COMPUTATIONAL GAME THEORY

Exercises

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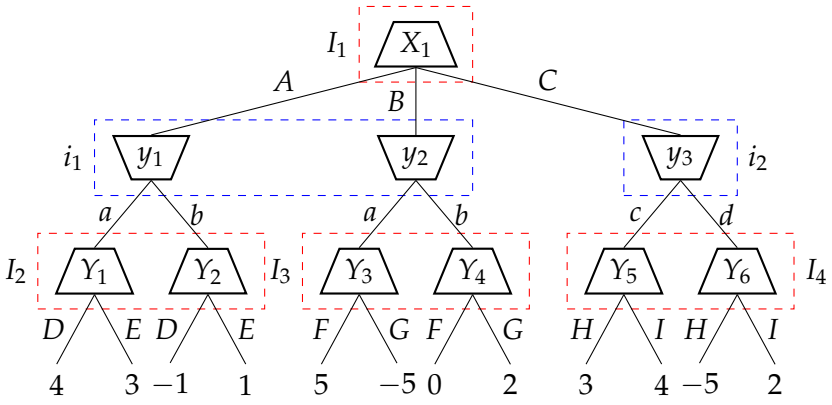
October 14, 2022

1 EXTENSIVE FORM GAMES

Exercise 1.
Consider following Extensive Form Game



Transform this game into a normal-form

Exercise 2.
Formulate the following game of a small poker as an EFG

- There is an ante of $1
- Deck is composed of these card J, J, Q, Q all with same color
- Each player receives a card at the beginning
- Player 1 either folds or bets $2
- Player 2 either calls or folds
- Player with the higher card wins

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Additionally write down amount of pure strategies in this game

Exercise 3.
Consider following map of Counter-Strike map Dust 2. There are 2 players, Terrorist (T) and Counter Terrorist (CT). The goal of the T is either to kill the CT or plant a bomb, which explodes after some time. Goal of the CT is to either kill the T and if the bomb was planted, the other goal is to defuse the bomb.

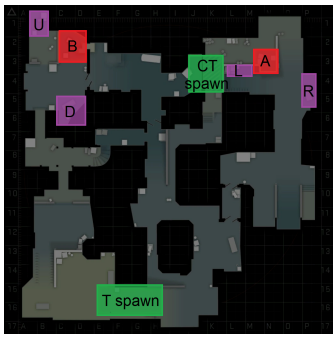


Figure 1: Counter-Strike map Dust 2

This map has 2 bomb sites A and B. Site B has two camping sites U and D, while A has camping sites L and R. The game then have following rules

- CT picks which site to defend at the beginning.
- T observes with probability p_{see} the CT, if the CT goes to the site B. If the CT is observed, then the T has p_{sn} probability that it will kill the CT.
- If T observed that CT goes to B, then it goes to the site A, otherwise it makes choice between A and B.
- When CT arrives at a site it camps at locations L or R on A or U or D on B
- If both CT and T picks the same site they engage in combat, in which one of them dies. If T knows which location CT picked it always wins the combat otherwise the CT is killed with probability p_c
- If T picked undefended site it freely plants a bomb, which informs CT that the bomb has been planted
- When CT is informed that bomb has been planted it runs to the other site and kills T with probability p_{st} and then he tries to defuse a bomb, otherwise he is killed.
- If CT ran from R to the B or from U to the A, he cannot defuse a bomb in time even when he kills the T. If CT ran from L to the B or from D to the A and kills the T, it defuses the bomb in time.
- T always gets reward 1 if it kills the CT. If CT kills T before he plants the bomb, then he gets reward of 1. If the bomb has been planted and

CT kills T and is able to defuse the bomb it again gets reward of 1, but when the bomb explodes he gets reward of $-\frac{1}{2}$

Additionally write down amount of pure strategies in this game

SOLUTIONS

Solution 1.

When transforming a game from extensive-form into the normal-form, each pure action for player must take into account what the player would play in each infoset. Therefore the pure actions in normal-form game are a Cartesian product across infoset actions.

$$\mathcal{A}_j^{\text{NFG}} = \bigtimes_{\mathcal{I}_i \in \mathcal{I}} \mathcal{A}_j(\mathcal{I}_i)$$

We will name the pure actions in NFG by concatenating the names of all actions, which create this action. For example first player actions, which is composed out of actions A, D, F, H will be called $ADFH$.

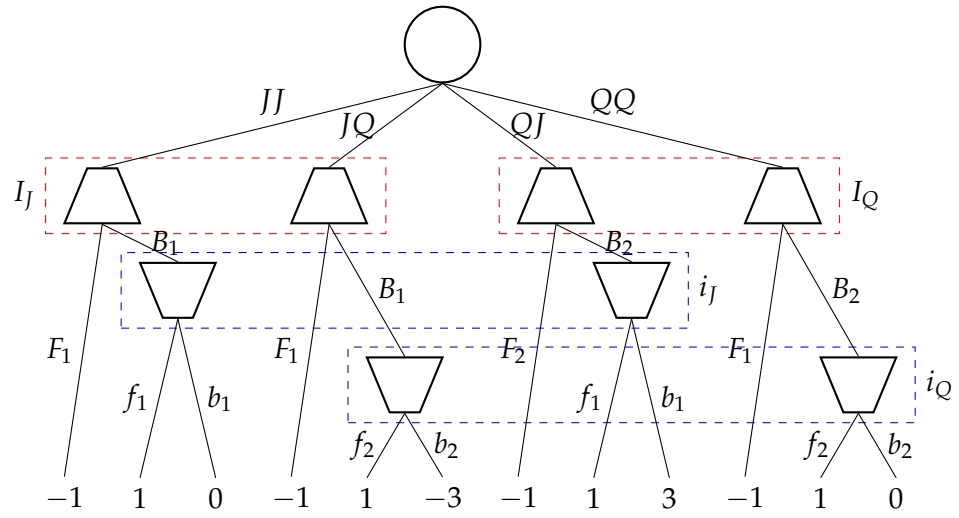
Pure actions of the player 1 are *ADFH, ADFI, ADGH, ADGI, AEFH, AEFI, AEGH, AEGI, BDFH, BDFI, BDGH, BDGI, BEFH, BEFI, BEGH, BEGI, CDFH, CDFI, CDGH, CDGI, CEFH, CEFI, CEGH, CEGI.*

Pure action of the player 2 are ac, ad, bc, bd. The utility matrix is then

		P2			
		ac	*ad*	*bc*	*bd*
P1	*ADFH*	4	4	−1	−1
	ADFI	4	4	−1	−1
	ADGH	4	4	−1	−1
	ADGI	4	4	−1	−1
	AEFH	3	3	1	1
	AEFI	3	3	1	1
	AEGH	3	3	1	1
	AEGI	3	3	1	1
	BDFH	5	5	0	0
	BDFI	5	5	0	0
	BDGH	−5	−5	2	2
	BDGI	−5	−5	2	2
	BEFH	5	5	0	0
	BEFI	5	5	0	0
	BEGH	−5	−5	2	2
	BEGI	−5	−5	2	2
	CDFH	3	−5	3	−5
	CDFI	4	2	4	2
	CDGH	3	−5	3	−5
	CDGI	4	2	4	2
	CEFH	3	−5	3	−5
	CEFI	4	2	4	2
	CEGH	3	−5	3	−5
	CEGI	4	2	4	2

Solution 2.

The game starts with dealing a card. We can either create 2 chance nodes (one for each card), or just a single one, with 4 possible outcomes. We will show the second variant.



Since both players have just two information sets, with just two actions each, we have to take each possible combination of a single action from each information set. Therefore there are $2 \cdot 2$ pure strategies. Namely for first player we have $F_1F_2, F_1B_2, B_1F_2, B_1B_2$ and for the second player $f_1f_2, f_1b_2, b_1f_2, b_1b_2$

Solution 3.
This game may be modelled in a multiple ways and we will use just one of them. In this representation we consider the game as a zero-sum and we show only the reward to the T. Left subtree under chance node p is taken with probability $1 - p$ and the right with p.