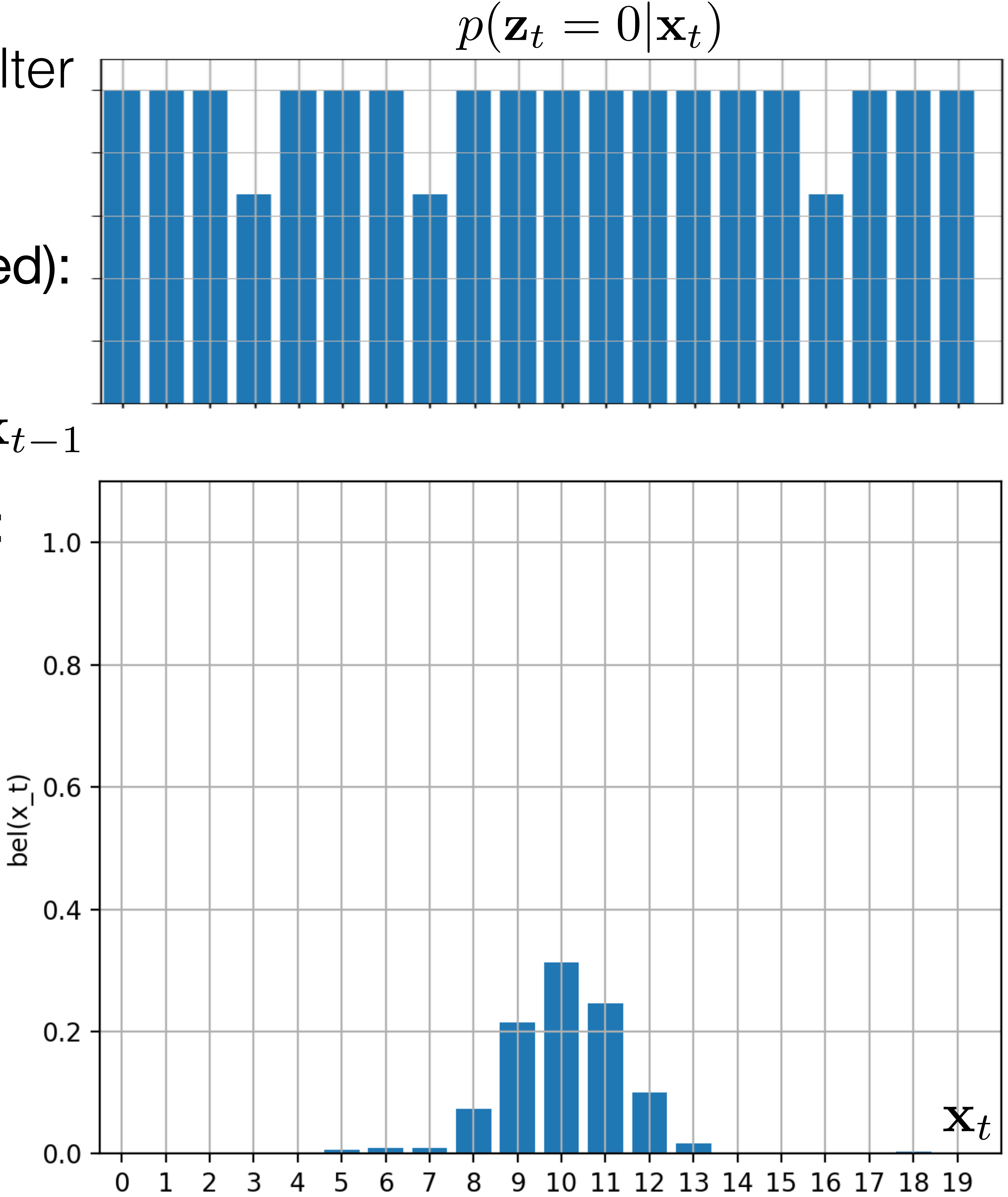

Summary: Bayes filter

- Drawbacks / Advantages Bayes filter?
- Markov assumption (complete states)
- Next: Kalman filter

https://cs.wikipedia.org/wiki/Thomas_Bayes

https://cs.wikipedia.org/wiki/Andrej_Markov

https://cs.wikipedia.org/wiki/Rudolf_Emil_K%C3%A1lm%C3%A1n