



DCGI

DEPARTMENT OF COMPUTER GRAPHICS AND INTERACTION

GEOMETRIC SEARCHING

PART 1: POINT LOCATION in 2D

PETR FELKEL

FEL CTU PRAGUE

felkel@fel.cvut.cz

<https://cw.felk.cvut.cz/doku.php/courses/a4m39vg/start>

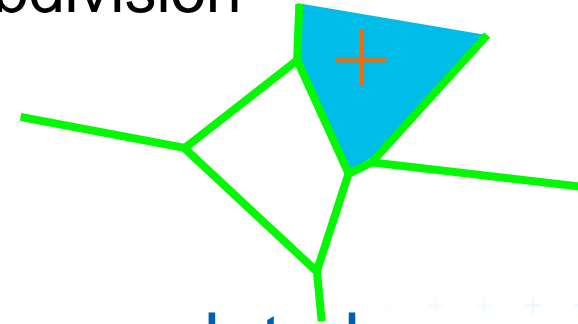
Based on [Berg] and [Mount]

Version from 05.10.2022

Geometric searching problems

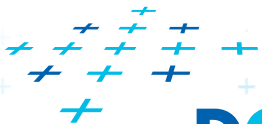
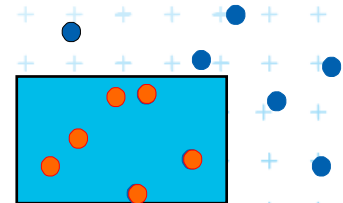
1. Point location (static) – Where am I?

- (Find the name of the state, pointed by mouse cursor)
- Search space S : a planar (spatial) subdivision
- Query: **point** Q
- Answer: **region** containing Q



2. Orthogonal range searching – Query a data base (Find points, located in d -dimensional axis-parallel box)

- Search space S : a set of points
- Query: set of orthogonal **intervals** q
- Answer: subset of **points** in the box
- (Was studied in DPG)



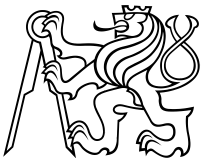
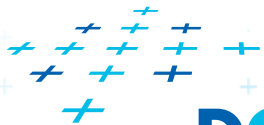
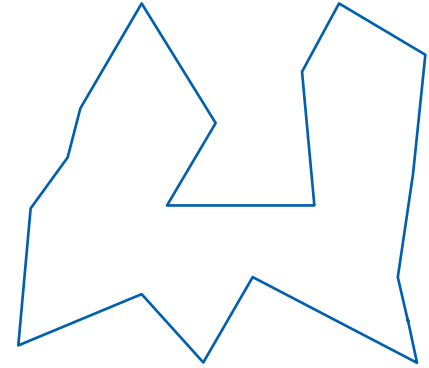
Part 1: Point location

- Point location in polygon
- Planar subdivision
- DCEL data structure
- Point location in planar subdivision
 - slabs
 - monotone sequence
 - trapezoidal map



Point location in polygon by ray crossing

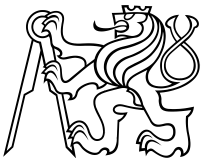
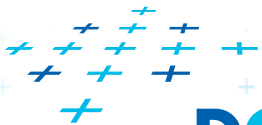
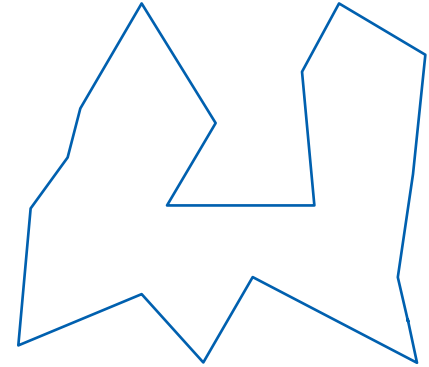
1. Ray crossing - $O(n)$



Point location in polygon by ray crossing

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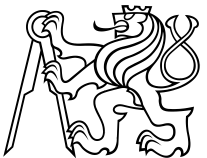
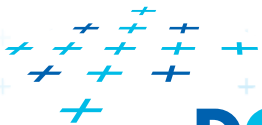
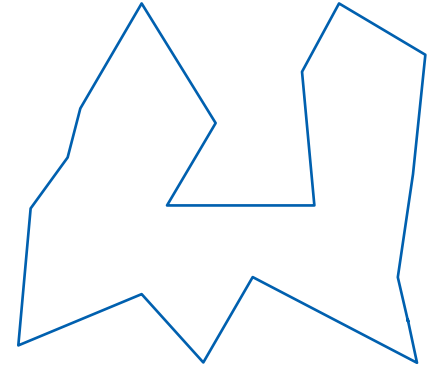
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Point location in polygon by ray crossing

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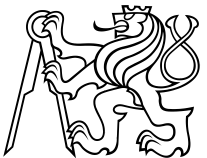
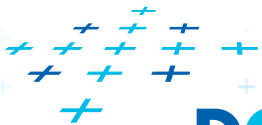
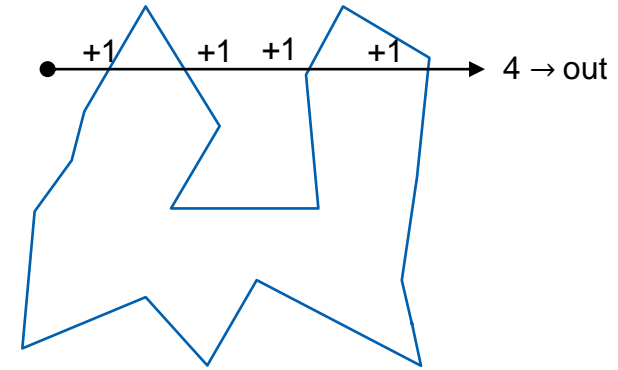
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Point location in polygon by ray crossing

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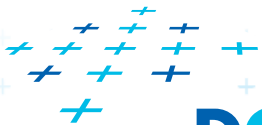
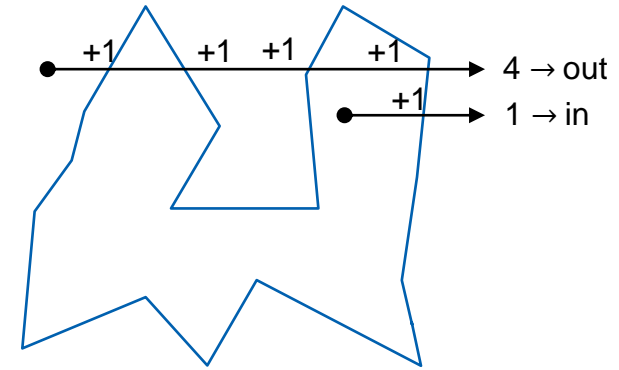
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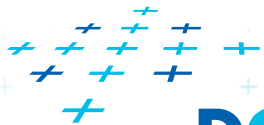
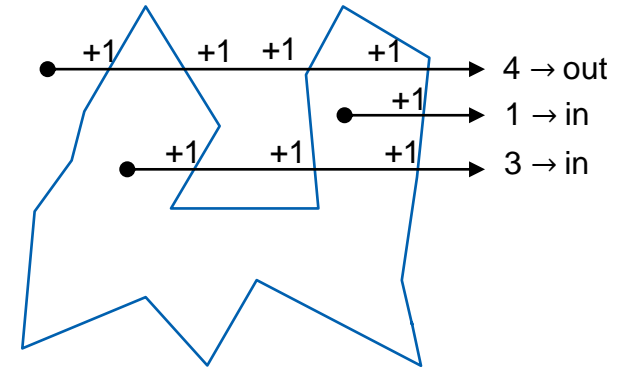
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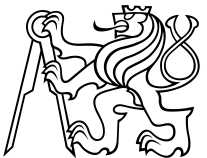
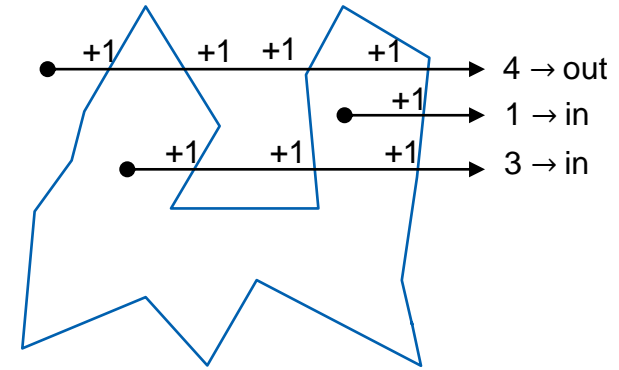
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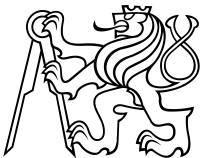
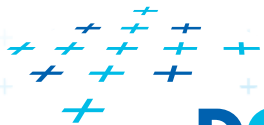
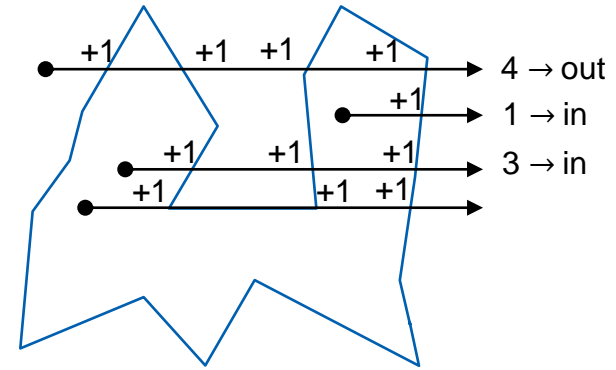
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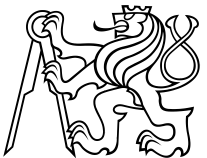
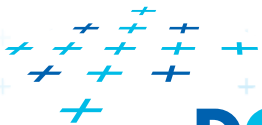
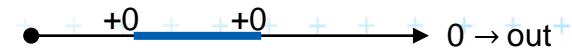
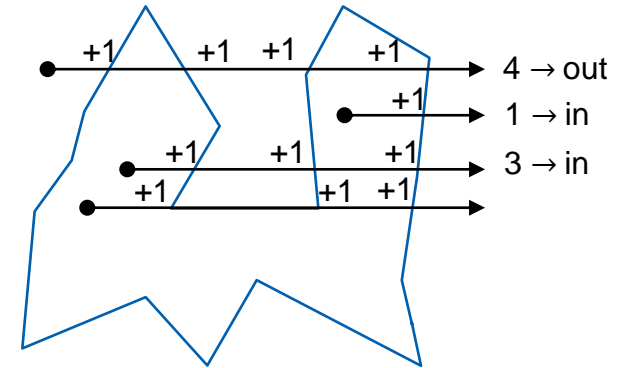
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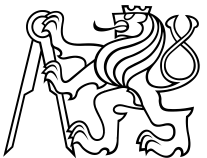
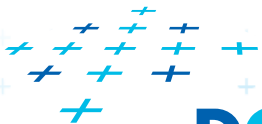
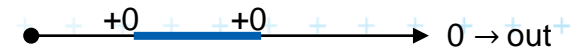
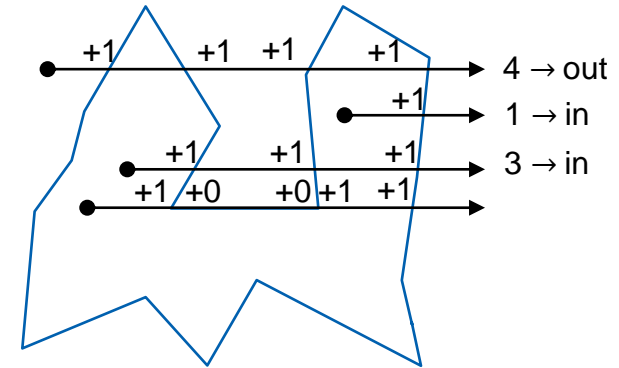
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Point location in polygon by ray crossing

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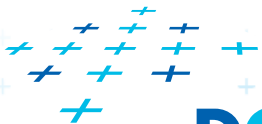
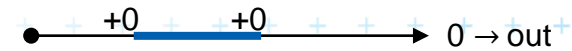
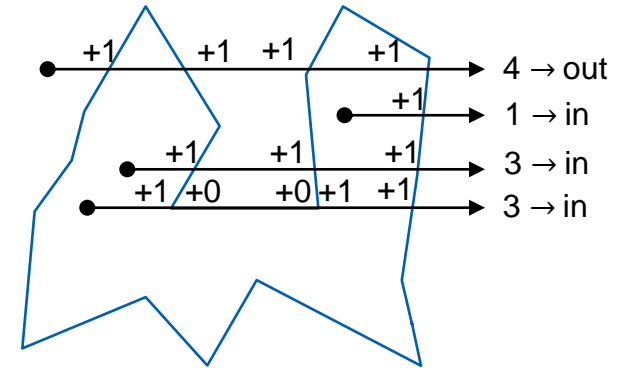
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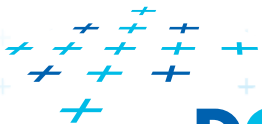
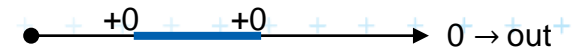
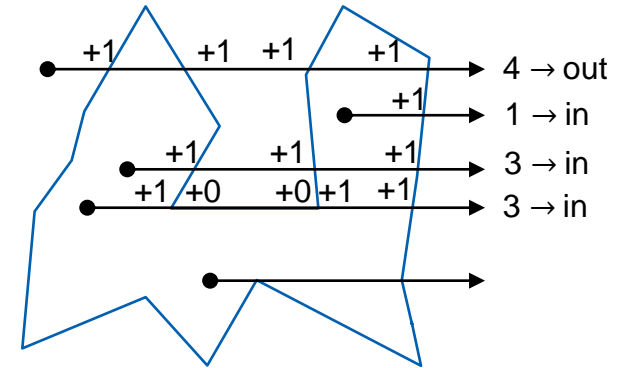
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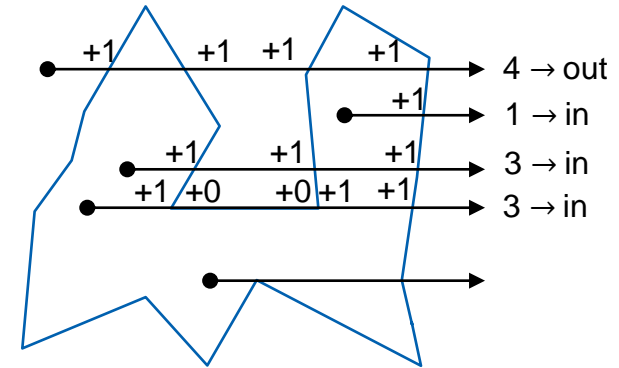
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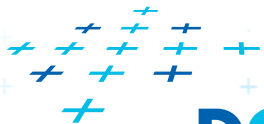
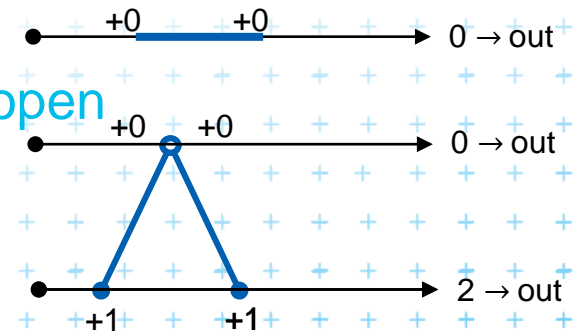
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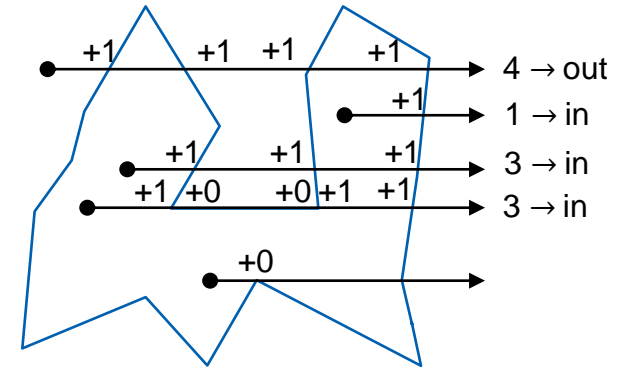
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Point location in polygon by ray crossing

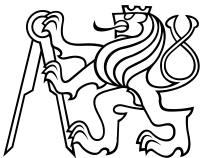
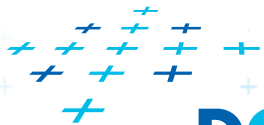
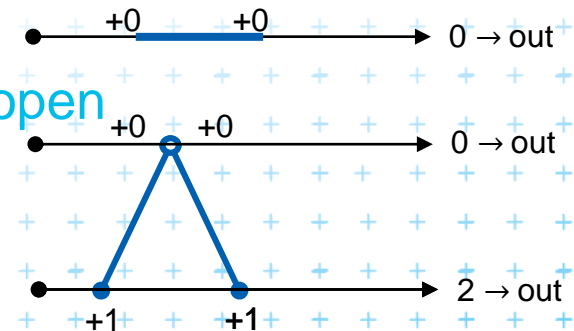
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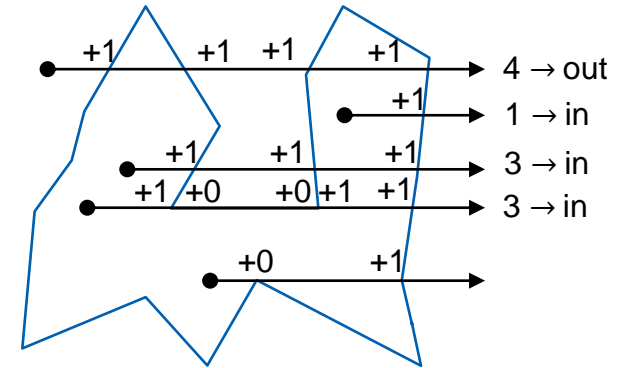
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Point location in polygon by ray crossing

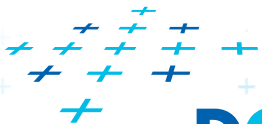
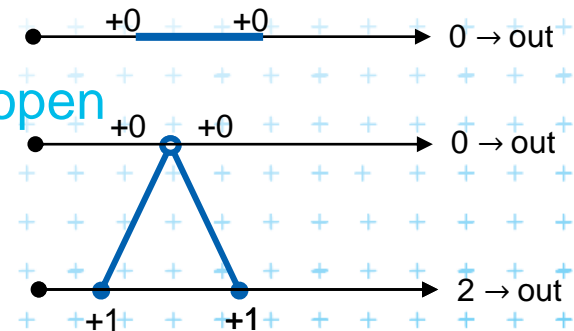
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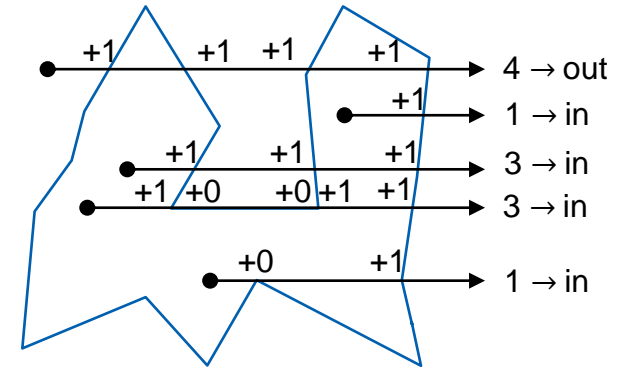
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Point location in polygon by ray crossing

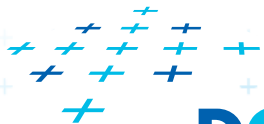
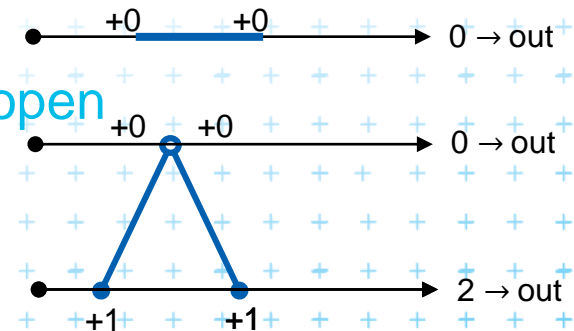
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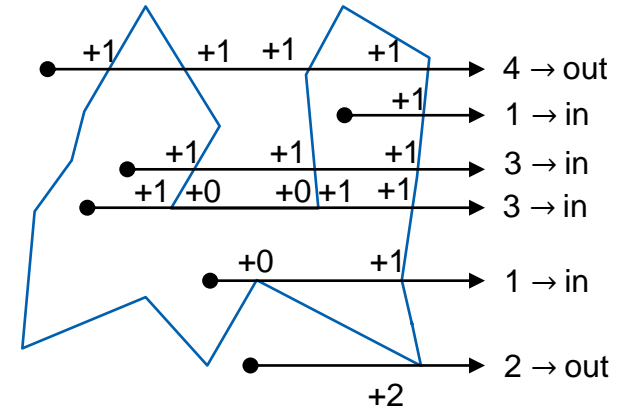
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Point location in polygon by ray crossing

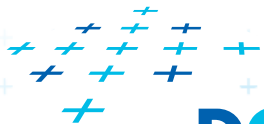
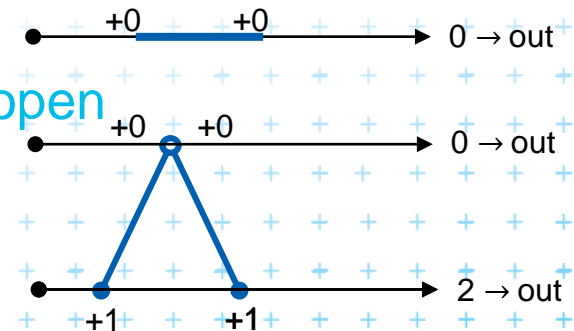
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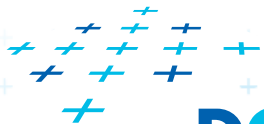
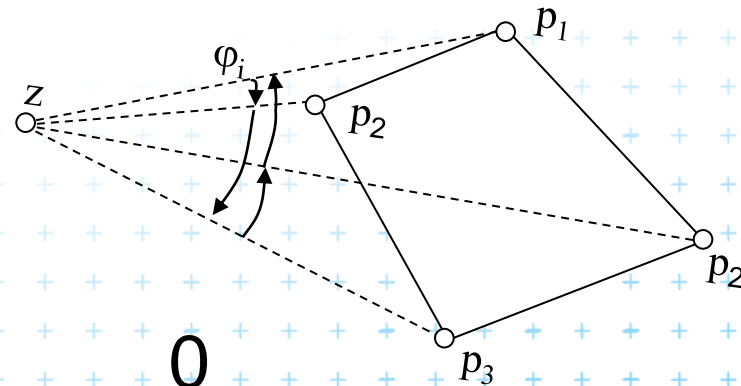
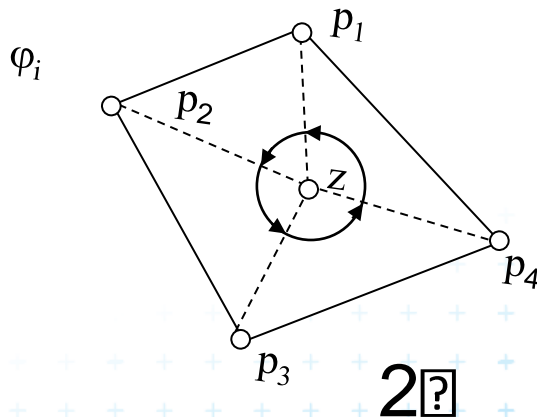


Point location in polygon

2. Winding number - $O(n)$

(number of turns around the point)

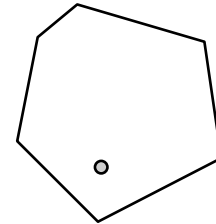
- Sum oriented angles $\varphi_i = \sphericalangle(p_i, z, p_{i+1})$
- If $(\sum \varphi_i = 2\pi)$ then inside (1 turn)
- If $(\sum \varphi_i = 0)$ then outside (no turn)
- About 20-times slower than ray crossing



Point location in convex polygon

3. Position relative to all edges

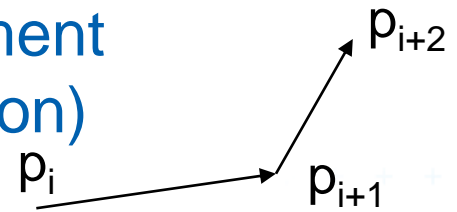
- For **convex** polygons
- If (left from all edges) then inside



■ Position of point in relation to the line segment (Determination of convex polygon orientation)

Convex polygon, non-collinear points

$$p_i = [x_i, y_i, 1], \quad p_{i+1} = [x_{i+1}, y_{i+1}, 1], \quad p_{i+2} = [x_{i+2}, y_{i+2}, 1]$$



$$\begin{vmatrix} x_i & y_i & 1 \\ x_{i+1} & y_{i+1} & 1 \\ x_{i+2} & y_{i+2} & 1 \end{vmatrix} > 0 \Rightarrow \text{point left from edge (for CCW polygon)}$$
$$\begin{vmatrix} x_i & y_i & 1 \\ x_{i+1} & y_{i+1} & 1 \\ x_{i+2} & y_{i+2} & 1 \end{vmatrix} < 0 \Rightarrow \text{point right from edge (for CW polygon)}$$

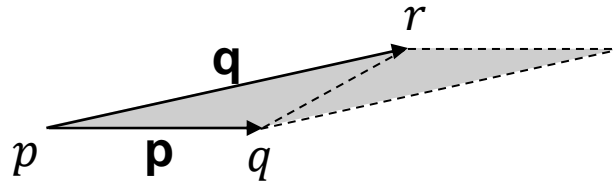


Area of Triangle

$$T = \frac{1}{2} |\mathbf{p} \times \mathbf{q}|$$

$$\mathbf{p} = q - p$$

$$\mathbf{q} = r - p$$



$$2T = p_x q_y - p_y q_x$$

using vector product $\mathbf{p} \times \mathbf{q}$

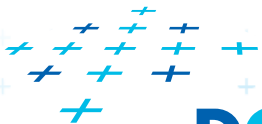
$$2T = \begin{vmatrix} p_x & p_y & 1 \\ q_x & q_y & 1 \\ r_x & r_y & 1 \end{vmatrix}$$

using coordinates of points

Orientation is computed as $\text{sign}(2T) =$

$$= \text{sign}(p_x q_y + q_x r_y + r_x p_y - p_x r_y - q_x p_y - r_x q_y)$$

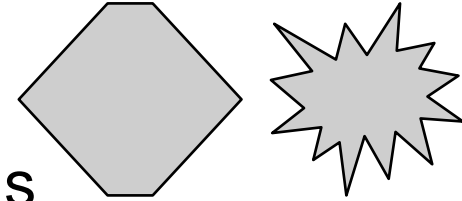
$$= \text{sign} \left((q_x - p_x)(r_y - p_y) - (q_y - p_y)(r_x - p_x) \right) \text{ for pivot } p$$



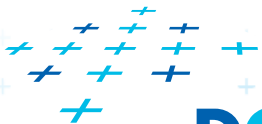
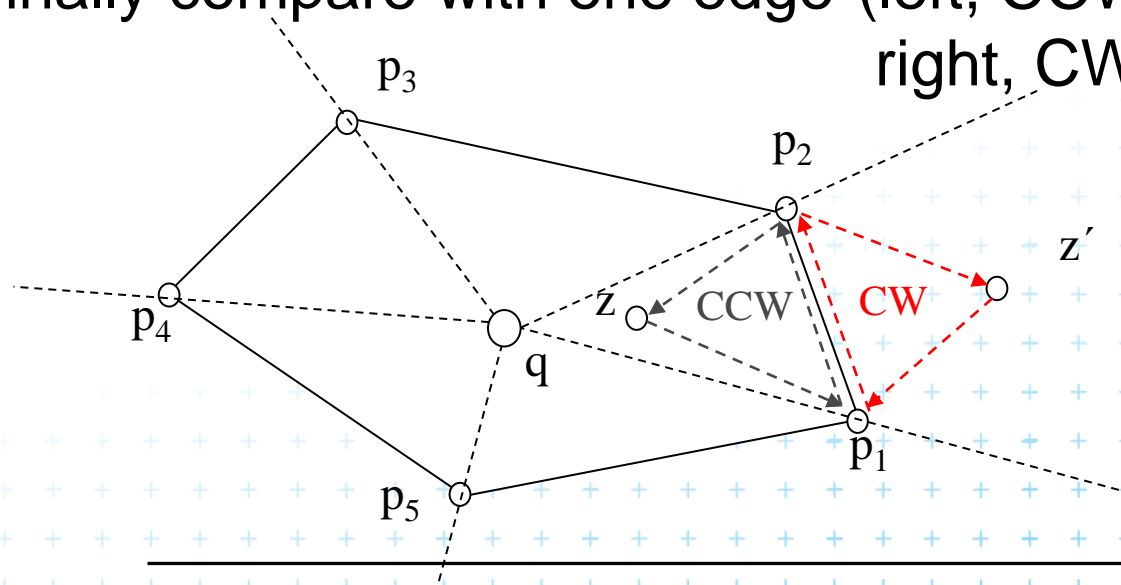
Point location in polygon

4. Binary search in angles

Works for convex and star-shaped polygons



1. Choose any point q inside / in the polygon core
2. q forms wedges with polygon edges
3. Binary search of **wedge** výseč based on angle
4. Finally compare with one edge (left, CCW \Rightarrow in, right, CW \Rightarrow out)

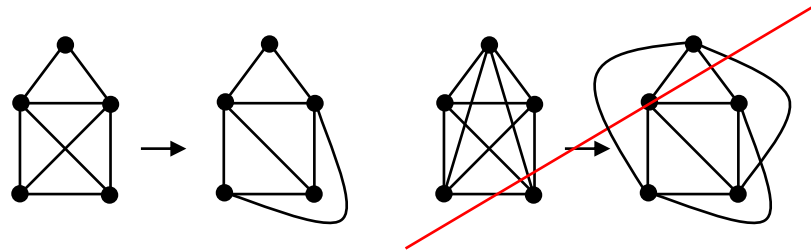


Planar graph

Planar graph

U =set of nodes, H =set of arcs

= Graph $G = (U, H)$ is planar, if it can be embedded into plane without crossings

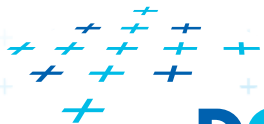


Planar embedding of planar graph $G = (U, H)$

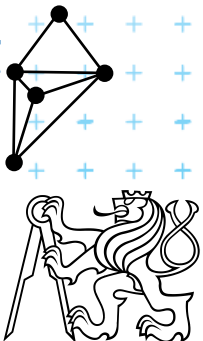
= mapping of each *node in U* to *vertex* in the plane and each *arc in H* into *simple curve (edge)* between the two images of extreme nodes of the arc, so that **no two images of arc intersect** except at their endpoints

Every planar graph can be embedded in such a way that arcs map to straight line segments [Fáry 1948]

=> Planar Straight Line Graph

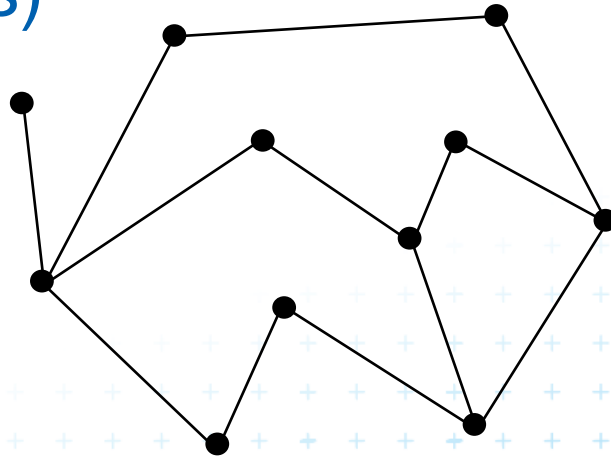


DCGI

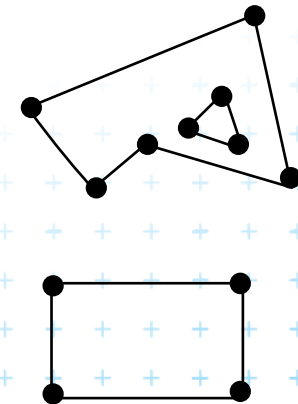


Planar subdivision

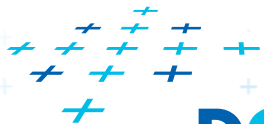
- = Partition of the plane determined by straight line planar embedding of a planar graph.
Also called PSLG – Planar Straight Line Graph
- (embedding of a planar graph in the plane such that its arcs are mapped into straight line segments)



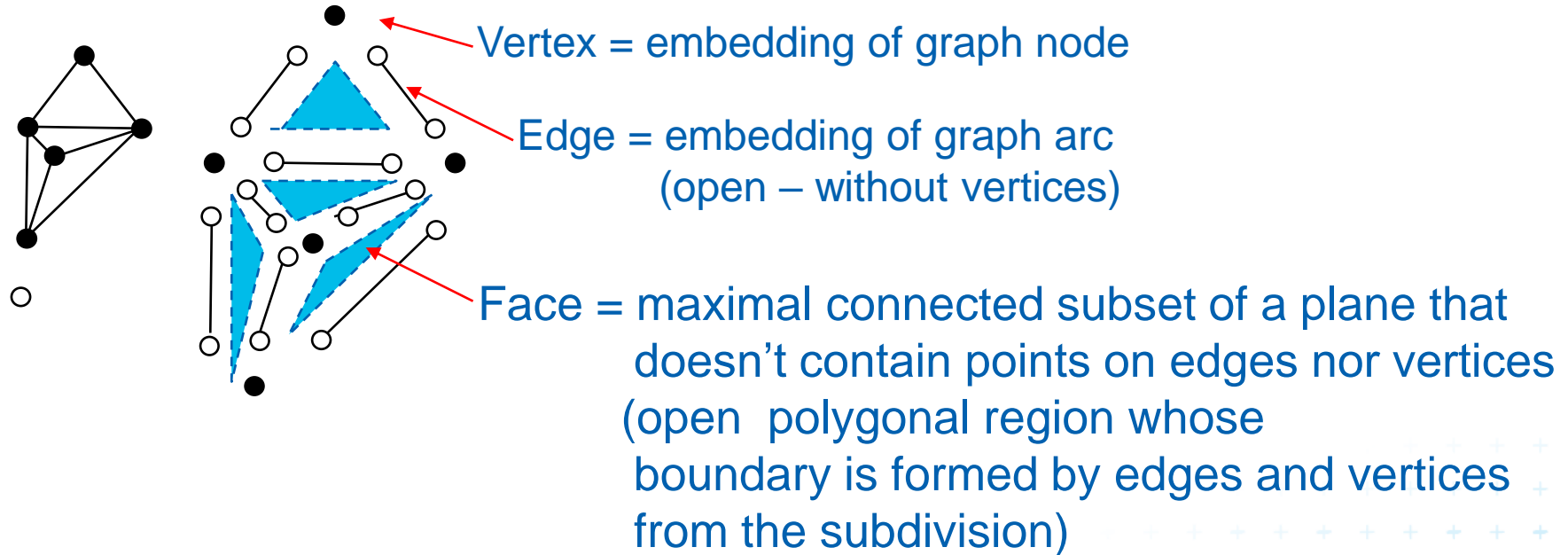
connected



disconnected



Planar subdivision



Complexity (size) of a subdivision = sum of number of vertices +
+ number of edges +
+ number of faces it consists of

Euler's formula: $|V| - |E| + |F| \geq 2$

$|V| = n, |E| \leq 3v - 6, |F| \leq 2v - 4$

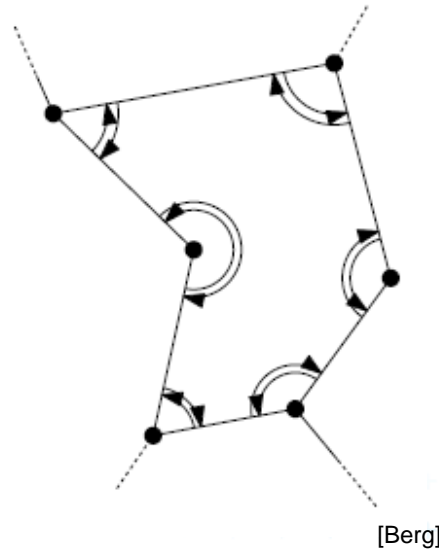
} $O(n)$ data structure



DCEL = Double Connected Edge List [Eastman 1982]

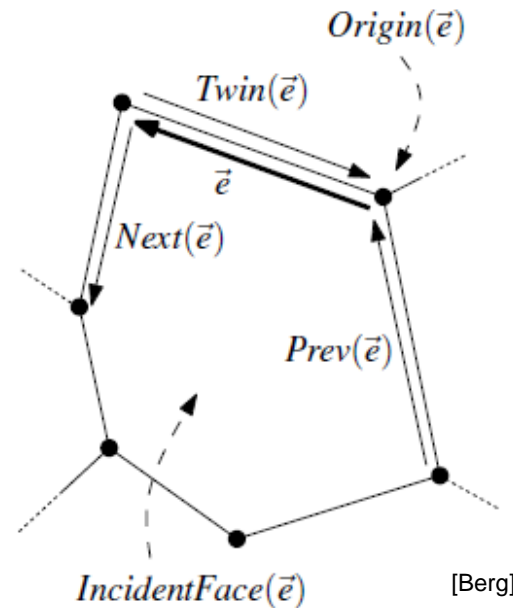
- A structure for storage of planar subdivision
- Operations like:

Walk around boundary of a given face

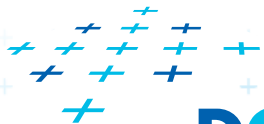


Pointers to next and prev edge

Get incident face

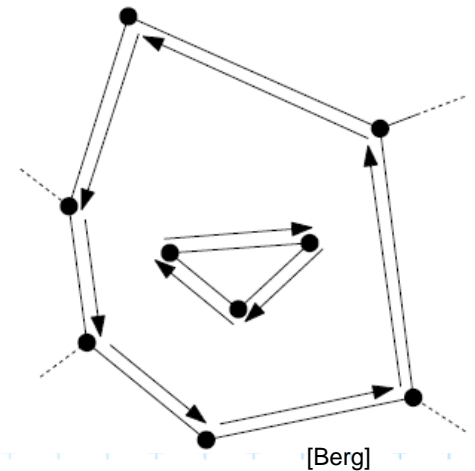


Half-edge, op. $Twin(e)$, unique $Next(e)$, $Prev(e)$

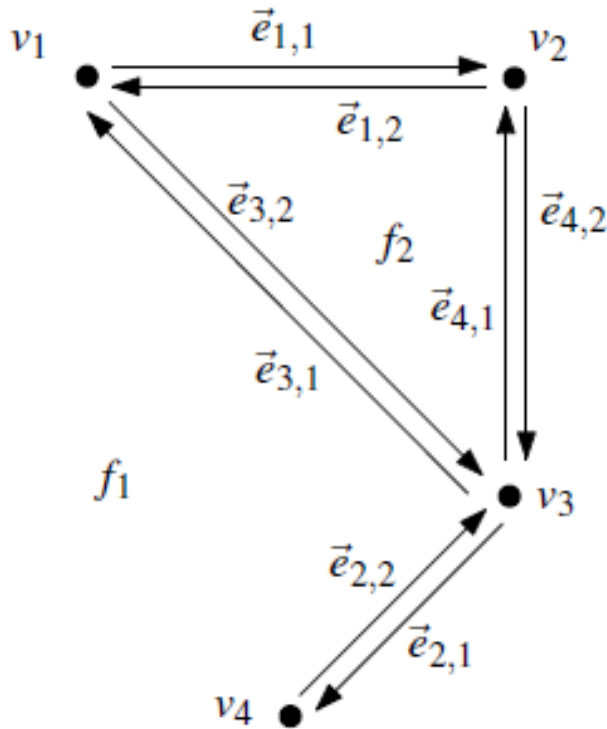


DCEL = Double Connected Edge List

- Vertex record v
 - Coordinates(v) and pointer to one IncidentEdge(v)
- Face record f
 - OuterComponent(f) pointer (boundary)
 - List of holes – InnerComponent(f)
- Half-edge record e
 - Origin(e), Twin(e), IncidentFace(e)
 - Next(e), Prev(e)
 - [Dest(e) = Origin(Twin(e))]
- Possible attribute data for each



DCEL = Double Connected Edge List

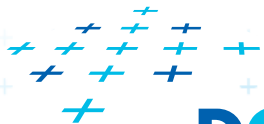


Vertex	Coordinates	IncidentEdge
v_1	(0, 4)	$\vec{e}_{1,1}$
v_2	(2, 4)	$\vec{e}_{4,2}$
v_3	(2, 2)	$\vec{e}_{2,1}$
v_4	(1, 1)	$\vec{e}_{2,2}$

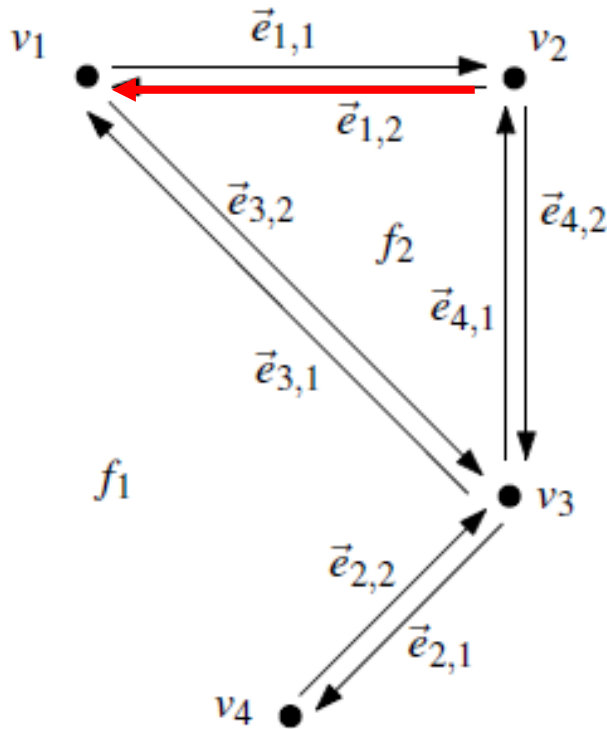
Face	OuterComponent	InnerComponents
f_1	nil	$\vec{e}_{1,1}$
f_2	$\vec{e}_{4,1}$	nil

Half-edge	Origin	Twin	IncidentFace	Next	Prev
$\vec{e}_{1,1}$	v_1	$\vec{e}_{1,2}$	f_1	$\vec{e}_{4,2}$	$\vec{e}_{3,1}$
$\vec{e}_{1,2}$	v_2	$\vec{e}_{1,1}$	f_2	$\vec{e}_{3,2}$	$\vec{e}_{4,1}$
$\vec{e}_{2,1}$	v_3	$\vec{e}_{2,2}$	f_1	$\vec{e}_{2,2}$	$\vec{e}_{4,2}$
$\vec{e}_{2,2}$	v_4	$\vec{e}_{2,1}$	f_1	$\vec{e}_{3,1}$	$\vec{e}_{2,1}$
$\vec{e}_{3,1}$	v_3	$\vec{e}_{3,2}$	f_1	$\vec{e}_{1,1}$	$\vec{e}_{2,2}$
$\vec{e}_{3,2}$	v_1	$\vec{e}_{3,1}$	f_2	$\vec{e}_{4,1}$	$\vec{e}_{1,2}$
$\vec{e}_{4,1}$	v_3	$\vec{e}_{4,2}$	f_2	$\vec{e}_{1,2}$	$\vec{e}_{3,2}$
$\vec{e}_{4,2}$	v_2	$\vec{e}_{4,1}$	f_1	$\vec{e}_{2,1}$	$\vec{e}_{1,1}$

[Berg]



DCEL = Double Connected Edge List

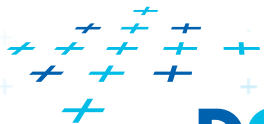


Vertex	Coordinates	IncidentEdge
v_1	(0,4)	$\vec{e}_{1,1}$
v_2	(2,4)	$\vec{e}_{4,2}$
v_3	(2,2)	$\vec{e}_{2,1}$
v_4	(1,1)	$\vec{e}_{2,2}$

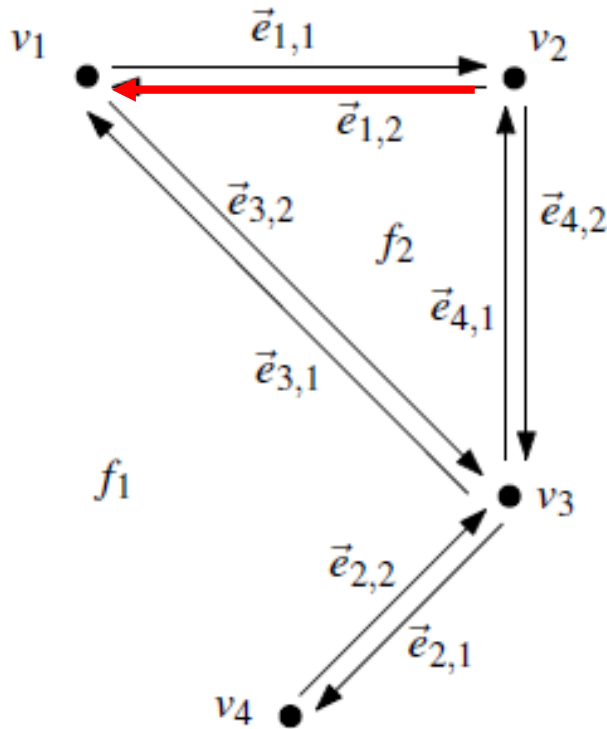
Face	OuterComponent	InnerComponents
f_1	nil	$\vec{e}_{1,1}$
f_2	$\vec{e}_{4,1}$	nil

Half-edge	Origin	Twin	IncidentFace	Next	Prev
$\vec{e}_{1,1}$	v_1	$\vec{e}_{1,2}$	f_1	$\vec{e}_{4,2}$	$\vec{e}_{3,1}$
$\vec{e}_{1,2}$	v_2	$\vec{e}_{1,1}$	f_2	$\vec{e}_{3,2}$	$\vec{e}_{4,1}$
$\vec{e}_{2,1}$	v_3	$\vec{e}_{2,2}$	f_1	$\vec{e}_{2,2}$	$\vec{e}_{4,2}$
$\vec{e}_{2,2}$	v_4	$\vec{e}_{2,1}$	f_1	$\vec{e}_{3,1}$	$\vec{e}_{2,1}$
$\vec{e}_{3,1}$	v_3	$\vec{e}_{3,2}$	f_1	$\vec{e}_{1,1}$	$\vec{e}_{2,2}$
$\vec{e}_{3,2}$	v_1	$\vec{e}_{3,1}$	f_2	$\vec{e}_{4,1}$	$\vec{e}_{1,2}$
$\vec{e}_{4,1}$	v_3	$\vec{e}_{4,2}$	f_2	$\vec{e}_{1,2}$	$\vec{e}_{3,2}$
$\vec{e}_{4,2}$	v_2	$\vec{e}_{4,1}$	f_1	$\vec{e}_{2,1}$	$\vec{e}_{1,1}$

[Berg]



DCEL = Double Connected Edge List

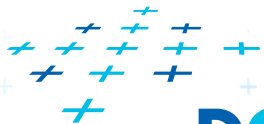


Vertex	Coordinates	IncidentEdge
v_1	(0, 4)	$\vec{e}_{1,1}$
v_2	(2, 4)	$\vec{e}_{4,2}$
v_3	(2, 2)	$\vec{e}_{2,1}$
v_4	(1, 1)	$\vec{e}_{2,2}$

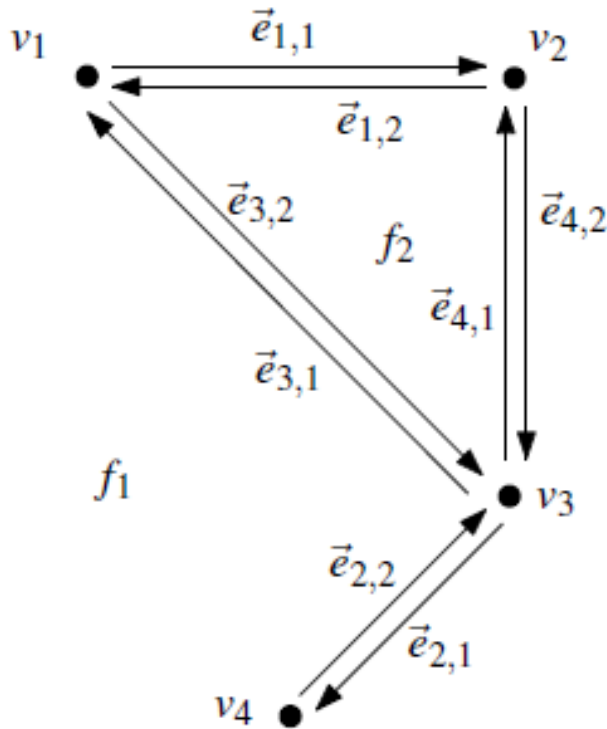
Face	OuterComponent	InnerComponents
f_1	nil	$\vec{e}_{1,1}$
f_2	$\vec{e}_{4,1}$	nil

Half-edge	Origin	Twin	IncidentFace	Next	Prev
$\vec{e}_{1,1}$	v_1	$\vec{e}_{1,2}$	f_1	$\vec{e}_{4,2}$	$\vec{e}_{3,1}$
$\vec{e}_{1,2}$	v_2	$\vec{e}_{1,1}$	f_2	$\vec{e}_{3,2}$	$\vec{e}_{4,1}$
$\vec{e}_{2,1}$	v_3	$\vec{e}_{2,2}$	f_1	$\vec{e}_{2,2}$	$\vec{e}_{4,2}$
$\vec{e}_{2,2}$	v_4	$\vec{e}_{2,1}$	f_1	$\vec{e}_{3,1}$	$\vec{e}_{2,1}$
$\vec{e}_{3,1}$	v_3	$\vec{e}_{3,2}$	f_1	$\vec{e}_{1,1}$	$\vec{e}_{2,2}$
$\vec{e}_{3,2}$	v_1	$\vec{e}_{3,1}$	f_2	$\vec{e}_{4,1}$	$\vec{e}_{1,2}$
$\vec{e}_{4,1}$	v_3	$\vec{e}_{4,2}$	f_2	$\vec{e}_{1,2}$	$\vec{e}_{3,2}$
$\vec{e}_{4,2}$	v_2	$\vec{e}_{4,1}$	f_1	$\vec{e}_{2,1}$	$\vec{e}_{1,1}$

[Berg]



DCEL = Double Connected Edge List

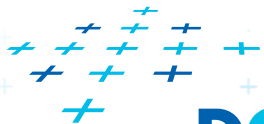


Vertex	Coordinates	IncidentEdge
v_1	(0, 4)	$\vec{e}_{1,1}$
v_2	(2, 4)	$\vec{e}_{4,2}$
v_3	(2, 2)	$\vec{e}_{2,1}$
v_4	(1, 1)	$\vec{e}_{2,2}$

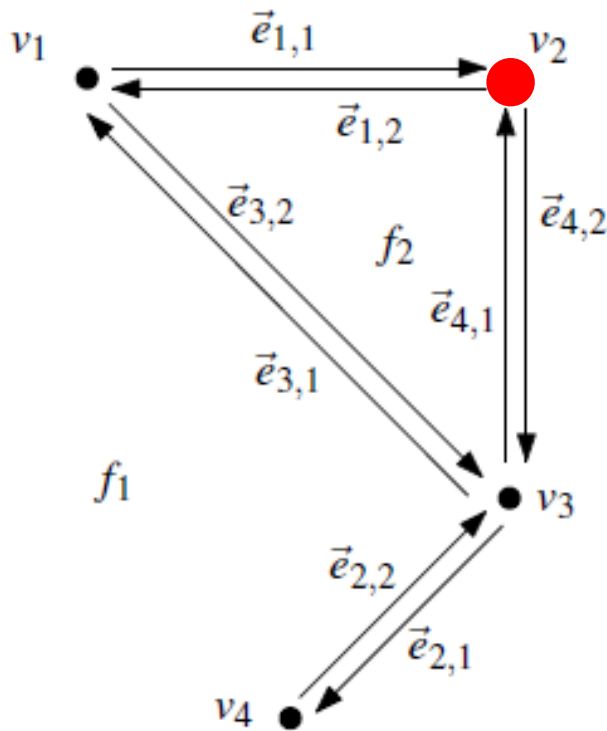
Face	OuterComponent	InnerComponents
f_1	nil	$\vec{e}_{1,1}$
f_2	$\vec{e}_{4,1}$	nil

Half-edge	Origin	Twin	IncidentFace	Next	Prev
$\vec{e}_{1,1}$	v_1	$\vec{e}_{1,2}$	f_1	$\vec{e}_{4,2}$	$\vec{e}_{3,1}$
$\vec{e}_{1,2}$	v_2	$\vec{e}_{1,1}$	f_2	$\vec{e}_{3,2}$	$\vec{e}_{4,1}$
$\vec{e}_{2,1}$	v_3	$\vec{e}_{2,2}$	f_1	$\vec{e}_{2,2}$	$\vec{e}_{4,2}$
$\vec{e}_{2,2}$	v_4	$\vec{e}_{2,1}$	f_1	$\vec{e}_{3,1}$	$\vec{e}_{2,1}$
$\vec{e}_{3,1}$	v_3	$\vec{e}_{3,2}$	f_1	$\vec{e}_{1,1}$	$\vec{e}_{2,2}$
$\vec{e}_{3,2}$	v_1	$\vec{e}_{3,1}$	f_2	$\vec{e}_{4,1}$	$\vec{e}_{1,2}$
$\vec{e}_{4,1}$	v_3	$\vec{e}_{4,2}$	f_2	$\vec{e}_{1,2}$	$\vec{e}_{3,2}$
$\vec{e}_{4,2}$	v_2	$\vec{e}_{4,1}$	f_1	$\vec{e}_{2,1}$	$\vec{e}_{1,1}$

[Berg]



DCEL = Double Connected Edge List

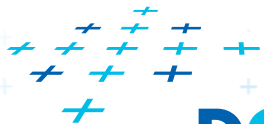


Vertex	Coordinates	IncidentEdge
v_1	(0, 4)	$\vec{e}_{1,1}$
v_2	(2, 4)	$\vec{e}_{4,2}$
v_3	(2, 2)	$\vec{e}_{2,1}$
v_4	(1, 1)	$\vec{e}_{2,2}$

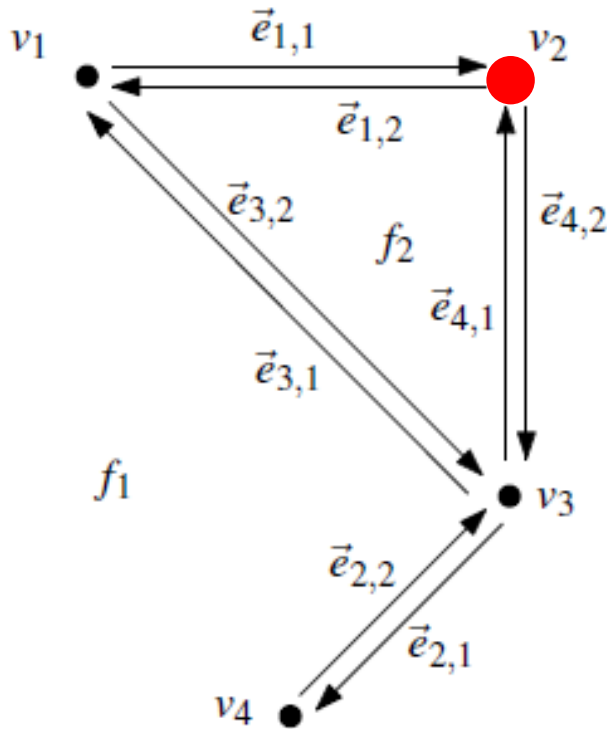
Face	OuterComponent	InnerComponents
f_1	nil	$\vec{e}_{1,1}$
f_2	$\vec{e}_{4,1}$	nil

Half-edge	Origin	Twin	IncidentFace	Next	Prev
$\vec{e}_{1,1}$	v_1	$\vec{e}_{1,2}$	f_1	$\vec{e}_{4,2}$	$\vec{e}_{3,1}$
$\vec{e}_{1,2}$	v_2	$\vec{e}_{1,1}$	f_2	$\vec{e}_{3,2}$	$\vec{e}_{4,1}$
$\vec{e}_{2,1}$	v_3	$\vec{e}_{2,2}$	f_1	$\vec{e}_{2,2}$	$\vec{e}_{4,2}$
$\vec{e}_{2,2}$	v_4	$\vec{e}_{2,1}$	f_1	$\vec{e}_{3,1}$	$\vec{e}_{2,1}$
$\vec{e}_{3,1}$	v_3	$\vec{e}_{3,2}$	f_1	$\vec{e}_{1,1}$	$\vec{e}_{2,2}$
$\vec{e}_{3,2}$	v_1	$\vec{e}_{3,1}$	f_2	$\vec{e}_{4,1}$	$\vec{e}_{1,2}$
$\vec{e}_{4,1}$	v_3	$\vec{e}_{4,2}$	f_2	$\vec{e}_{1,2}$	$\vec{e}_{3,2}$
$\vec{e}_{4,2}$	v_2	$\vec{e}_{4,1}$	f_1	$\vec{e}_{2,1}$	$\vec{e}_{1,1}$

[Berg]



DCEL = Double Connected Edge List

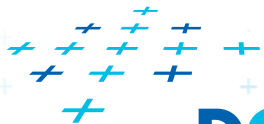


Vertex	Coordinates	IncidentEdge
v_1	(0, 4)	$\vec{e}_{1,1}$
v_2	(2, 4)	$\vec{e}_{4,2}$
v_3	(2, 2)	$\vec{e}_{2,1}$
v_4	(1, 1)	$\vec{e}_{2,2}$

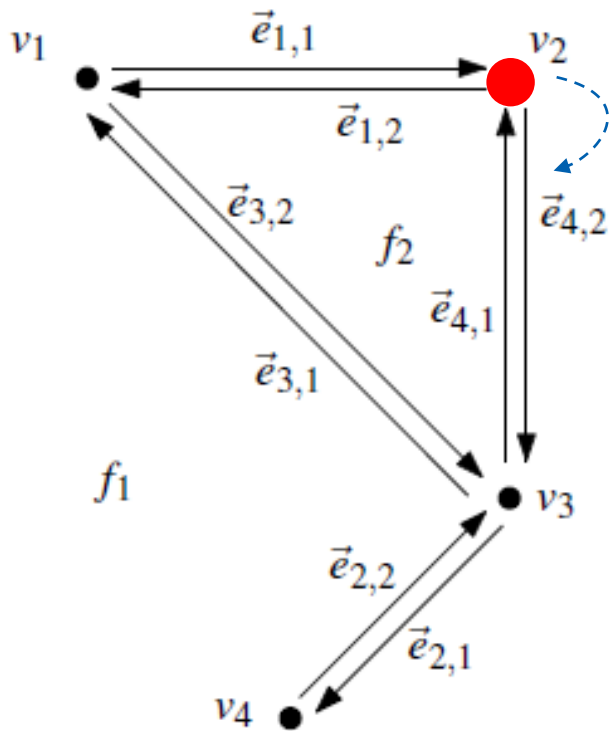
Face	OuterComponent	InnerComponents
f_1	nil	$\vec{e}_{1,1}$
f_2	$\vec{e}_{4,1}$	nil

Half-edge	Origin	Twin	IncidentFace	Next	Prev
$\vec{e}_{1,1}$	v_1	$\vec{e}_{1,2}$	f_1	$\vec{e}_{4,2}$	$\vec{e}_{3,1}$
$\vec{e}_{1,2}$	v_2	$\vec{e}_{1,1}$	f_2	$\vec{e}_{3,2}$	$\vec{e}_{4,1}$
$\vec{e}_{2,1}$	v_3	$\vec{e}_{2,2}$	f_1	$\vec{e}_{2,2}$	$\vec{e}_{4,2}$
$\vec{e}_{2,2}$	v_4	$\vec{e}_{2,1}$	f_1	$\vec{e}_{3,1}$	$\vec{e}_{2,1}$
$\vec{e}_{3,1}$	v_3	$\vec{e}_{3,2}$	f_1	$\vec{e}_{1,1}$	$\vec{e}_{2,2}$
$\vec{e}_{3,2}$	v_1	$\vec{e}_{3,1}$	f_2	$\vec{e}_{4,1}$	$\vec{e}_{1,2}$
$\vec{e}_{4,1}$	v_3	$\vec{e}_{4,2}$	f_2	$\vec{e}_{1,2}$	$\vec{e}_{3,2}$
$\vec{e}_{4,2}$	v_2	$\vec{e}_{4,1}$	f_1	$\vec{e}_{2,1}$	$\vec{e}_{1,1}$

[Berg]



DCEL = Double Connected Edge List



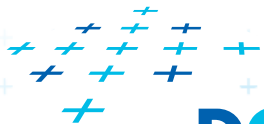
Vertex	Coordinates	IncidentEdge
v_1	(0, 4)	$\vec{e}_{1,1}$
v_2	(2, 4)	$\vec{e}_{4,2}$
v_3	(2, 2)	$\vec{e}_{2,1}$
v_4	(1, 1)	$\vec{e}_{2,2}$

← One of edges

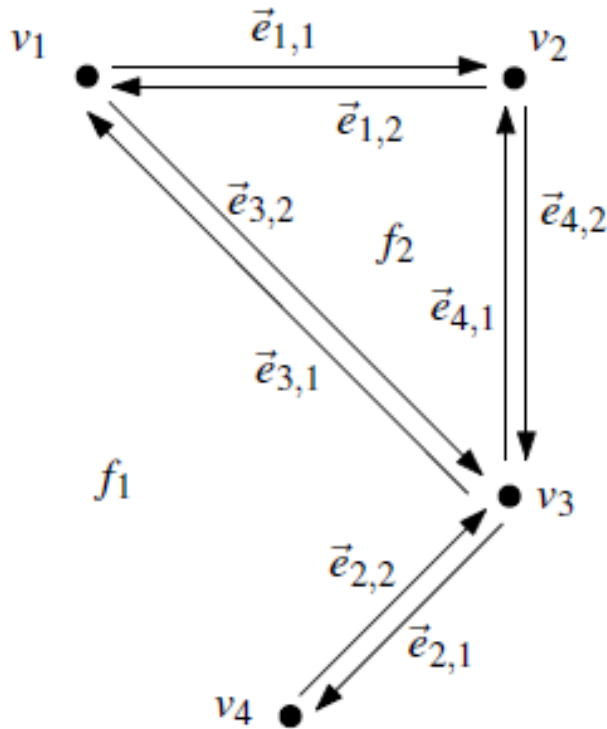
Face	OuterComponent	InnerComponents
f_1	nil	$\vec{e}_{1,1}$
f_2	$\vec{e}_{4,1}$	nil

Half-edge	Origin	Twin	IncidentFace	Next	Prev
$\vec{e}_{1,1}$	v_1	$\vec{e}_{1,2}$	f_1	$\vec{e}_{4,2}$	$\vec{e}_{3,1}$
$\vec{e}_{1,2}$	v_2	$\vec{e}_{1,1}$	f_2	$\vec{e}_{3,2}$	$\vec{e}_{4,1}$
$\vec{e}_{2,1}$	v_3	$\vec{e}_{2,2}$	f_1	$\vec{e}_{2,2}$	$\vec{e}_{4,2}$
$\vec{e}_{2,2}$	v_4	$\vec{e}_{2,1}$	f_1	$\vec{e}_{3,1}$	$\vec{e}_{2,1}$
$\vec{e}_{3,1}$	v_3	$\vec{e}_{3,2}$	f_1	$\vec{e}_{1,1}$	$\vec{e}_{2,2}$
$\vec{e}_{3,2}$	v_1	$\vec{e}_{3,1}$	f_2	$\vec{e}_{4,1}$	$\vec{e}_{1,2}$
$\vec{e}_{4,1}$	v_3	$\vec{e}_{4,2}$	f_2	$\vec{e}_{1,2}$	$\vec{e}_{3,2}$
$\vec{e}_{4,2}$	v_2	$\vec{e}_{4,1}$	f_1	$\vec{e}_{2,1}$	$\vec{e}_{1,1}$

[Berg]



DCEL = Double Connected Edge List



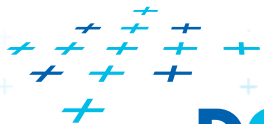
Vertex	Coordinates	IncidentEdge
v_1	(0, 4)	$\vec{e}_{1,1}$
v_2	(2, 4)	$\vec{e}_{4,2}$
v_3	(2, 2)	$\vec{e}_{2,1}$
v_4	(1, 1)	$\vec{e}_{2,2}$

← One of edges

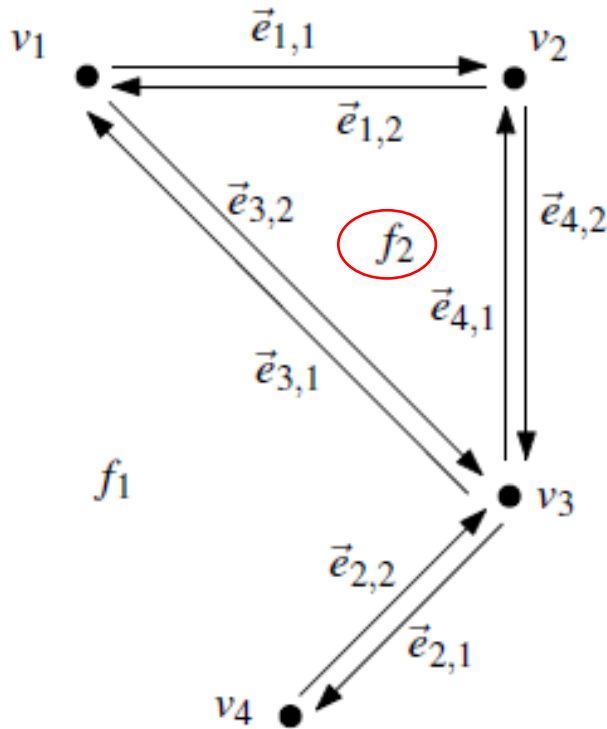
Face	OuterComponent	InnerComponents
f_1	nil	$\vec{e}_{1,1}$
f_2	$\vec{e}_{4,1}$	nil

Half-edge	Origin	Twin	IncidentFace	Next	Prev
$\vec{e}_{1,1}$	v_1	$\vec{e}_{1,2}$	f_1	$\vec{e}_{4,2}$	$\vec{e}_{3,1}$
$\vec{e}_{1,2}$	v_2	$\vec{e}_{1,1}$	f_2	$\vec{e}_{3,2}$	$\vec{e}_{4,1}$
$\vec{e}_{2,1}$	v_3	$\vec{e}_{2,2}$	f_1	$\vec{e}_{2,2}$	$\vec{e}_{4,2}$
$\vec{e}_{2,2}$	v_4	$\vec{e}_{2,1}$	f_1	$\vec{e}_{3,1}$	$\vec{e}_{2,1}$
$\vec{e}_{3,1}$	v_3	$\vec{e}_{3,2}$	f_1	$\vec{e}_{1,1}$	$\vec{e}_{2,2}$
$\vec{e}_{3,2}$	v_1	$\vec{e}_{3,1}$	f_2	$\vec{e}_{4,1}$	$\vec{e}_{1,2}$
$\vec{e}_{4,1}$	v_3	$\vec{e}_{4,2}$	f_2	$\vec{e}_{1,2}$	$\vec{e}_{3,2}$
$\vec{e}_{4,2}$	v_2	$\vec{e}_{4,1}$	f_1	$\vec{e}_{2,1}$	$\vec{e}_{1,1}$

[Berg]



DCEL = Double Connected Edge List



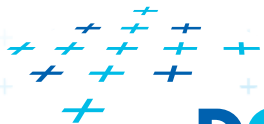
Vertex	Coordinates	IncidentEdge
v_1	(0, 4)	$\vec{e}_{1,1}$
v_2	(2, 4)	$\vec{e}_{4,2}$
v_3	(2, 2)	$\vec{e}_{2,1}$
v_4	(1, 1)	$\vec{e}_{2,2}$

← One of edges

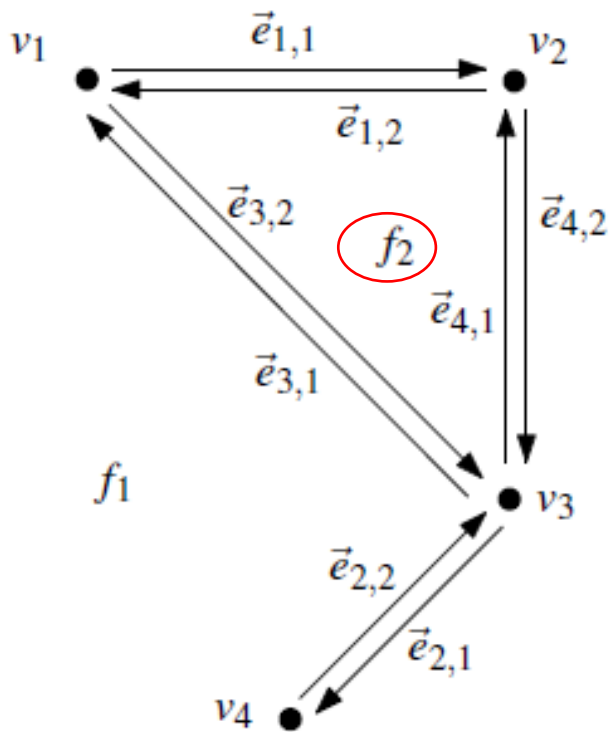
Face	OuterComponent	InnerComponents
f_1	nil	$\vec{e}_{1,1}$
f_2	$\vec{e}_{4,1}$	nil

Half-edge	Origin	Twin	IncidentFace	Next	Prev
$\vec{e}_{1,1}$	v_1	$\vec{e}_{1,2}$	f_1	$\vec{e}_{4,2}$	$\vec{e}_{3,1}$
$\vec{e}_{1,2}$	v_2	$\vec{e}_{1,1}$	f_2	$\vec{e}_{3,2}$	$\vec{e}_{4,1}$
$\vec{e}_{2,1}$	v_3	$\vec{e}_{2,2}$	f_1	$\vec{e}_{2,2}$	$\vec{e}_{4,2}$
$\vec{e}_{2,2}$	v_4	$\vec{e}_{2,1}$	f_1	$\vec{e}_{3,1}$	$\vec{e}_{2,1}$
$\vec{e}_{3,1}$	v_3	$\vec{e}_{3,2}$	f_1	$\vec{e}_{1,1}$	$\vec{e}_{2,2}$
$\vec{e}_{3,2}$	v_1	$\vec{e}_{3,1}$	f_2	$\vec{e}_{4,1}$	$\vec{e}_{1,2}$
$\vec{e}_{4,1}$	v_3	$\vec{e}_{4,2}$	f_2	$\vec{e}_{1,2}$	$\vec{e}_{3,2}$
$\vec{e}_{4,2}$	v_2	$\vec{e}_{4,1}$	f_1	$\vec{e}_{2,1}$	$\vec{e}_{1,1}$

[Berg]



DCEL = Double Connected Edge List



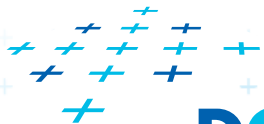
Vertex	Coordinates	IncidentEdge
v_1	(0, 4)	$\vec{e}_{1,1}$
v_2	(2, 4)	$\vec{e}_{4,2}$
v_3	(2, 2)	$\vec{e}_{2,1}$
v_4	(1, 1)	$\vec{e}_{2,2}$

← One of edges

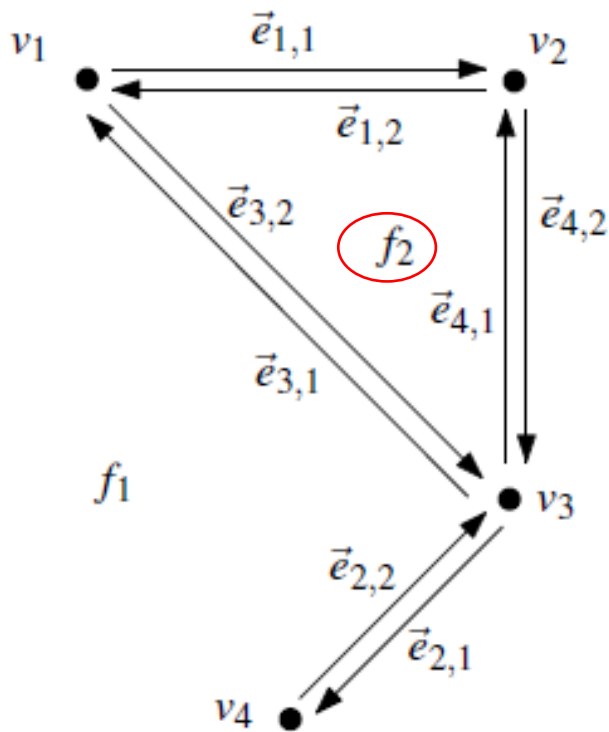
Face	OuterComponent	InnerComponents
f_1	nil	$\vec{e}_{1,1}$
f_2	$\vec{e}_{4,1}$	nil

Half-edge	Origin	Twin	IncidentFace	Next	Prev
$\vec{e}_{1,1}$	v_1	$\vec{e}_{1,2}$	f_1	$\vec{e}_{4,2}$	$\vec{e}_{3,1}$
$\vec{e}_{1,2}$	v_2	$\vec{e}_{1,1}$	f_2	$\vec{e}_{3,2}$	$\vec{e}_{4,1}$
$\vec{e}_{2,1}$	v_3	$\vec{e}_{2,2}$	f_1	$\vec{e}_{2,2}$	$\vec{e}_{4,2}$
$\vec{e}_{2,2}$	v_4	$\vec{e}_{2,1}$	f_1	$\vec{e}_{3,1}$	$\vec{e}_{2,1}$
$\vec{e}_{3,1}$	v_3	$\vec{e}_{3,2}$	f_1	$\vec{e}_{1,1}$	$\vec{e}_{2,2}$
$\vec{e}_{3,2}$	v_1	$\vec{e}_{3,1}$	f_2	$\vec{e}_{4,1}$	$\vec{e}_{1,2}$
$\vec{e}_{4,1}$	v_3	$\vec{e}_{4,2}$	f_2	$\vec{e}_{1,2}$	$\vec{e}_{3,2}$
$\vec{e}_{4,2}$	v_2	$\vec{e}_{4,1}$	f_1	$\vec{e}_{2,1}$	$\vec{e}_{1,1}$

[Berg]



DCEL = Double Connected Edge List



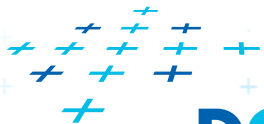
Vertex	Coordinates	IncidentEdge
v_1	(0, 4)	$\vec{e}_{1,1}$
v_2	(2, 4)	$\vec{e}_{4,2}$
v_3	(2, 2)	$\vec{e}_{2,1}$
v_4	(1, 1)	$\vec{e}_{2,2}$

One of edges

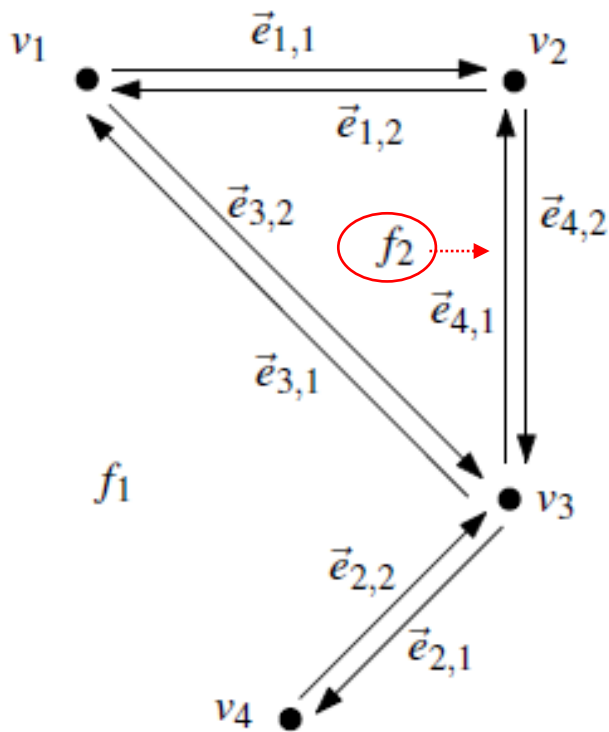
Face	OuterComponent	InnerComponents
f_1	nil	$\vec{e}_{1,1}$
f_2	$\vec{e}_{4,1}$	nil

Half-edge	Origin	Twin	IncidentFace	Next	Prev
$\vec{e}_{1,1}$	v_1	$\vec{e}_{1,2}$	f_1	$\vec{e}_{4,2}$	$\vec{e}_{3,1}$
$\vec{e}_{1,2}$	v_2	$\vec{e}_{1,1}$	f_2	$\vec{e}_{3,2}$	$\vec{e}_{4,1}$
$\vec{e}_{2,1}$	v_3	$\vec{e}_{2,2}$	f_1	$\vec{e}_{2,2}$	$\vec{e}_{4,2}$
$\vec{e}_{2,2}$	v_4	$\vec{e}_{2,1}$	f_1	$\vec{e}_{3,1}$	$\vec{e}_{2,1}$
$\vec{e}_{3,1}$	v_3	$\vec{e}_{3,2}$	f_1	$\vec{e}_{1,1}$	$\vec{e}_{2,2}$
$\vec{e}_{3,2}$	v_1	$\vec{e}_{3,1}$	f_2	$\vec{e}_{4,1}$	$\vec{e}_{1,2}$
$\vec{e}_{4,1}$	v_3	$\vec{e}_{4,2}$	f_2	$\vec{e}_{1,2}$	$\vec{e}_{3,2}$
$\vec{e}_{4,2}$	v_2	$\vec{e}_{4,1}$	f_1	$\vec{e}_{2,1}$	$\vec{e}_{1,1}$

[Berg]



DCEL = Double Connected Edge List



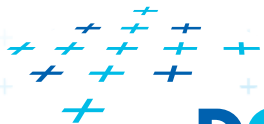
Vertex	Coordinates	IncidentEdge
v_1	(0, 4)	$\vec{e}_{1,1}$
v_2	(2, 4)	$\vec{e}_{4,2}$
v_3	(2, 2)	$\vec{e}_{2,1}$
v_4	(1, 1)	$\vec{e}_{2,2}$

One of edges

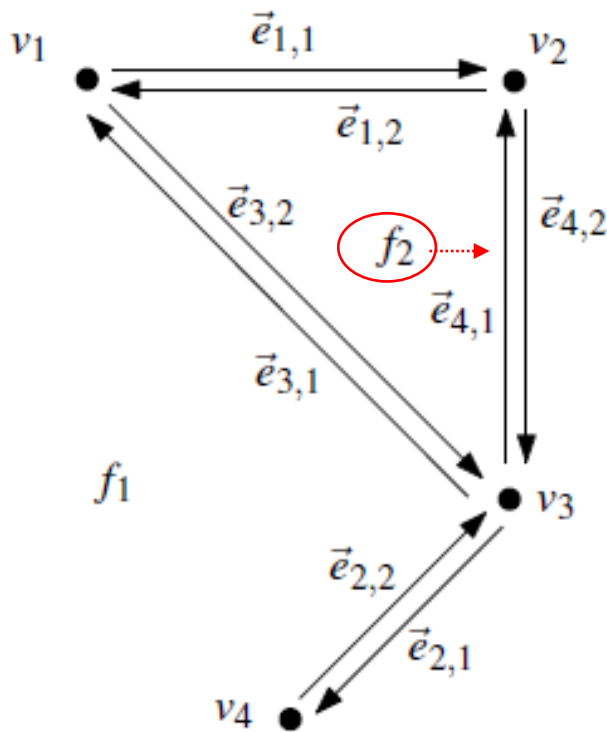
Face	OuterComponent	InnerComponents
f_1	nil	$\vec{e}_{1,1}$
f_2	$\vec{e}_{4,1}$	nil

Half-edge	Origin	Twin	IncidentFace	Next	Prev
$\vec{e}_{1,1}$	v_1	$\vec{e}_{1,2}$	f_1	$\vec{e}_{4,2}$	$\vec{e}_{3,1}$
$\vec{e}_{1,2}$	v_2	$\vec{e}_{1,1}$	f_2	$\vec{e}_{3,2}$	$\vec{e}_{4,1}$
$\vec{e}_{2,1}$	v_3	$\vec{e}_{2,2}$	f_1	$\vec{e}_{2,2}$	$\vec{e}_{4,2}$
$\vec{e}_{2,2}$	v_4	$\vec{e}_{2,1}$	f_1	$\vec{e}_{3,1}$	$\vec{e}_{2,1}$
$\vec{e}_{3,1}$	v_3	$\vec{e}_{3,2}$	f_1	$\vec{e}_{1,1}$	$\vec{e}_{2,2}$
$\vec{e}_{3,2}$	v_1	$\vec{e}_{3,1}$	f_2	$\vec{e}_{4,1}$	$\vec{e}_{1,2}$
$\vec{e}_{4,1}$	v_3	$\vec{e}_{4,2}$	f_2	$\vec{e}_{1,2}$	$\vec{e}_{3,2}$
$\vec{e}_{4,2}$	v_2	$\vec{e}_{4,1}$	f_1	$\vec{e}_{2,1}$	$\vec{e}_{1,1}$

[Berg]



DCEL = Double Connected Edge List



Vertex	Coordinates	IncidentEdge
v_1	(0, 4)	$\vec{e}_{1,1}$
v_2	(2, 4)	$\vec{e}_{4,2}$
v_3	(2, 2)	$\vec{e}_{2,1}$
v_4	(1, 1)	$\vec{e}_{2,2}$

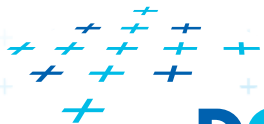
One of edges

List of holes

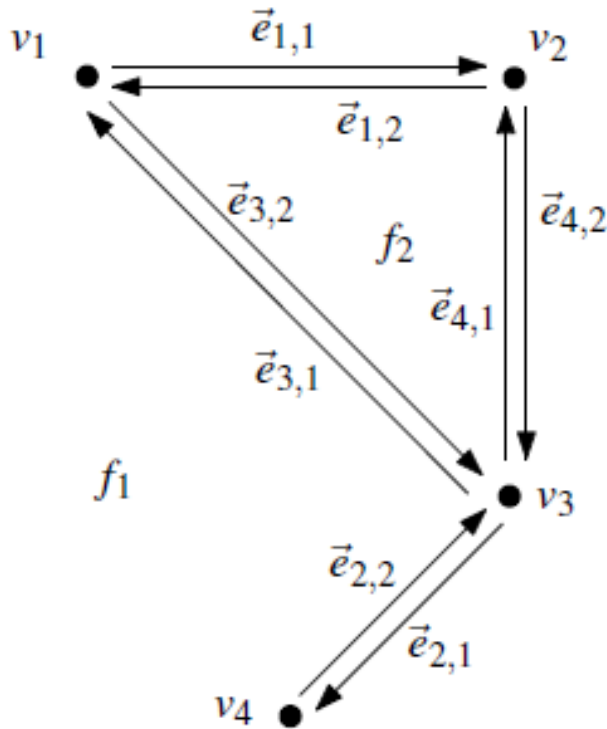
Face	OuterComponent	InnerComponents
f_1	nil	$\vec{e}_{1,1}$
f_2	$\vec{e}_{4,1}$	nil

Half-edge	Origin	Twin	IncidentFace	Next	Prev
$\vec{e}_{1,1}$	v_1	$\vec{e}_{1,2}$	f_1	$\vec{e}_{4,2}$	$\vec{e}_{3,1}$
$\vec{e}_{1,2}$	v_2	$\vec{e}_{1,1}$	f_2	$\vec{e}_{3,2}$	$\vec{e}_{4,1}$
$\vec{e}_{2,1}$	v_3	$\vec{e}_{2,2}$	f_1	$\vec{e}_{2,2}$	$\vec{e}_{4,2}$
$\vec{e}_{2,2}$	v_4	$\vec{e}_{2,1}$	f_1	$\vec{e}_{3,1}$	$\vec{e}_{2,1}$
$\vec{e}_{3,1}$	v_3	$\vec{e}_{3,2}$	f_1	$\vec{e}_{1,1}$	$\vec{e}_{2,2}$
$\vec{e}_{3,2}$	v_1	$\vec{e}_{3,1}$	f_2	$\vec{e}_{4,1}$	$\vec{e}_{1,2}$
$\vec{e}_{4,1}$	v_3	$\vec{e}_{4,2}$	f_2	$\vec{e}_{1,2}$	$\vec{e}_{3,2}$
$\vec{e}_{4,2}$	v_2	$\vec{e}_{4,1}$	f_1	$\vec{e}_{2,1}$	$\vec{e}_{1,1}$

[Berg]



DCEL = Double Connected Edge List



Vertex	Coordinates	IncidentEdge
v_1	(0, 4)	$\vec{e}_{1,1}$
v_2	(2, 4)	$\vec{e}_{4,2}$
v_3	(2, 2)	$\vec{e}_{2,1}$
v_4	(1, 1)	$\vec{e}_{2,2}$

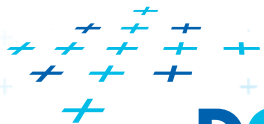
One of edges

List of holes

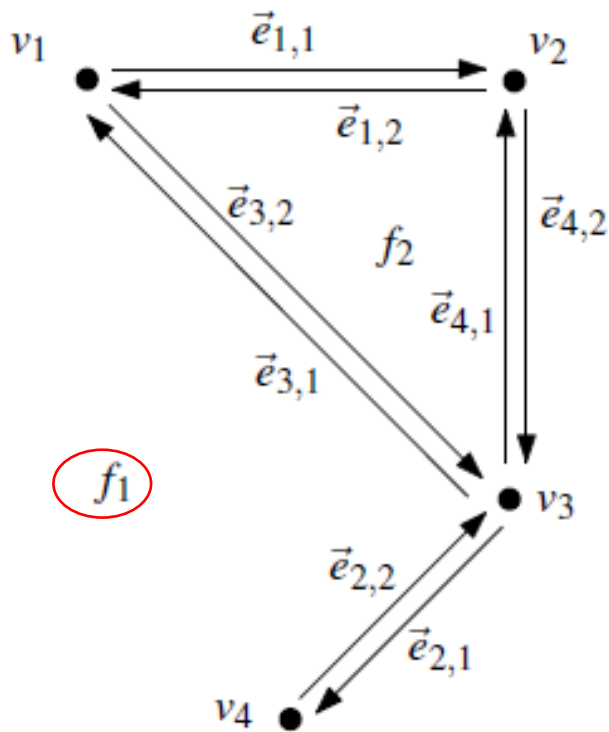
Face	OuterComponent	InnerComponents
f_1	nil	$\vec{e}_{1,1}$
f_2	$\vec{e}_{4,1}$	nil

Half-edge	Origin	Twin	IncidentFace	Next	Prev
$\vec{e}_{1,1}$	v_1	$\vec{e}_{1,2}$	f_1	$\vec{e}_{4,2}$	$\vec{e}_{3,1}$
$\vec{e}_{1,2}$	v_2	$\vec{e}_{1,1}$	f_2	$\vec{e}_{3,2}$	$\vec{e}_{4,1}$
$\vec{e}_{2,1}$	v_3	$\vec{e}_{2,2}$	f_1	$\vec{e}_{2,2}$	$\vec{e}_{4,2}$
$\vec{e}_{2,2}$	v_4	$\vec{e}_{2,1}$	f_1	$\vec{e}_{3,1}$	$\vec{e}_{2,1}$
$\vec{e}_{3,1}$	v_3	$\vec{e}_{3,2}$	f_1	$\vec{e}_{1,1}$	$\vec{e}_{2,2}$
$\vec{e}_{3,2}$	v_1	$\vec{e}_{3,1}$	f_2	$\vec{e}_{4,1}$	$\vec{e}_{1,2}$
$\vec{e}_{4,1}$	v_3	$\vec{e}_{4,2}$	f_2	$\vec{e}_{1,2}$	$\vec{e}_{3,2}$
$\vec{e}_{4,2}$	v_2	$\vec{e}_{4,1}$	f_1	$\vec{e}_{2,1}$	$\vec{e}_{1,1}$

[Berg]



DCEL = Double Connected Edge List



Vertex	Coordinates	IncidentEdge
v_1	(0, 4)	$\vec{e}_{1,1}$
v_2	(2, 4)	$\vec{e}_{4,2}$
v_3	(2, 2)	$\vec{e}_{2,1}$
v_4	(1, 1)	$\vec{e}_{2,2}$

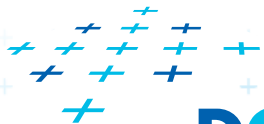
One of edges

List of holes

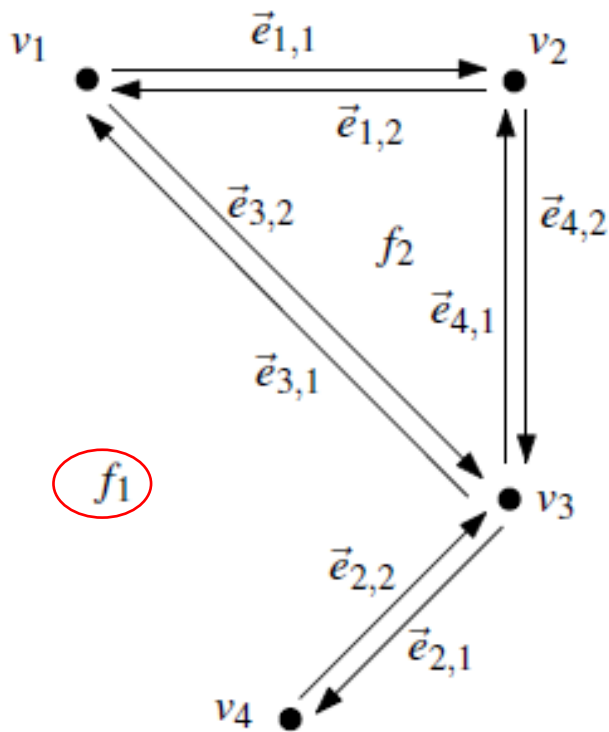
Face	OuterComponent	InnerComponents
f_1	nil	$\vec{e}_{1,1}$
f_2	$\vec{e}_{4,1}$	nil

Half-edge	Origin	Twin	IncidentFace	Next	Prev
$\vec{e}_{1,1}$	v_1	$\vec{e}_{1,2}$	f_1	$\vec{e}_{4,2}$	$\vec{e}_{3,1}$
$\vec{e}_{1,2}$	v_2	$\vec{e}_{1,1}$	f_2	$\vec{e}_{3,2}$	$\vec{e}_{4,1}$
$\vec{e}_{2,1}$	v_3	$\vec{e}_{2,2}$	f_1	$\vec{e}_{2,2}$	$\vec{e}_{4,2}$
$\vec{e}_{2,2}$	v_4	$\vec{e}_{2,1}$	f_1	$\vec{e}_{3,1}$	$\vec{e}_{2,1}$
$\vec{e}_{3,1}$	v_3	$\vec{e}_{3,2}$	f_1	$\vec{e}_{1,1}$	$\vec{e}_{2,2}$
$\vec{e}_{3,2}$	v_1	$\vec{e}_{3,1}$	f_2	$\vec{e}_{4,1}$	$\vec{e}_{1,2}$
$\vec{e}_{4,1}$	v_3	$\vec{e}_{4,2}$	f_2	$\vec{e}_{1,2}$	$\vec{e}_{3,2}$
$\vec{e}_{4,2}$	v_2	$\vec{e}_{4,1}$	f_1	$\vec{e}_{2,1}$	$\vec{e}_{1,1}$

[Berg]



DCEL = Double Connected Edge List



Vertex	Coordinates	IncidentEdge
v_1	(0, 4)	$\vec{e}_{1,1}$
v_2	(2, 4)	$\vec{e}_{4,2}$
v_3	(2, 2)	$\vec{e}_{2,1}$
v_4	(1, 1)	$\vec{e}_{2,2}$

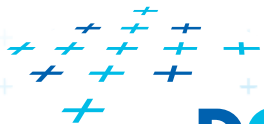
One of edges

List of holes

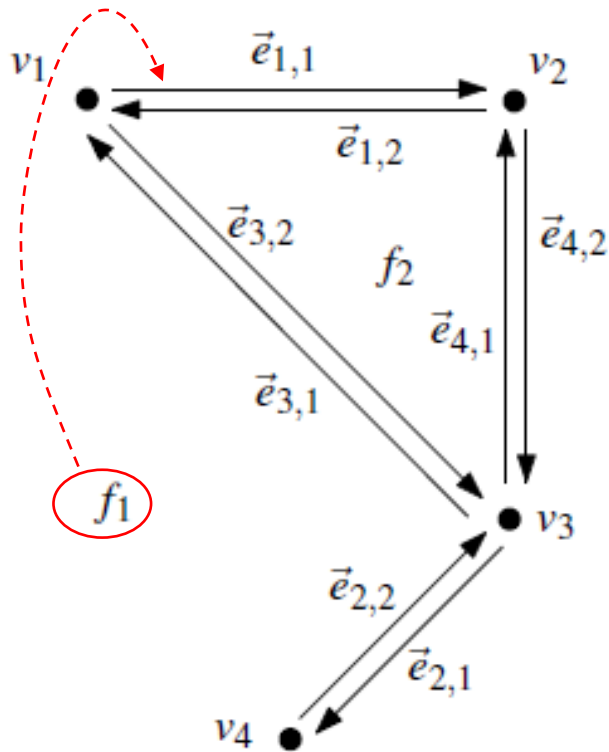
Face	OuterComponent	InnerComponents
f_1	nil	$\vec{e}_{1,1}$
f_2	$\vec{e}_{4,1}$	nil

Half-edge	Origin	Twin	IncidentFace	Next	Prev
$\vec{e}_{1,1}$	v_1	$\vec{e}_{1,2}$	f_1	$\vec{e}_{4,2}$	$\vec{e}_{3,1}$
$\vec{e}_{1,2}$	v_2	$\vec{e}_{1,1}$	f_2	$\vec{e}_{3,2}$	$\vec{e}_{4,1}$
$\vec{e}_{2,1}$	v_3	$\vec{e}_{2,2}$	f_1	$\vec{e}_{2,2}$	$\vec{e}_{4,2}$
$\vec{e}_{2,2}$	v_4	$\vec{e}_{2,1}$	f_1	$\vec{e}_{3,1}$	$\vec{e}_{2,1}$
$\vec{e}_{3,1}$	v_3	$\vec{e}_{3,2}$	f_1	$\vec{e}_{1,1}$	$\vec{e}_{2,2}$
$\vec{e}_{3,2}$	v_1	$\vec{e}_{3,1}$	f_2	$\vec{e}_{4,1}$	$\vec{e}_{1,2}$
$\vec{e}_{4,1}$	v_3	$\vec{e}_{4,2}$	f_2	$\vec{e}_{1,2}$	$\vec{e}_{3,2}$
$\vec{e}_{4,2}$	v_2	$\vec{e}_{4,1}$	f_1	$\vec{e}_{2,1}$	$\vec{e}_{1,1}$

[Berg]



DCEL = Double Connected Edge List



Vertex	Coordinates	IncidentEdge
v_1	(0, 4)	$\vec{e}_{1,1}$
v_2	(2, 4)	$\vec{e}_{4,2}$
v_3	(2, 2)	$\vec{e}_{2,1}$
v_4	(1, 1)	$\vec{e}_{2,2}$

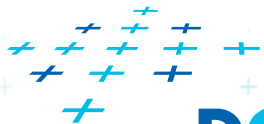
One of edges

List of holes

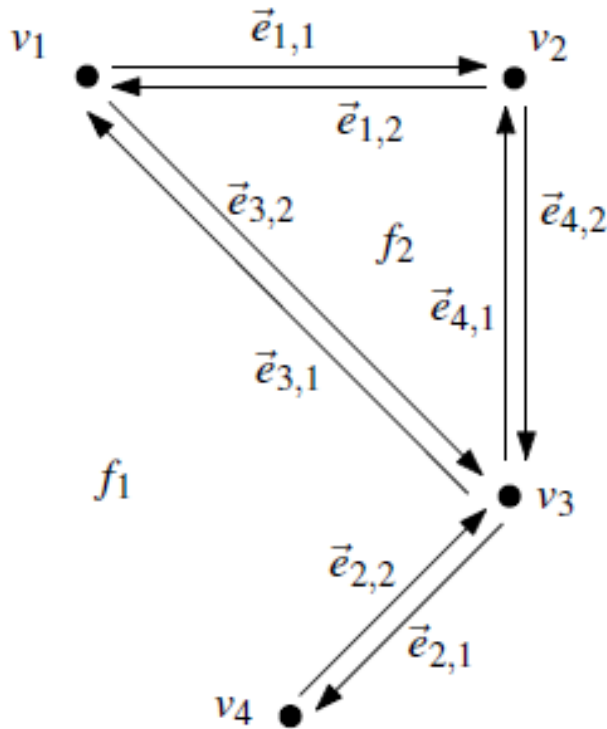
Face	OuterComponent	InnerComponents
f_1	nil	$\vec{e}_{1,1}$
f_2	$\vec{e}_{4,1}$	nil

Half-edge	Origin	Twin	IncidentFace	Next	Prev
$\vec{e}_{1,1}$	v_1	$\vec{e}_{1,2}$	f_1	$\vec{e}_{4,2}$	$\vec{e}_{3,1}$
$\vec{e}_{1,2}$	v_2	$\vec{e}_{1,1}$	f_2	$\vec{e}_{3,2}$	$\vec{e}_{4,1}$
$\vec{e}_{2,1}$	v_3	$\vec{e}_{2,2}$	f_1	$\vec{e}_{2,2}$	$\vec{e}_{4,2}$
$\vec{e}_{2,2}$	v_4	$\vec{e}_{2,1}$	f_1	$\vec{e}_{3,1}$	$\vec{e}_{2,1}$
$\vec{e}_{3,1}$	v_3	$\vec{e}_{3,2}$	f_1	$\vec{e}_{1,1}$	$\vec{e}_{2,2}$
$\vec{e}_{3,2}$	v_1	$\vec{e}_{3,1}$	f_2	$\vec{e}_{4,1}$	$\vec{e}_{1,2}$
$\vec{e}_{4,1}$	v_3	$\vec{e}_{4,2}$	f_2	$\vec{e}_{1,2}$	$\vec{e}_{3,2}$
$\vec{e}_{4,2}$	v_2	$\vec{e}_{4,1}$	f_1	$\vec{e}_{2,1}$	$\vec{e}_{1,1}$

[Berg]



DCEL = Double Connected Edge List



Vertex	Coordinates	IncidentEdge
v_1	(0, 4)	$\vec{e}_{1,1}$
v_2	(2, 4)	$\vec{e}_{4,2}$
v_3	(2, 2)	$\vec{e}_{2,1}$
v_4	(1, 1)	$\vec{e}_{2,2}$

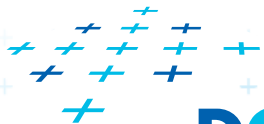
One of edges

List of holes

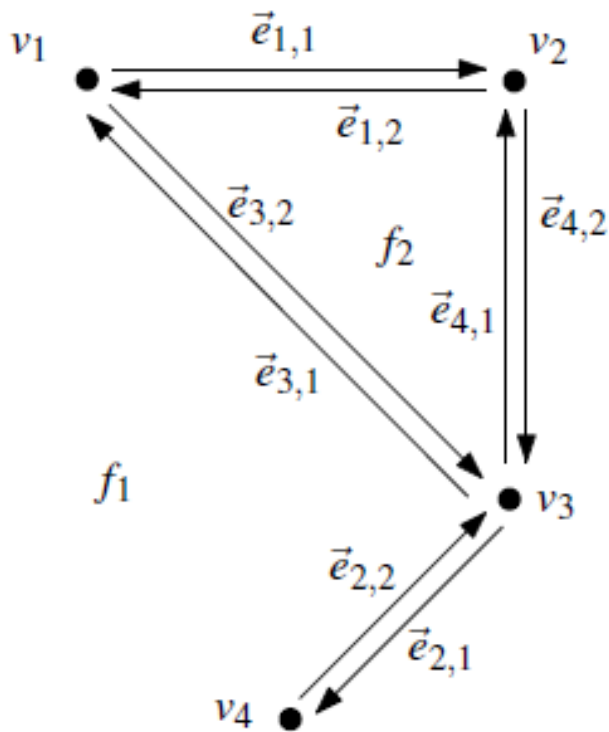
Face	OuterComponent	InnerComponents
f_1	nil	$\vec{e}_{1,1}$
f_2	$\vec{e}_{4,1}$	nil

Half-edge	Origin	Twin	IncidentFace	Next	Prev
$\vec{e}_{1,1}$	v_1	$\vec{e}_{1,2}$	f_1	$\vec{e}_{4,2}$	$\vec{e}_{3,1}$
$\vec{e}_{1,2}$	v_2	$\vec{e}_{1,1}$	f_2	$\vec{e}_{3,2}$	$\vec{e}_{4,1}$
$\vec{e}_{2,1}$	v_3	$\vec{e}_{2,2}$	f_1	$\vec{e}_{2,2}$	$\vec{e}_{4,2}$
$\vec{e}_{2,2}$	v_4	$\vec{e}_{2,1}$	f_1	$\vec{e}_{3,1}$	$\vec{e}_{2,1}$
$\vec{e}_{3,1}$	v_3	$\vec{e}_{3,2}$	f_1	$\vec{e}_{1,1}$	$\vec{e}_{2,2}$
$\vec{e}_{3,2}$	v_1	$\vec{e}_{3,1}$	f_2	$\vec{e}_{4,1}$	$\vec{e}_{1,2}$
$\vec{e}_{4,1}$	v_3	$\vec{e}_{4,2}$	f_2	$\vec{e}_{1,2}$	$\vec{e}_{3,2}$
$\vec{e}_{4,2}$	v_2	$\vec{e}_{4,1}$	f_1	$\vec{e}_{2,1}$	$\vec{e}_{1,1}$

[Berg]



DCEL = Double Connected Edge List



G – geometry
T – topology

G

Vertex	Coordinates	IncidentEdge
v_1	(0, 4)	$\vec{e}_{1,1}$
v_2	(2, 4)	$\vec{e}_{4,2}$
v_3	(2, 2)	$\vec{e}_{2,1}$
v_4	(1, 1)	$\vec{e}_{2,2}$

One of edges

List of holes

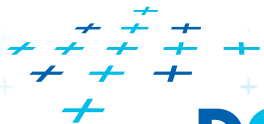
T

Face	OuterComponent	InnerComponents
f_1	nil	$\vec{e}_{1,1}$
f_2	$\vec{e}_{4,1}$	nil

T

Half-edge	Origin	Twin	IncidentFace	Next	Prev
$\vec{e}_{1,1}$	v_1	$\vec{e}_{1,2}$	f_1	$\vec{e}_{4,2}$	$\vec{e}_{3,1}$
$\vec{e}_{1,2}$	v_2	$\vec{e}_{1,1}$	f_2	$\vec{e}_{3,2}$	$\vec{e}_{4,1}$
$\vec{e}_{2,1}$	v_3	$\vec{e}_{2,2}$	f_1	$\vec{e}_{2,2}$	$\vec{e}_{4,2}$
$\vec{e}_{2,2}$	v_4	$\vec{e}_{2,1}$	f_1	$\vec{e}_{3,1}$	$\vec{e}_{2,1}$
$\vec{e}_{3,1}$	v_3	$\vec{e}_{3,2}$	f_1	$\vec{e}_{1,1}$	$\vec{e}_{2,2}$
$\vec{e}_{3,2}$	v_1	$\vec{e}_{3,1}$	f_2	$\vec{e}_{4,1}$	$\vec{e}_{1,2}$
$\vec{e}_{4,1}$	v_3	$\vec{e}_{4,2}$	f_2	$\vec{e}_{1,2}$	$\vec{e}_{3,2}$
$\vec{e}_{4,2}$	v_2	$\vec{e}_{4,1}$	f_1	$\vec{e}_{2,1}$	$\vec{e}_{1,1}$

[Berg]

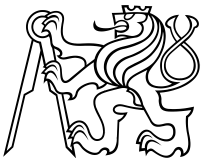
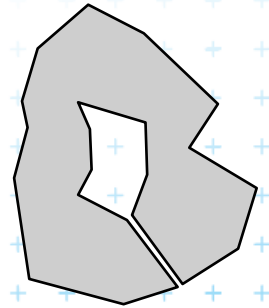


DCGI



DCEL simplifications

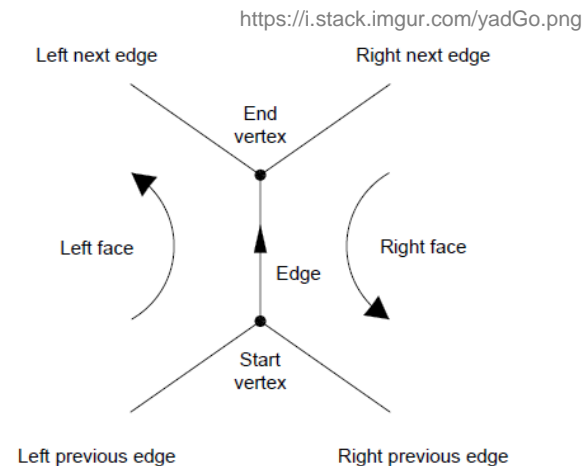
- If no operations with vertices and no attributes
 - No vertex table (no separate vertex records)
 - Store vertex coords in half-edge origin (in the half-edge table)
- If no need for faces (e.g. river network)
 - No face record and no IncidentFace() field (in the half-edge table)
- If only connected subdivision allowed
 - Join holes with rest by dummy edges
 - Visit all half-edges by simple graph traversal
 - No InnerComponent() list for faces



Other structures for representing PSLG

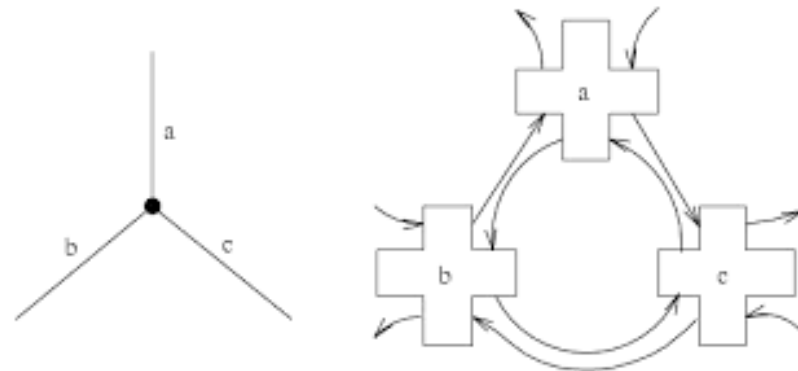
■ Winged edge [Baumgart 1975]

- The oldest, complicated manipulation
- Randomly stored edge direction around faces

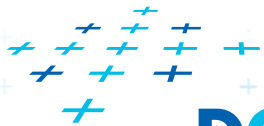
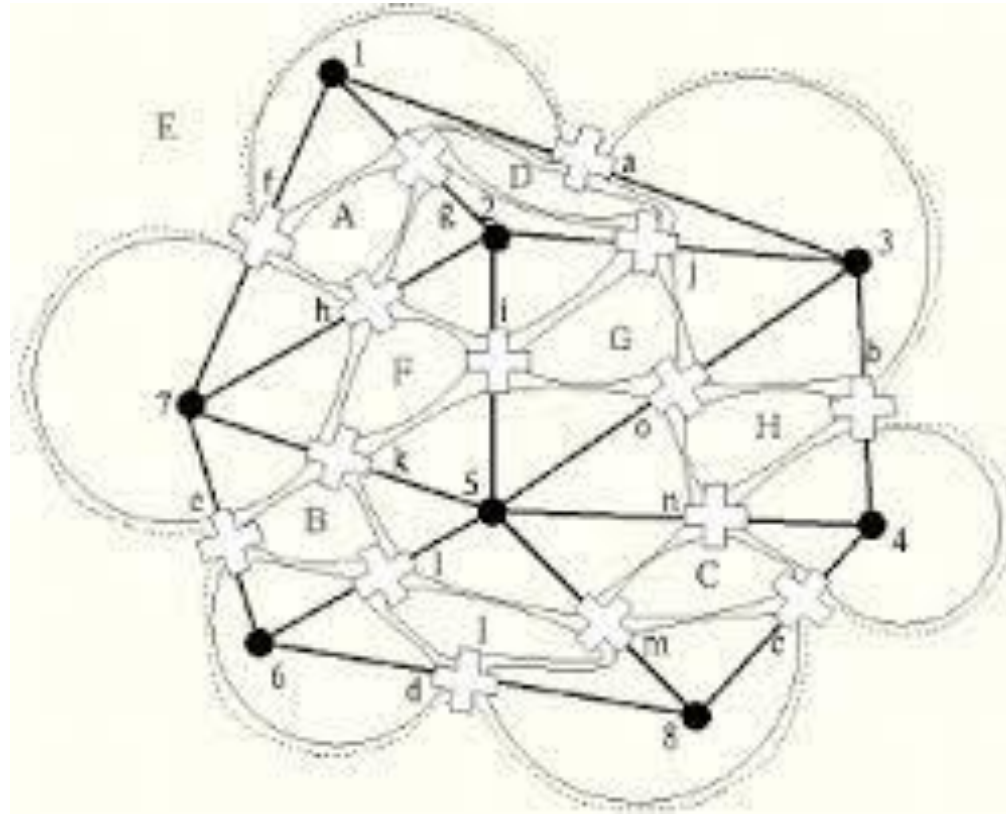


■ Quad edge [Guibas & Stolfi 1985]

- Stores PSLG and its dual
- Pointers to edges
 - Around vertex
 - Around face
- E.g., for Voronoi diagrams & Delaunay triangulations



Quad edge

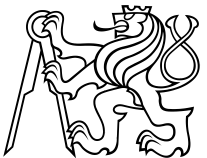


DCGI



Point location in planar subdivision

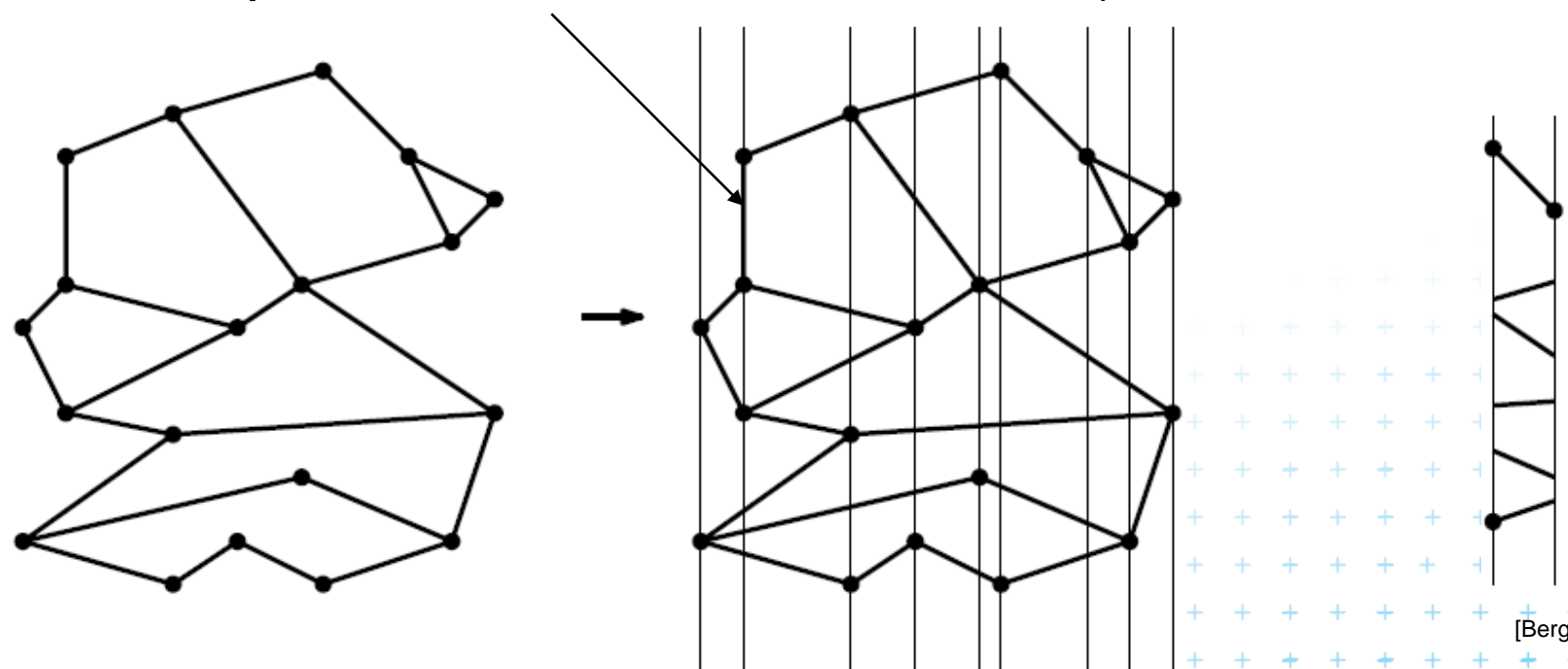
- Using special search structures
an optimal algorithm can be made with
 - $O(n)$ preprocessing,
 - $O(n)$ memory and
 - $O(\log n)$ query time.
- Simpler methods
 1. Slabs $O(\log n)$ query, $O(n^2)$ memory
 2. monotone chain tree $O(\log^2 n)$ query, $O(n^2)$ memory
 3. trapezoidal map $O(\log n)$ query expected time
 $O(n)$ expected memory



1. Vertical (horizontal) slabs

[Dobkin and Lipton, 1976]

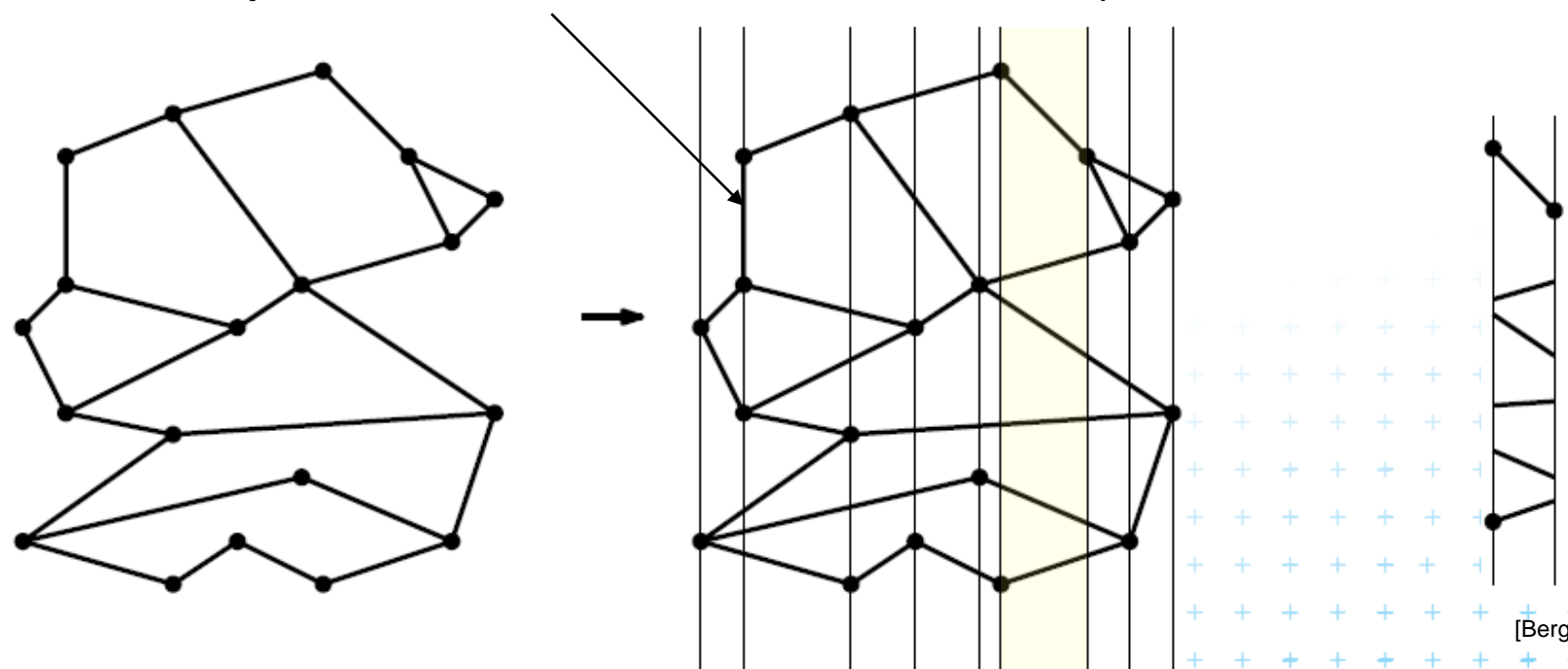
- Draw vertical or horizontal lines through vertices
- It partitions the plane into vertical slabs
 - Avoid points with same x coordinate (to be solved later)



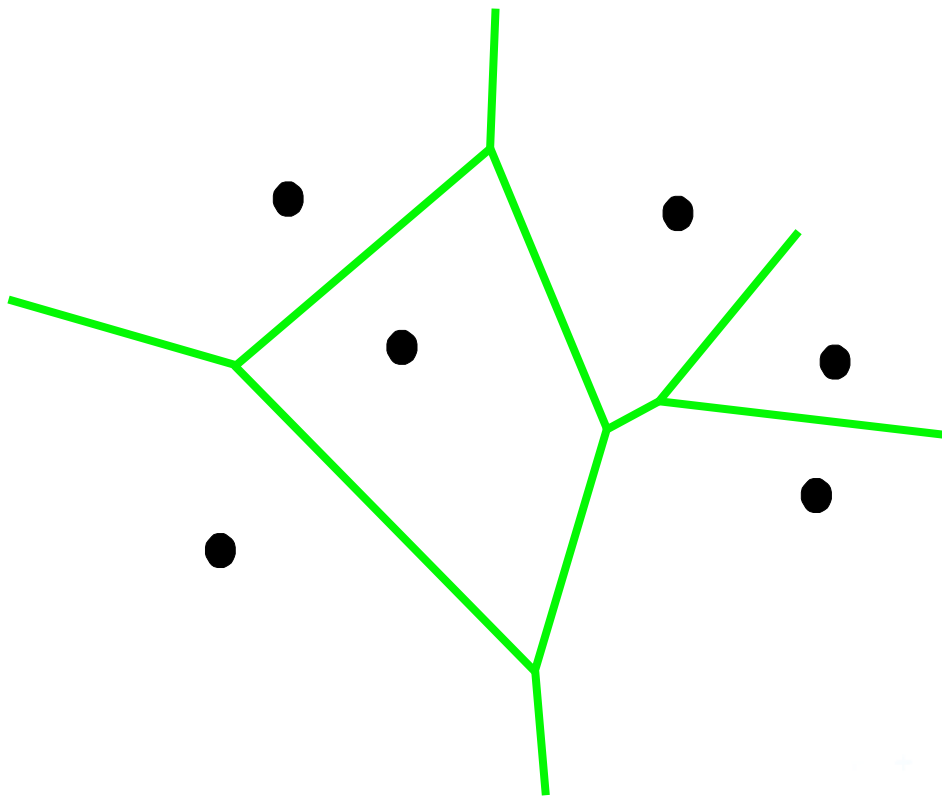
1. Vertical (horizontal) slabs

[Dobkin and Lipton, 1976]

- Draw vertical or horizontal lines through vertices
- It partitions the plane into vertical slabs
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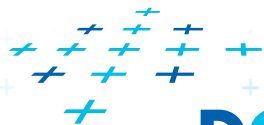
Horizontal slabs example



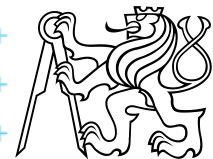
1. Find slab
in T_y for y

T_x and T_y are arrays

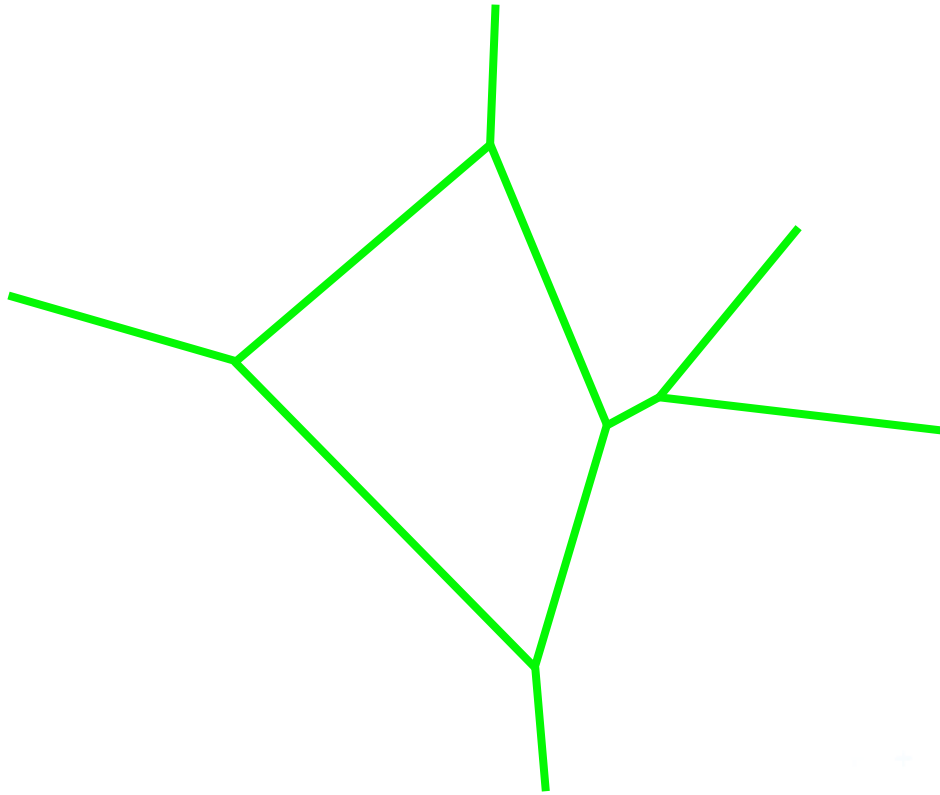
2. Find slab part in T_x for x



DCGI



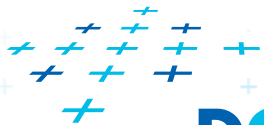
Horizontal slabs example



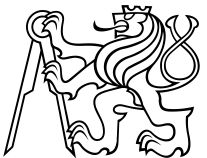
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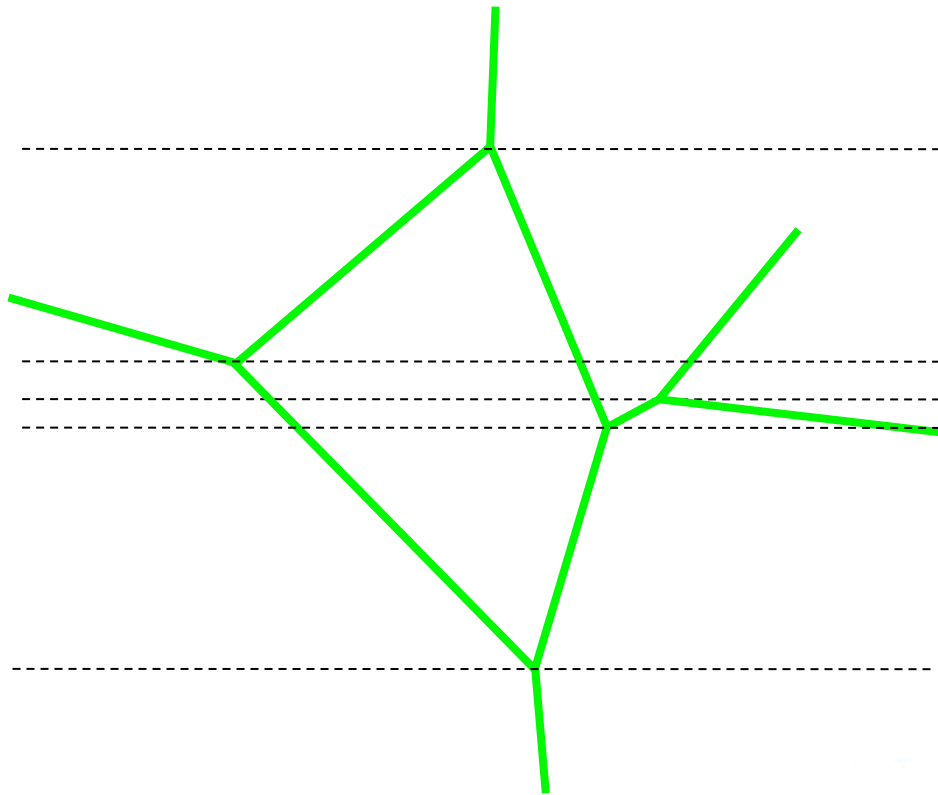
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DCGI



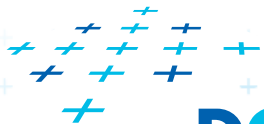
Horizontal slabs example



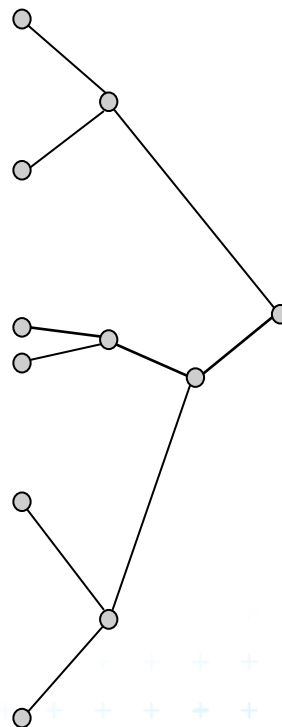
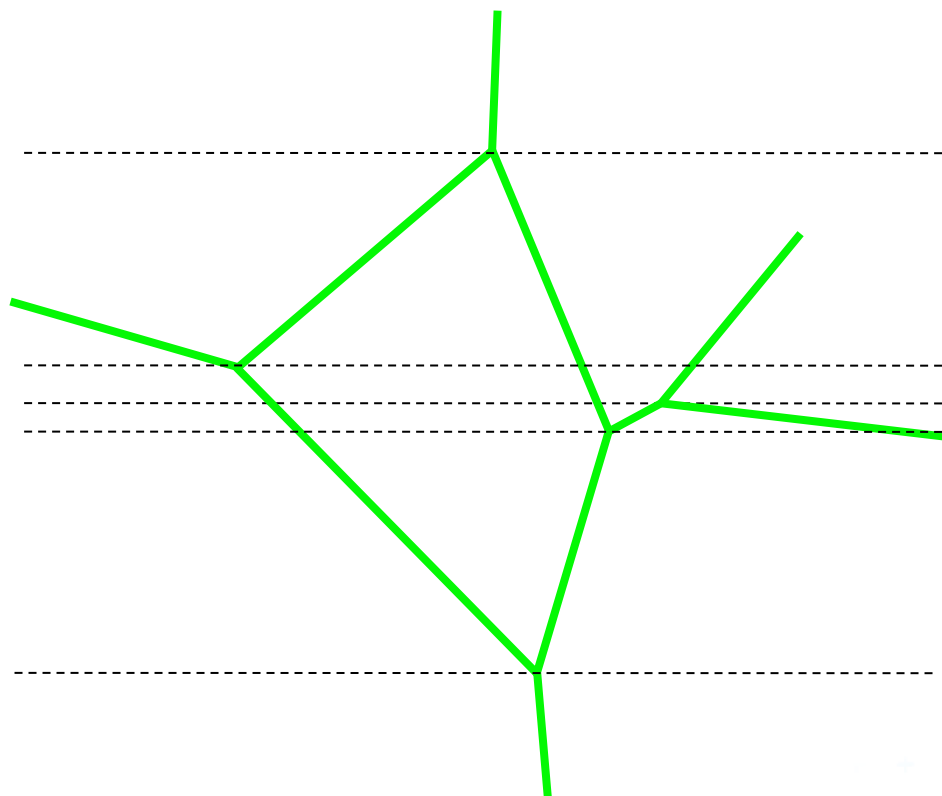
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T_x and T_y are arrays

2. Find slab part in T_x for x



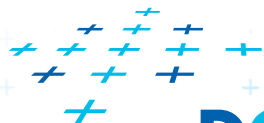
Horizontal slabs example



1. Find slab
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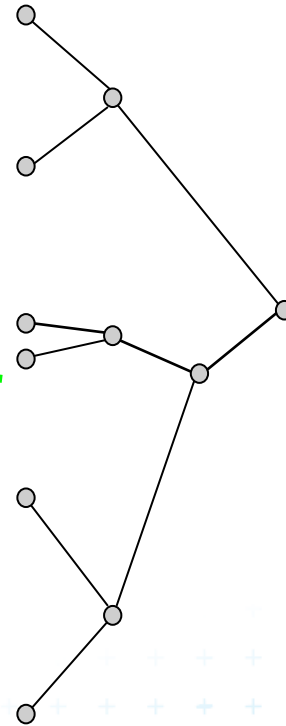
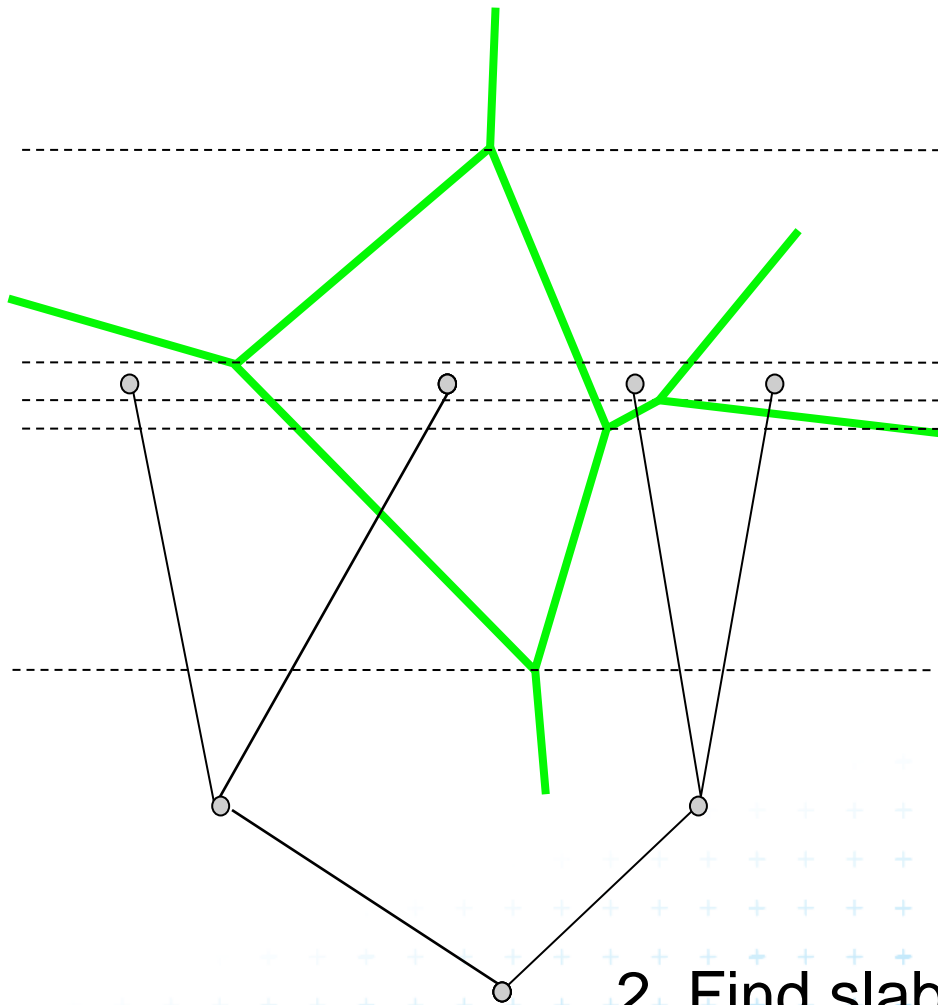
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DCGI



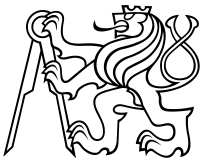
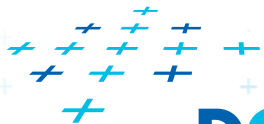
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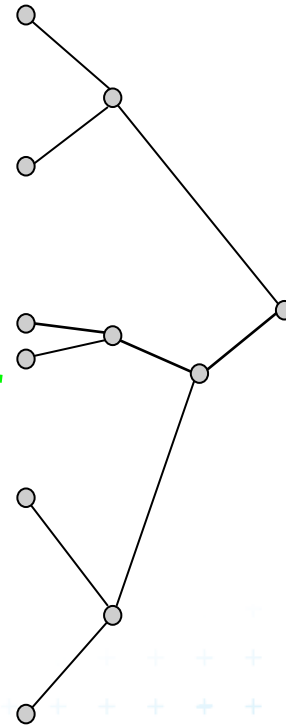
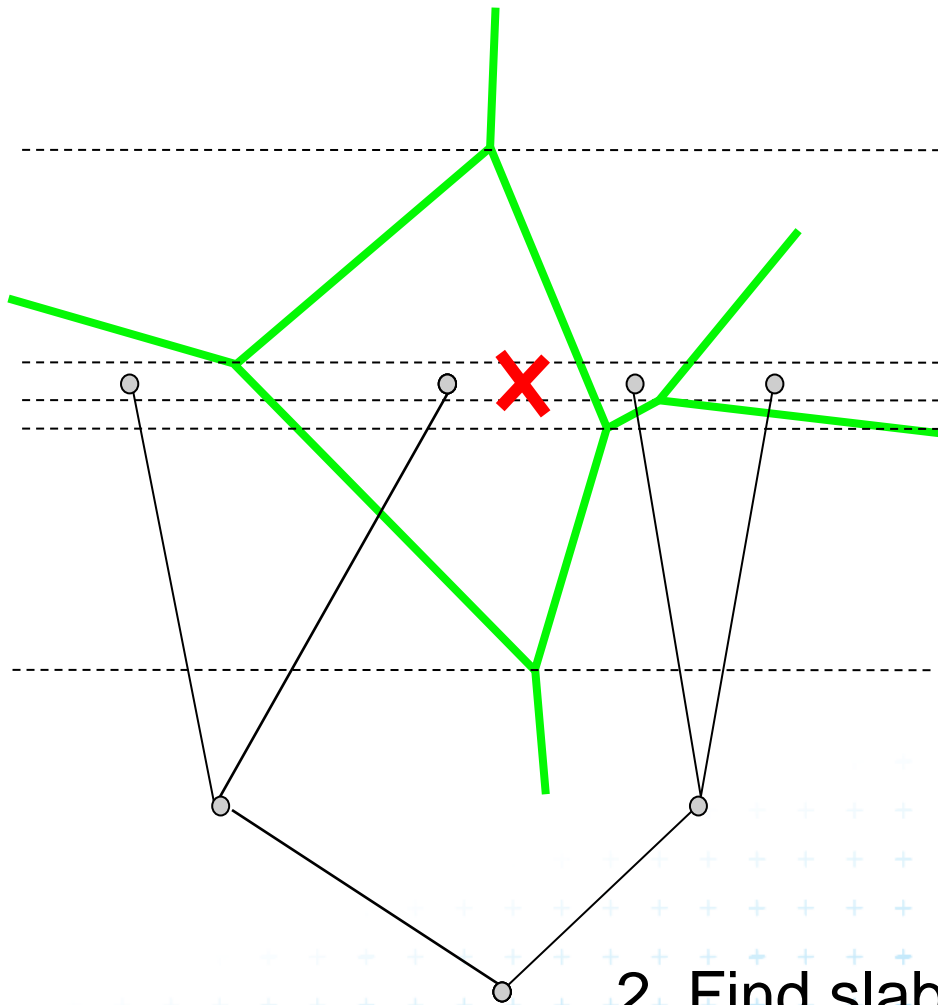
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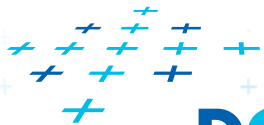
Horizontal slabs example



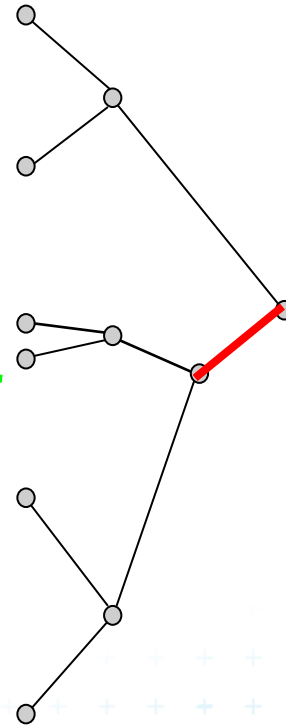
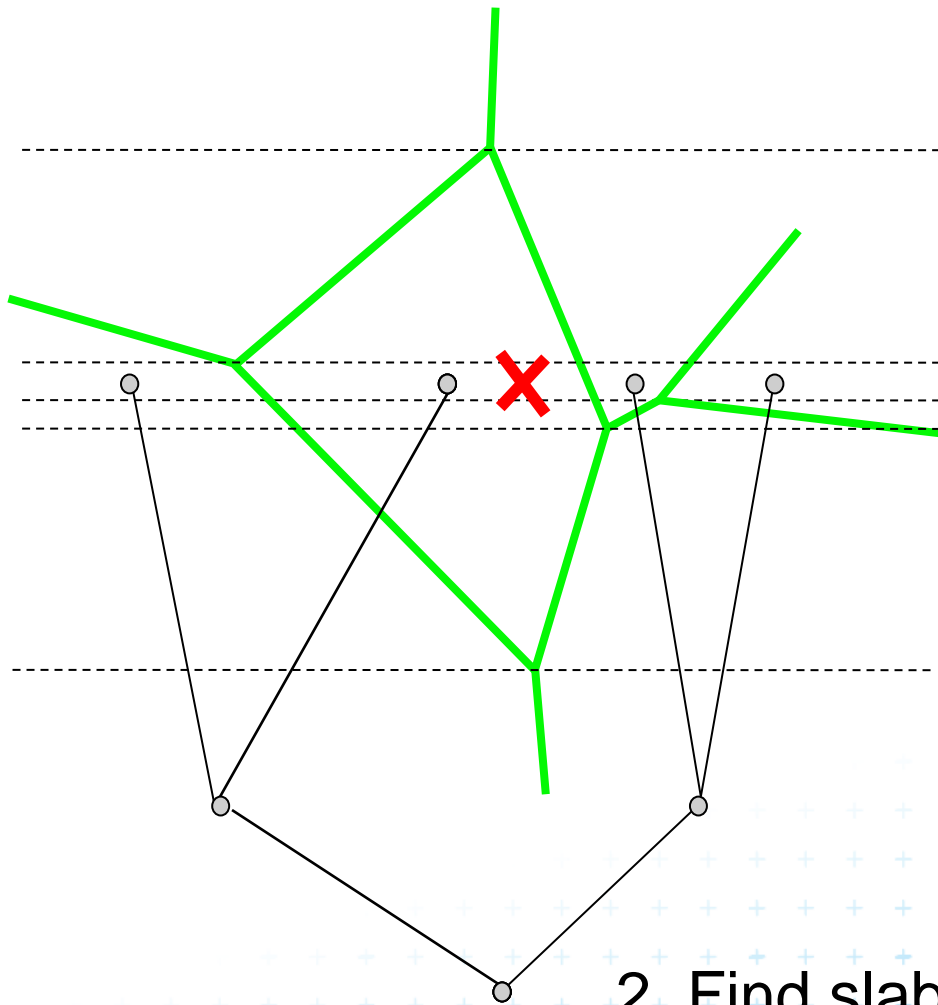
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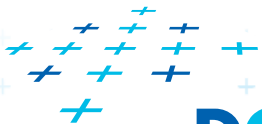
Horizontal slabs example



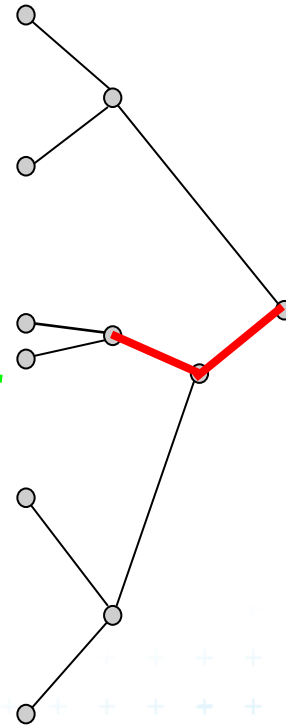
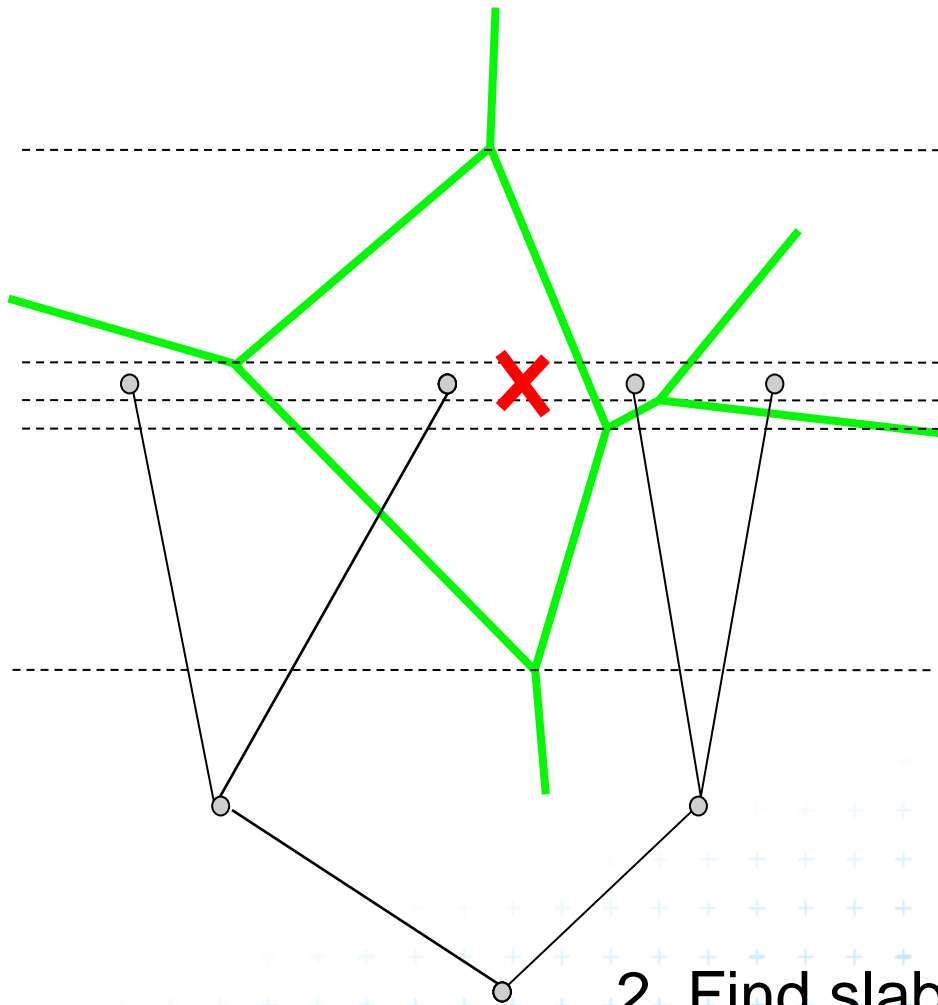
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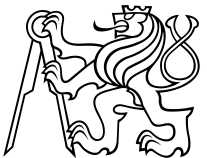
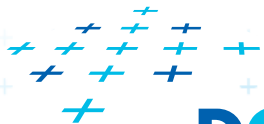
Horizontal slabs example



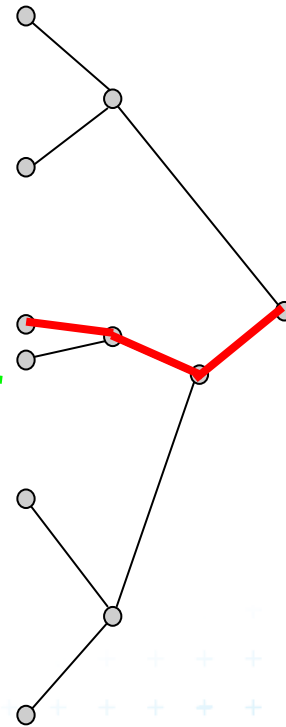
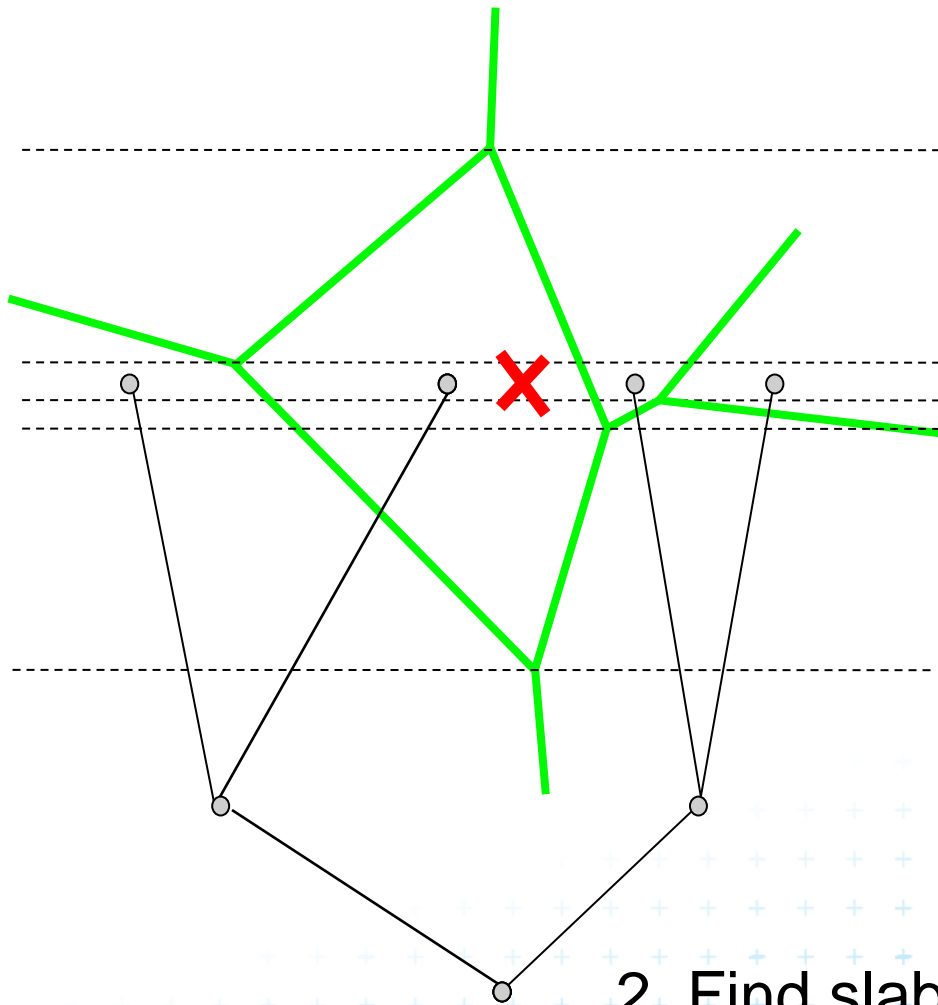
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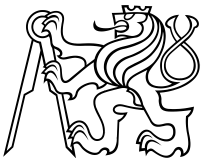
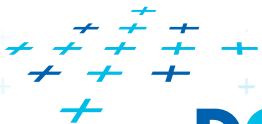
Horizontal slabs example



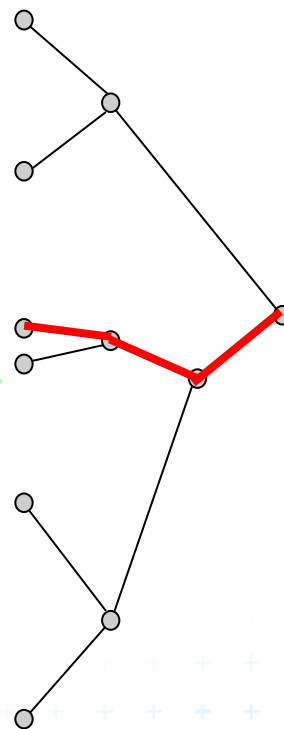
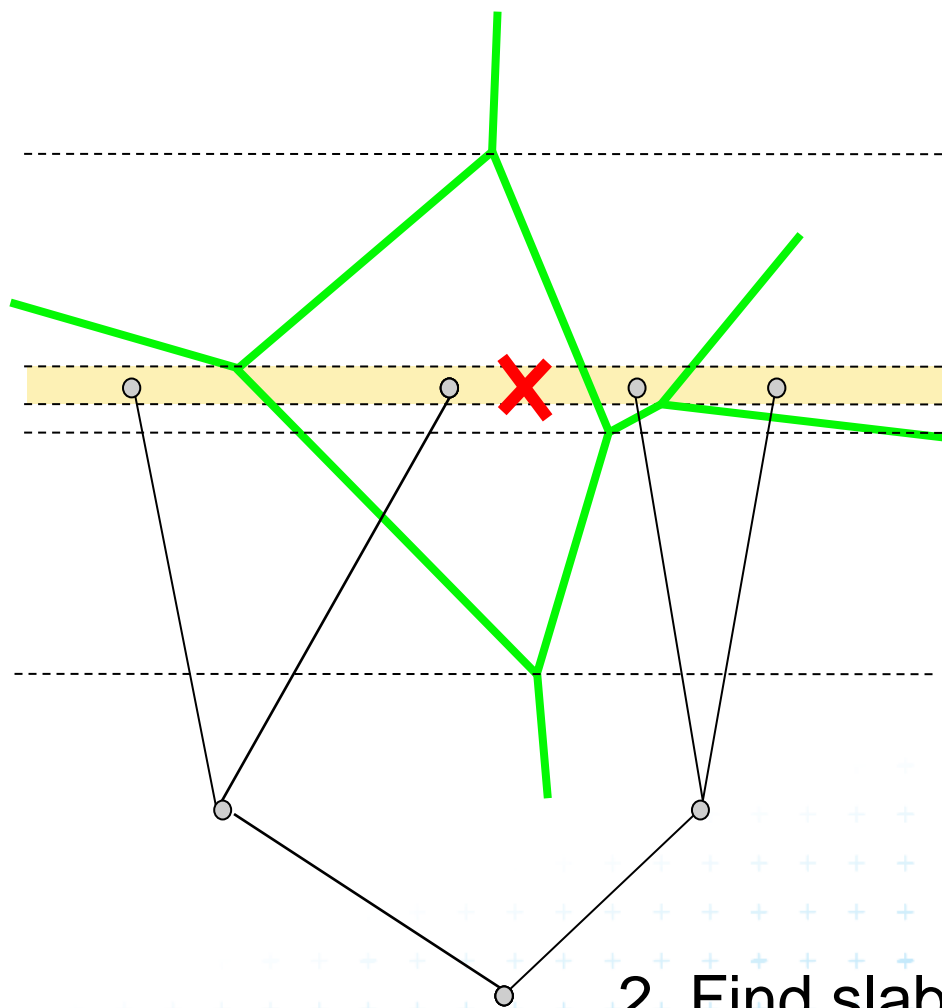
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T_x and T_y are arrays

2. Find slab part in T_x for x



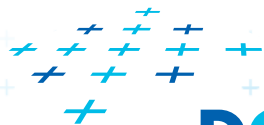
Horizontal slabs example



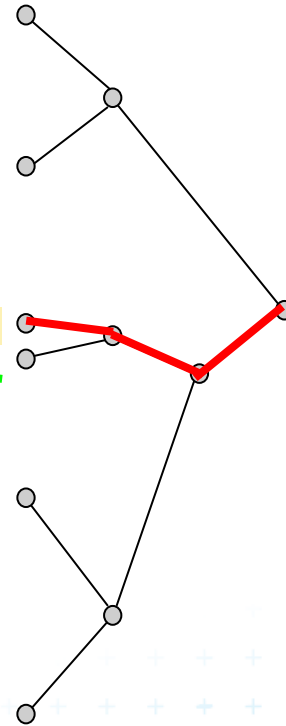
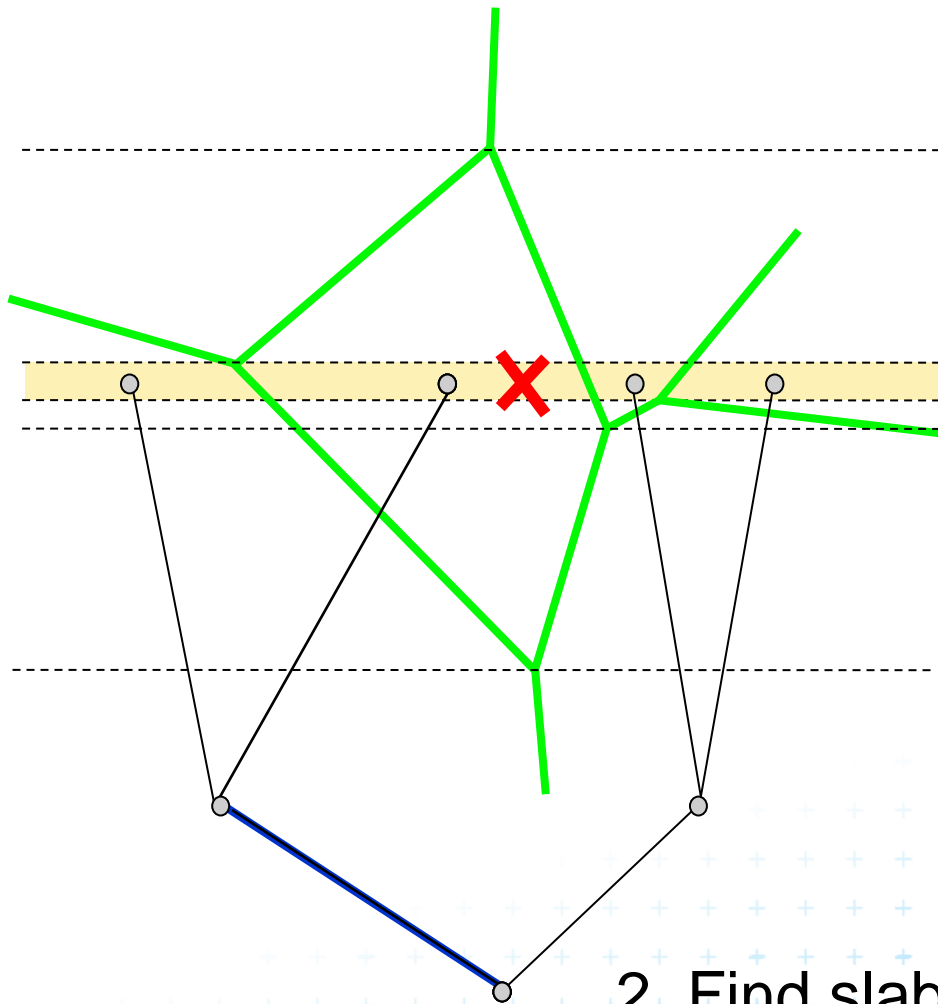
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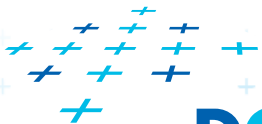
Horizontal slabs example



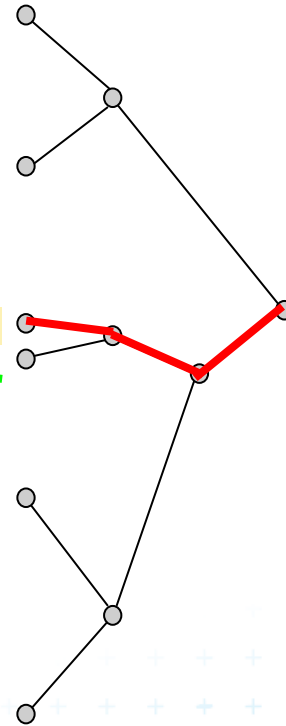
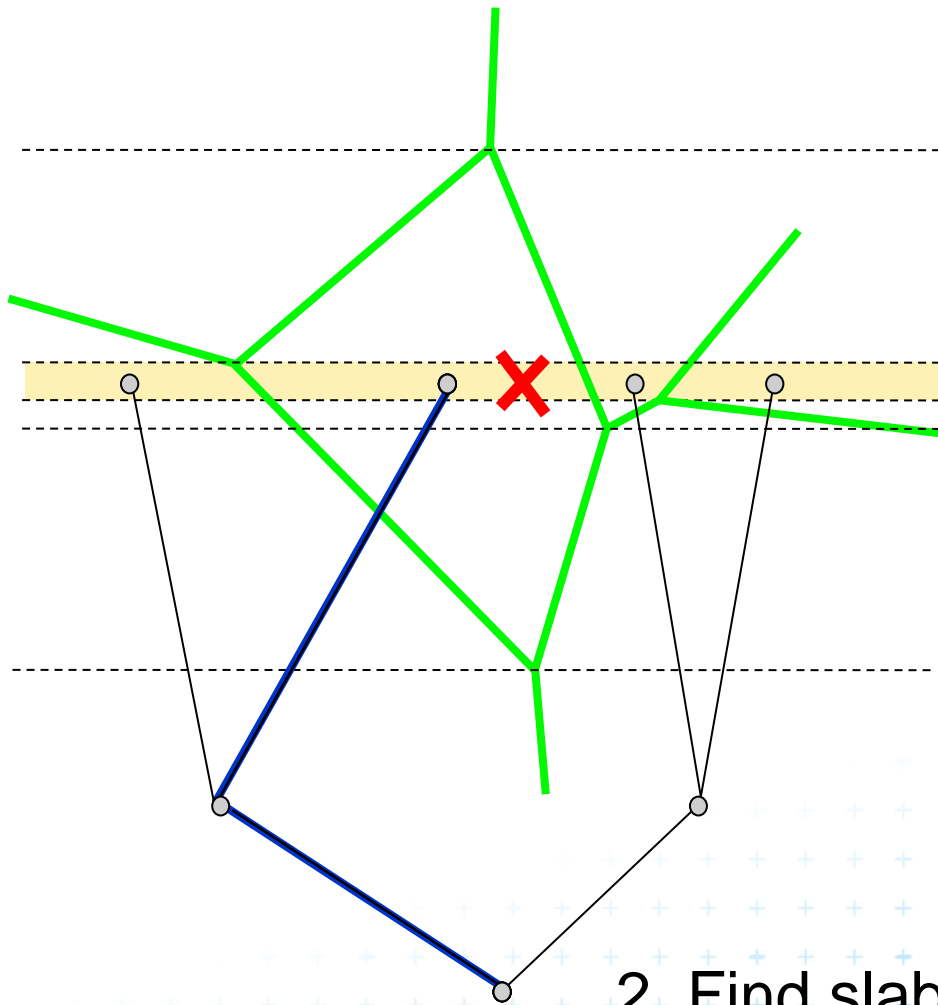
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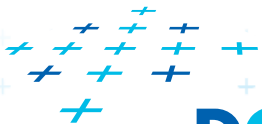
Horizontal slabs example



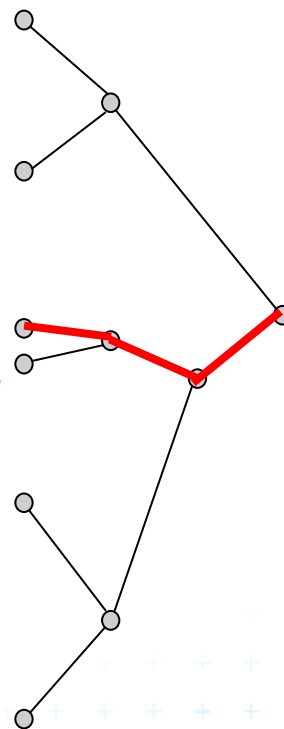
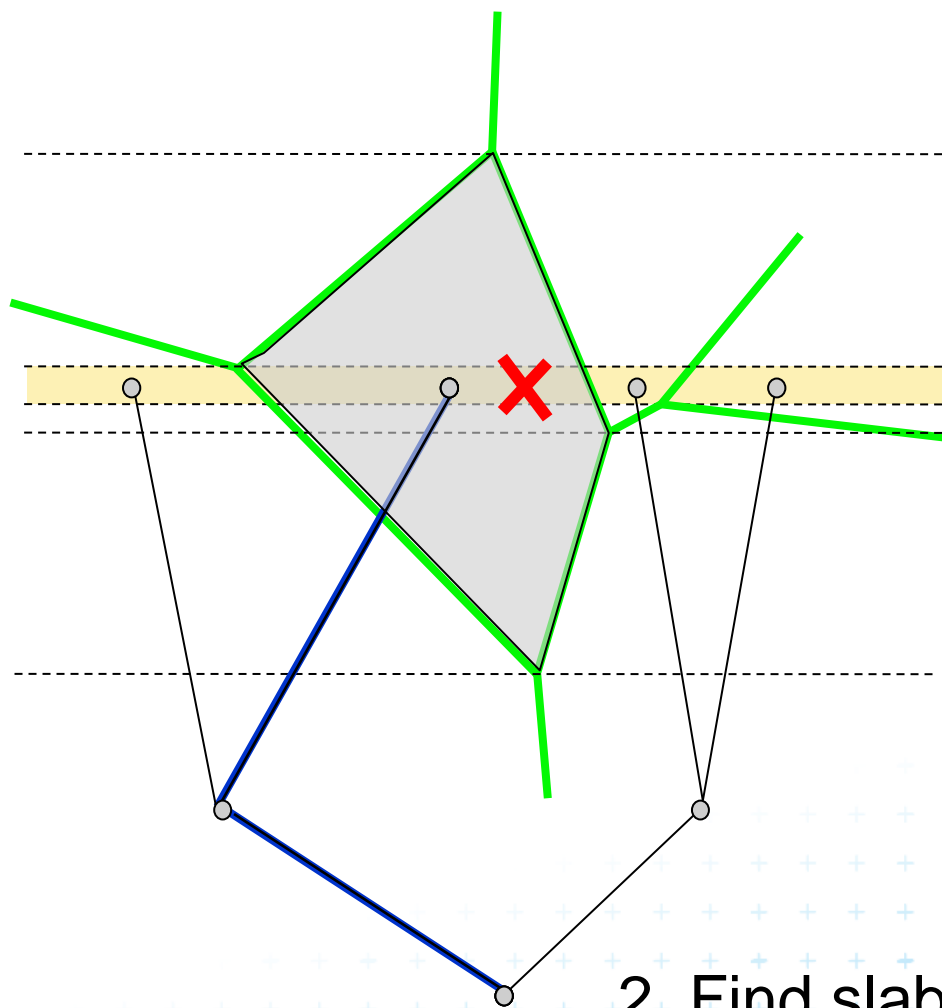
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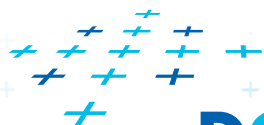
Horizontal slabs example



1. Find slab in T_y for y

T_x and T_y are arrays

2. Find slab part in T_x for x



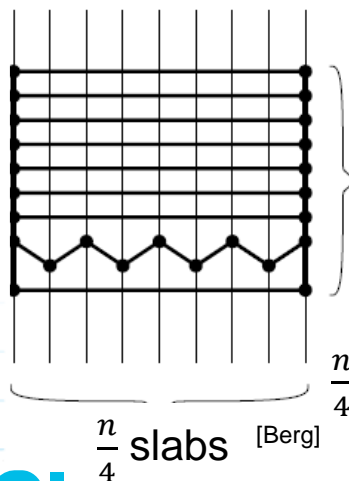
Horizontal slabs complexity

- Query time $O(\log n)$

$O(\log n)$ time in slab array T_y (size max $2n$ endpoints)
 + $O(\log n)$ time in slab array T_x (slab crossed max by n edges)

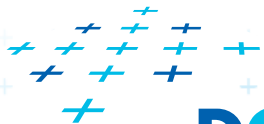
- Memory $O(n^2)$

- Slabs: Array with y-coordinates of vertices ... $O(n)$
- For each slab $O(n)$ edges intersecting the slab



$$\frac{n}{4} + 2 \frac{n}{4} + \frac{n}{4} = O(n) \text{ edges}$$

$$\frac{n}{4} * \frac{n}{4} = O(n^2) \text{ faces}$$



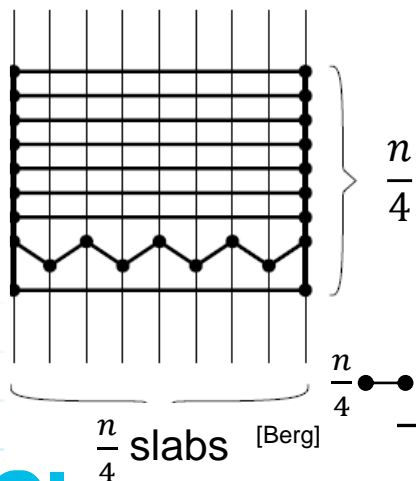
Horizontal slabs complexity

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$O(\log n)$ time in slab array T_y (size max $2n$ endpoints)
 + $O(\log n)$ time in slab array T_x (slab crossed max by n edges)

- Memory $O(n^2)$

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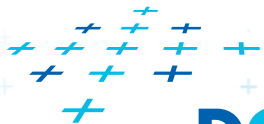


$O(n^2)$ construction

$O(\log n)$ query

$O(n^2)$ memory

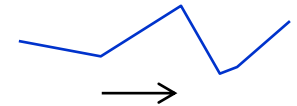
$$\frac{n}{4} \bullet \bullet + 2 \frac{n}{4} \bullet \bullet + \frac{n}{4} \bullet \bullet = O(n) \text{ edges} \quad \frac{n}{4} * \frac{n}{4} = O(n^2) \text{ faces}$$



2. Monotone chain tree

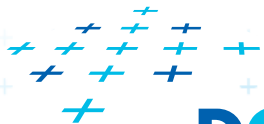
[Lee and Preparata, 1977]

- Construct monotone planar subdivision
 - The edges are all monotone in the same direction
- Each separator chain
 - is monotone (can be projected to line and searched)
 - splits the plane into two parts – allows binary search

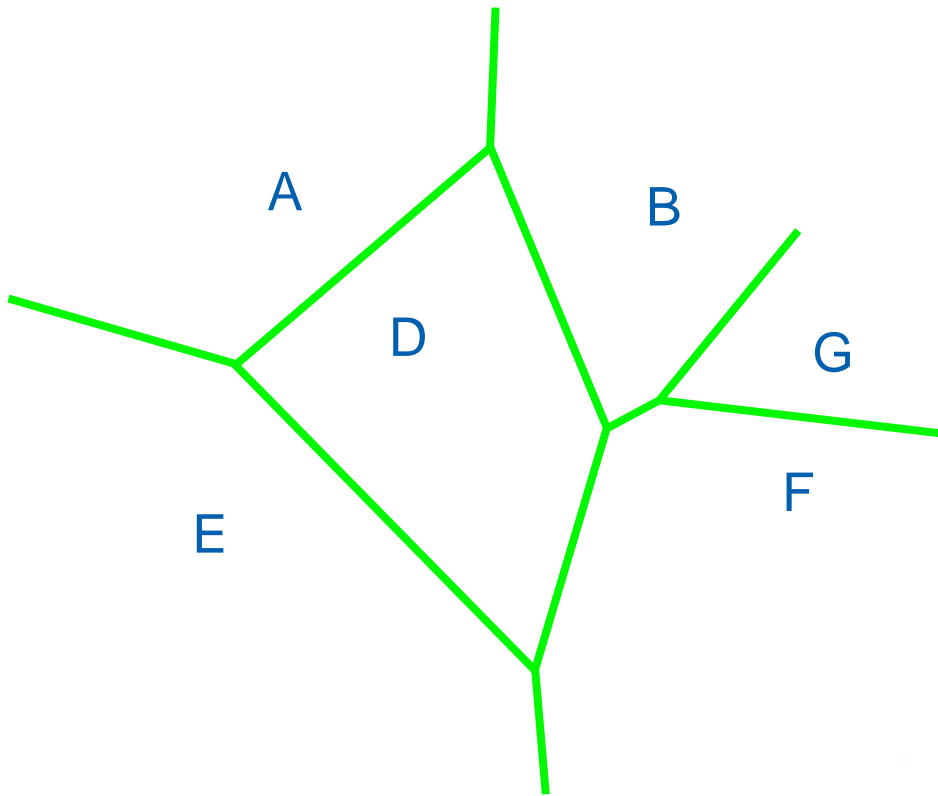


■ Algorithm

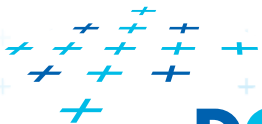
- Preprocess: Find the separators (e.g., horizontal)
- Search:
 - Binary search among separators (Y) ... $O(\log n)$ times
 - Binary search along the separator (X) ... $O(\log n)$
- Not optimal, but simple $O(\log^2 n)$ query
- Can be made optimal, but the algorithm and data structures are complicated $O(n^2)$ memory



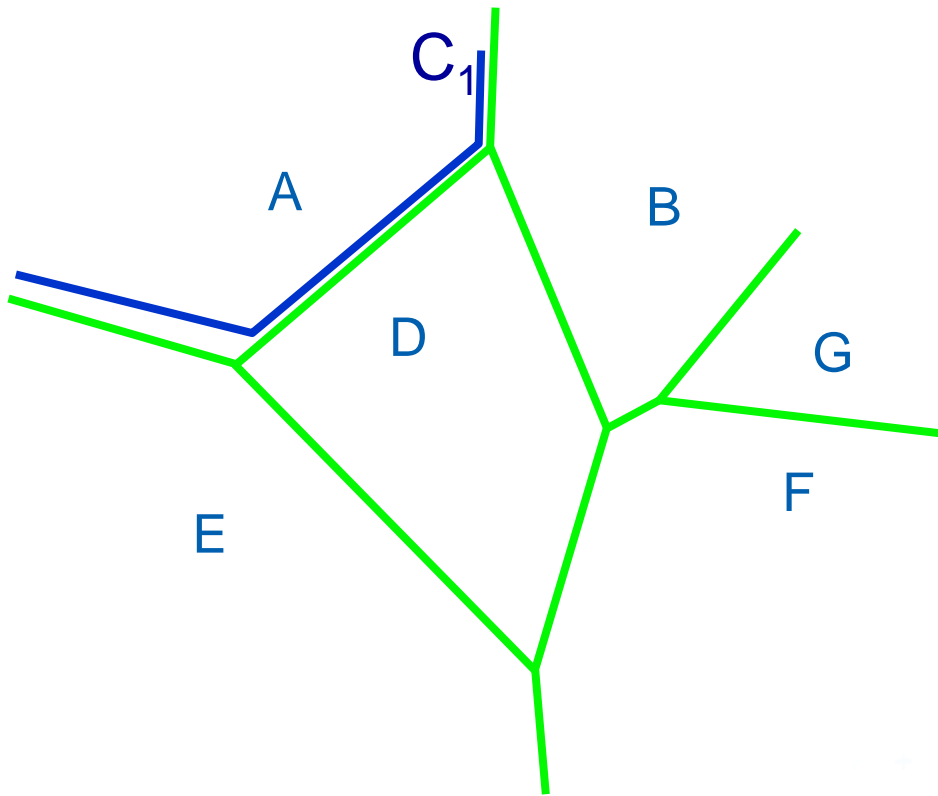
Monotone chain tree example



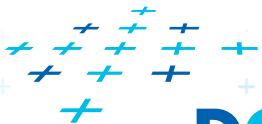
0. Construct the chains and the chain tree
1. Start with the middle chain
2. Find projection of x in the projection of the chain – determine the segment
3. Identify position of x in relation to the segment – Left or Right
(This is the position of x relatively to the whole chain)
4. Continue in L or R chain -> goto 2.
or stop if in the leaf



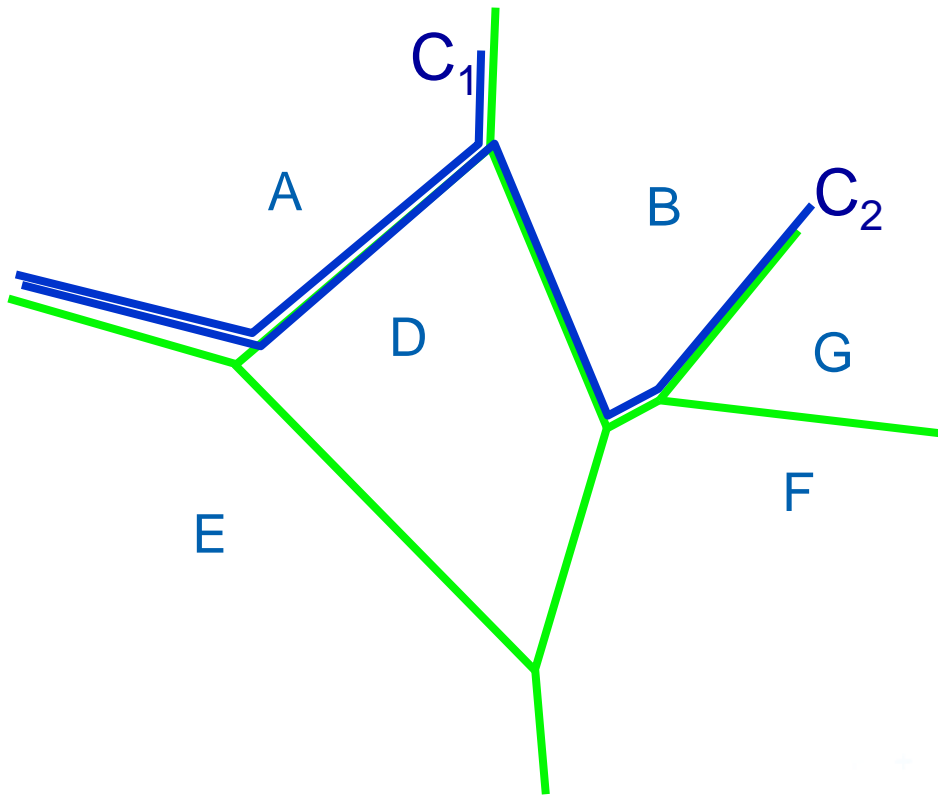
Monotone chain tree example



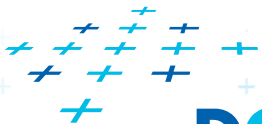
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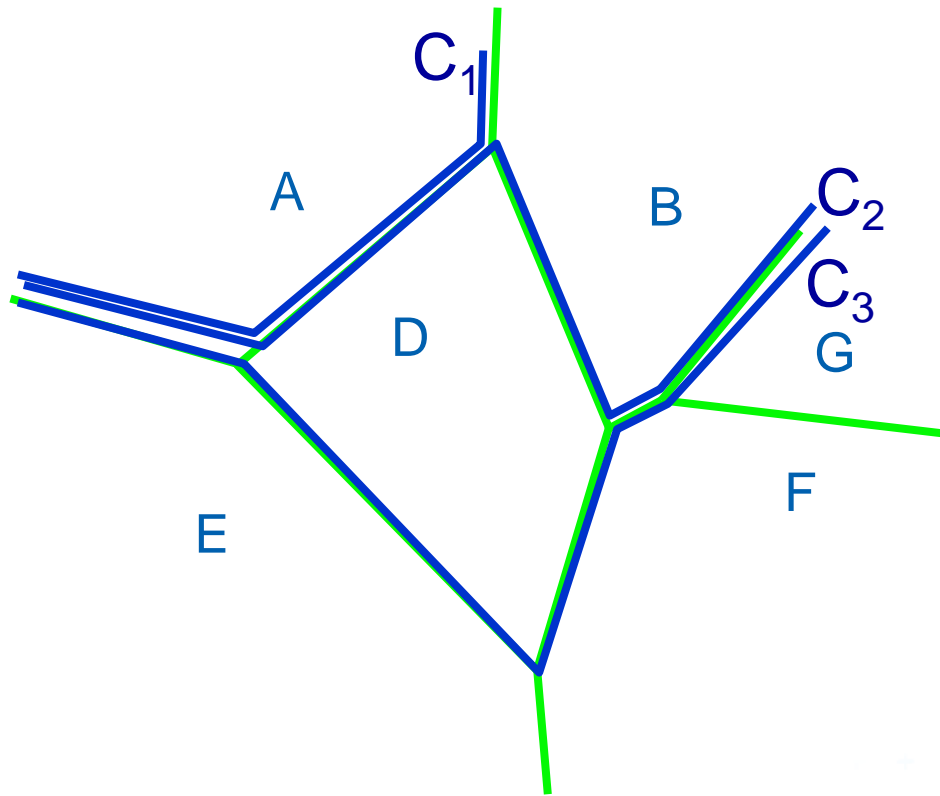
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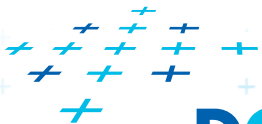
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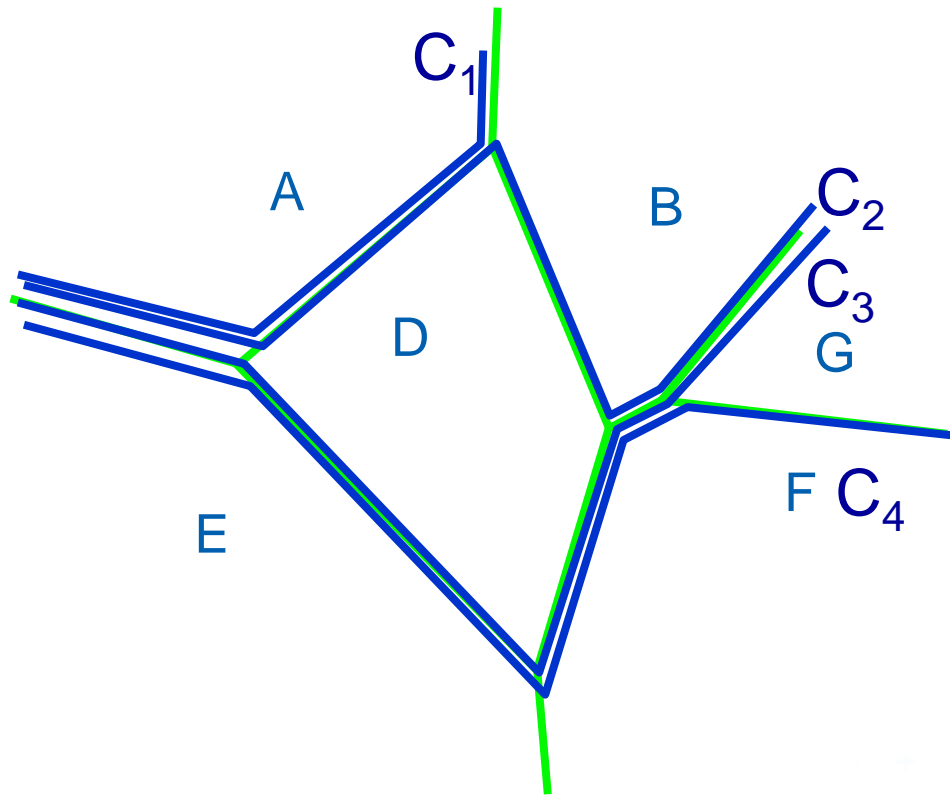
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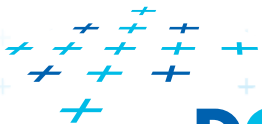
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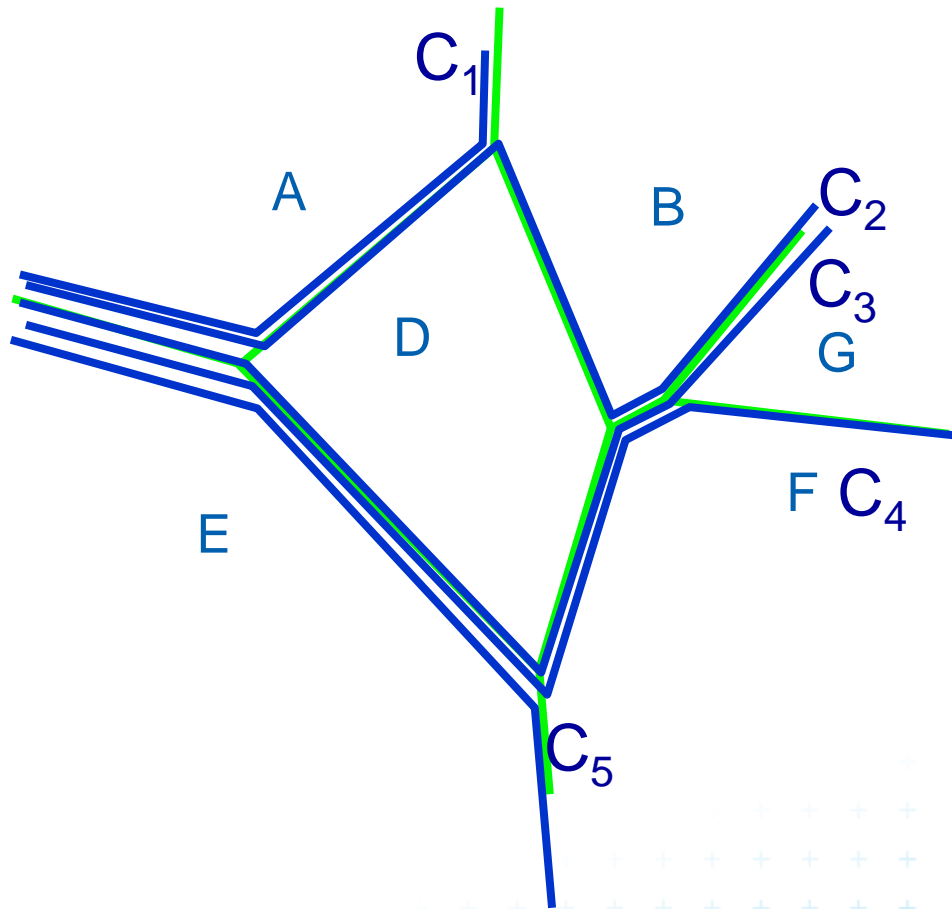
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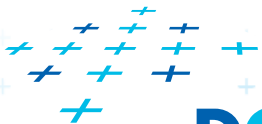
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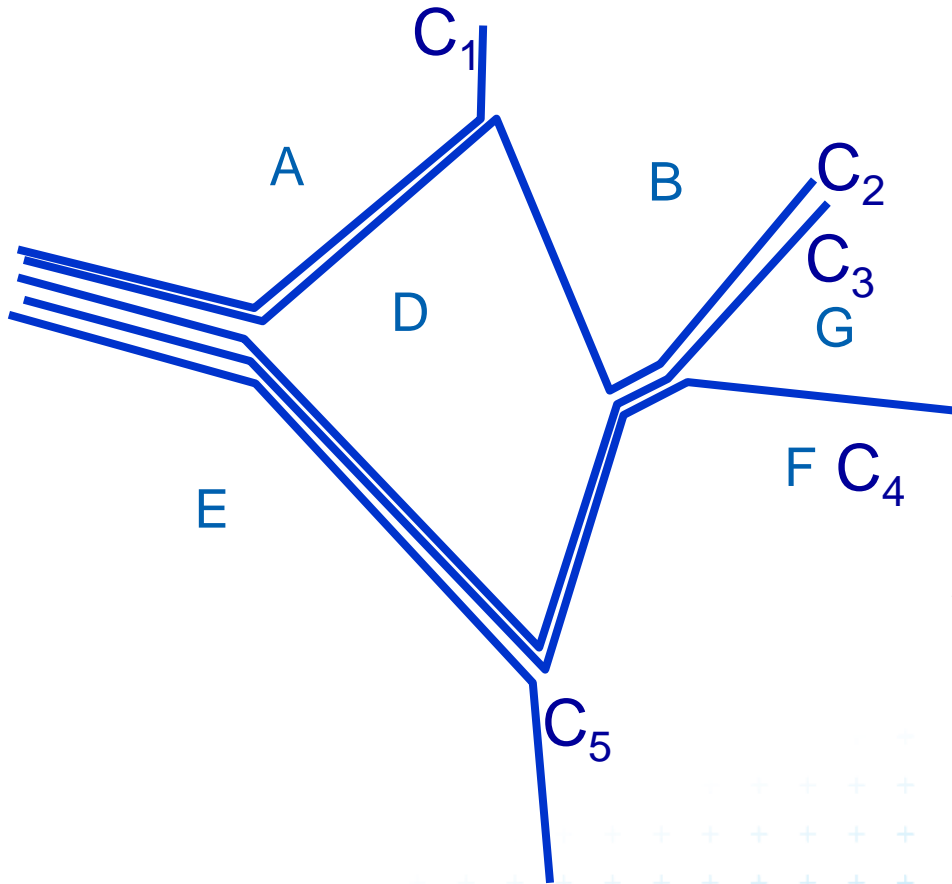
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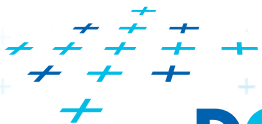
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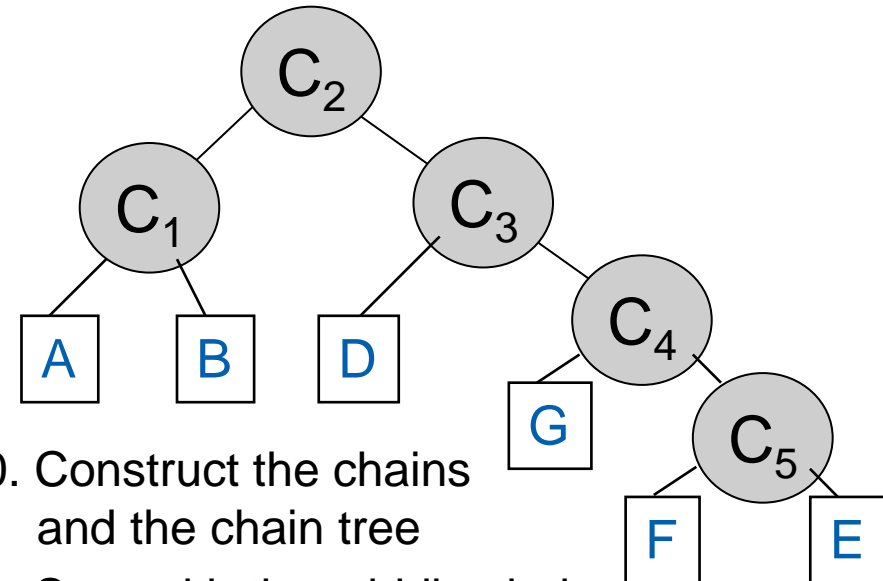
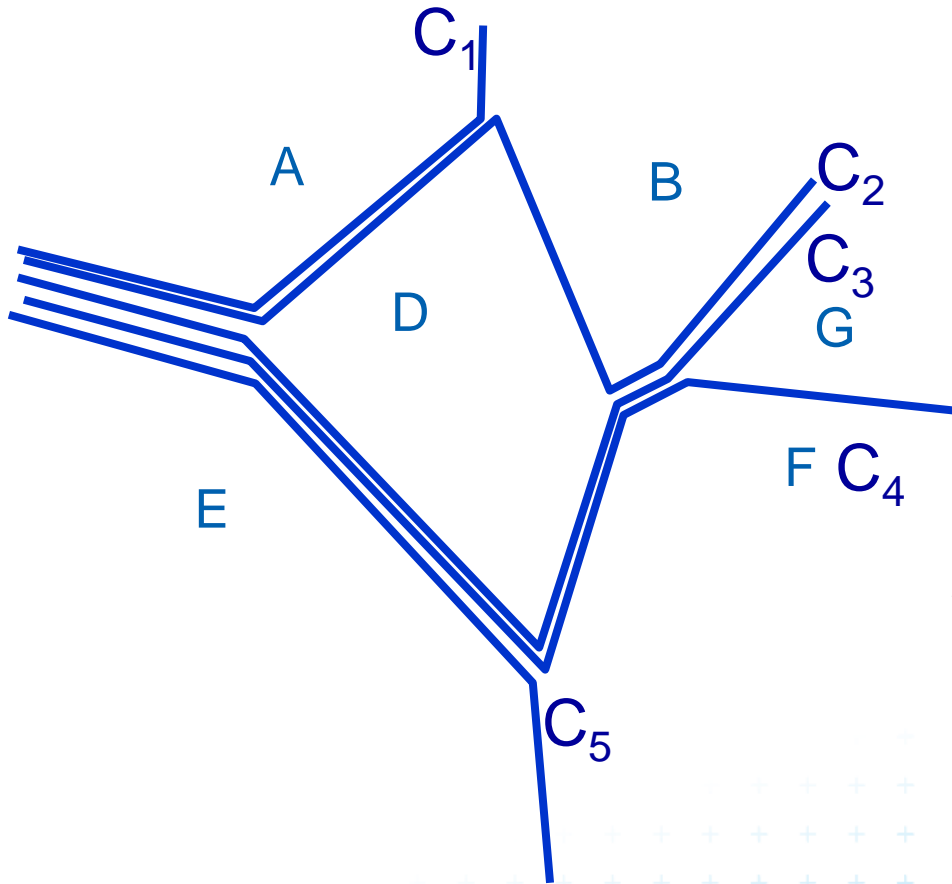
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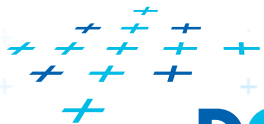
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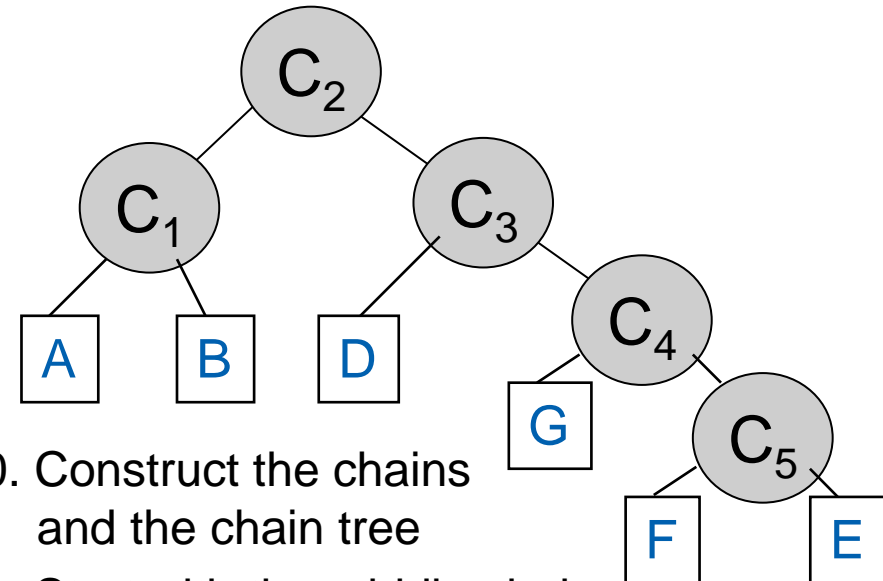
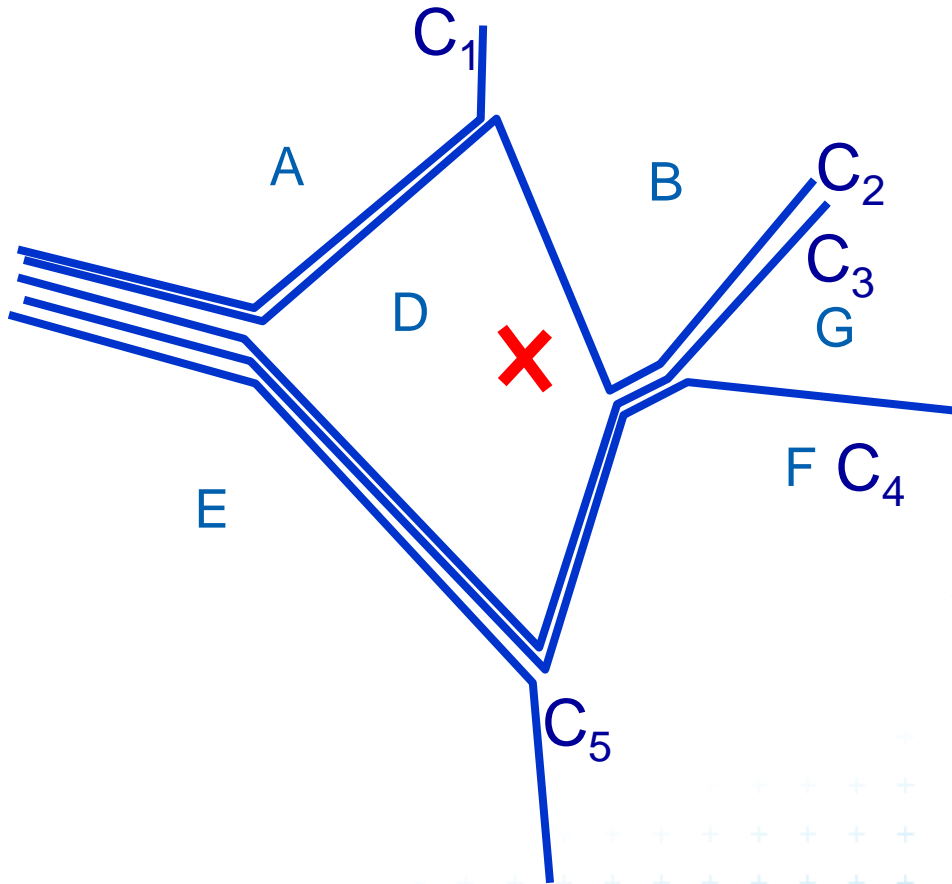
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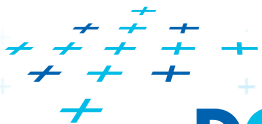
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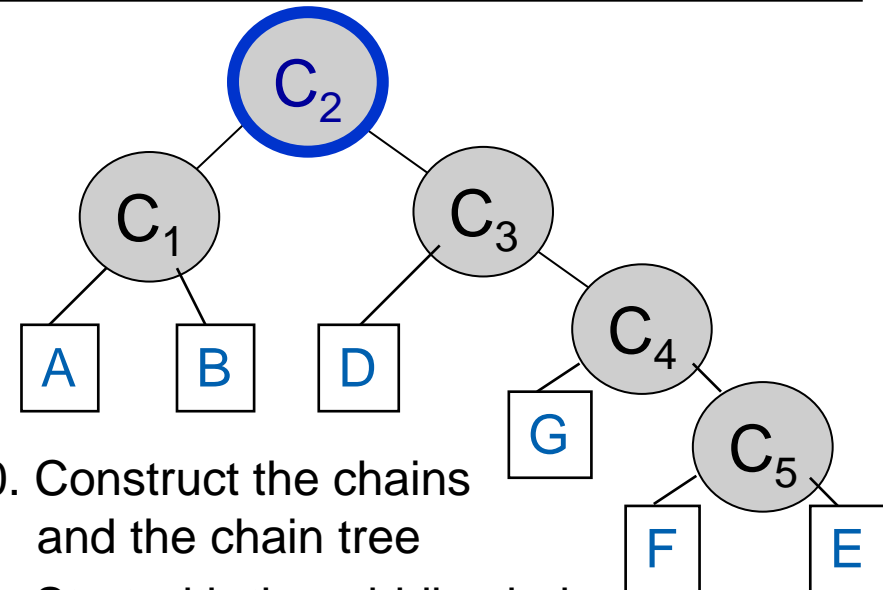
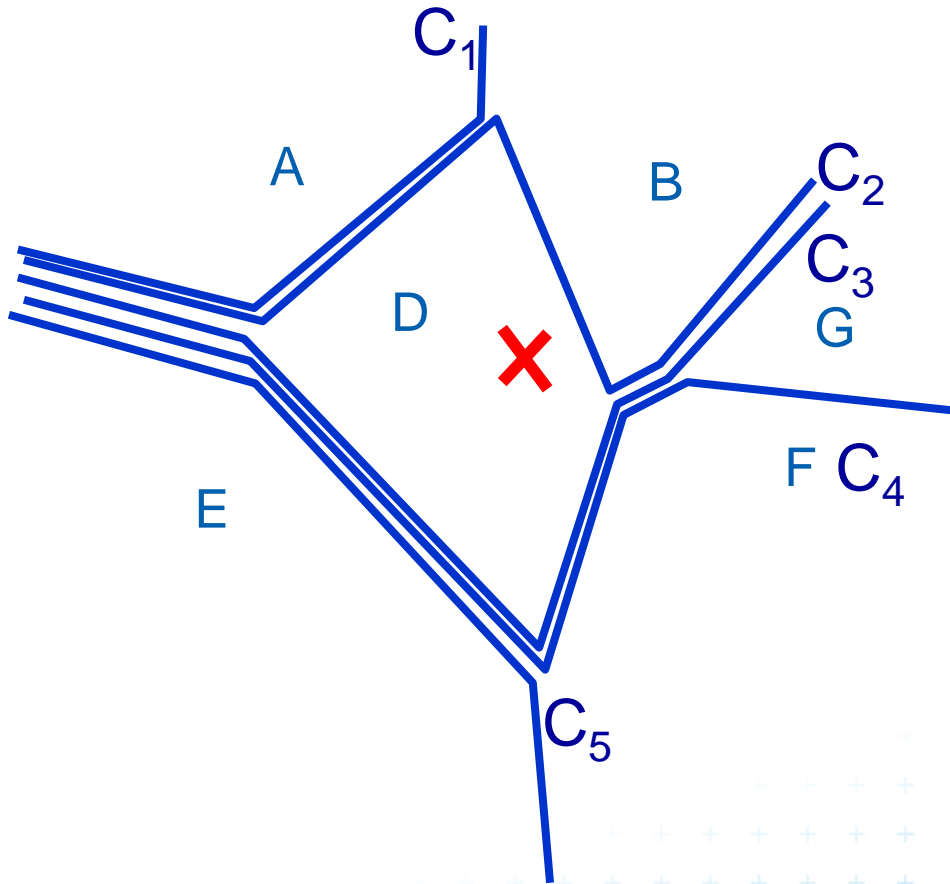
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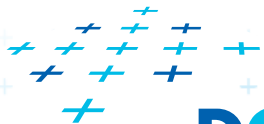
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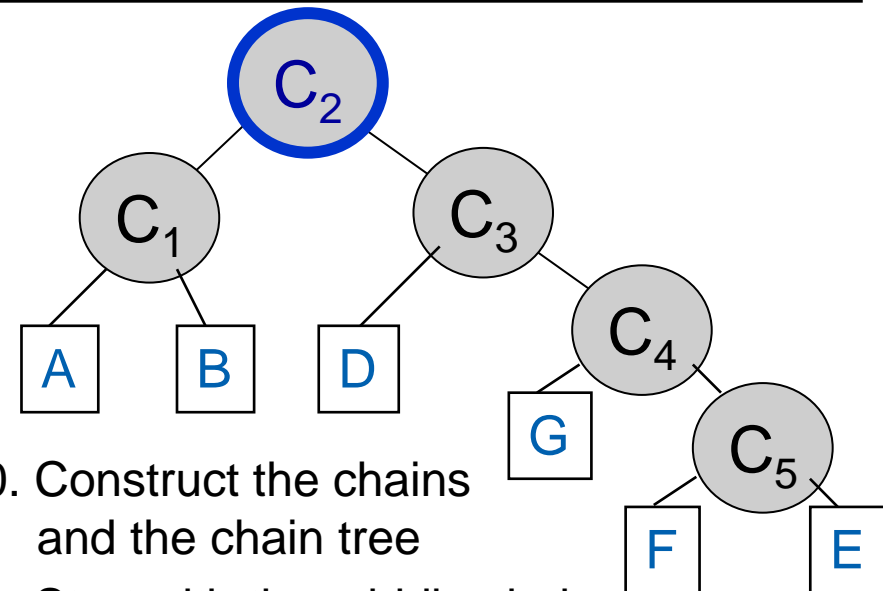
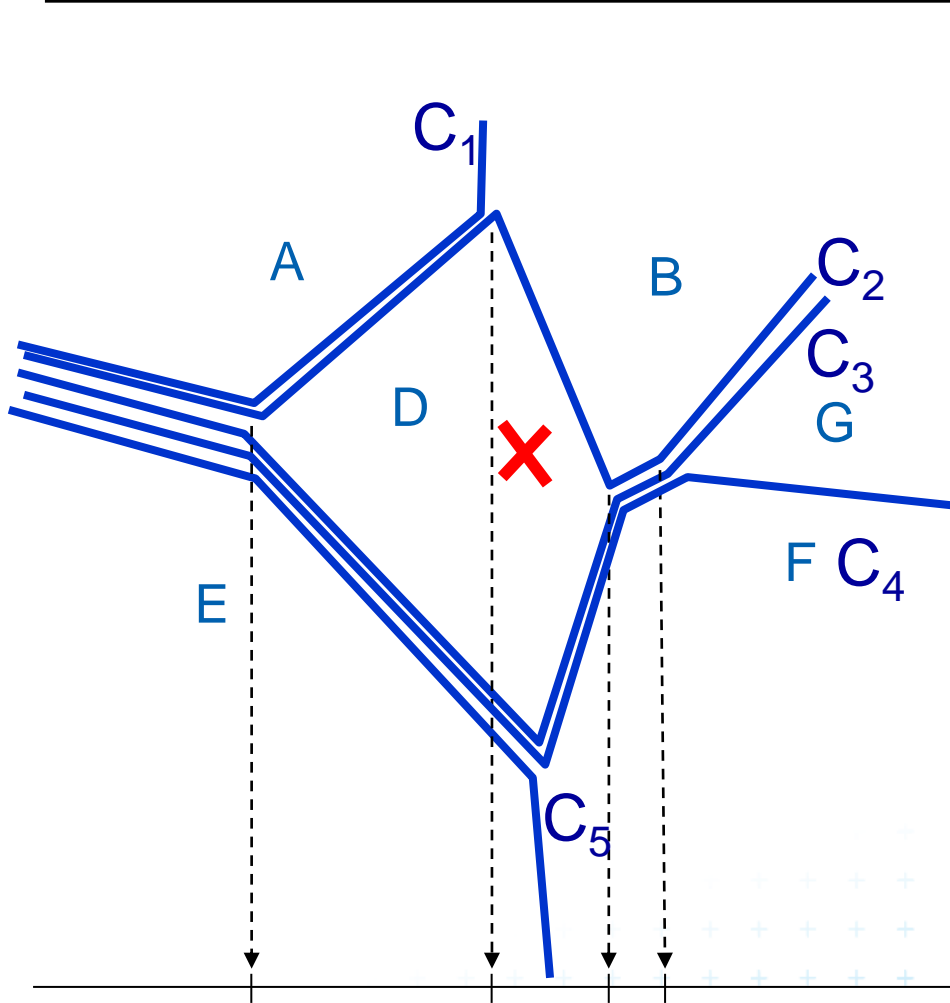
Monotone chain tree example



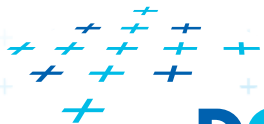
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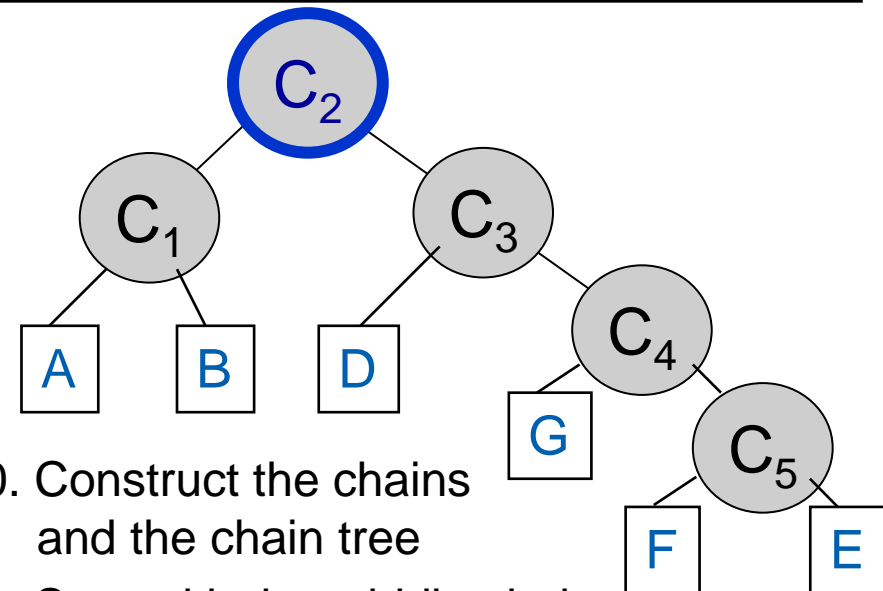
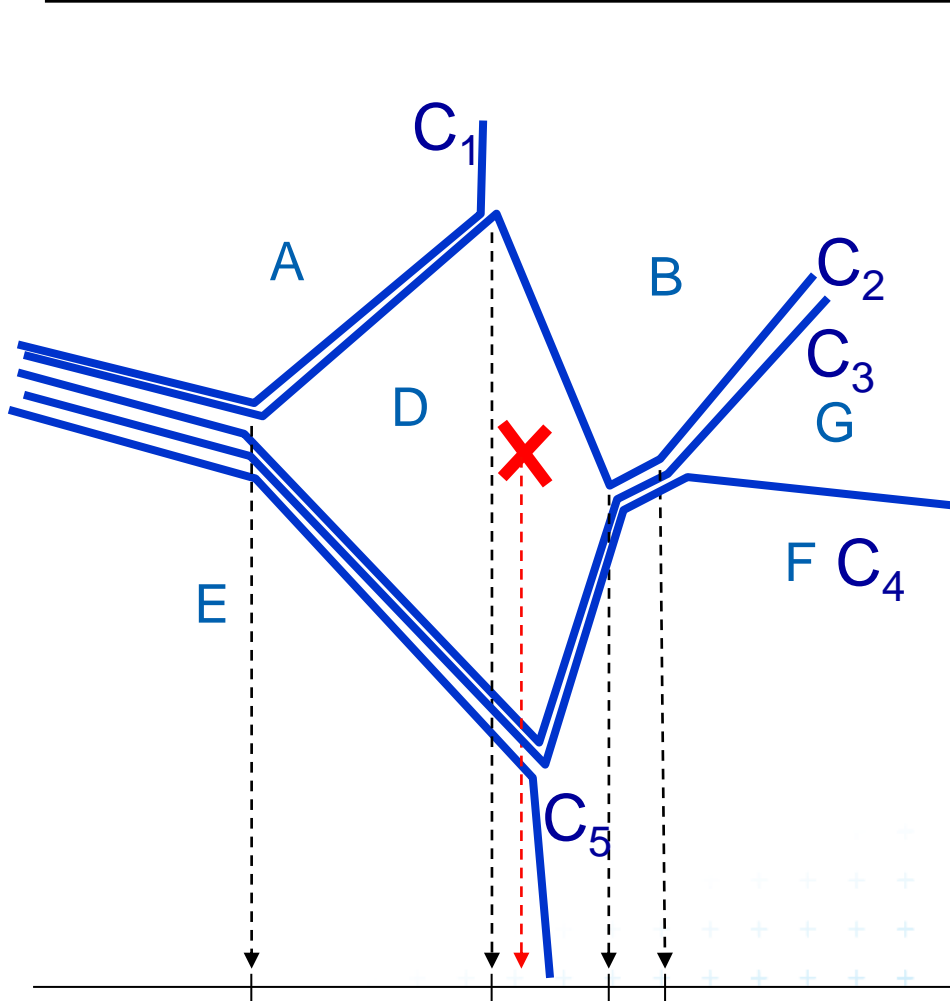
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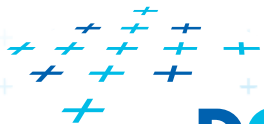
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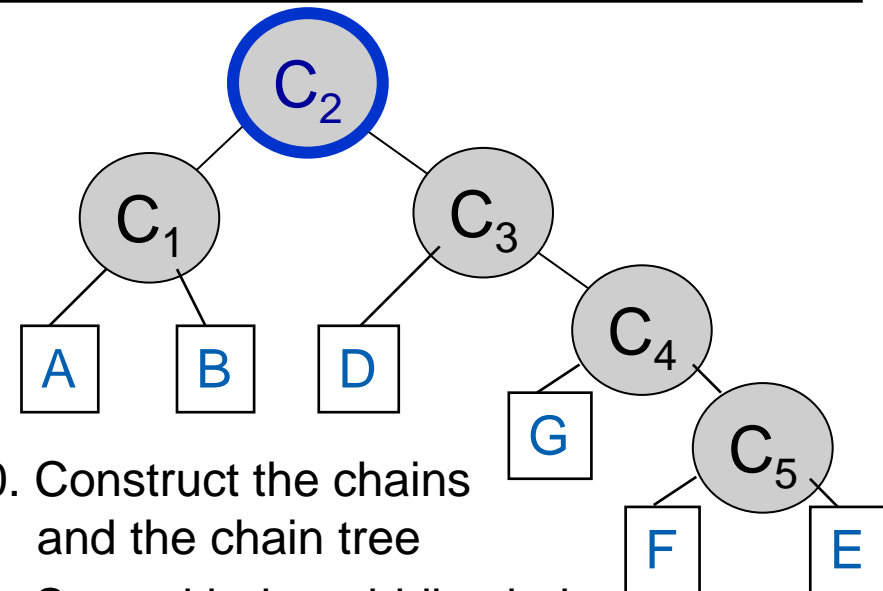
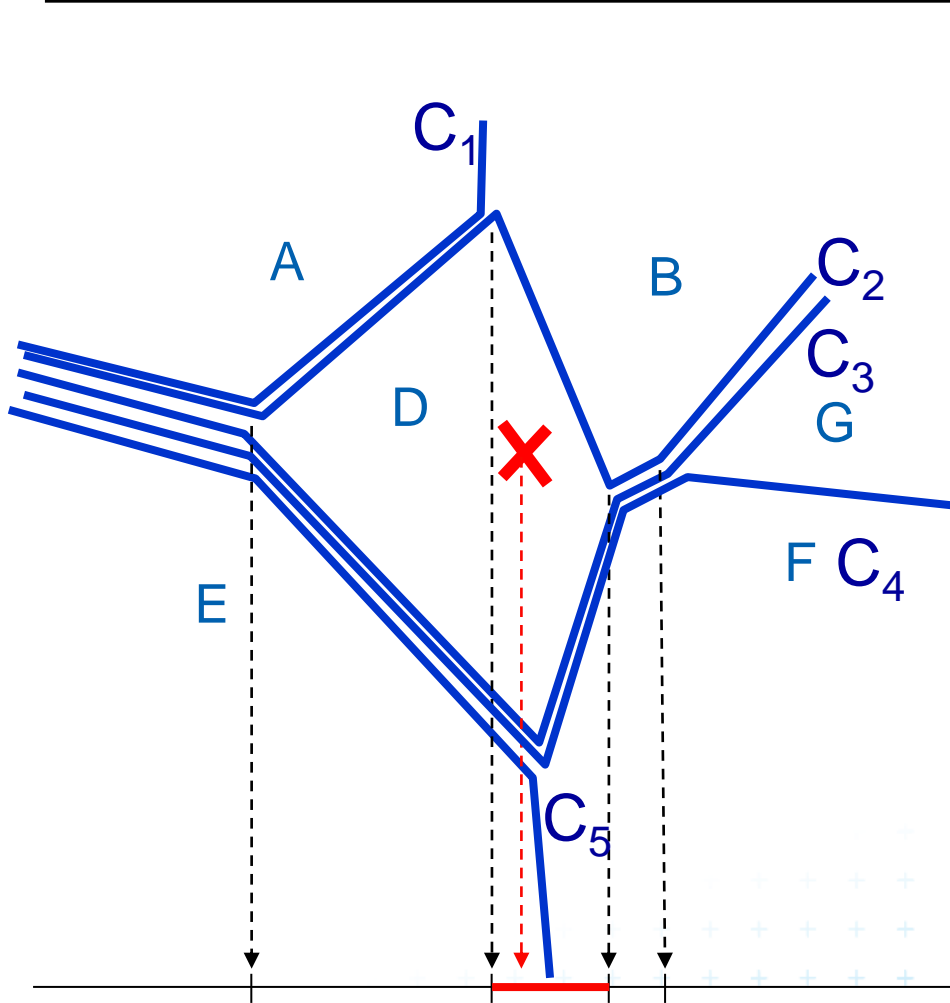
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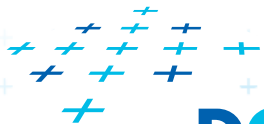
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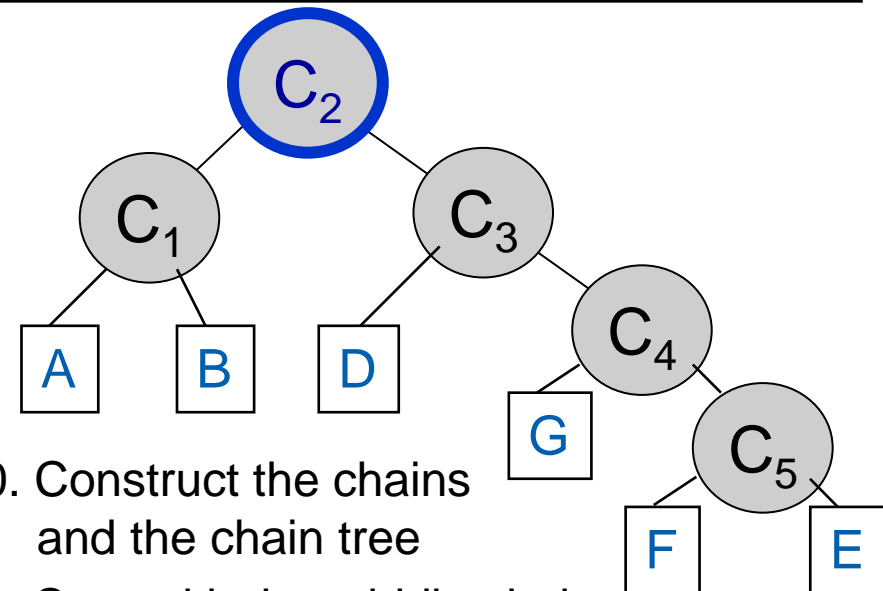
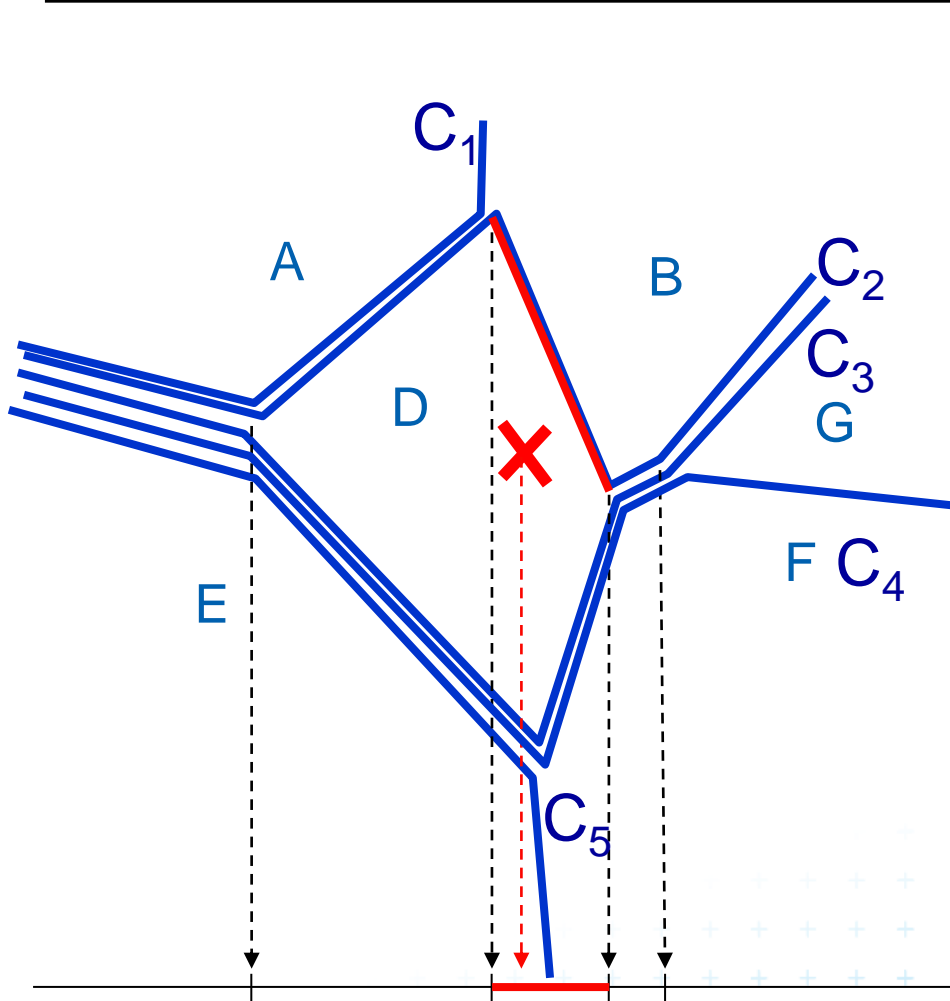
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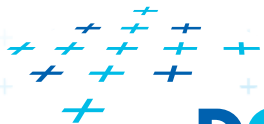
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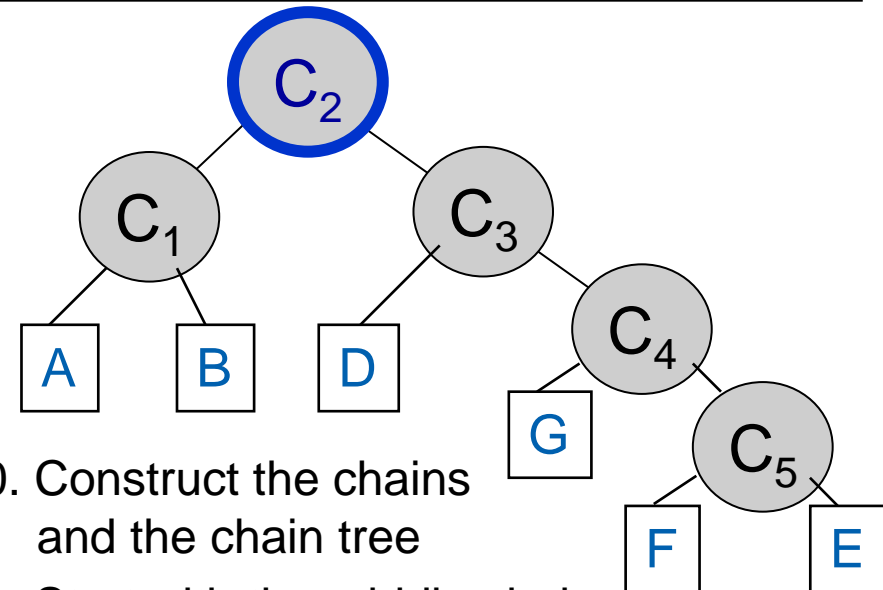
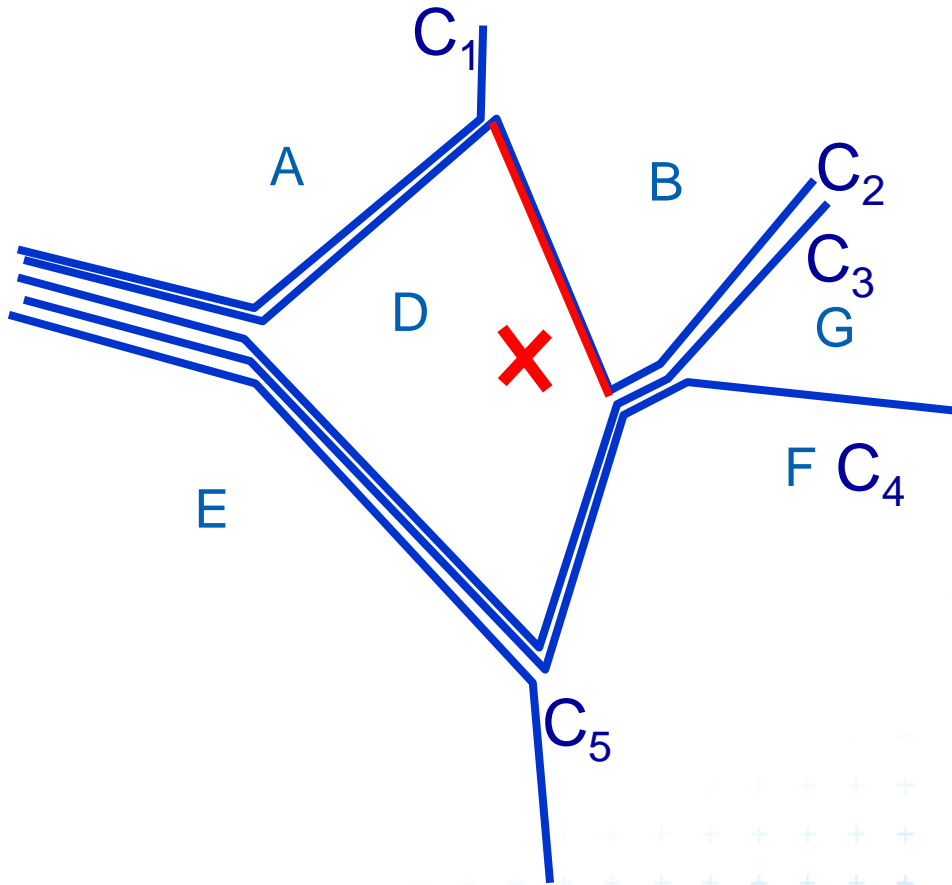
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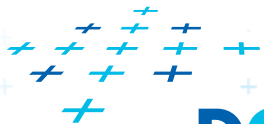
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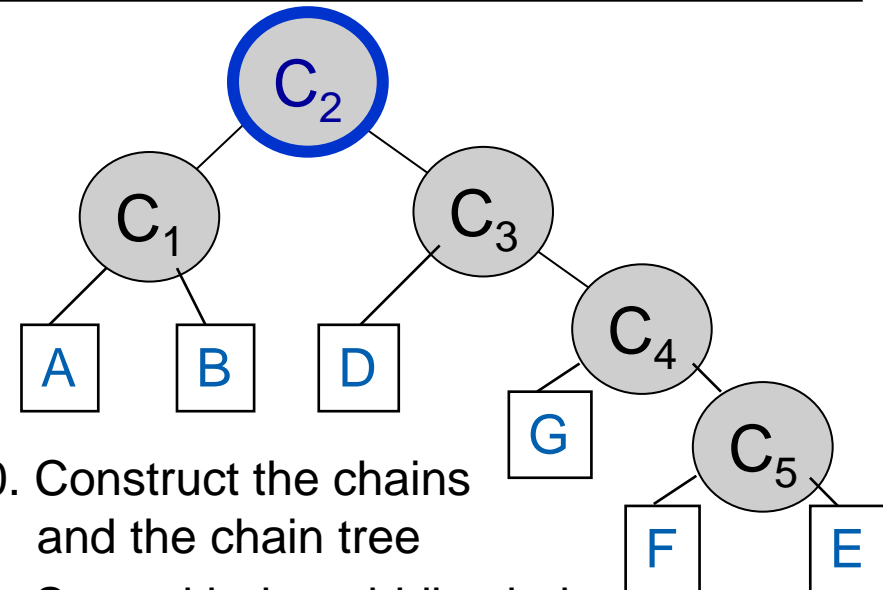
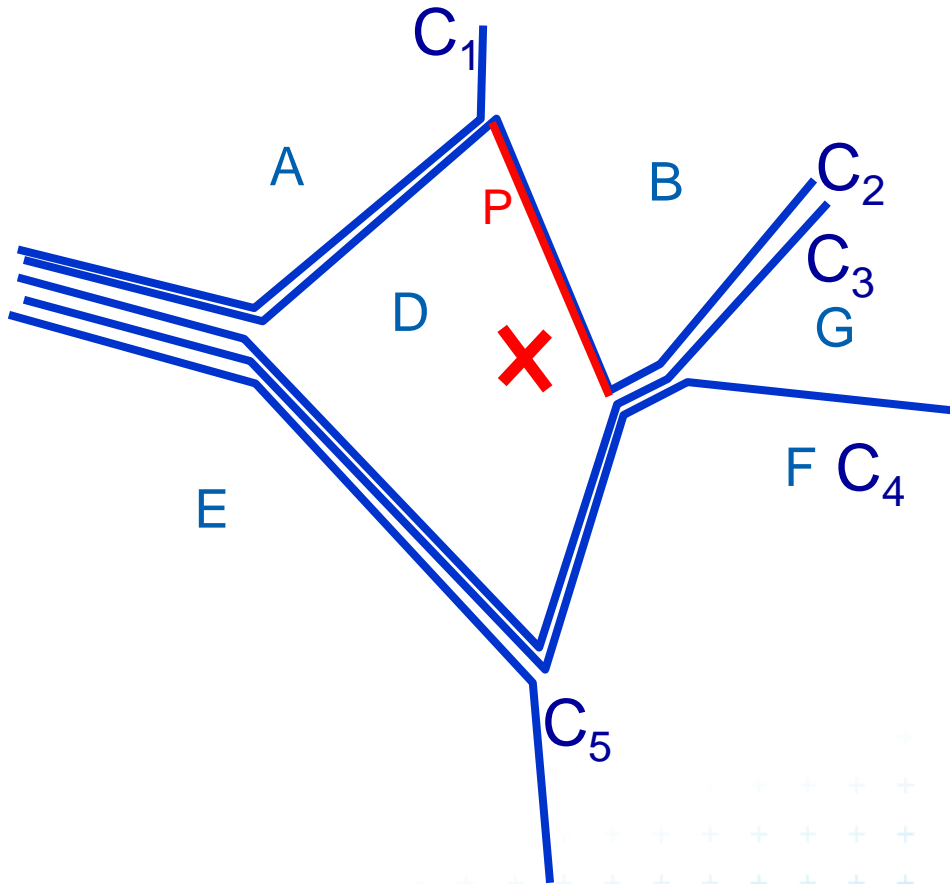
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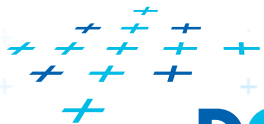
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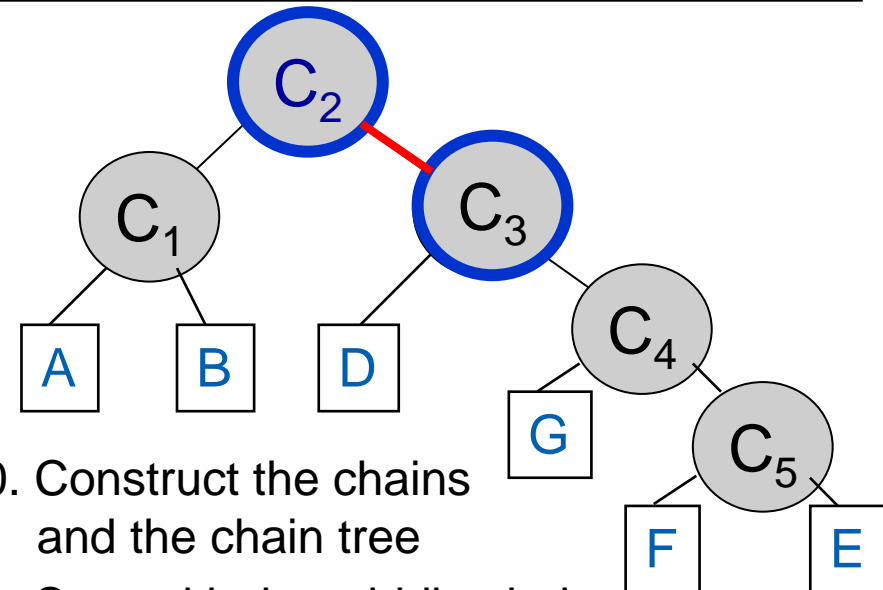
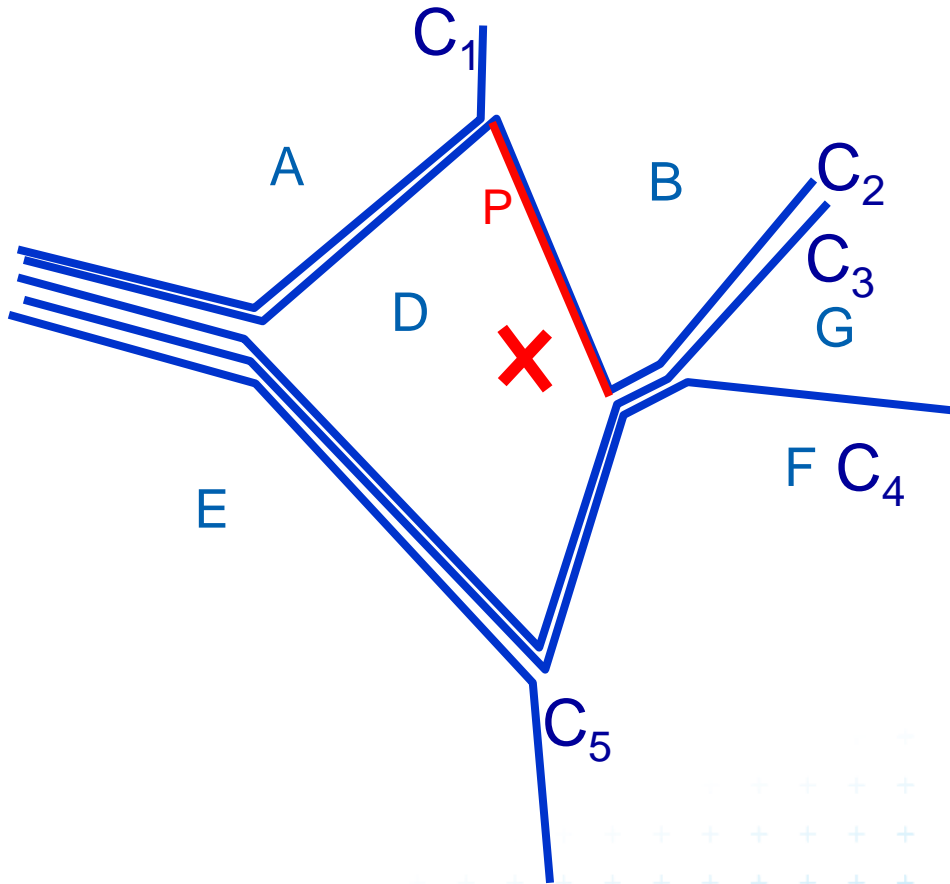
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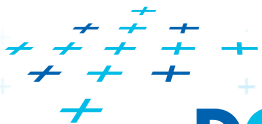
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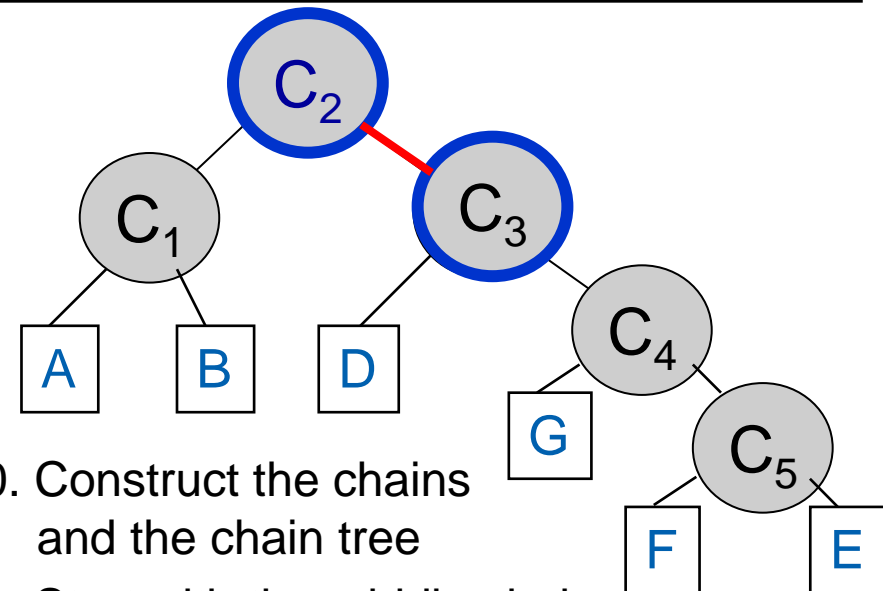
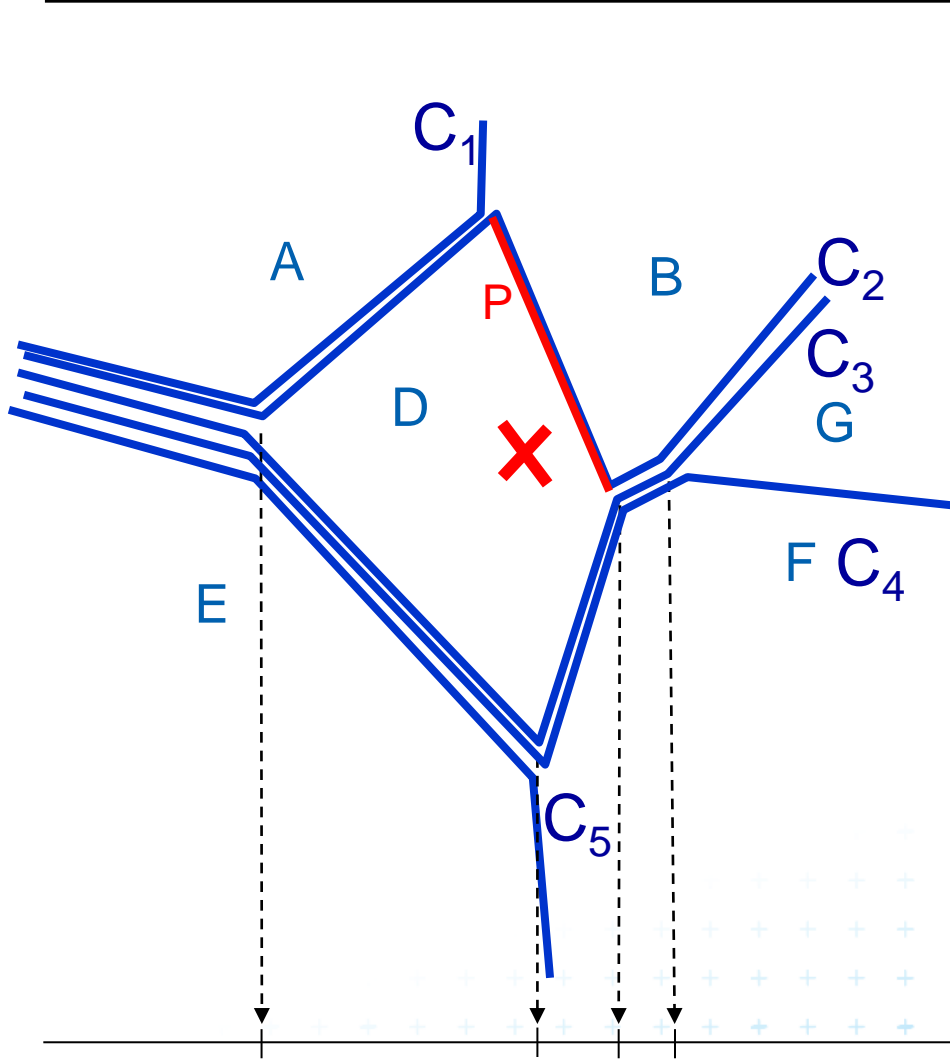
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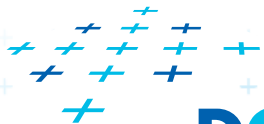
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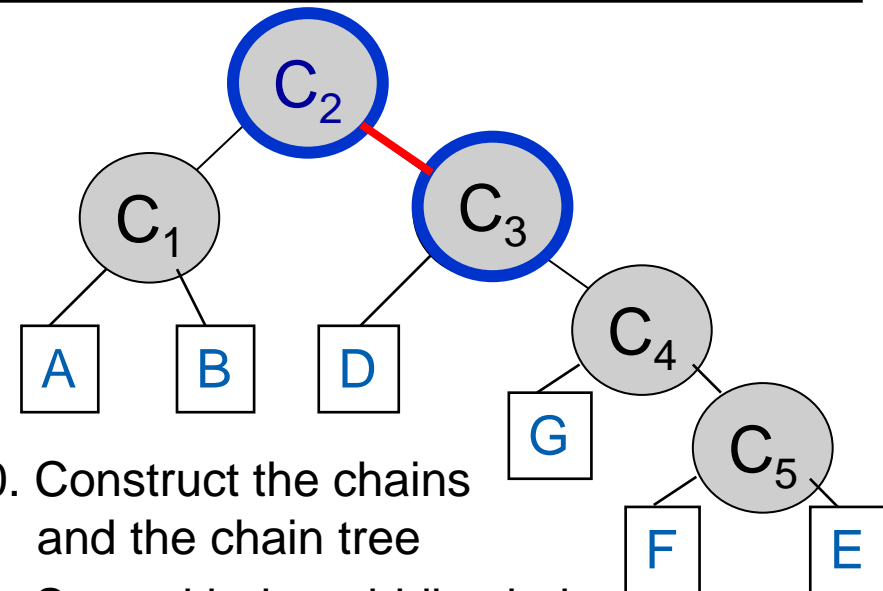
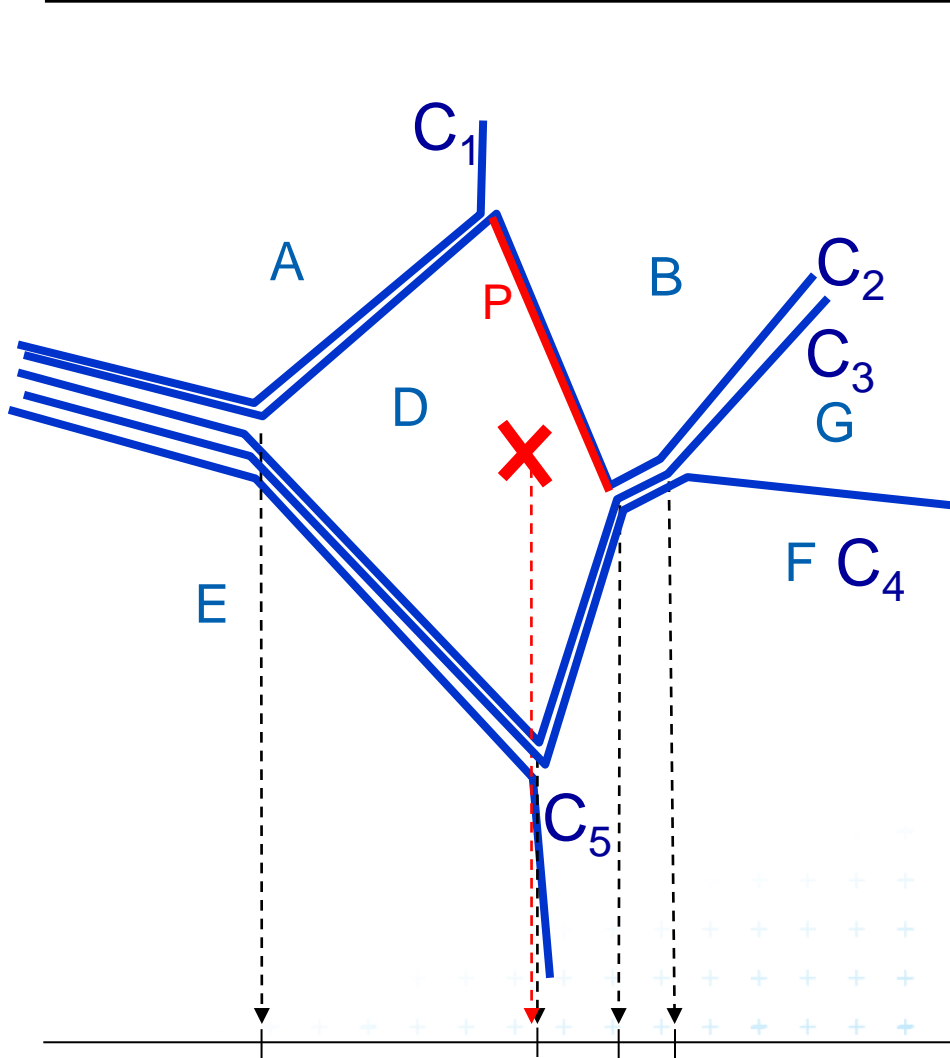
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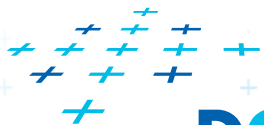
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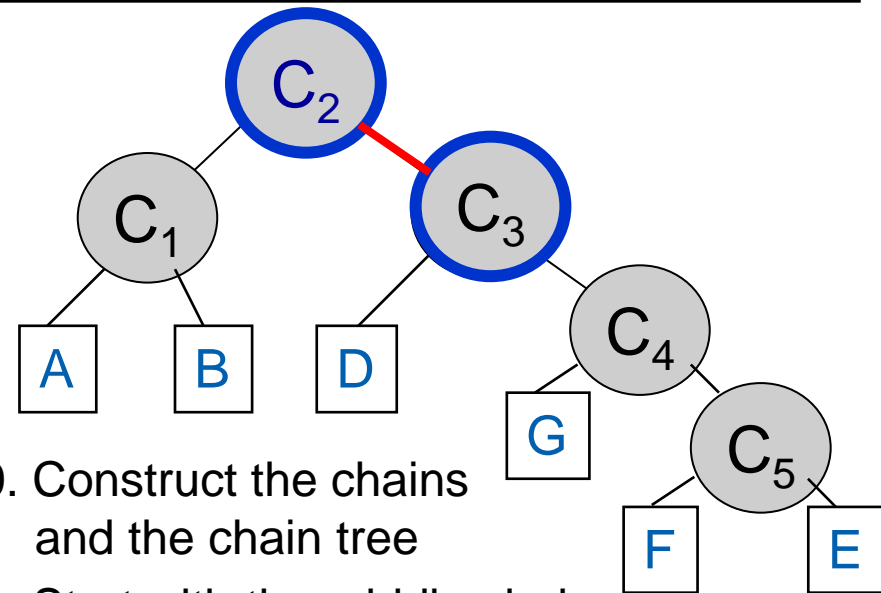
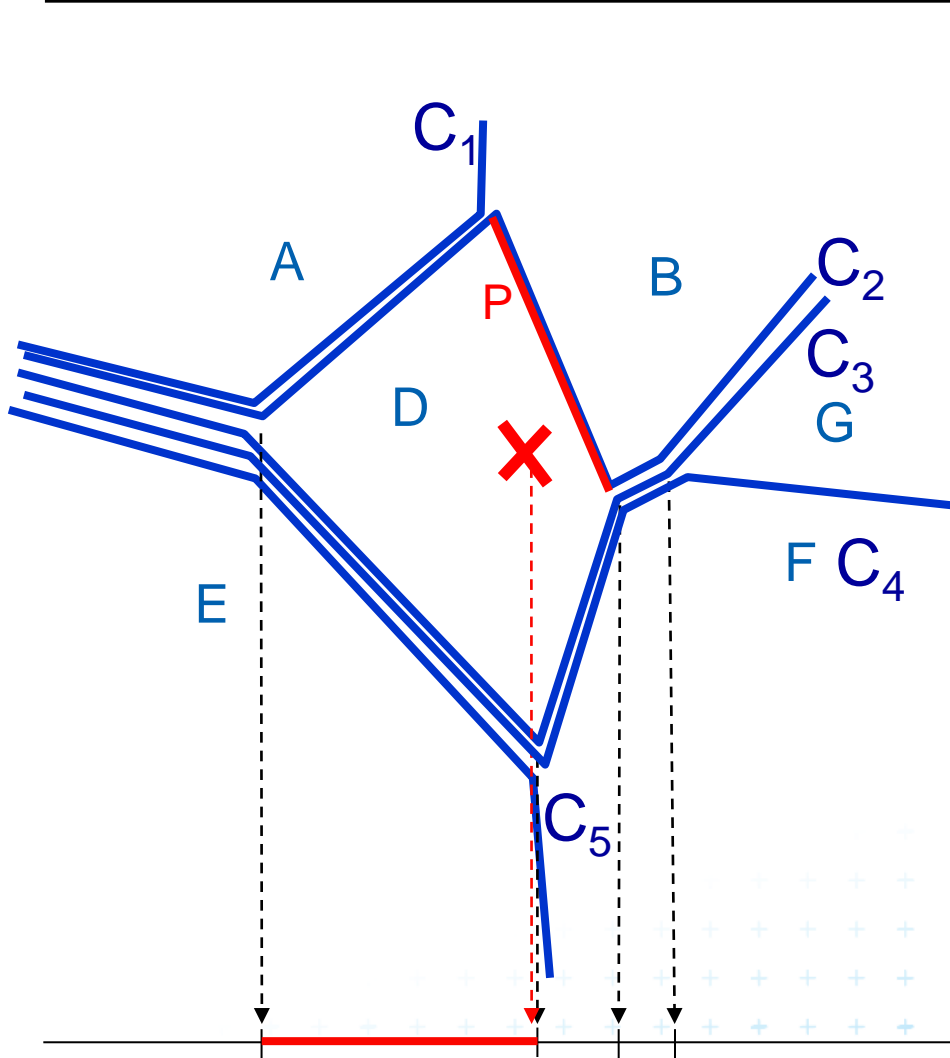
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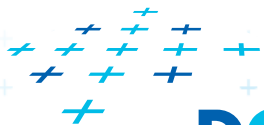
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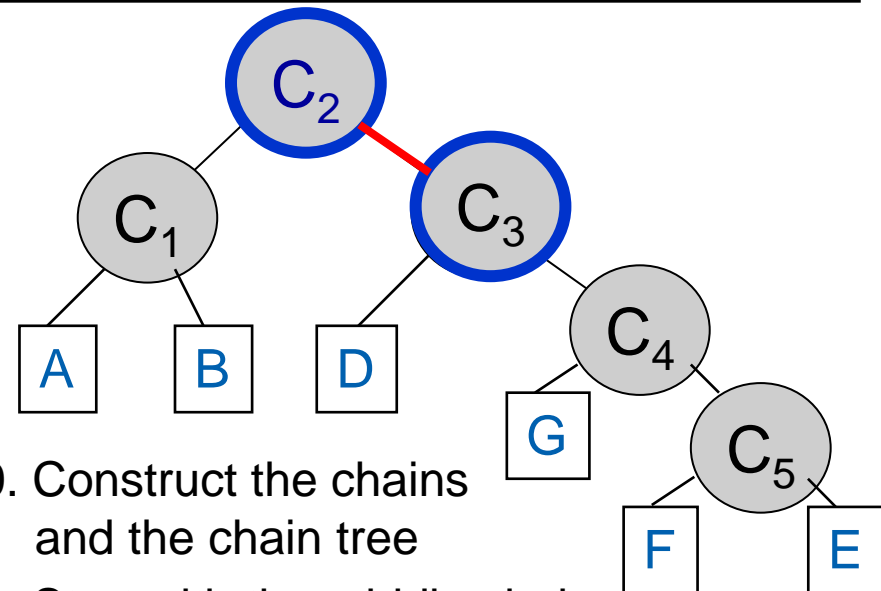
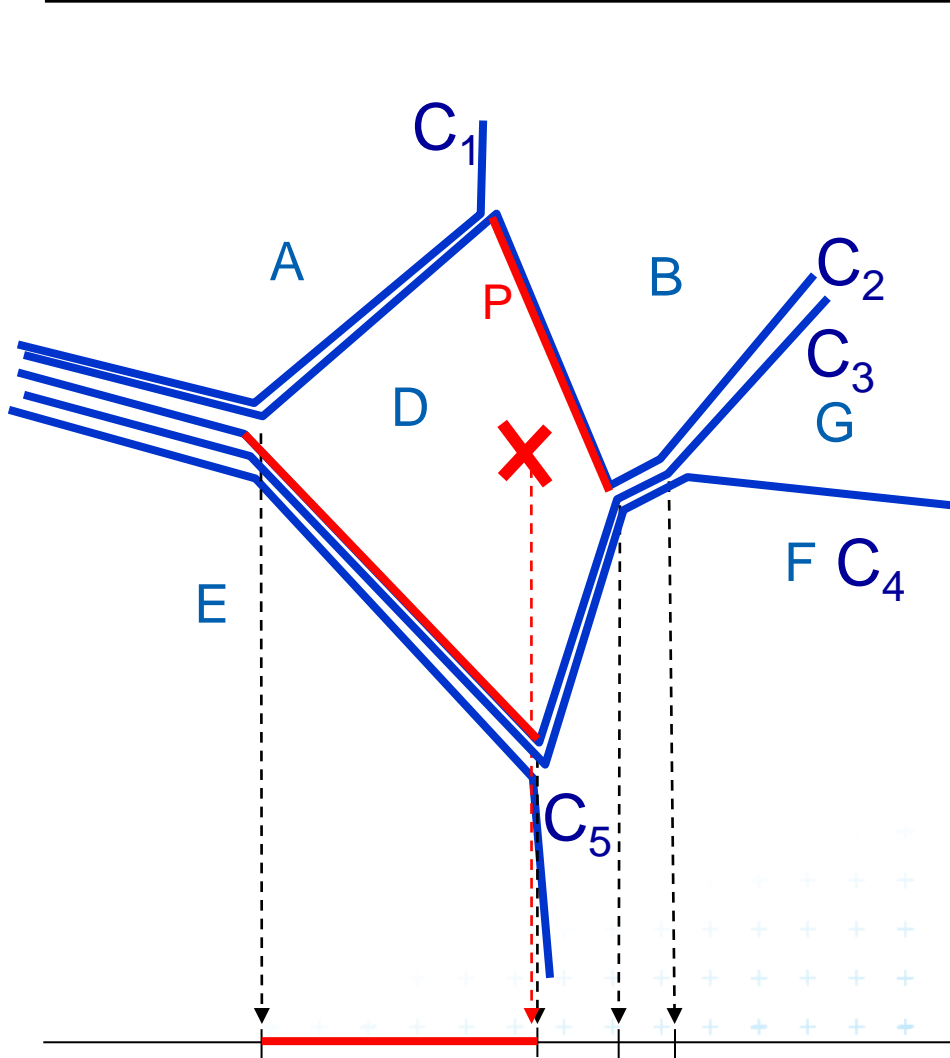
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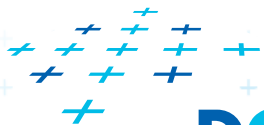
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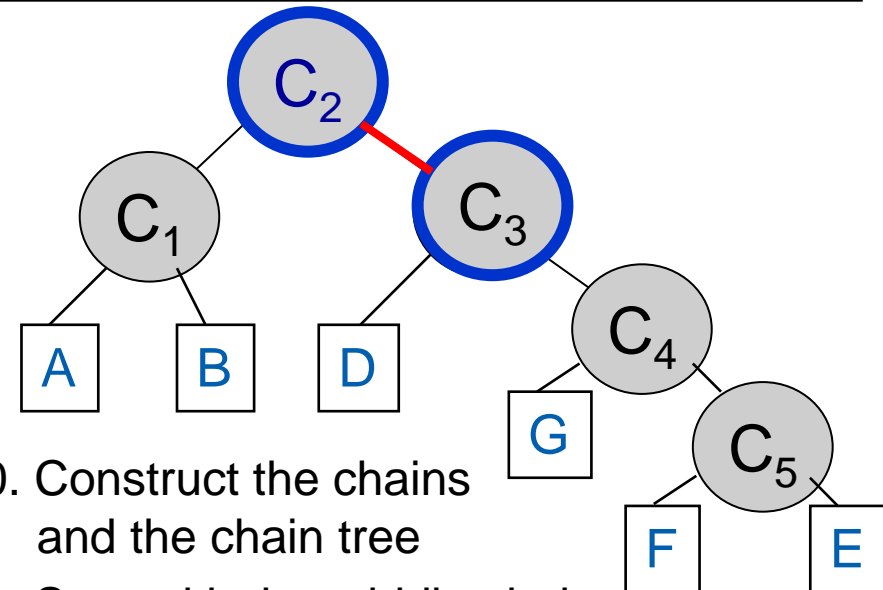
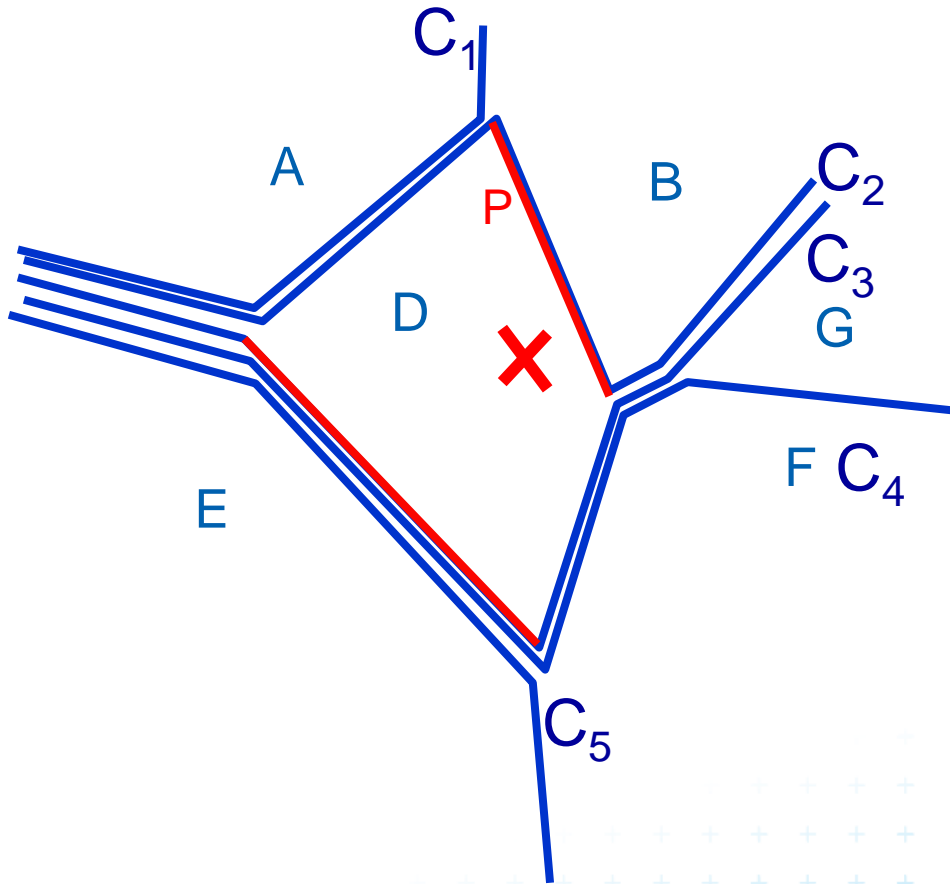
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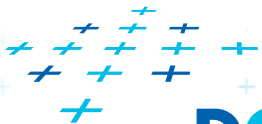
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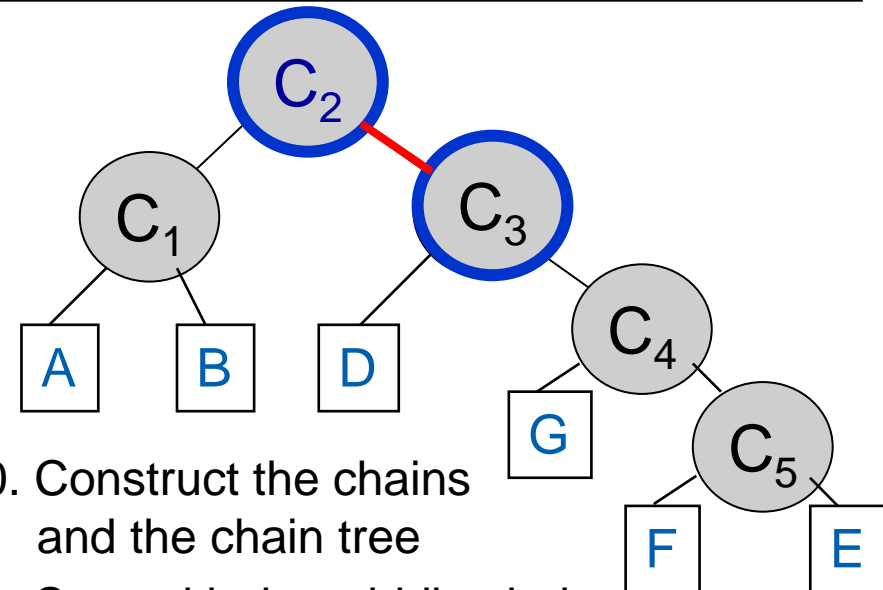
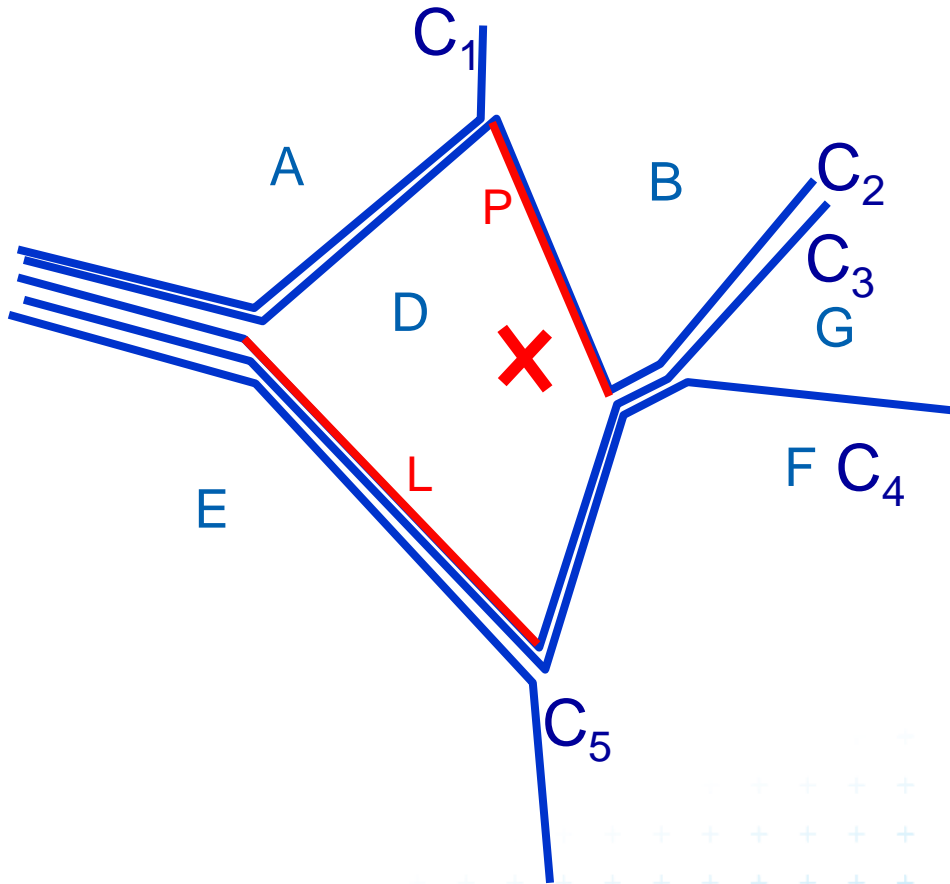
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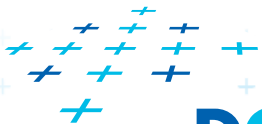
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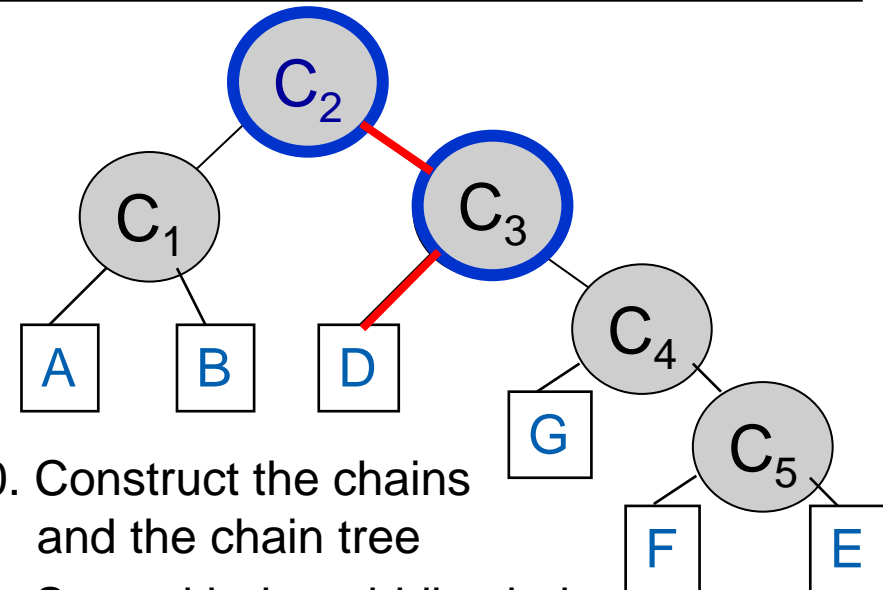
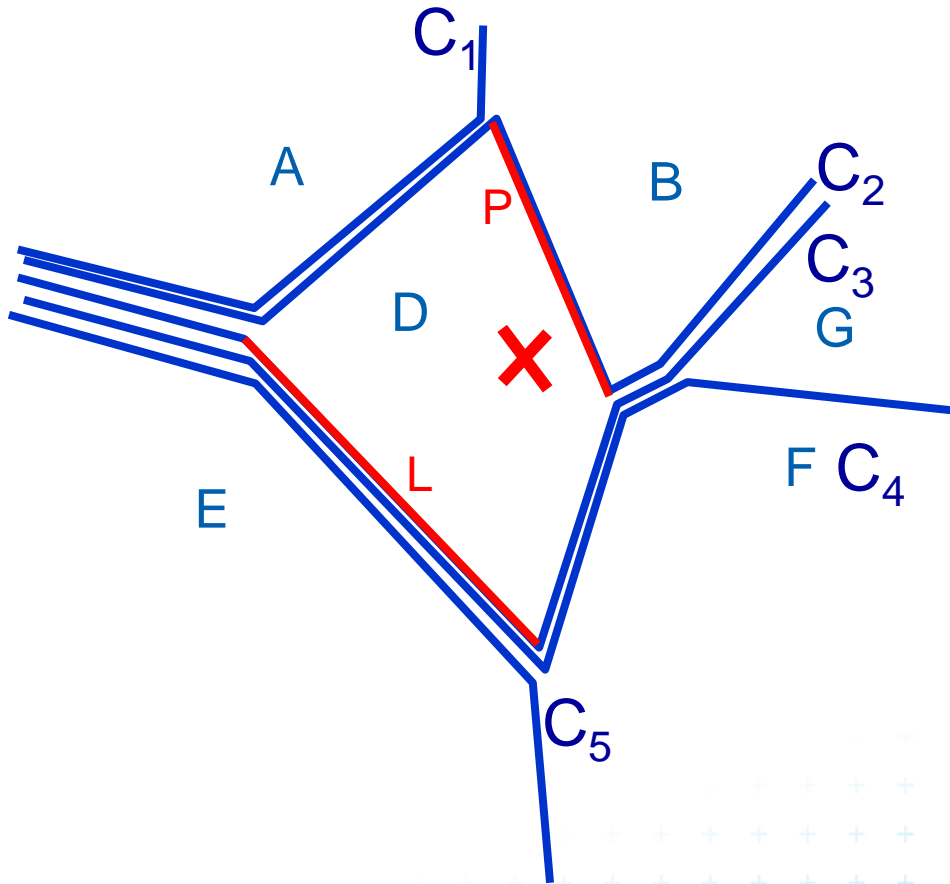
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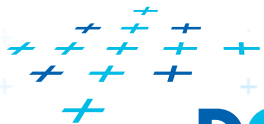
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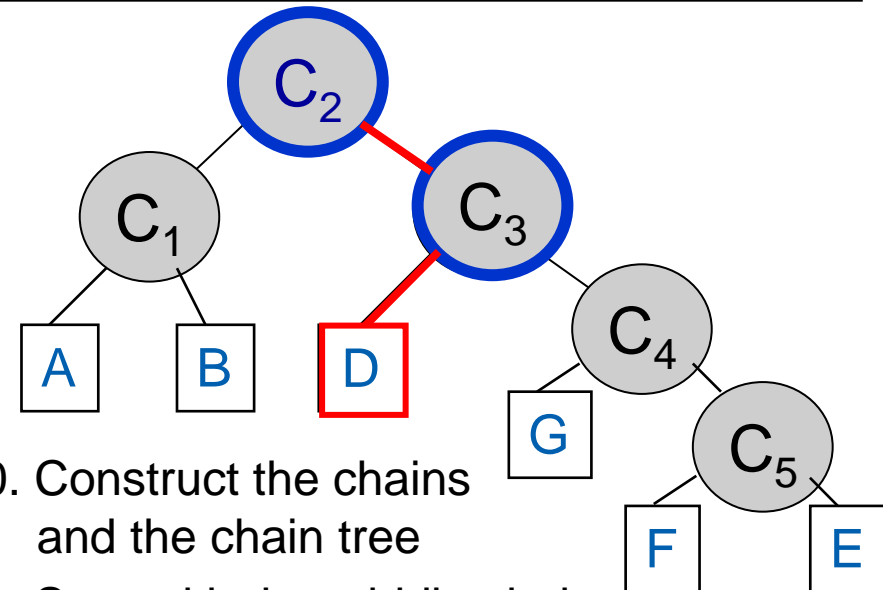
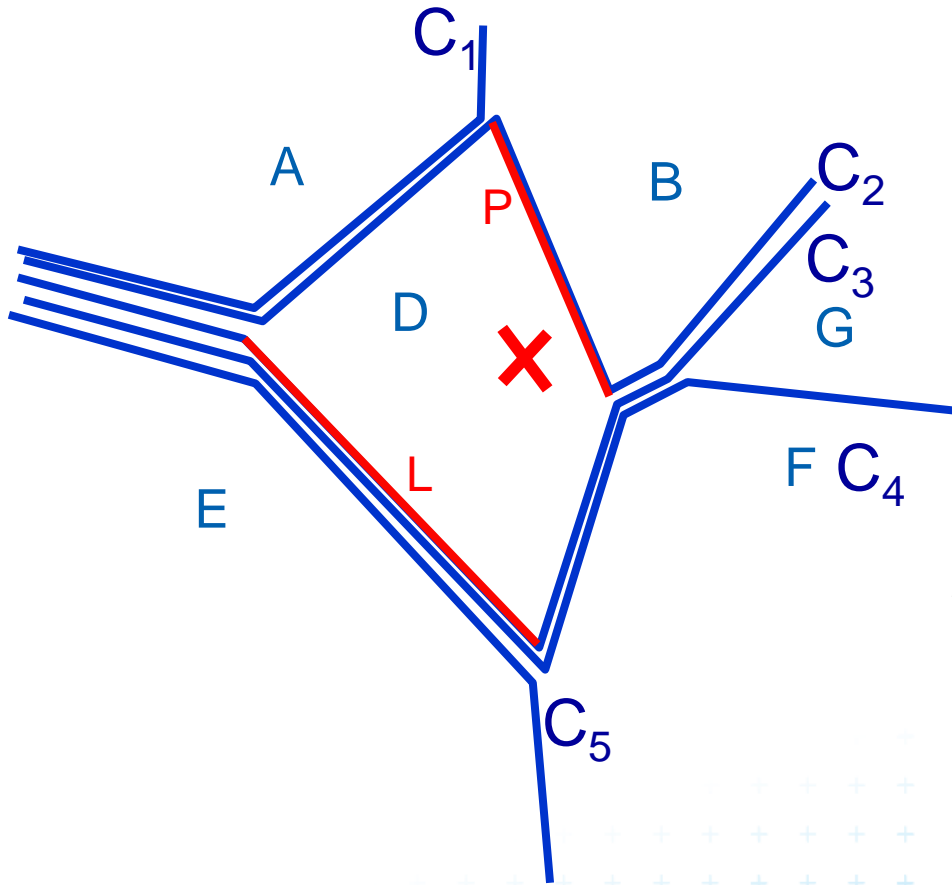
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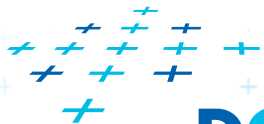
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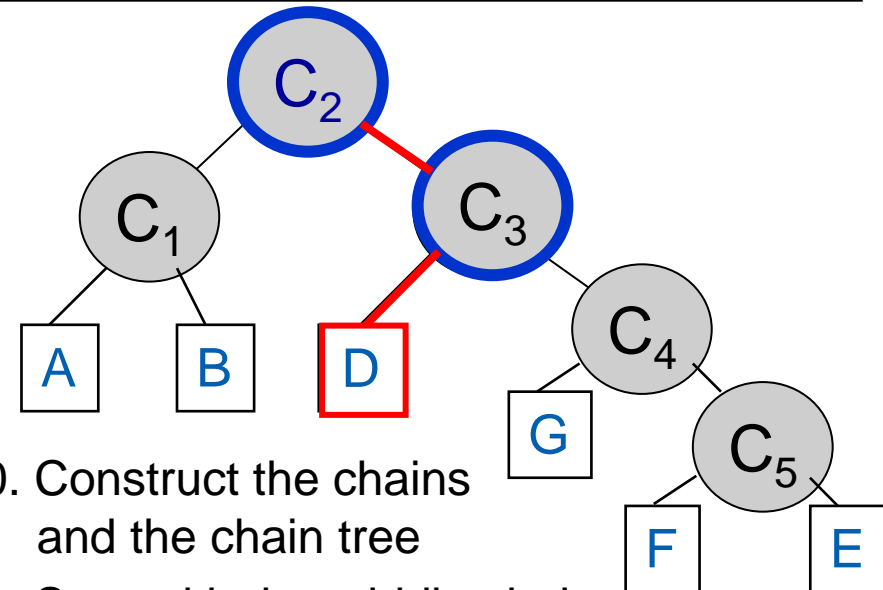
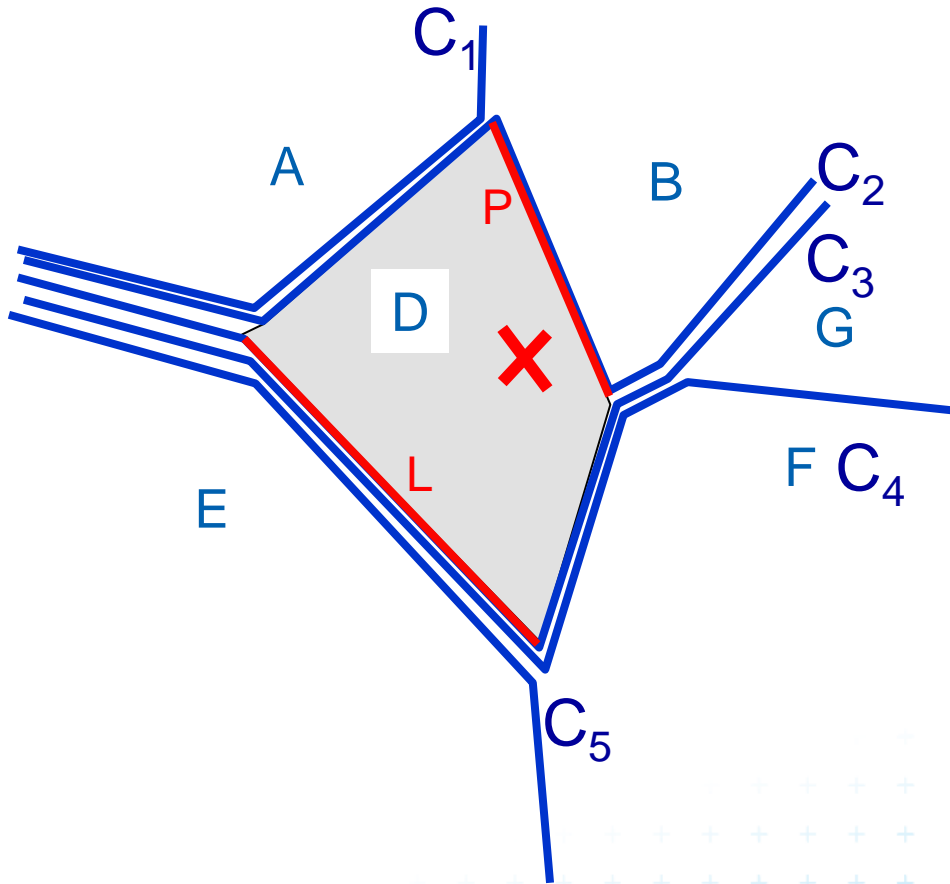
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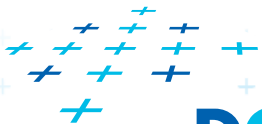
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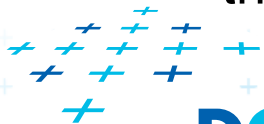
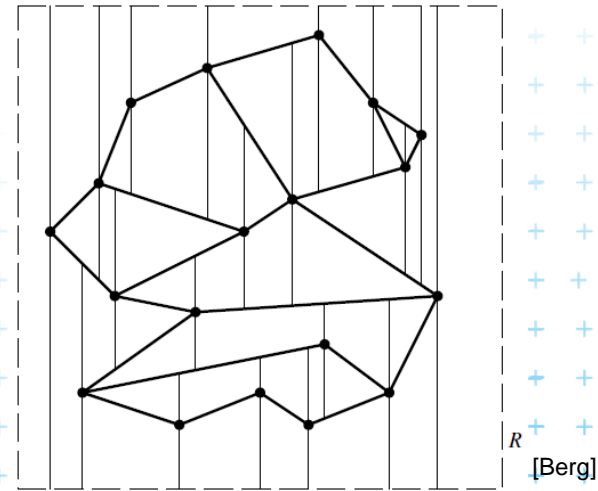


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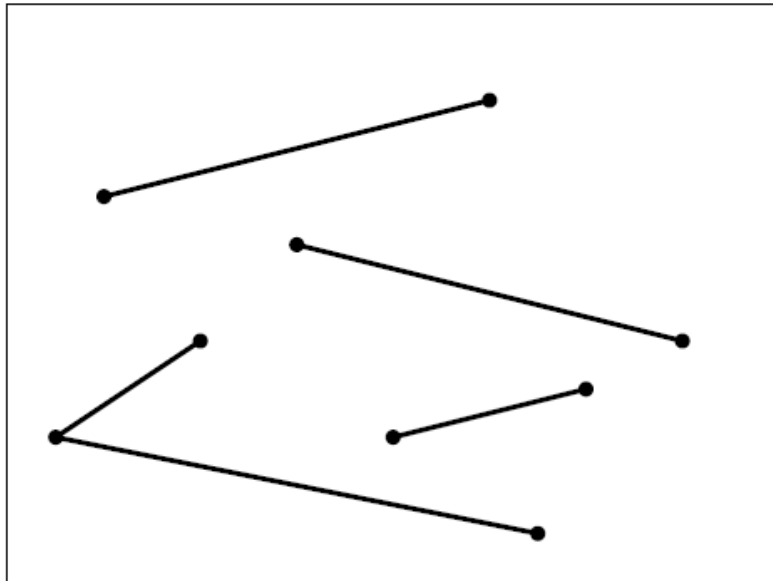
3. Trapezoidal map (TM) search

- The simplest and most practical known optimal algorithm
- Randomized algorithm with $O(n)$ expected storage and $O(\log n)$ expected query time
- Expectation depends on the random order of segments during construction, not on the position of the segments
- TM is refinement of original subdivision
- Converts complex shapes into simple ones
- Weaker assumption on input:
 - Input individual segments, not polygons
 - $S = \{s_1, s_2, \dots, s_n\}$
 - S_i subset of first i segments
 - Answer: segment below the pointed trapezoid (□)



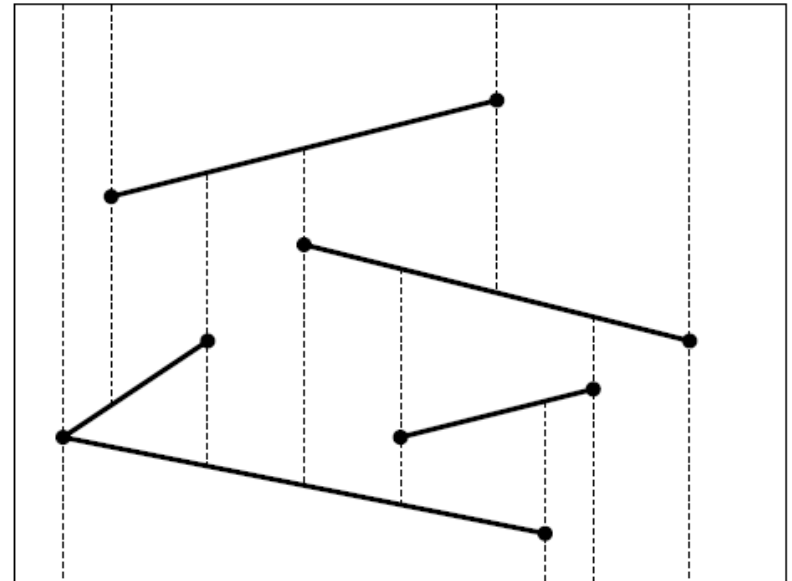
Trapezoidal map of line segments in general position

Input: individual segments S



Construction →

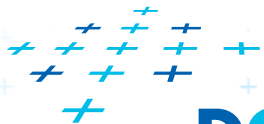
Trapezoidal map T



- They do not intersect, except in endpoints
- No vertical segments
- No 2 distinct endpoints with the same x-coordinate

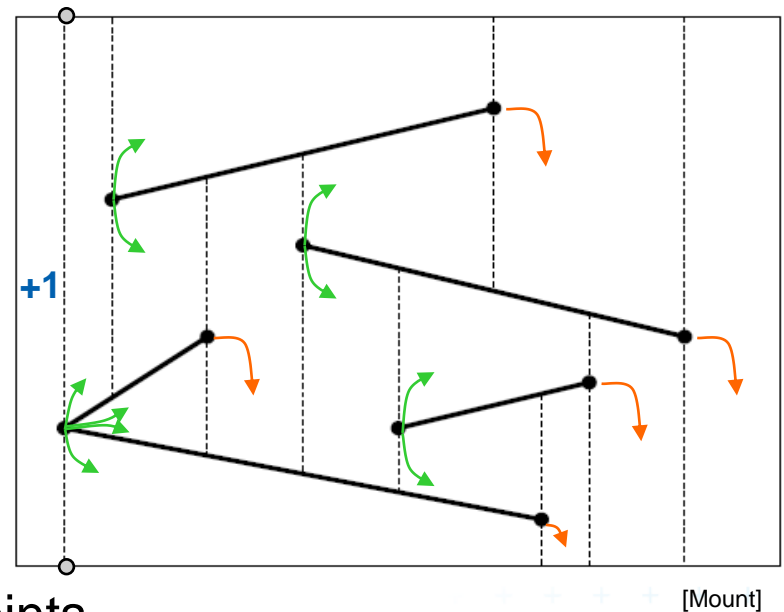
- Bounding rectangle
- 4 Bullets up and down
- Stop on input segment or on bounding rectangle

[Mount]



Trapezoidal map of line segments in general position

- Faces are trapezoids Δ with vertical sides
- Given n segments, TM has
 - at most $6n+4$ vertices
 - at most $3n+1$ trapezoids

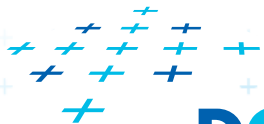


■ Proof:

- each endpoint 2 bullets $\rightarrow 1+2$ points
- $2n$ endpoints $\cdot 3 + 4 = 6n+4$ vertices



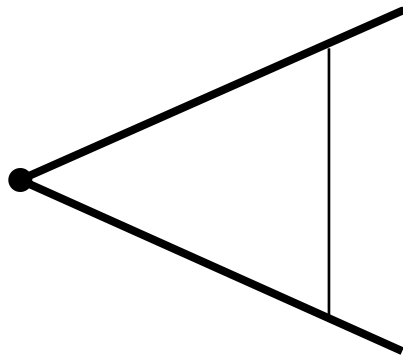
- start point \rightarrow max 2 trapezoids Δ
- end point \rightarrow 1 trapezoid Δ
- $3 \cdot (n \text{ segments}) + 1 \text{ left } \Delta \Rightarrow \text{max } 3n+1 \Delta$



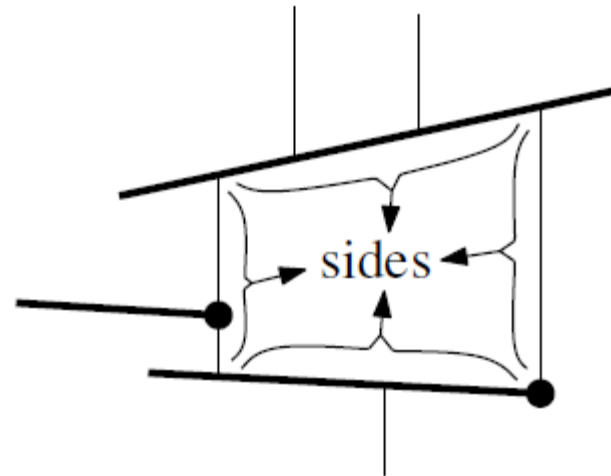
Trapezoidal map of line segments in general position

Each face has

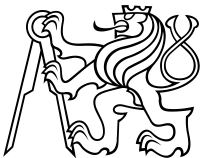
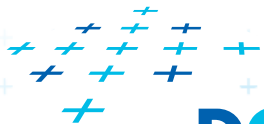
- one or two **vertical sides** (trapezoid or triangle) and
- exactly two **non-vertical sides**



One vertical side



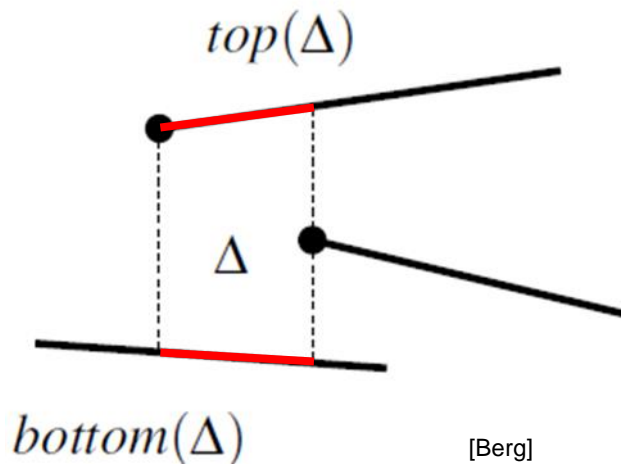
Two vertical sides



Two non-vertical sides

Non-vertical side  or 

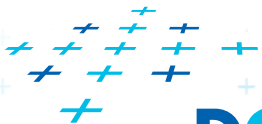
- is contained in one of the segments of set S
- or in the horizontal edge of bounding rectangle R



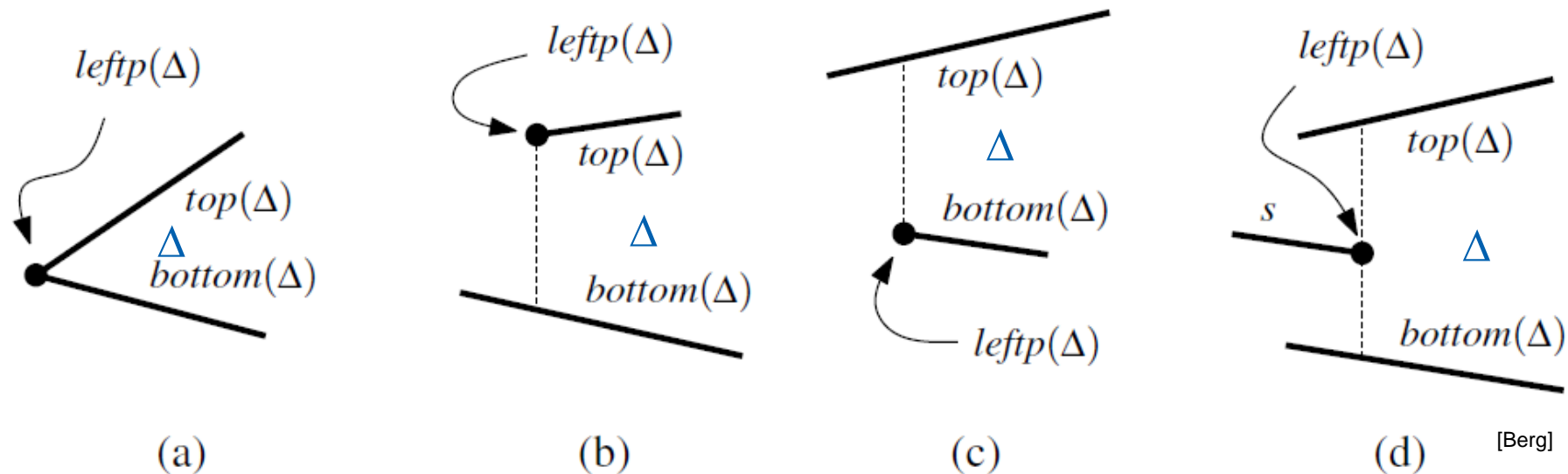
segments:

$top(\Delta)$ - bounds from above

$bottom(\Delta)$ - bounds from below



Vertical sides – left vertical side of Δ



Left vertical side is defined by the segment end-point $p = \text{leftp}(\Delta)$

- (a) common left point p itself
- (b) by the lower vert. extension of left point p ending at $\text{bottom}(\Delta)$
- (c) by the upper vert. extension of left point p ending at $\text{top}(\Delta)$
- (d) by both vert. extensions of the right point p
- (e) the left edge of the bounding rectangle R (leftmost Δ only)



Vertical sides - summary

Vertical edges are defined by segment endpoints

- $leftp(\Delta)$ = the end point defining the left edge of Δ
- $rightp(\Delta)$ = the end point defining the right edge of Δ

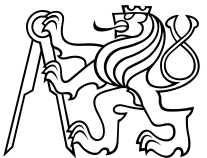
$leftp(\Delta)$ is

- the **left endpoint** of $top()$ or $bottom()$ or both (b, c, a)
- the **right point** of a third segment (d)
- the **lower left corner of** the bounding rectangle **R** (e)



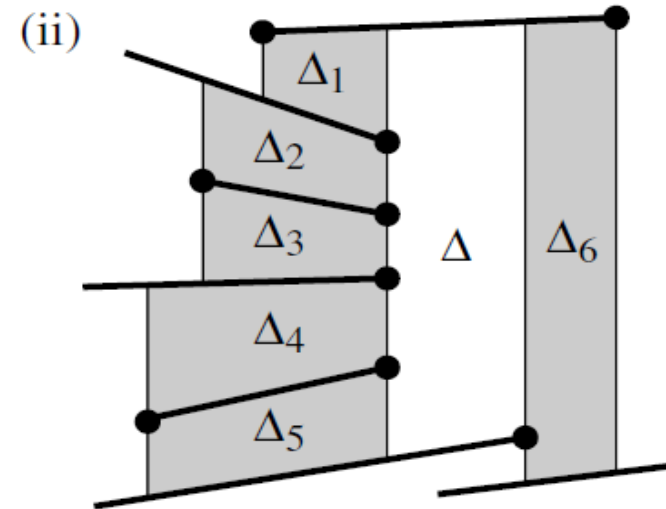
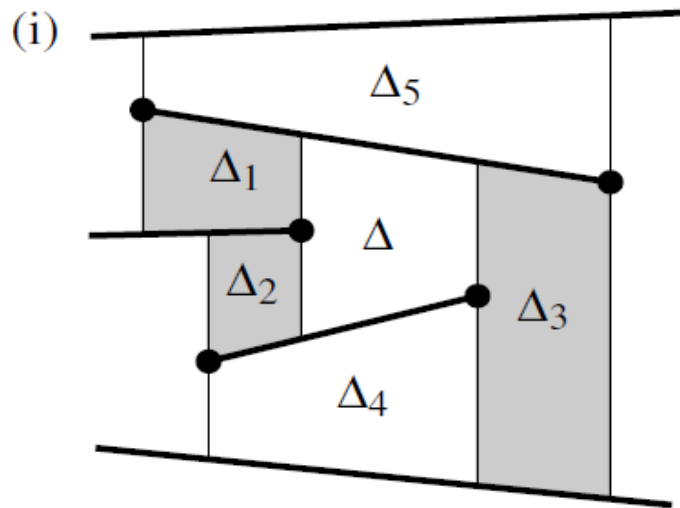
Trapezoid Δ

- Trapezoid Δ is uniquely defined by
 - the segments $top(\Delta)$, $bottom(\Delta)$
 - And by the endpoints $leftp(\Delta)$, $rightp(\Delta)$



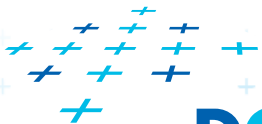
Adjacency of trapezoids segments in general position

- Trapezoids Δ and Δ' are **adjacent**, if they meet along a vertical edge



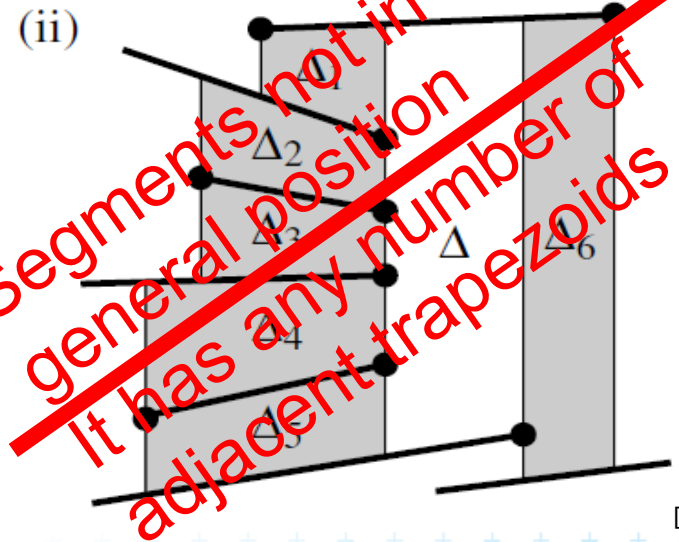
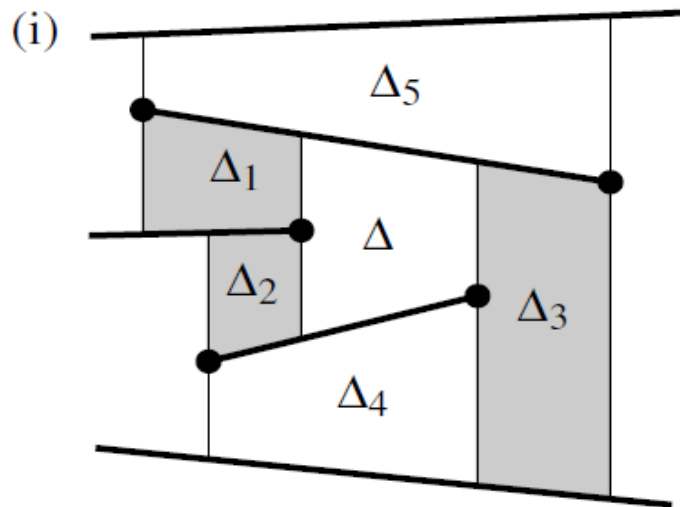
[Berg]

- Δ_1 = upper left neighbor of Δ (common *top*(Δ) edge)
- Δ_2 = lower left neighbor of Δ (common *bottom*(Δ))
- Δ_3 is a right neighbor of Δ (common *top*(Δ) or *bottom*(Δ))



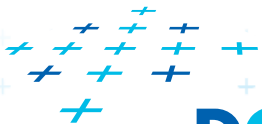
Adjacency of trapezoids segments in general position

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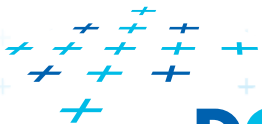
- Δ_1 = upper left neighbor of Δ (common $top(\Delta)$ edge)
- Δ_2 = lower left neighbor of Δ (common $bottom(\Delta)$)
- Δ_3 is a right neighbor of Δ (common $top(\Delta)$ or $bottom(\Delta)$)



Representation of the trapezoidal map T

Special trapezoidal map structure $T(S)$ stores:

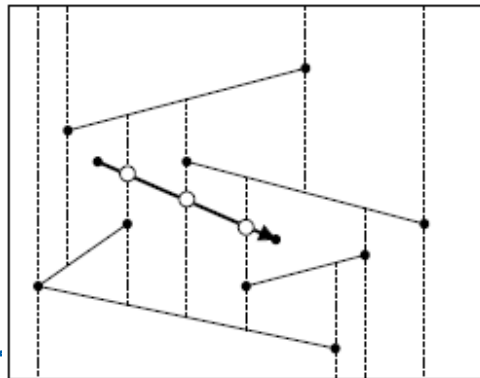
- Records for all **line segments** and **end points**
- Records for each **trapezoid** $\Delta \in T(S)$
 - Definition of Δ - pointers to segments $top(\Delta)$, $bottom(\Delta)$,
- pointers to points $leftp(\Delta)$, $rightp(\Delta)$
 - Pointers to its max **four neighboring trapezoids**
 - Pointer to the **leaf \boxtimes in the search structure D** (see below)
- Does not store the geometry explicitly!
- Geometry of trapezoids is computed in $O(1)$



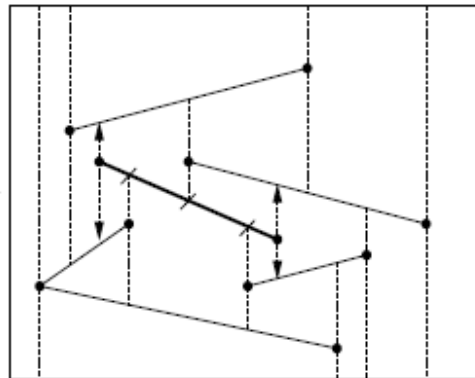
Construction of trapezoidal map

■ Randomized incremental algorithm

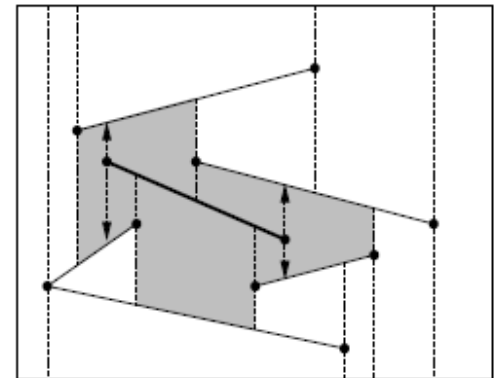
1. Create the initial bounding rectangle ($T_0 = 1\Delta$) ... $O(n)$
2. Randomize the order of segments in S
3. for $i = 1$ to n do
4. Add segment S_i to trapezoidal map T_i
5. locate left endpoint of S_i in $T_{i-1} \Rightarrow$ start trapezoid
6. find intersected trapezoids
7. shoot 4 bullets from endpoints of $S_i \Rightarrow$ create new trapezoids
8. trim intersected vertical bullet paths



Locate left endpoint and determine intersections



Shoot new bullet paths and trim intersecting rays



Newly created trapezoids

[Mount]

Trapezoidal map point location

- While creating the trapezoidal map T construct the *Point location data structure* D
- Query this data structure



Point location data structure D

- Rooted directed **acyclic graph** (not a tree!!)

- Leaves \square – trapezoids, each appears exactly once
- Internal nodes – 2 outgoing edges, guide the search

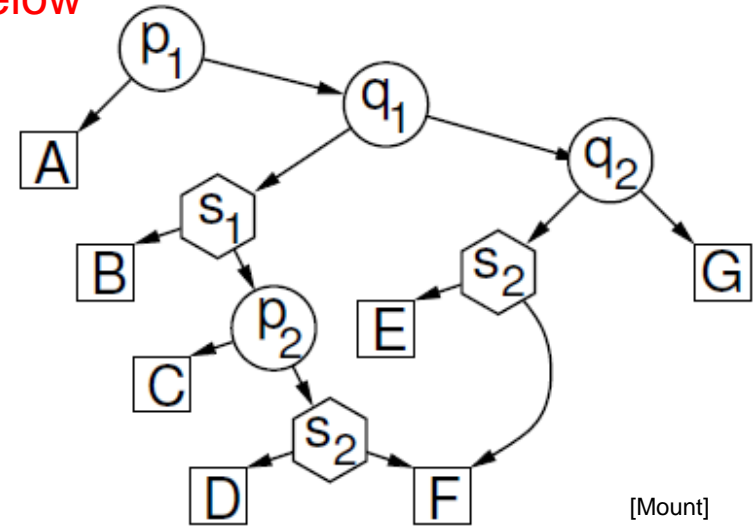
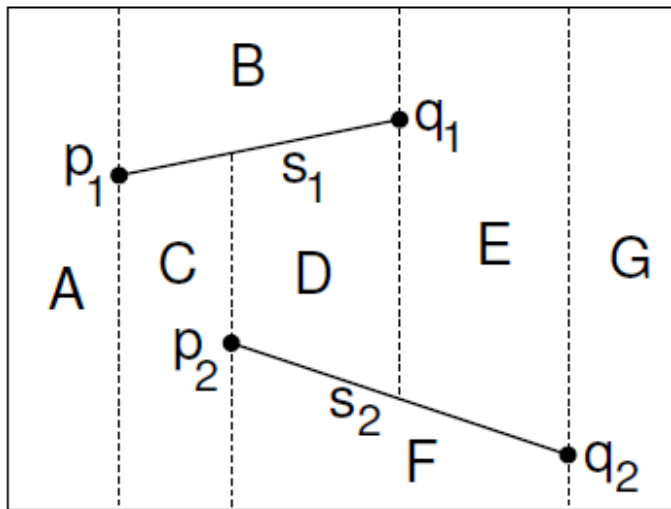
$\circledast p_1$ x-node – x-coord x_0 of segment start- or end-point

left child lies left of vertical line $x=x_0$

right child lies right of vertical line $x=x_0$

– used first to detect the vertical slab

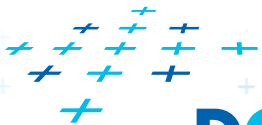
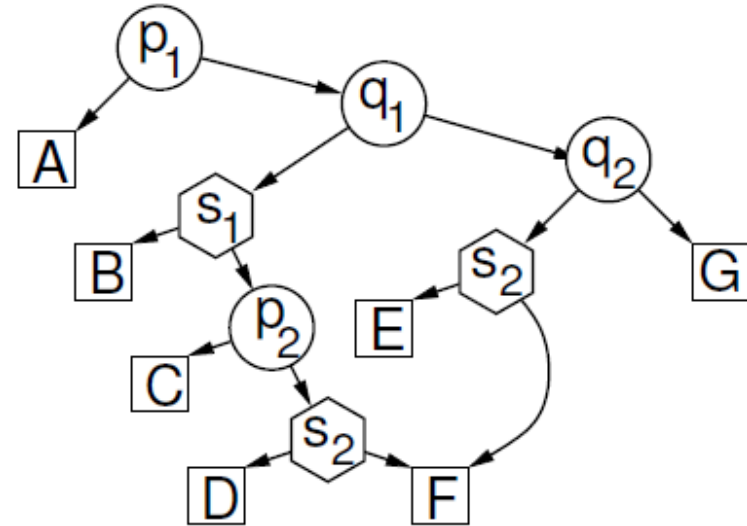
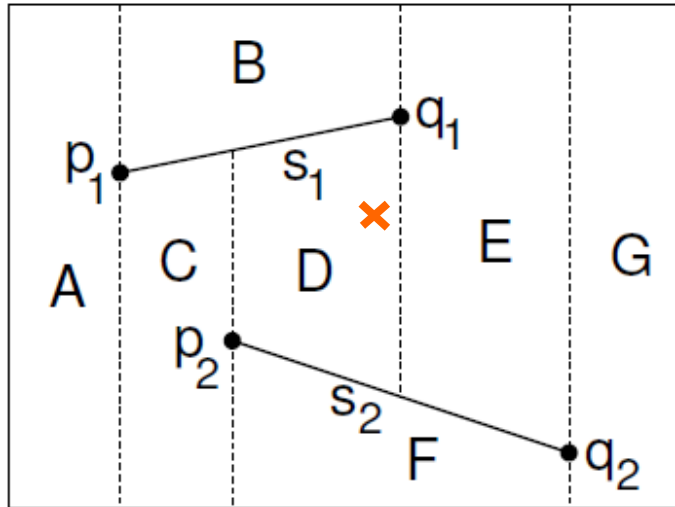
$\hexagon s_1$ y-node – pointer to the line segment of the subdivision (not only its y!!!)
left – above, right – below



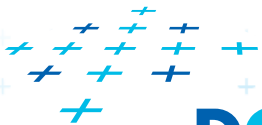
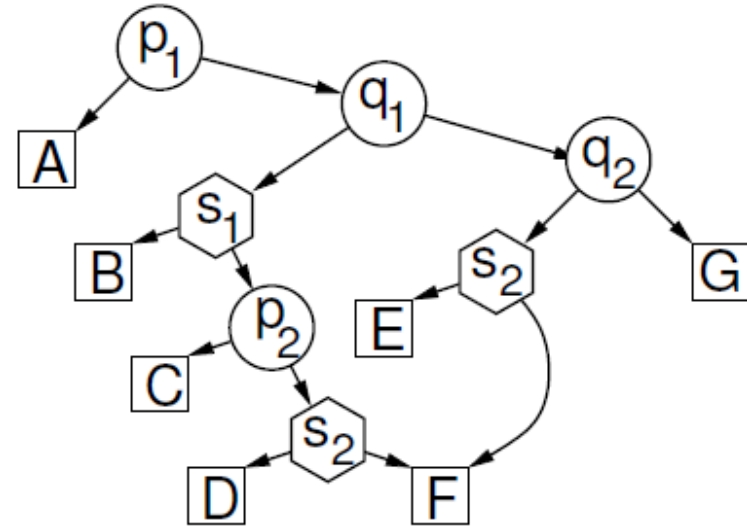
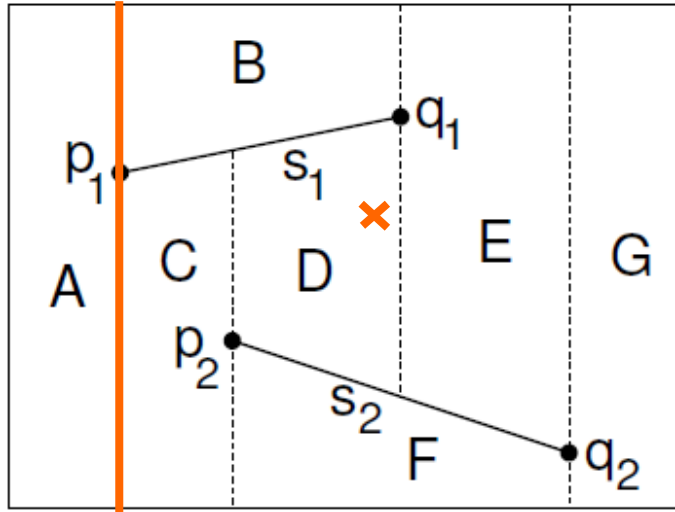
[Mount]



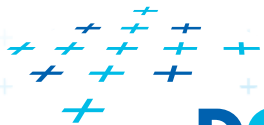
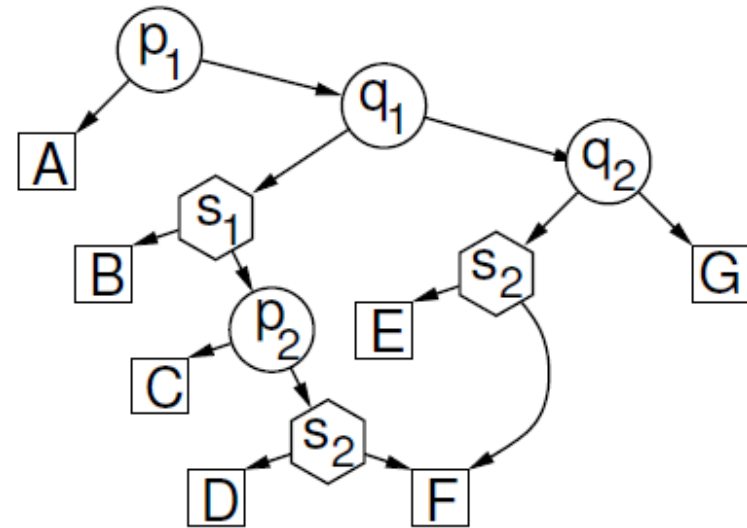
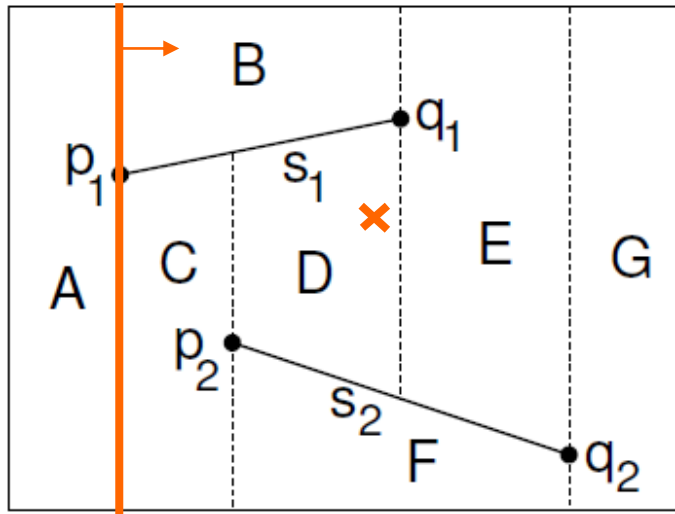
TM search example



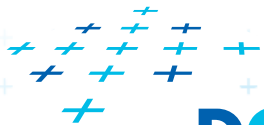
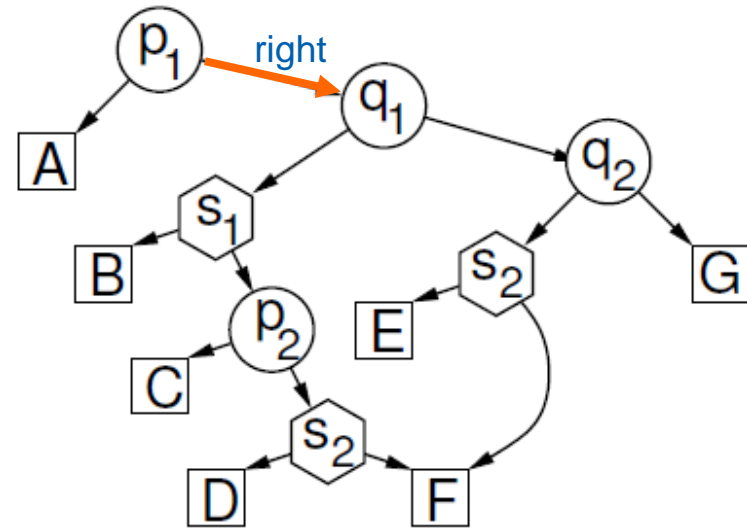
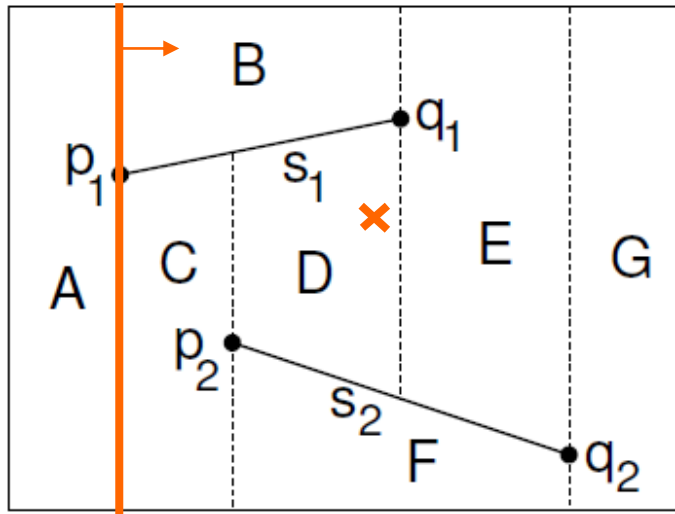
TM search example



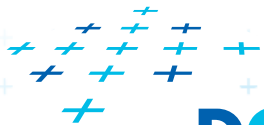
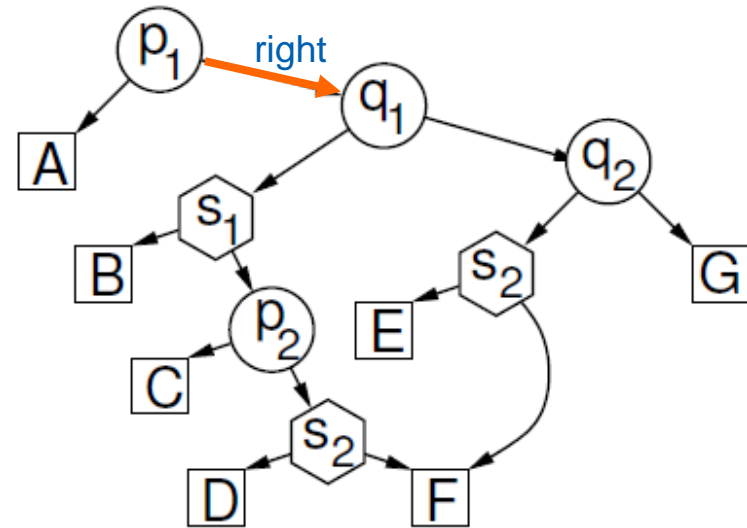
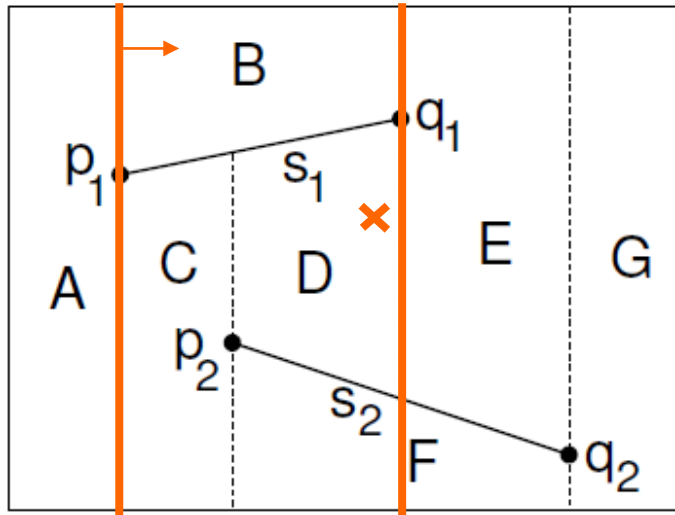
TM search example



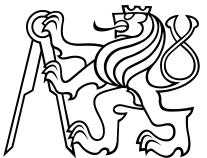
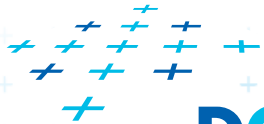
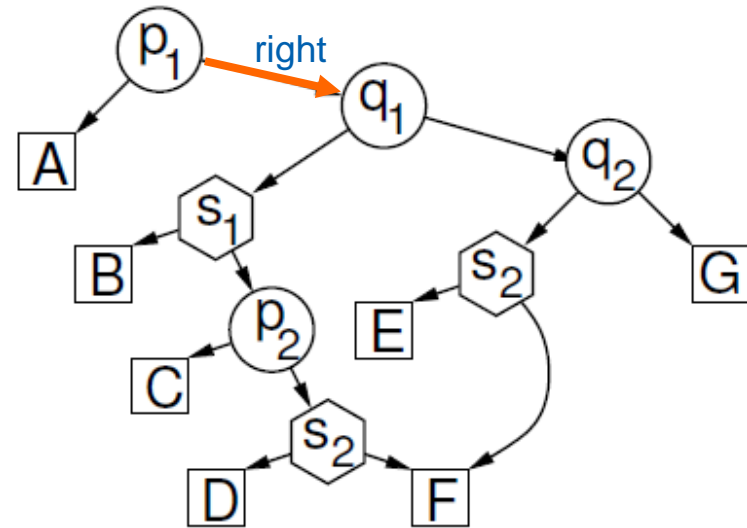
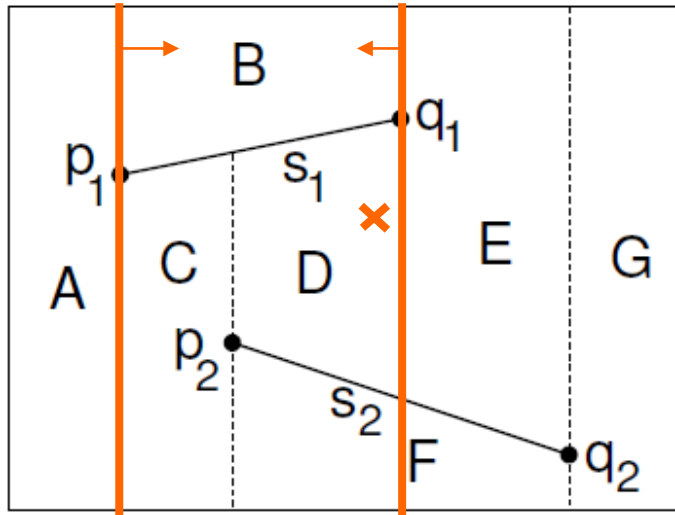
TM search example



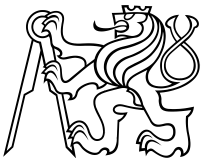
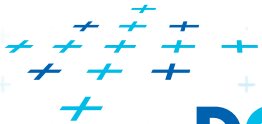
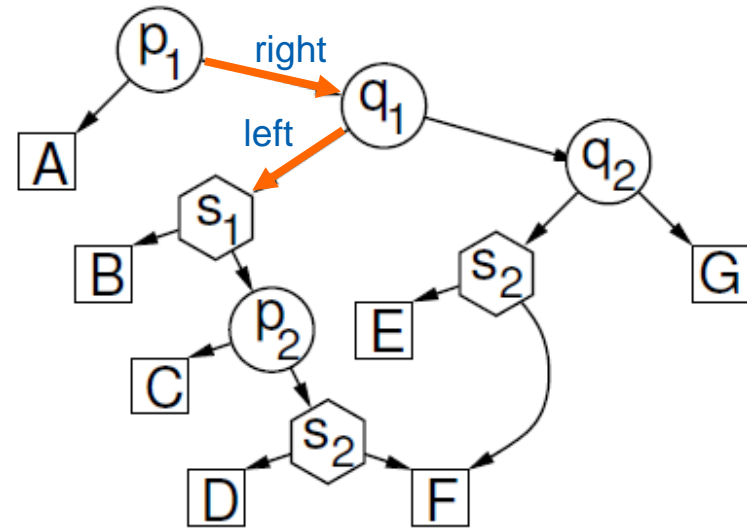
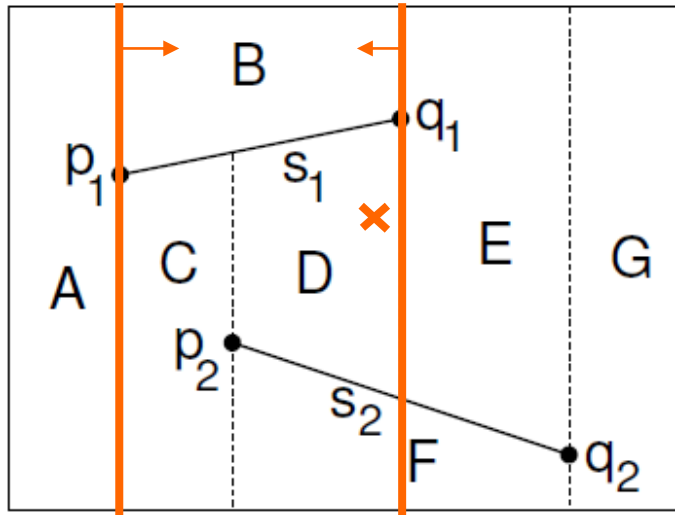
TM search example



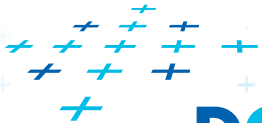
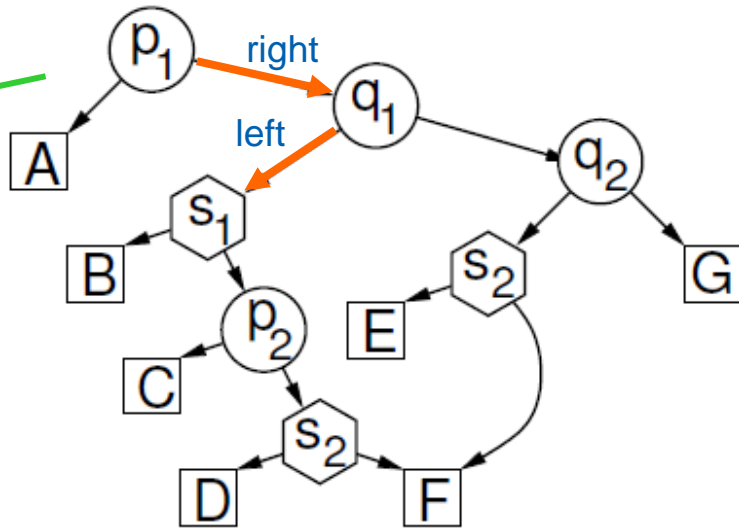
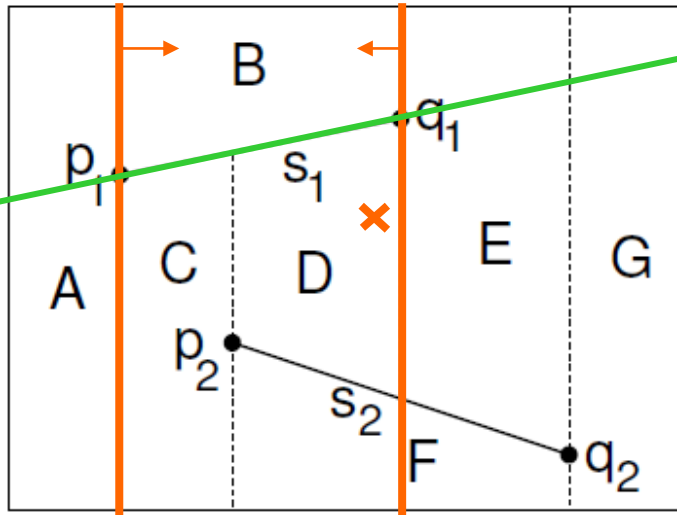
TM search example



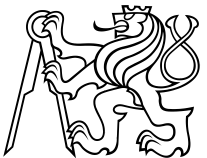
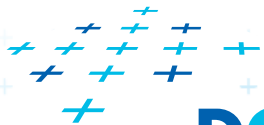
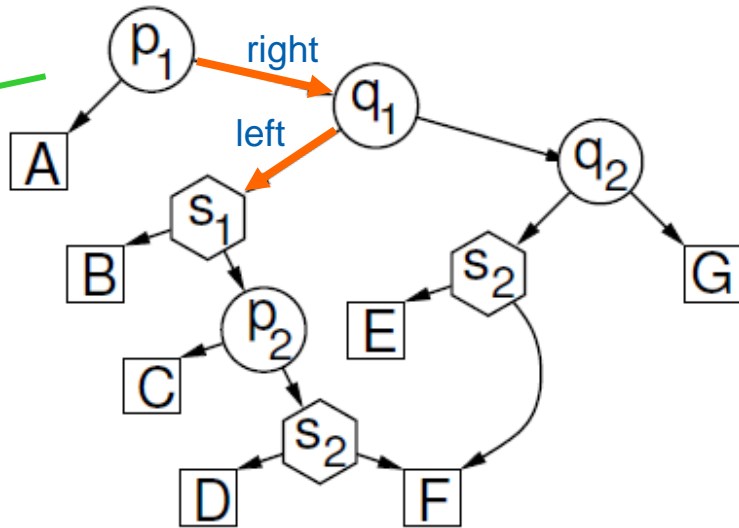
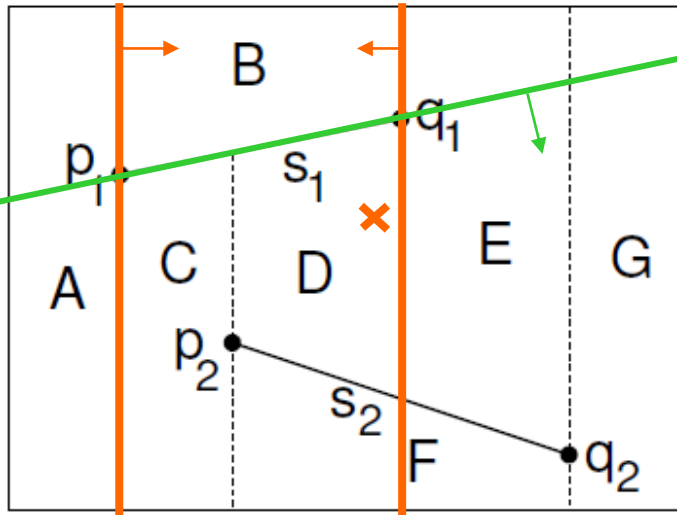
TM search example



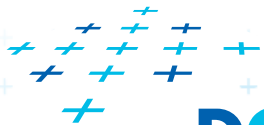
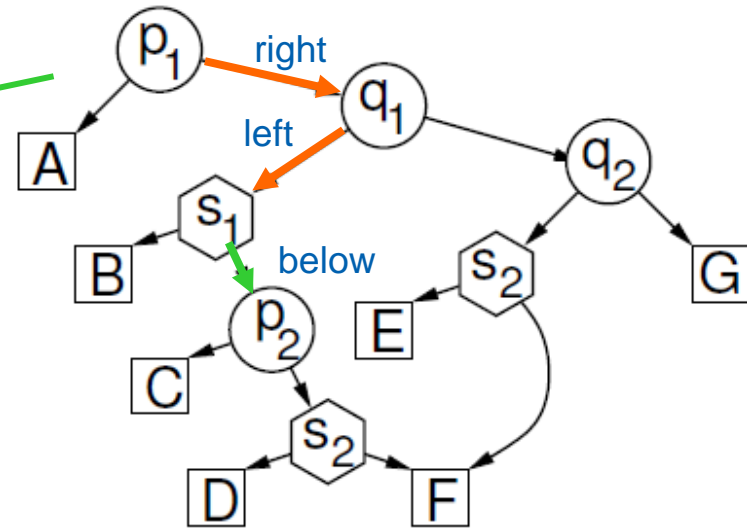
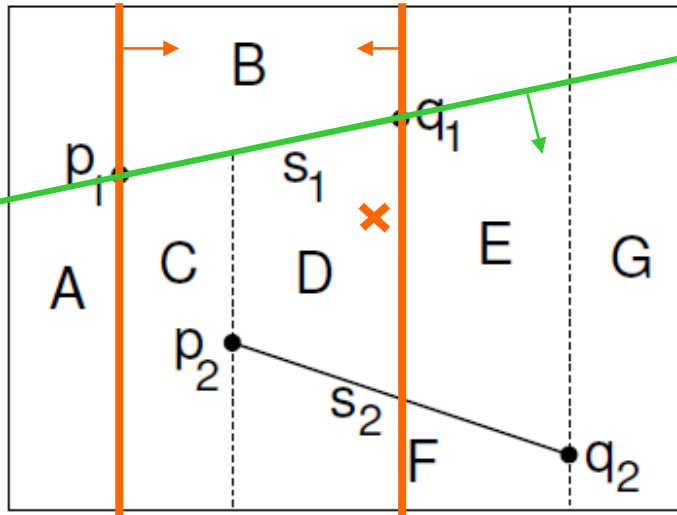
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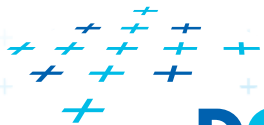
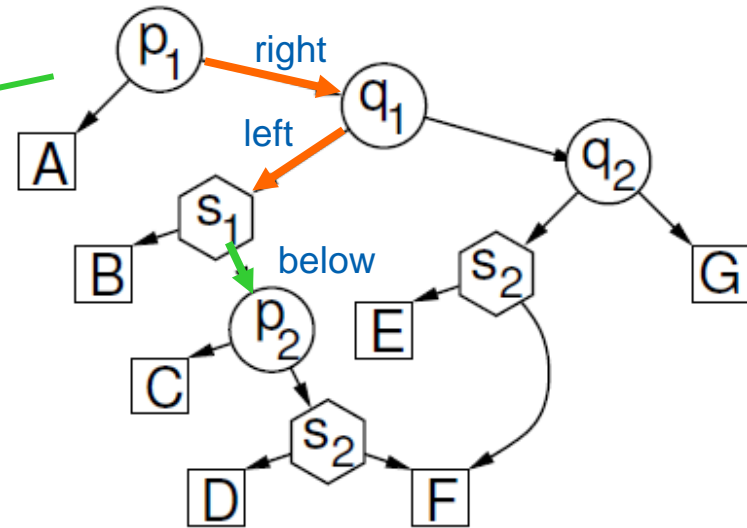
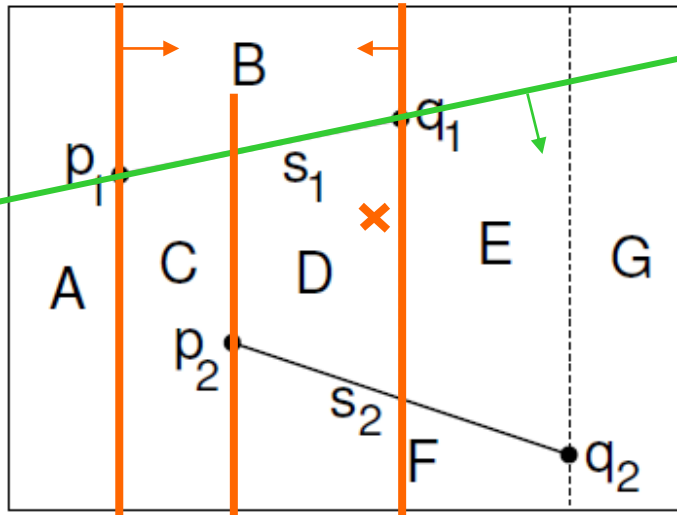
TM search example



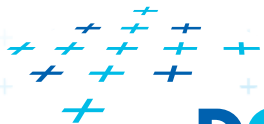
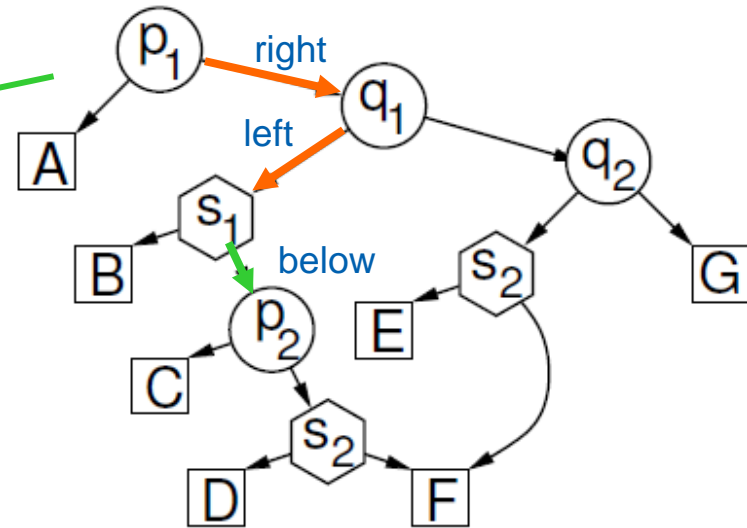
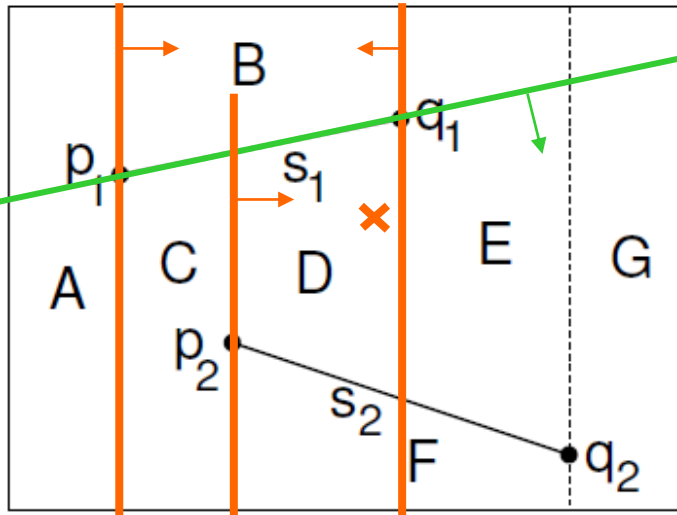
TM search example



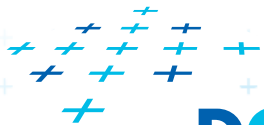
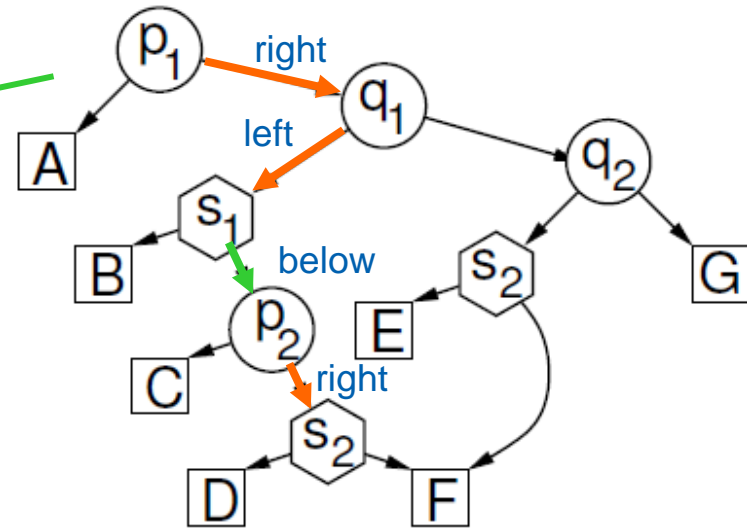
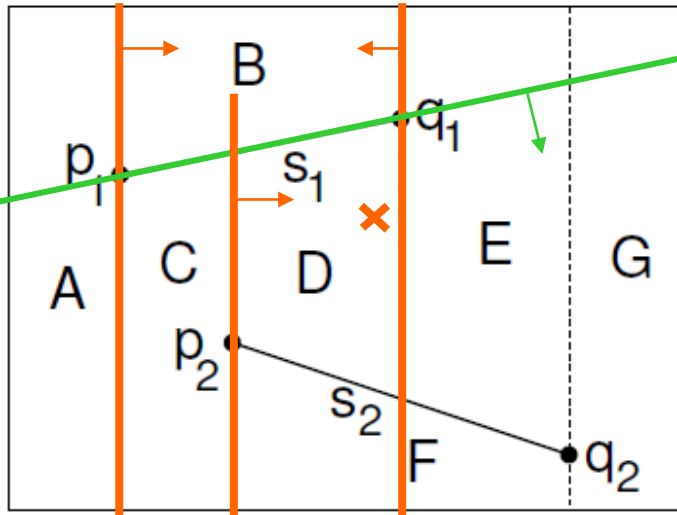
TM search example



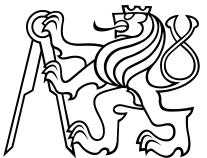
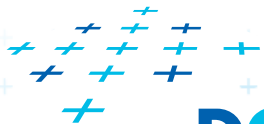
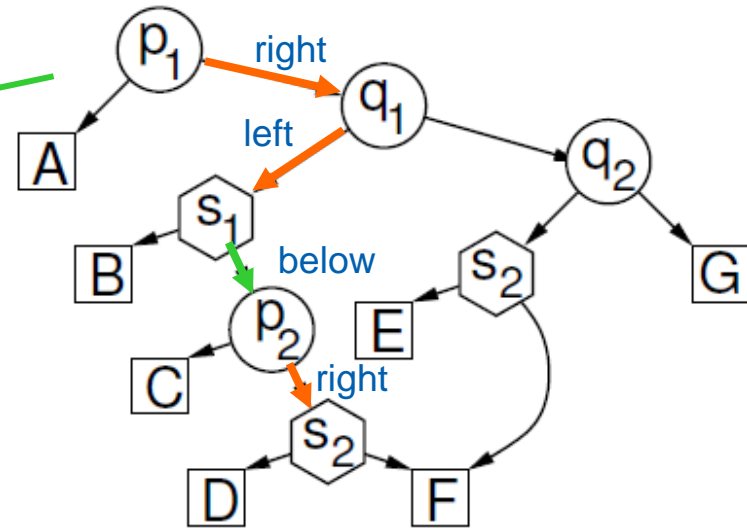
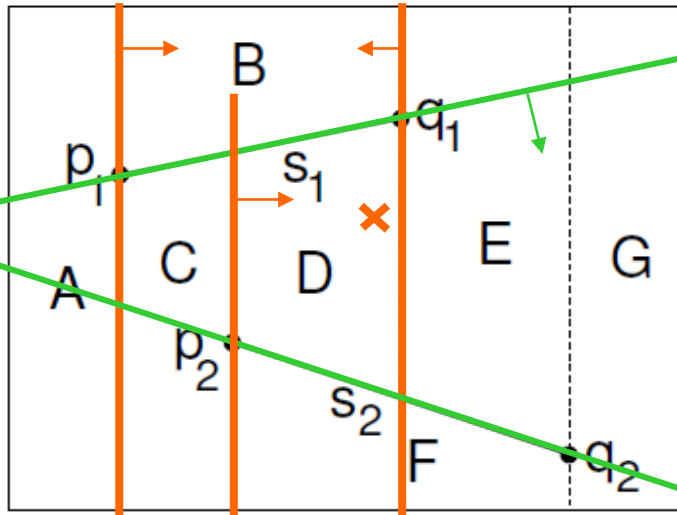
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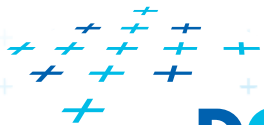
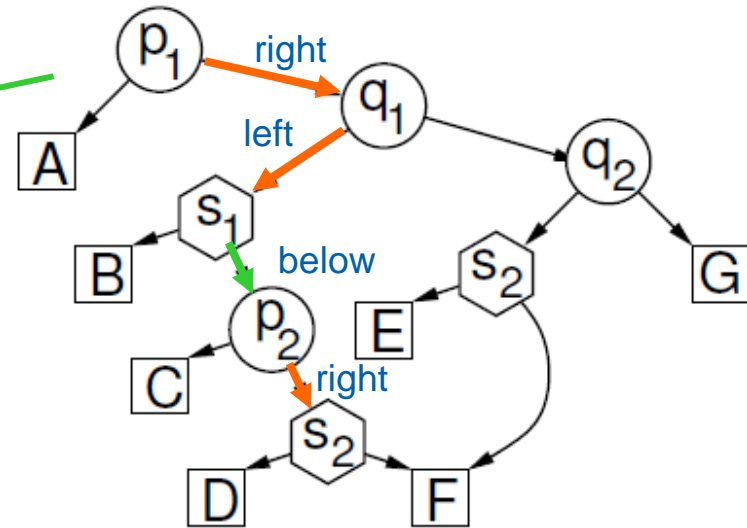
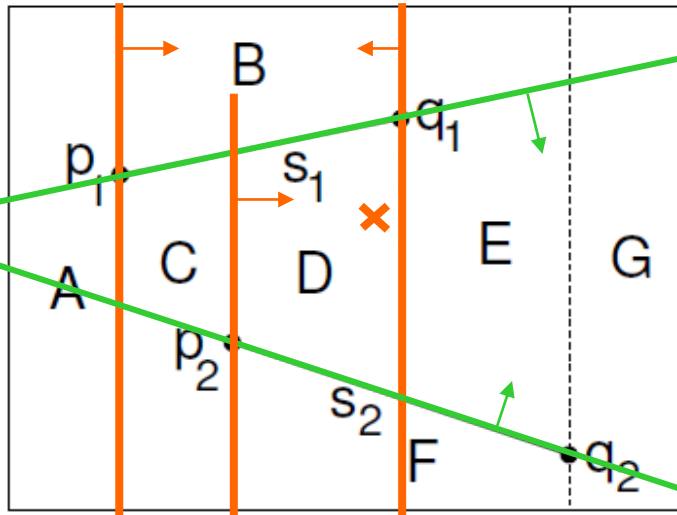
TM search example



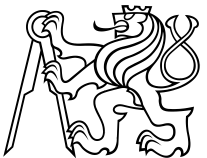
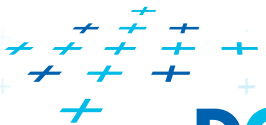
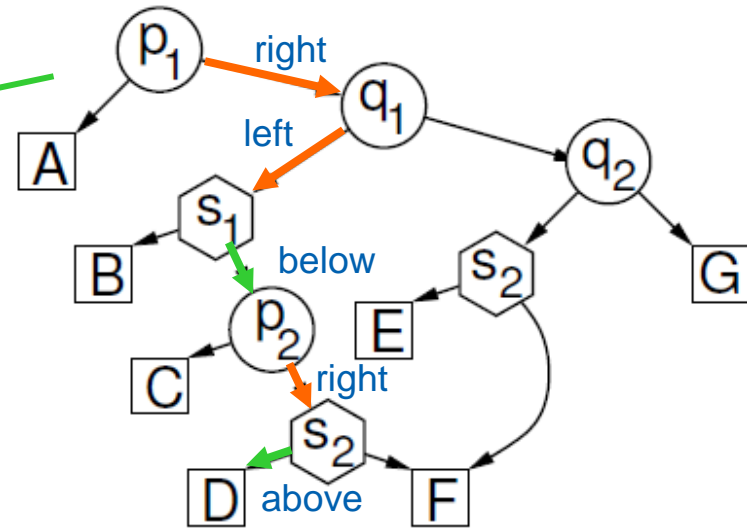
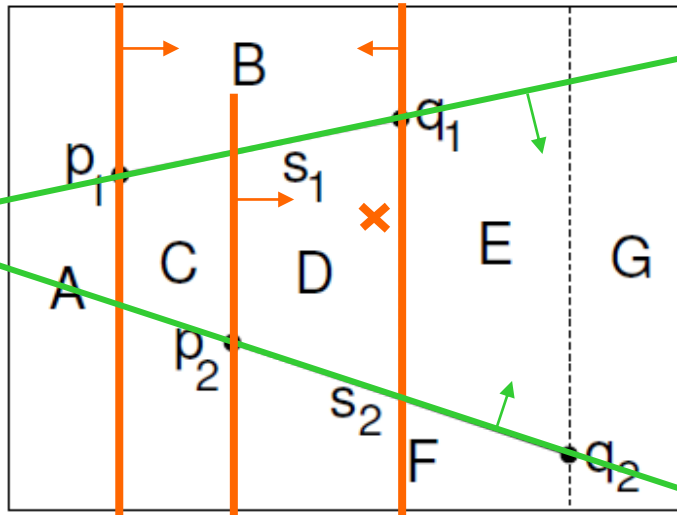
TM search example



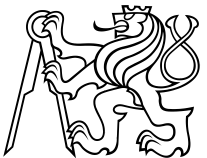
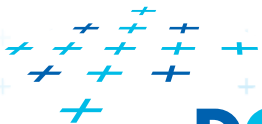
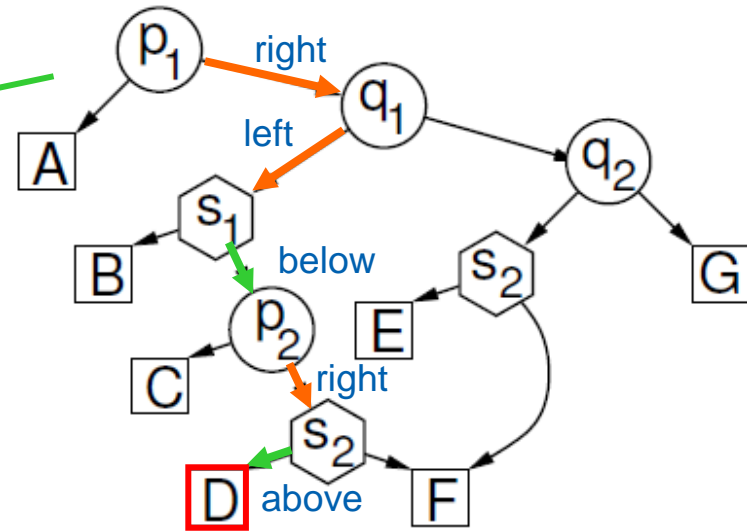
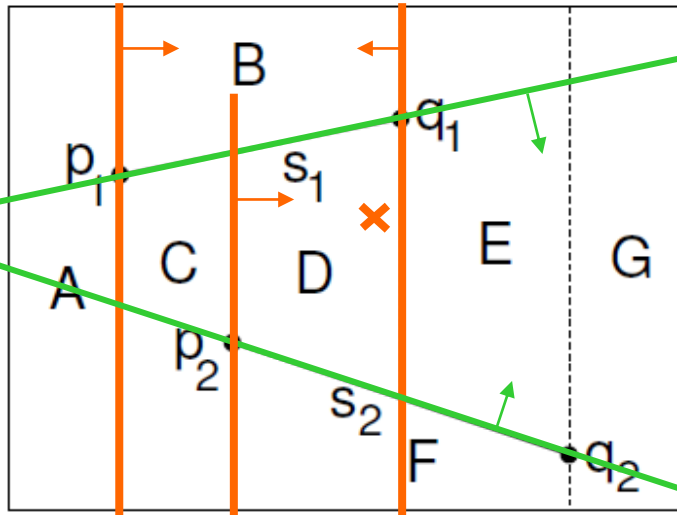
TM search example



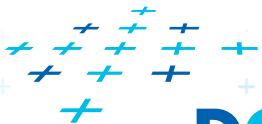
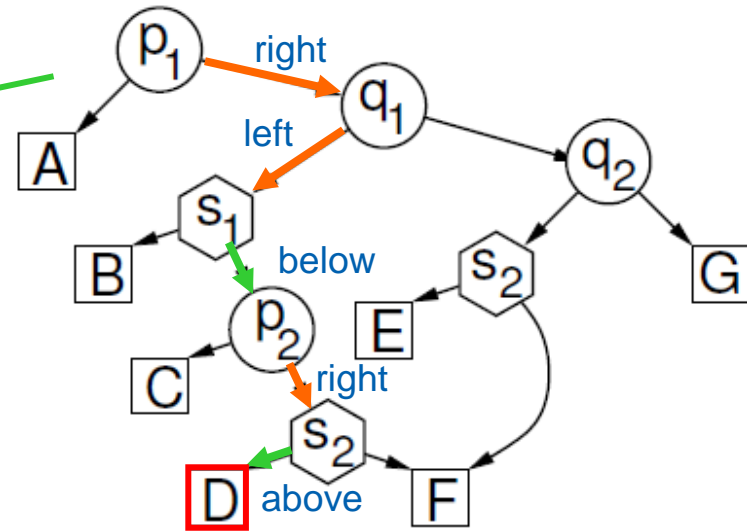
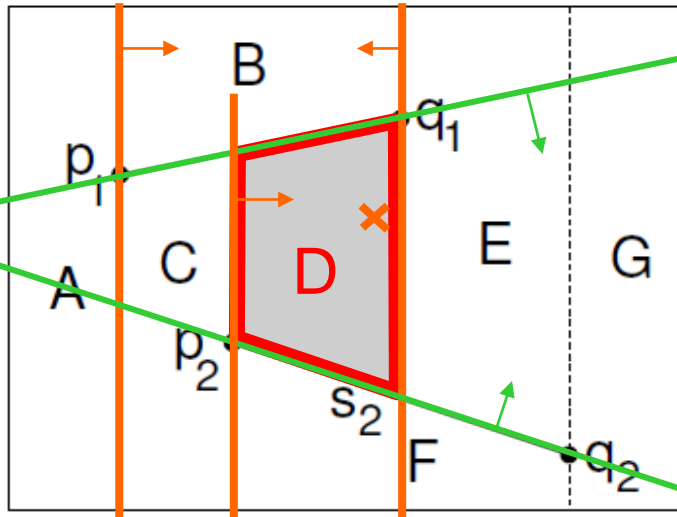
TM search example



TM search example

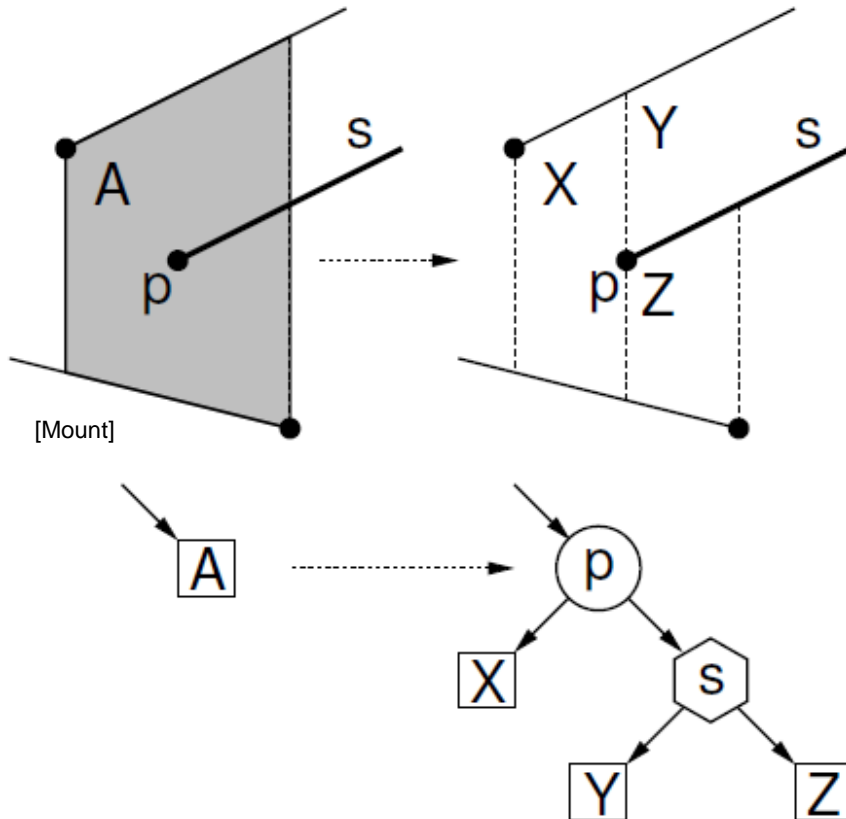


TM search example



Construction – addition of a segment

a) Single (left or right) endpoint - 3 new trapezoids



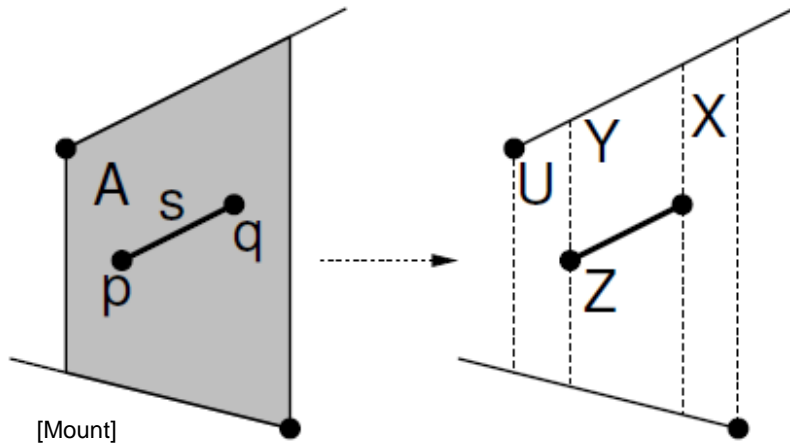
Trapezoid A replaced by

- * x-node for point p
- add left leaf for $X \Delta$
- add right subtree
- * y-node for segment s
- add left leaf for $Y \Delta$ above
- add right leaf $Z \Delta$ below



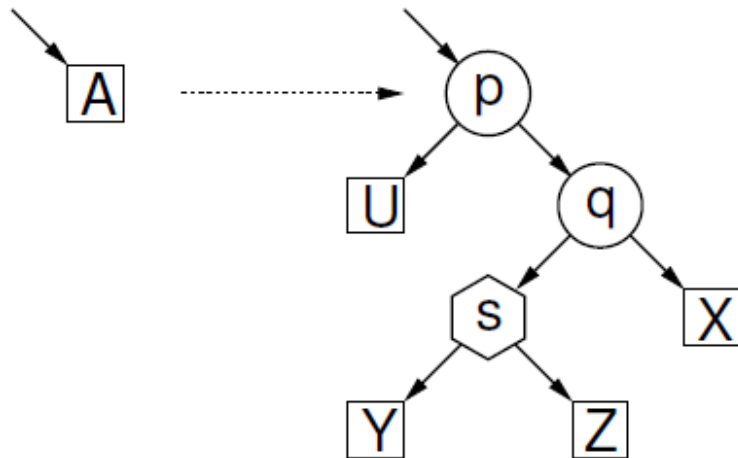
Construction – addition of a segment

b) Two segment endpoints – 4 new trapezoids



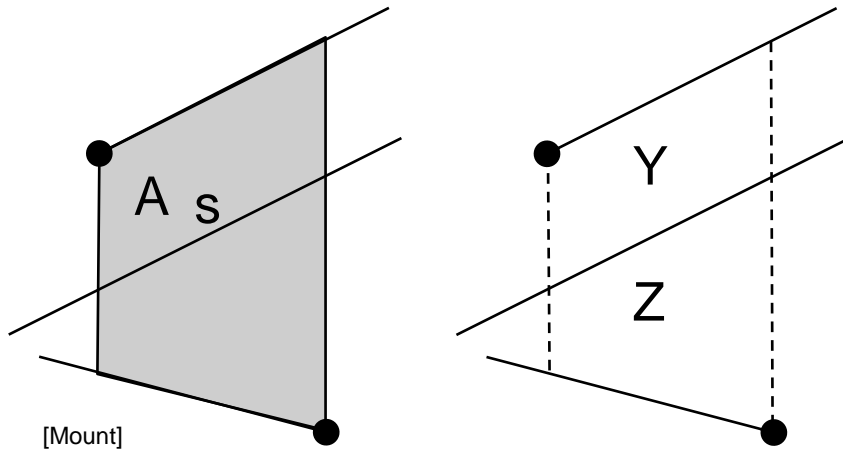
Trapezoid A replaced by

- * x-node for point p
- * x-node for point q
- * y-node for segment s
- add leaves for U, X, Y, Z



Construction – addition of a segment

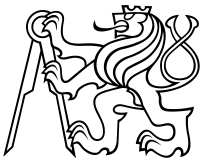
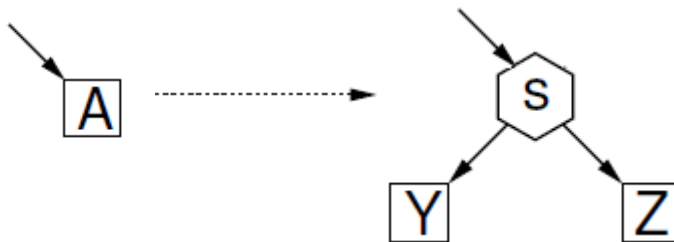
c) No segment endpoint – create 2 trapezoids



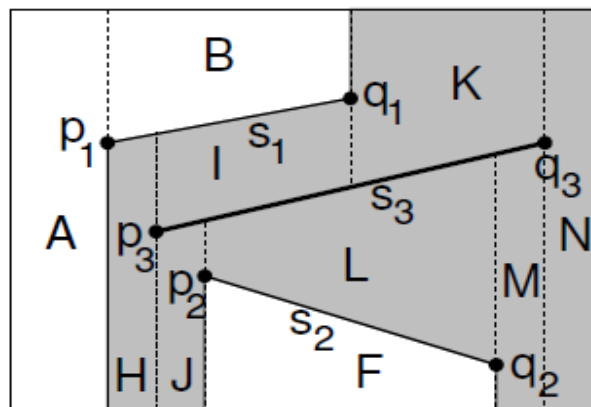
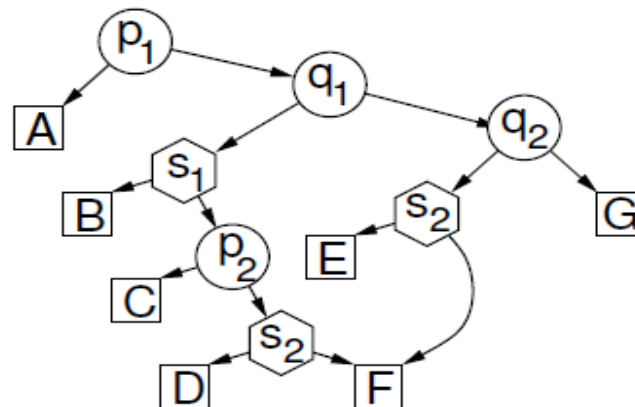
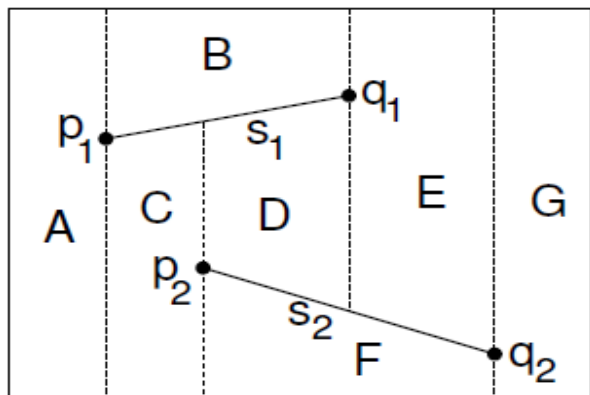
[Mount]

Trapezoid A replaced by

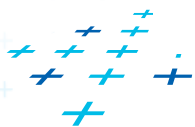
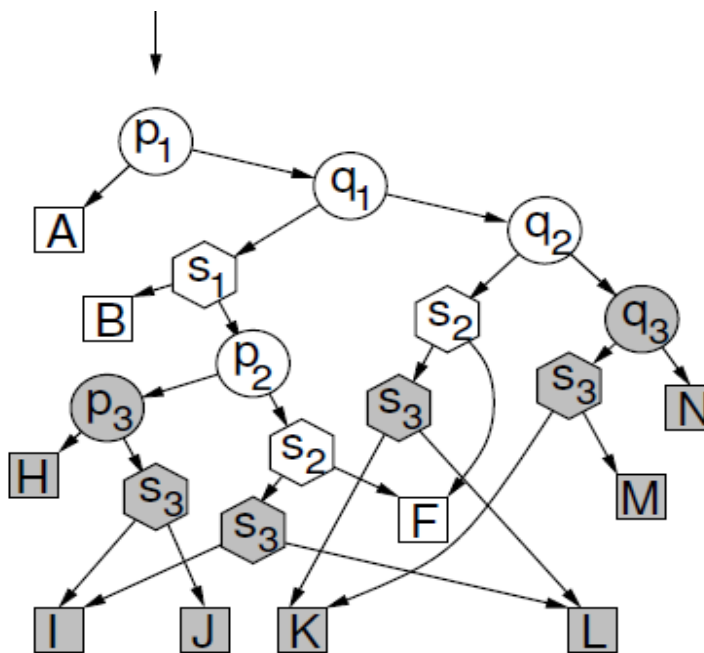
- * y-node for segment s
- add leaves for Y, Z



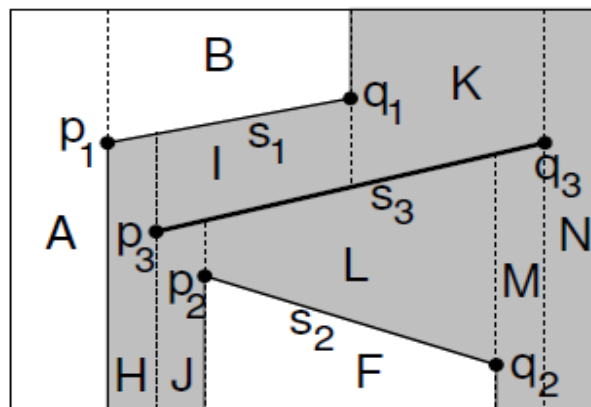
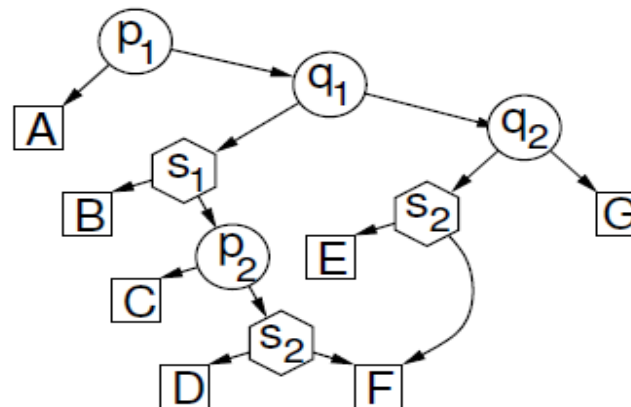
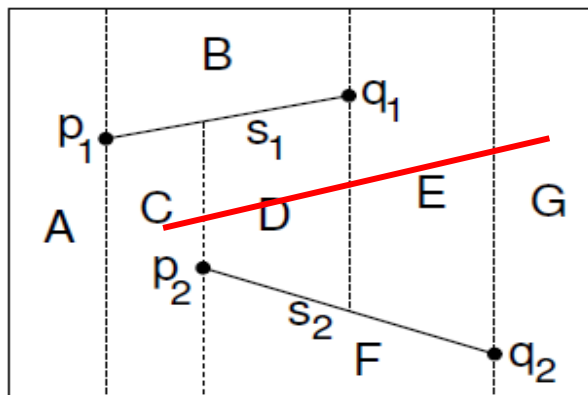
Segment insertion example



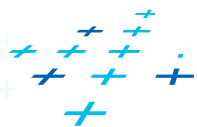
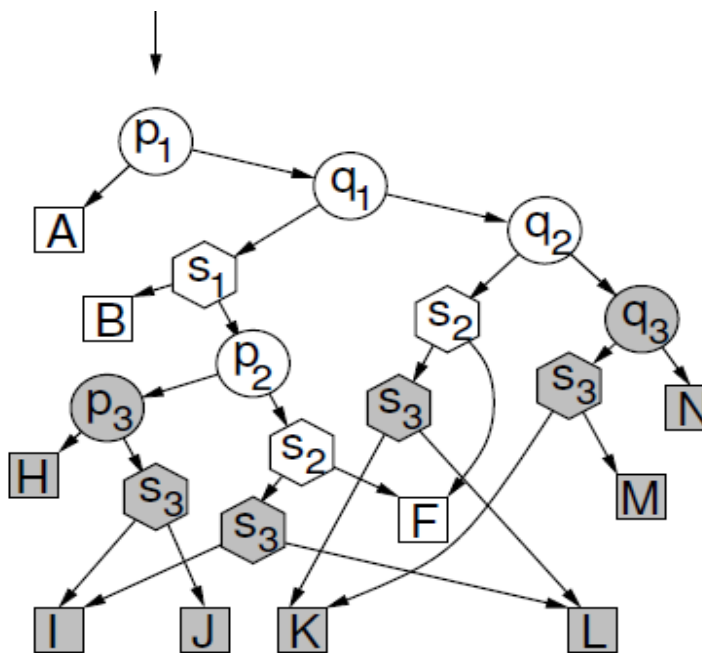
[Mount]



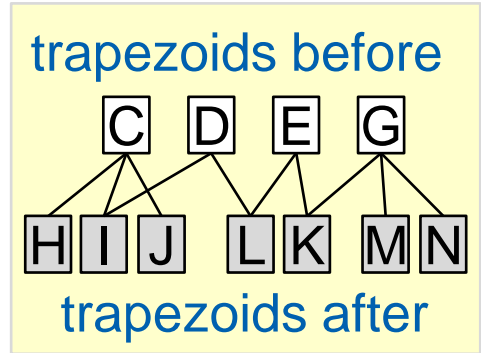
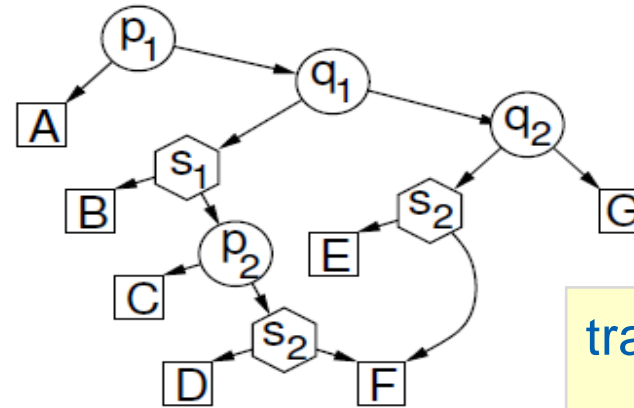
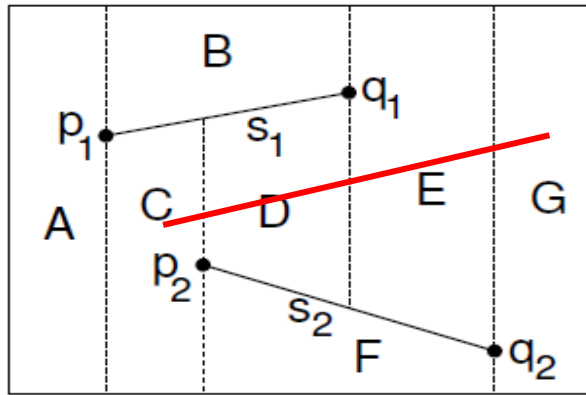
Segment insertion example



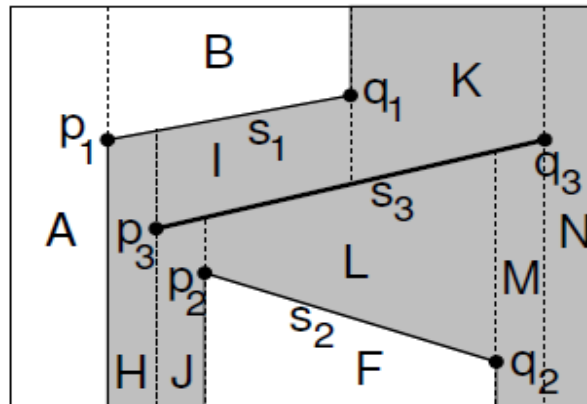
[Mount]



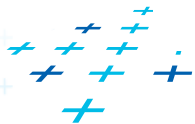
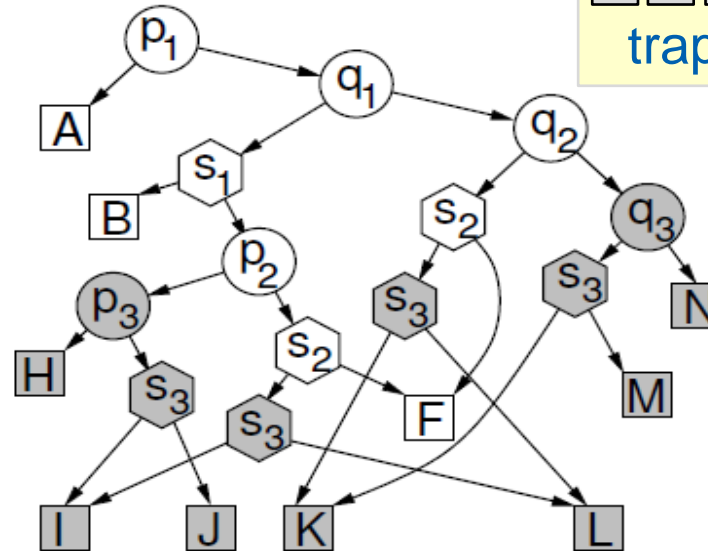
Segment insertion example



4 → 7

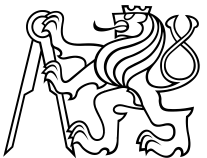


[Mount]



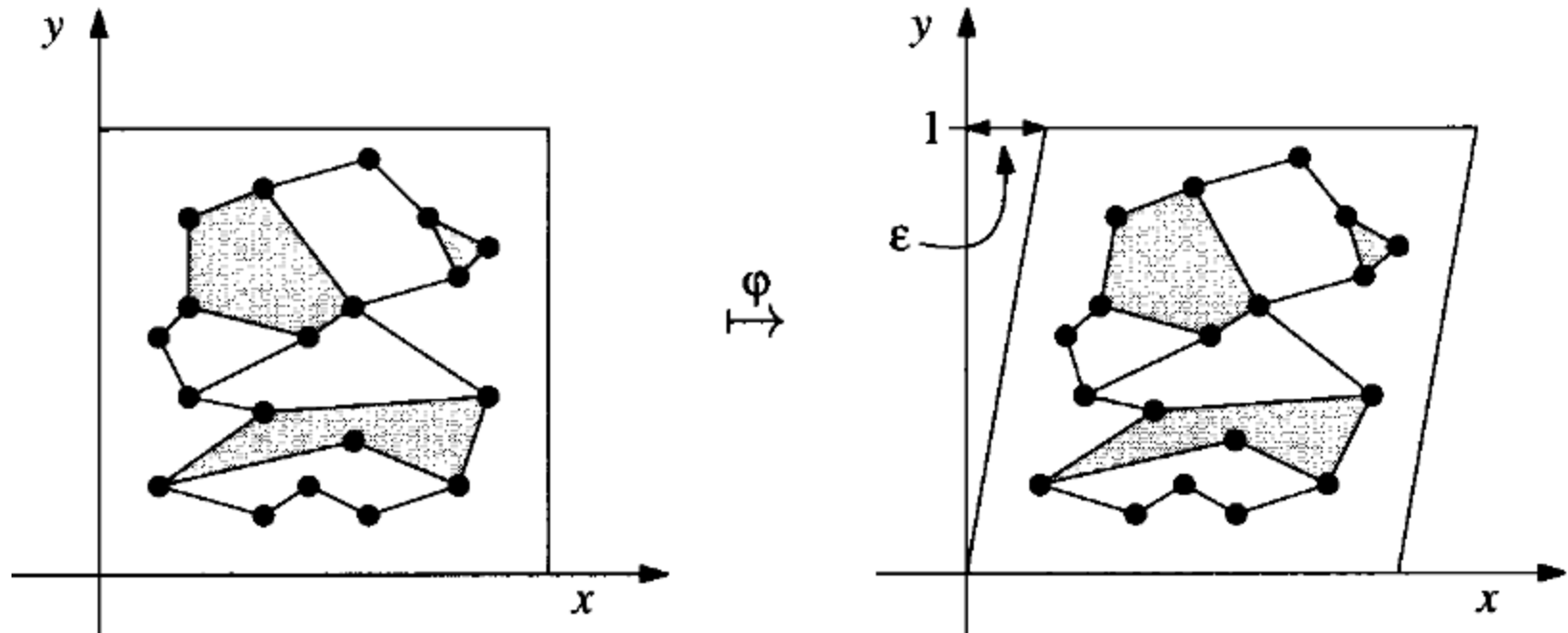
Analysis and proofs

- This holds:
 - Number of newly created Δ for inserted segment $O(1)$ (some added, some removed)
 - Search structure size is max $O(n^2)$, but $O(n)$ expected
 - Search point $O(\log n)$ in average
=> Expected construction $O(n(1 + \log n)) = O(n \log n)$
- For detailed analysis and proofs see
 - [Berg] or [Mount]

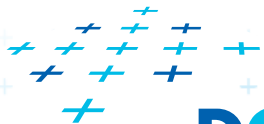


Handling of degenerate cases - principle

- No distinct endpoints lie on common vertical line
 - Rotate or shear the coordinates $x' = x + \varepsilon y$, $y' = y$



[Berg]



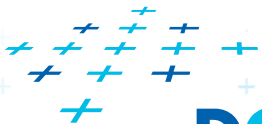
DCGI



Handling of degenerate cases - realization

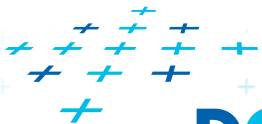
■ Trick

- store original (x, y) , not the sheared x', y'
- we need to perform just 2 operations:
 1. For two points p, q determine if transformed point q is to the **left**, to the **right** or **on** vertical line through point p
 - If $x_p = x_q$ then compare y_p and y_q (on only for $y_p = y_q$)
 - => use the original coords (x, y) and **lexicographic order**
 2. For segment given by two points decide if **3rd point q lies above, below, or on the segment $p_1 p_2$**
 - Mapping preserves this relation
 - => use the original coords (x, y)



Point location summary

- **Slab method** [Dobkin and Lipton, 1976]
 - $O(n^2)$ memory $O(\log n)$ time
- **Monotone chain tree in planar subdivision** [Lee and Preparata, 77]
 - $O(n^2)$ memory $O(\log^2 n)$ time
- **Layered directed acyclic graph (Layered DAG) in planar subdivision** [Chazelle, Guibas, 1986] [Edelsbrunner, Guibas, and Stolfi, 1986]
 - $O(n)$ memory $O(\log n)$ time => optimal algorithm of planar subdivision search (optimal but complex alg. => see elsewhere)
- **Trapezoidal map**
 - $O(n)$ expected memory $O(\log n)$ expected time
 - $O(n \log n)$ expected preprocessing (simple alg.)



References

- **[Berg]** Mark de Berg, Otfried Cheong, Marc van Kreveld, Mark Overmars: **Computational Geometry: *Algorithms and Applications***, Springer-Verlag, 3rd rev. ed. 2008. 386 pages, 370 fig. ISBN: 978-3-540-77973-5
<http://www.cs.uu.nl/geobook/>
- **[Mount]** Mount, D.: **Computational Geometry Lecture Notes for Fall 2016**, University of Maryland, Lectures 9, 10
<http://www.cs.umd.edu/class/fall2016/cmsc754/Lects/cmsc754-fall16-lects.pdf>

