COMPUTATIONAL GAME THEORY

Exercises

November 17, 2022

1 BAYESIAN GAMES

Exercise 1.
Consider a the following Bayesian simultaneous-move game involving two armies fighting for an island. Army 1 can be of type *weak*, or *strong*, both with equal probability. Army 2 is always *weak*. Both players can chose to *attack* or *not to attack*. Neither army can observe the other's action. Army 2 does not know which type of army it is fighting against.

The following matrices capture the payoffs. Army 1 is the row player.

Matrix for *Weak*

	A	N
A	$-3, -3$	$5, 2$
N	$0, 5$	$0, 0$

Matrix for *Strong*

	A	N
A	$3, -3$	$5, 0$
N	$0, 5$	$0, 0$

(1)

Which of the following strategy profiles are Bayesian equilibria?

1. Army 1: (Weak: Not attack, Strong: Attack);
   Army 2: Attack
2. Army 1: (Weak: Not attack, Strong: Attack);
   Army 2: Not attack
3. Army 1: (Weak: Attack, Strong: Attack);
   Army 2: Attack
4. There is no Bayesian equilibrium.

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Exercise 2.

BAYESIAN GAMES Consider the following Bayesian simultaneous-move game with players a_1 and a_2. Player a_1 has two types a_1^l and a_1^r, and two actions U and D. Player a_2 has only one type and actions L and R. Both types of player a_1 are equally likely.

Matrix for a_1^l

	L	R
U	1,0	0,2
D	0,3	1,0

Matrix for a_1^r

	L	R
U	0,2	1,1
D	1,0	0,2

(2)

Draw Game 2 in extensive form.

Exercise 3.

Calculate Bayesian equilibria of game 2.

Exercise 4.

Suppose there are two bidders a_1 and a_2, whose independent private values are either 1 or 3 with equal probability. Assume ties are broken randomly.

1. Find the expected revenue using second-price auction rules.
2. (Trick question?) What would be the revenue of a first-price auction?
3. Now suppose there were three bidders instead of two. How does the revenue change?

Exercise 5.

Consider a second-price auction involving two bidders a_1 and a_2 whose private values are either 0 or 1 with equal probability. Bidder a_1 sometimes makes a mistake about his value for the object: when his value is 1, he knows it is 1; however, when his value is 0, half of the time, he believes it is actually 1 by mistake. Assume that ties are broken randomly and that bids must be integers. Draw the full game tree of this situation.

Exercise 6.

Using the tree from the last exercise, calculate a_1's expected utility from bidding 0 compared to bidding 1, when a_1 thinks his value is 1. What is a_1's optimal strategy?