COMPUTATIONAL GAME THEORY

Exercises

November 17, 2022

1 OPTIMAL AUCTIONS

Exercise 1.
Suppose you had a single good to sell to a single agent with a private valuation which is drawn from an exponential distribution.

$$\text{cdf} : F(x) = 1 - e^{-x}$$
$$\text{pdf} : f(x) = e^{-x}$$

The agent knows her value and you must post a price that she can either take or leave. This is a monopoly question that is equivalent to an optimal auction design (setting a reserve price) with a single bidder. What price should you set to maximize your profit?

Hint: calculate the agent's virtual valuation.

Exercise 2.
Consider an optimal auction with two bidders A and B who have independent private values, but where A's valuation is drawn from uniform $[0,1]$ while B's valuation is drawn from uniform $[0,3]$. If A's valuation realized valuation is 0.8 and B's realized valuation is 1.6, who wins and what does she pay?

Hint: Use Meyerson's optimal auction and virtual valuations.

2 VCG

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Exercise 3.
Should we build a road?

	build	not build	payment under VCG
a_1	200	0	
a_2	100	0	
a_3	0	250	

Exercise 4.
What if both increase their bids?

	build	not build	payment
a_1	250	0	
a_2	150	0	
a_3	0	250	

Exercise 5.
What happens if bidder a_1 manages to submit two bids?

Compare

	build	not build	payment
a_1	20	0	
a_2	0	10	0

with

	build	not build	payment
a_1	20	0	
a_1	20	0	
a_2	0	10	0

Exercise 6.
Find the social wellfare maximizing allocation and the corresponding payments under VCG.

	v_i			
	A	B	AB	payment
$bidder_1$	10	5	15	
$bidder_2$	1	6	12	

Exercise 7.
Find the social wellfare maximizing allocation and the corresponding payments under VCG.

	v_i			
	A	B	AB	payment
$bidder_1$	10	5	15	
$bidder_2$	1	10	12	

Example from Game Theory Online.