ALG 09

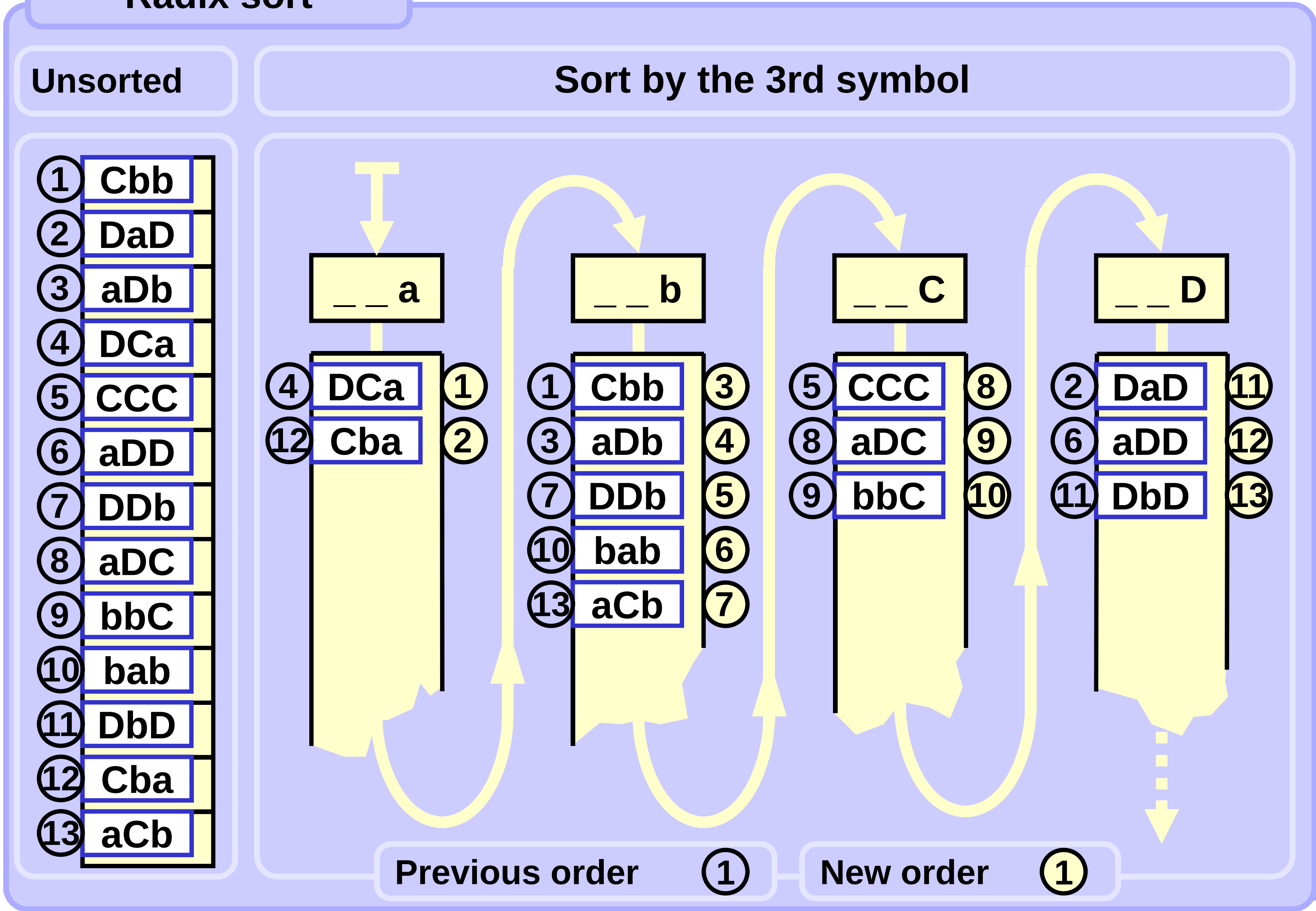
Radix sort

Counting sort

Overview of sorts asymptotic complexities

Sorting experiment

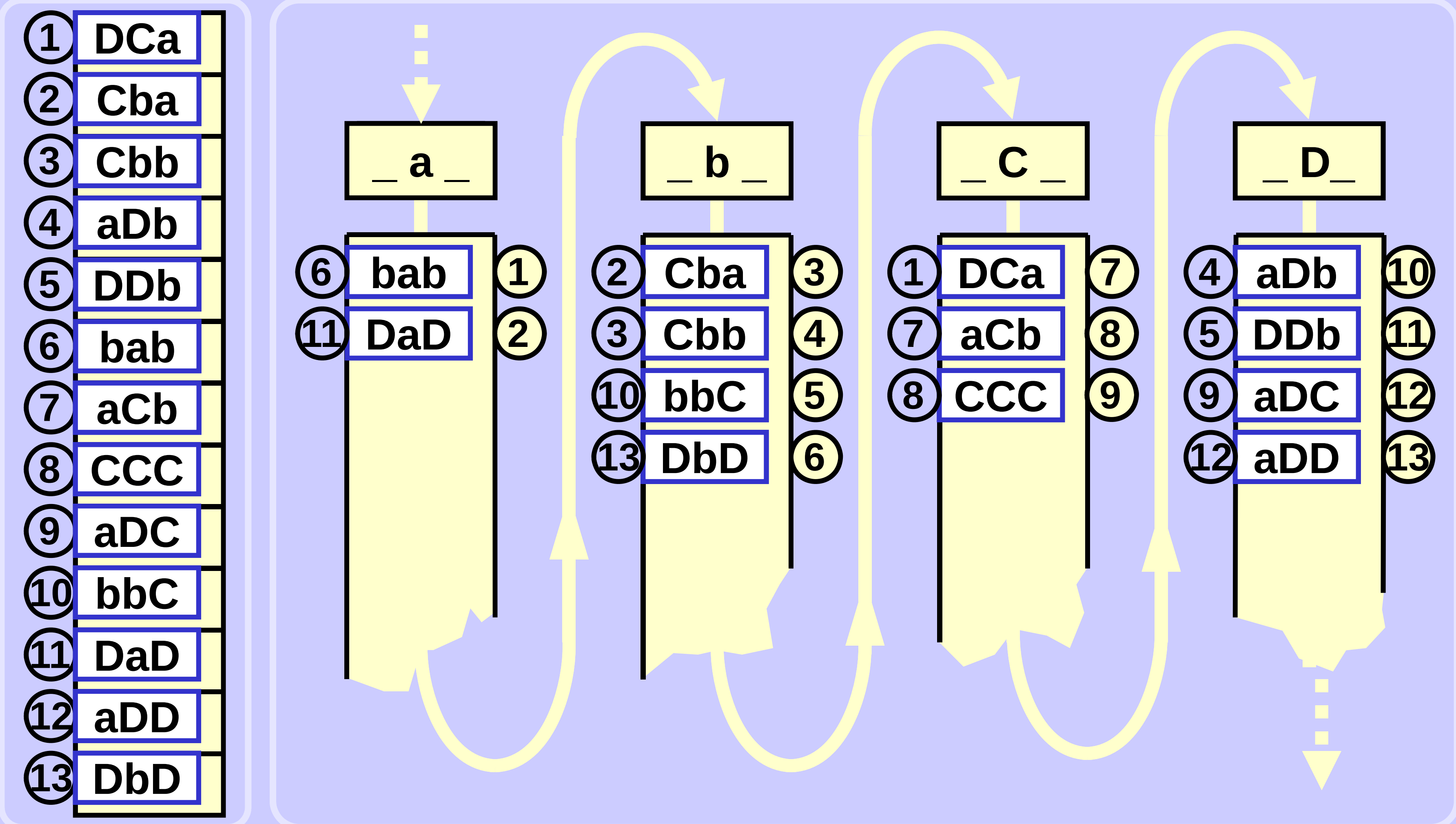
Radix sort


Unsorted
Sort by the 3rd symbol
1 Cbb
2 DaD
3 aDb
4 DCa
5 CCC
6 aDD
7 DDb
8 aDC
9 bbC
10 bab
11 DbD
12 Cba
13 aCb
_ _ a
4 DCa 1
12 Cba 2
_ _ b
1 Cbb 3
3 aDb 4
7 DDb 5
10 bab 6
13 aCb 7
_ _ C
5 CCC 8
8 aDC 9
9 bbC 10
_ _ D
2 DaD 11
6 aDD 12
11 DbD 13
Previous order 1
New order 1

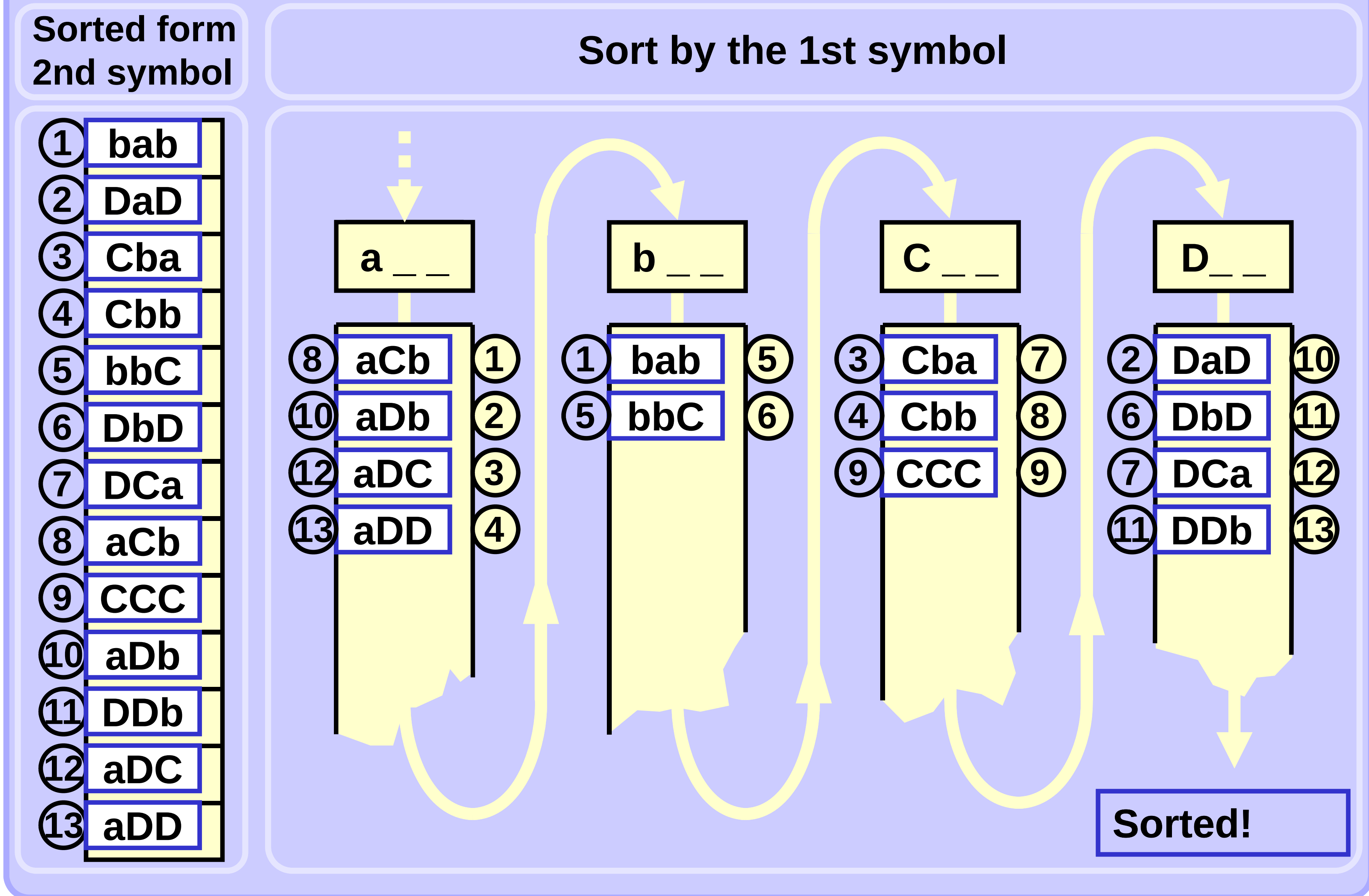
Radix sort

Sorted from 3rd symbol

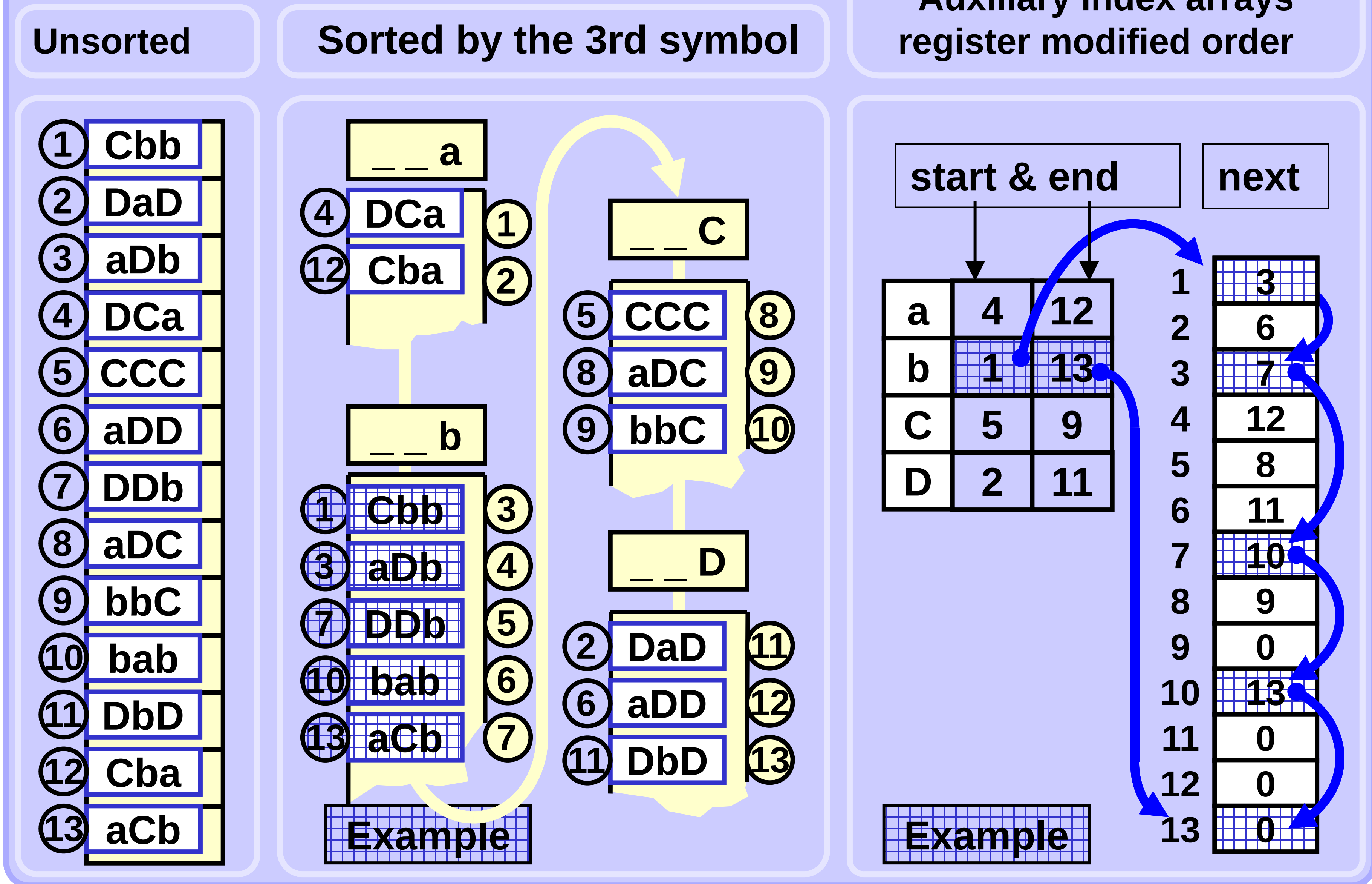
Sort by the 2nd symbol



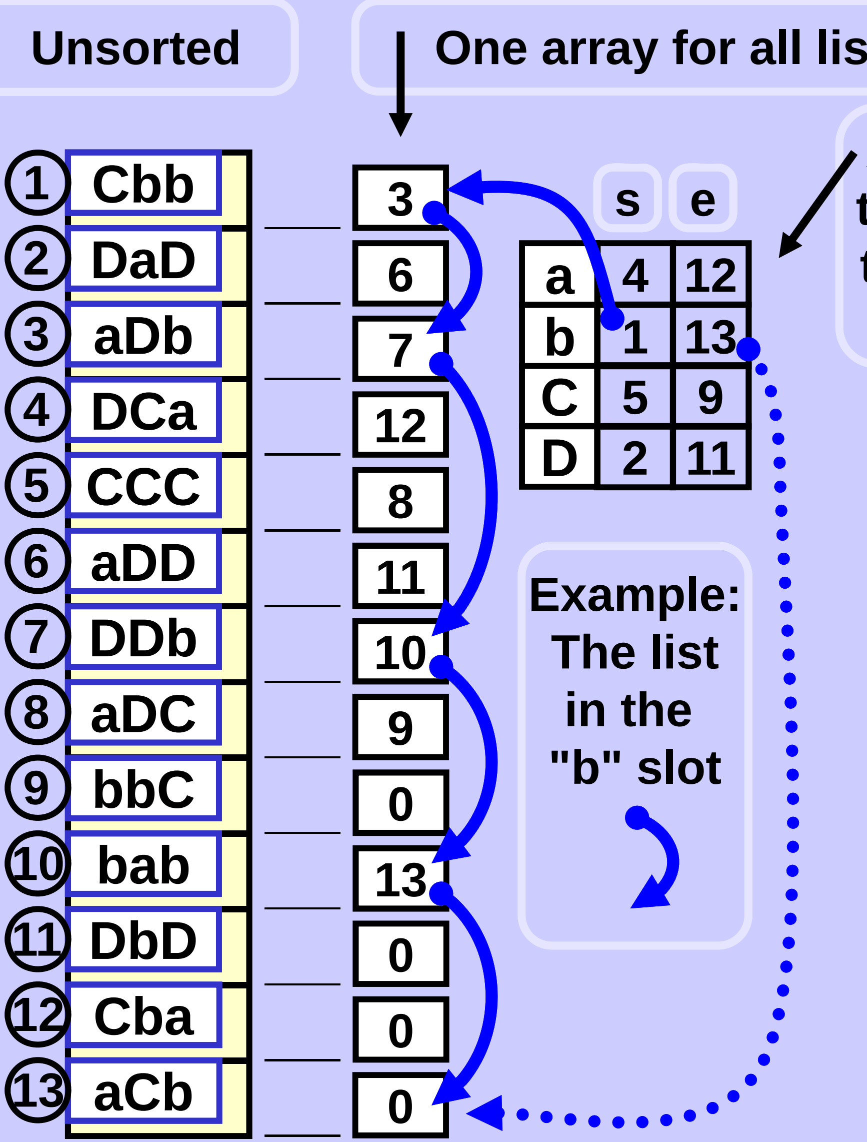
Radix sort


Sorted form 2nd symbol
1 bab
2 DaD
3 Cba
4 Cbb
5 bbC
6 DbD
7 DCa
8 aCb
9 CCC
10 aDb
11 DDb
12 aDC
13 aDD
Sort by the 1st symbol
a _ _
8 aCb 1
10 aDb 2
12 aDC 3
13 aDD 4
b _ _
1 bab 5
5 bbC 6
C _ _
3 Cba 7
4 Cbb 8
9 CCC 9
D _ _
2 DaD 10
6 DbD 11
7 DCa 12
11 DDb 13
Sorted!

Radix sort Implementation



Radix sort Implementation



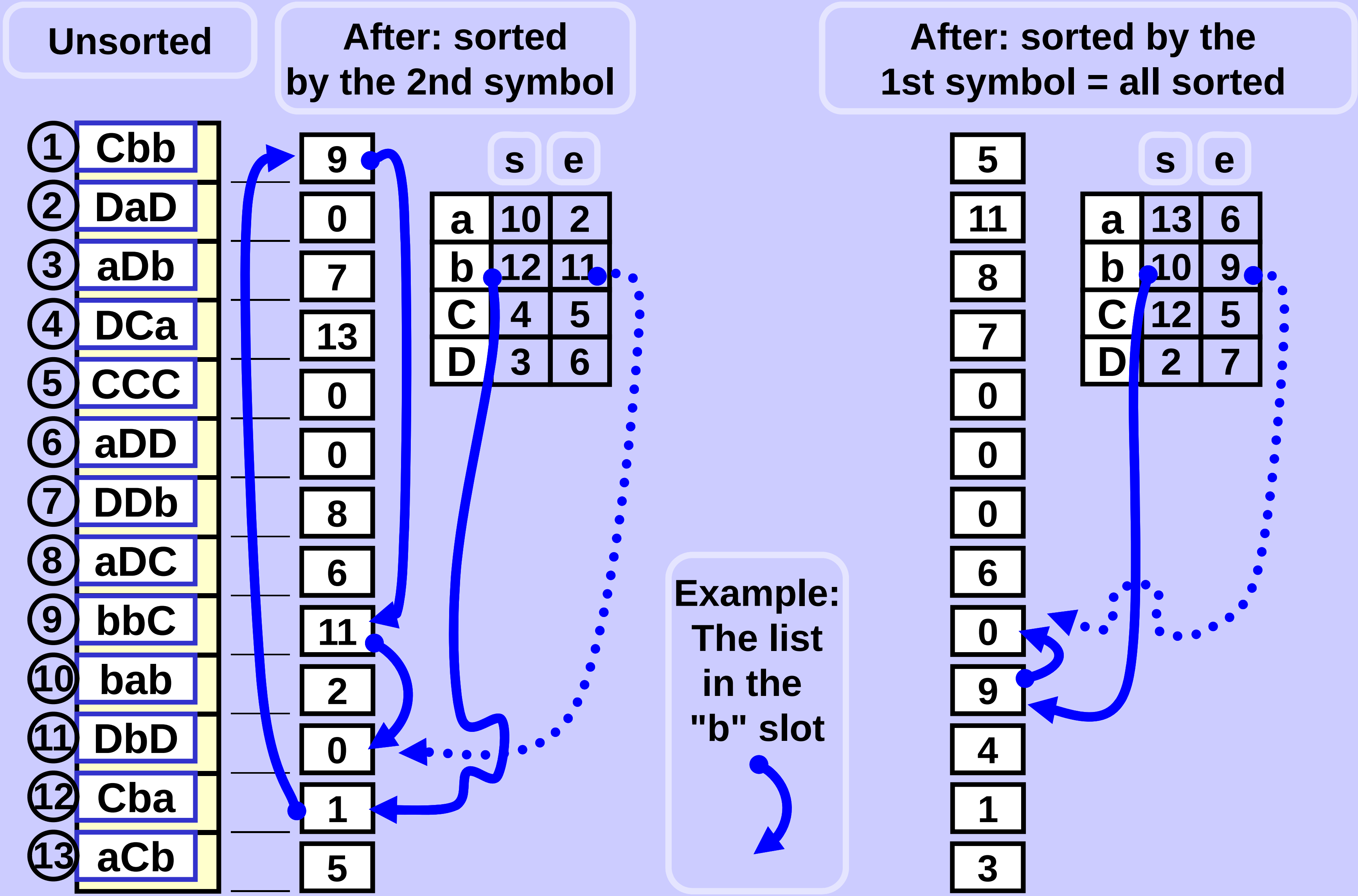
Array of pointers to the start and to the end of the list for each symbol

Here, both arrays reflect the status after sorting by the 3rd character.

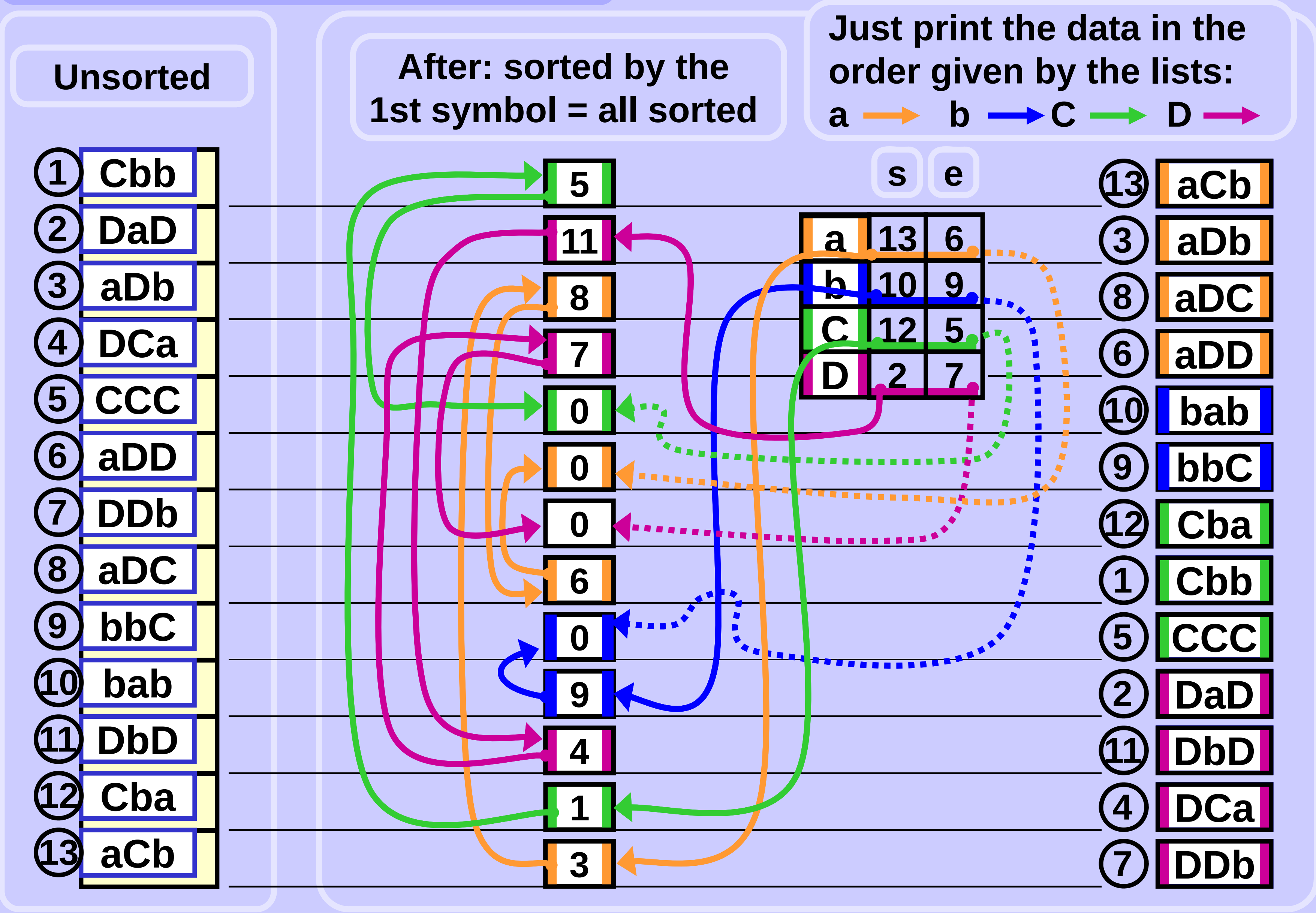
Radix sort can be performed without moving the original data.

It suffices just to manipulate the pointer arrays which contain all information about the current progress of the sort.

Radix sort Implementation



Radix sort Implementation



From sorted by 2nd symbol to sorted by 1st symbol

Arrays specify order after sorted by the 2nd symbol.

	s	e
a	10	2
b	12	11
C	4	5
D	3	6

		d	
1	Cbb	9	4
2	DaD	0	2
3	aDb	7	10
4	DCa	13	7
5	CCC	0	9
6	aDD	0	13
7	DDb	8	11
8	aDC	6	12
9	bbC	11	5
10	bab	2	1
11	DbD	0	6
12	Cba	1	3
13	aCb	5	8

	s1	e1
a	0	0
b	0	0
C	0	0
D	0	0

Arrays will specify order after sorted by the 1st symbol.

d1
0
0
0
0
0
0
0
0
0
0
0
0
0

Update arrays s, e, d:
Fill temporary arrrays
s1, e1, d1
and copy their contents
back to s, e, d.

Implementation will not copy anything, it will only swap the pointers to the original and temporary arrays.

From sorted by 2nd symbol to sorted by 1st symbol

Sorted by the 2nd symbol

	s	e
a	10	2
b	12	11
C	4	5
D	3	6

		d	
1	Cbb	9	4
2	DaD	0	2
3	aDb	7	10
4	DCa	13	7
5	CCC	0	9
6	aDD	0	13
7	DDb	8	11
8	aDC	6	12
9	bbC	11	5
10	bab	2	1
11	DbD	0	6
12	Cba	1	3
13	aCb	5	8

	s1	e1	s1	e1	s1	e1	s1	e1	s1	e1	s1	e1	s1	e1
a	0	0	0	0	0	0	0	0	0	0	0	0	0	0
b	0	0	10	10	10	10	10	10	10	10	10	9	10	9
C	0	0	0	0	0	0	12	12	12	1	12	1	12	1
D	0	0	0	0	2	2	2	2	2	2	2	2	2	11

d1	d1	d1	d1	d1	d1	d1
0	0	0	0	0	0	0
0	0	0	0	0	0	11
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	9	9
0	0	0	0	0	0	0
0	0	0	0	1	1	1
0	0	0	0	0	0	0

From sorted by 2nd symbol to sorted by 1st symbol

Sorted by the 2nd symbol

	s	e
a	10	2
b	12	11
C	4	5
D	3	6

		d	
1	Cbb	9	4
2	DaD	0	2
3	aDb	7	10
4	DCa	13	7
5	CCC	0	9
6	aDD	0	13
7	DDb	8	11
8	aDC	6	12
9	bbC	11	5
10	bab	2	1
11	DbD	0	6
12	Cba	1	3
13	aCb	5	8

	s1	e1	s1	e1	s1	e1	s1	e1	s1	e1	s1	e1	s1	e1
a	0	0	13	13	13	13	13	3	13	3	13	8	13	6
b	10	9	10	9	10	9	10	9	10	9	10	9	10	9
C	12	1	12	1	12	5	12	5	12	5	12	5	12	5
D	2	4	2	4	2	4	2	4	2	7	2	7	2	7

d1	d1	d1	d1	d1	d1	d1
0	0	5	5	5	5	5
11	11	11	11	11	11	11
0	0	0	0	0	8	8
0	0	0	0	7	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	6
0	0	0	0	0	0	0
9	9	9	9	9	9	9
4	4	4	4	4	4	4
1	1	1	1	1	1	1
0	0	0	3	3	3	3

Finished

Radix sort Implementation

```
def radixSort(A):
    alphabetsize = 128    # 2^16 in unicode
    S = [0] * alphabetsize    # all starts
    E = [0] * alphabetsize    # all ends
    D = [0] * len(A)          # all lists
    S1 = [0] * alphabetsize
    E1 = [0] * alphabetsize
    D1 = [0] * len(A)

    radixInit(A, S ,E, D)     # 1st pass with last char

    for p in range(len(A[0])-2, -1, -1):
        radixStep(A, p, S, E, D, S1, E1, D1)
        S, S1 = S1, S     # just swap arrays
        E, E1 = E1, E     #  ditto
        D, D1 = D1, D     #  ditto

    radixOutput(A, S, E, D)  # print sorted A
```

Radix sort Implementation

```
def radixInit(A, S, E, D):
    pos = len(A[0]) - 1                 #  last char in string
    for i in range(len(S)):
        S[i], E[i] = -1, -1
    for i in range(len(A)):
        c = ord(A[i][pos])              # char to index
        if S[c] == -1:
            S[c], E[c] = i, i           # start new list
        else:                           # extend existing list
            D[E[c]] = i
            E[c] = i
```

Add trailing spaces to shorter strings to make all strings of the same length.

Caution: The arrays in the code are indexed from 0, The arrays in the code are indexed from 1.

Radix sort Implementation

```
def radixStep(A, pos, S, E, D, S1, E1, D1):
    for i in range(len(S)):
        S1[i], E1[i] = -1, -1   # init arrays

    for i in range(len(S)):
        if S[i] != -1:          # unempty old list
            j = S[i]            # traverse the list
            while True:
                c = ord(A[j][pos])       # list index
                if S1[c] == -1:
                    S1[c], E1[c] = j, j  # start new list
                else:           # extend existing list
                    D1[E1[c]] = j
                    E1[c] = j
                if j == E[i]:
                    break
                j = D[j]        # next string index
```

Radix sort

Resume

d symbols .... d loops

loop .... $\Theta(n)$ operations

---

total .... $\Theta(d \cdot n)$ operations

d << n $\Rightarrow$ ...... $\Theta(n)$ operations

Radix sort does not change the order of equal values.

Asymptotic complexity of Radix sort is $\Theta(d \cdot n)$

It is a stable sort.

Counting sort

Input

input.length == N

1	2	3	4	5	6	7	8	9	10	11	12
10	3	8	8	6	3	8	10	6	10	8	5

Frequency

frequency.length == k

k = max(input) - min(input) + 1

3	4	5	6	7	8	9	10
2	0	1	2	0	4	0	3

Output

output.length == N

1	2	3	4	5	6	7	8	9	10	11	12
3	3	5	6	6	8	8	8	8	10	10	10

Counting sort

Step 1

Reset frequency array

3	4	5	6	7	8	9	10
0	0	0	0	0	0	0	0

One pass through the input array

10	3	8	8	6	3	8	10	6	10	8	5

Fill the frequency array

3	4	5	6	7	8	9	10
2	0	1	2	0	4	0	3

```
for x in inputArr:
      freq[x] += 1
```

Counting sort

Step 2

One pass

low = 3

3	4	5	6	7	8	9	10
2	0	1	2	0	4	0	3

high = 10

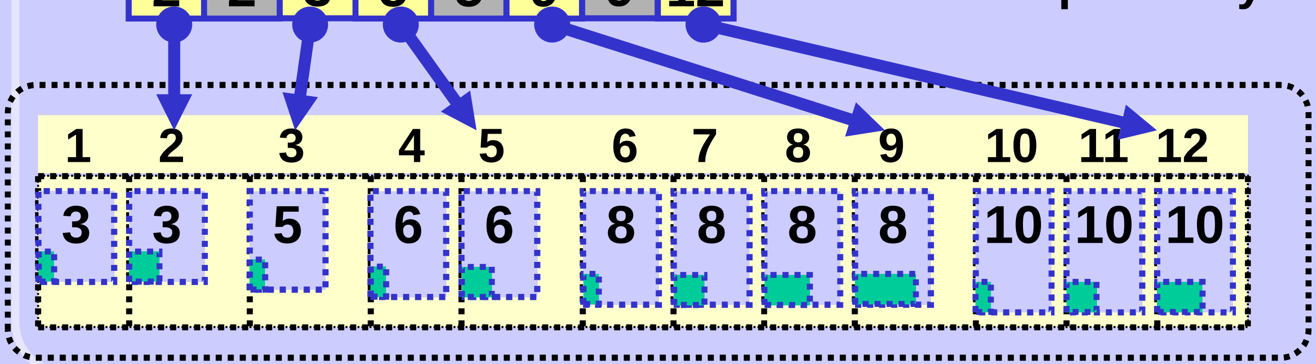
Frequency array changes its meaning

Update the frequncy array

```
for i in range(low+1, high+1):
    freq[i] += freq[i-1]
```

3	4	5	6	7	8	9	10
2	2	3	5	5	9	9	12

Elem freq[j] denotes position of the last elem with value j in the output array

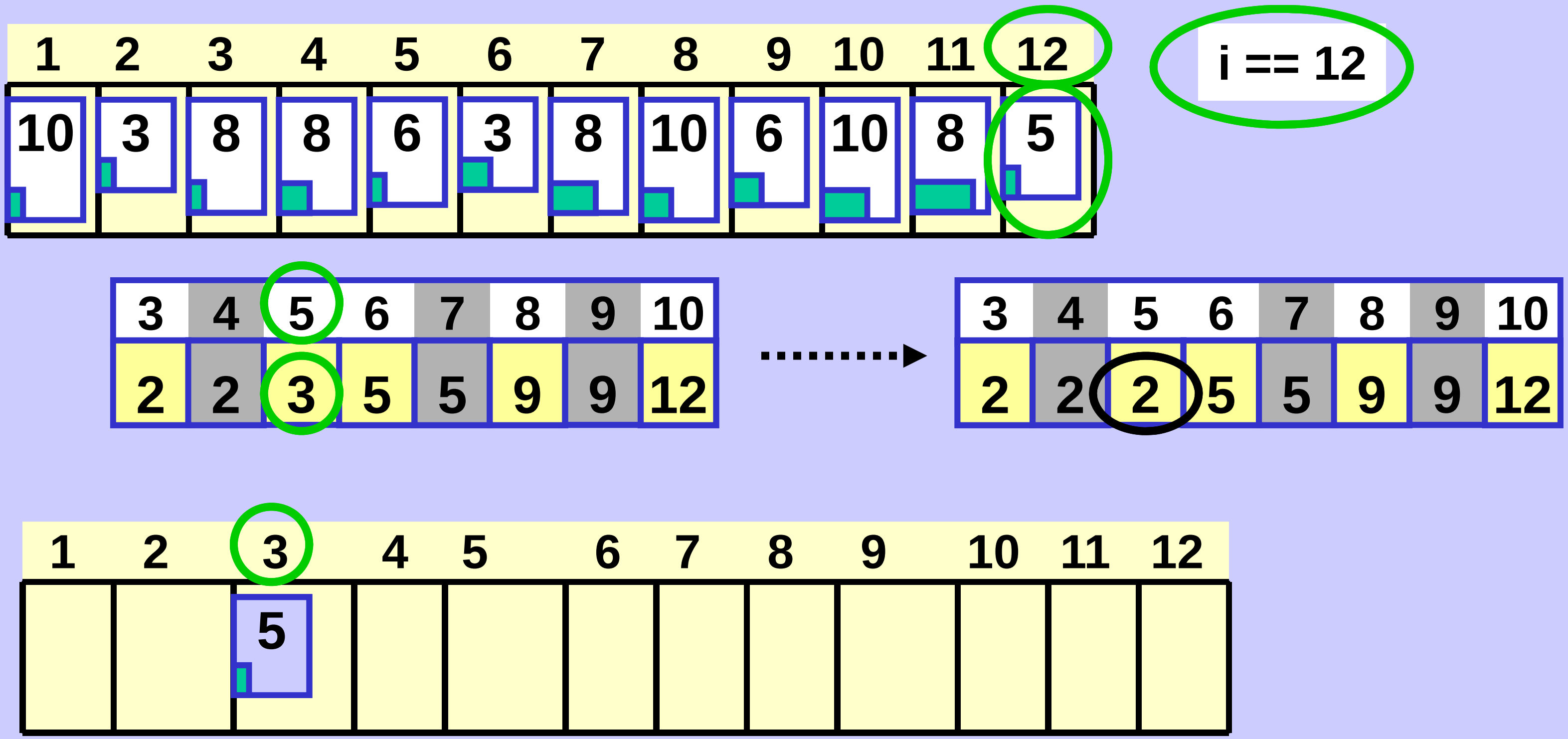


Counting sort

Step 3

```
i == N
```

```
for i in range(N, 0, -1):
    output[freq[input[i]]] = input[i]
    freq[input[i]] -= 1
```

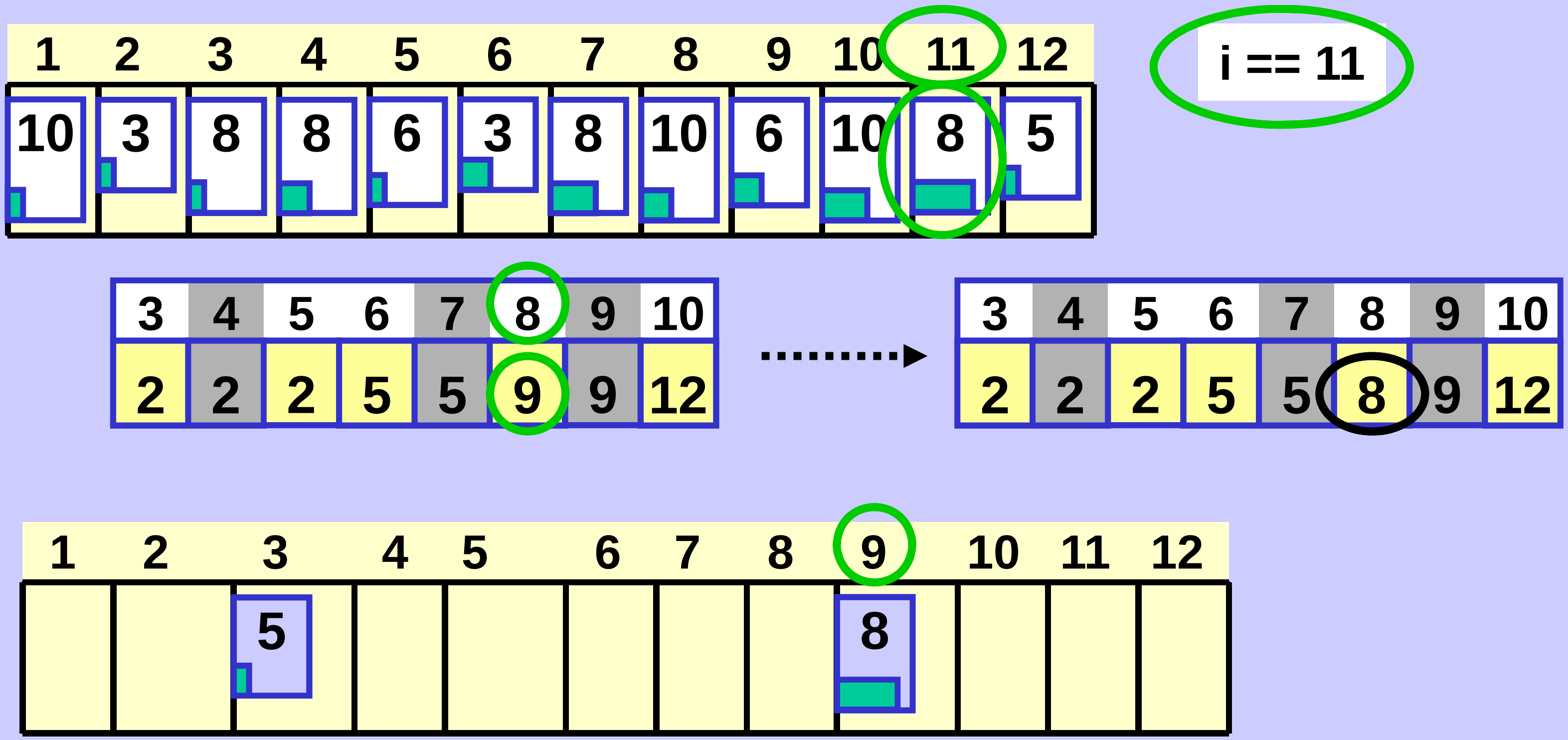


Counting sort

Step 3

`i == N-1`

```
for i in range(N, 0, -1):
    output[freq[input[i]]] = input[i]
    freq[input[i]] -= 1
```

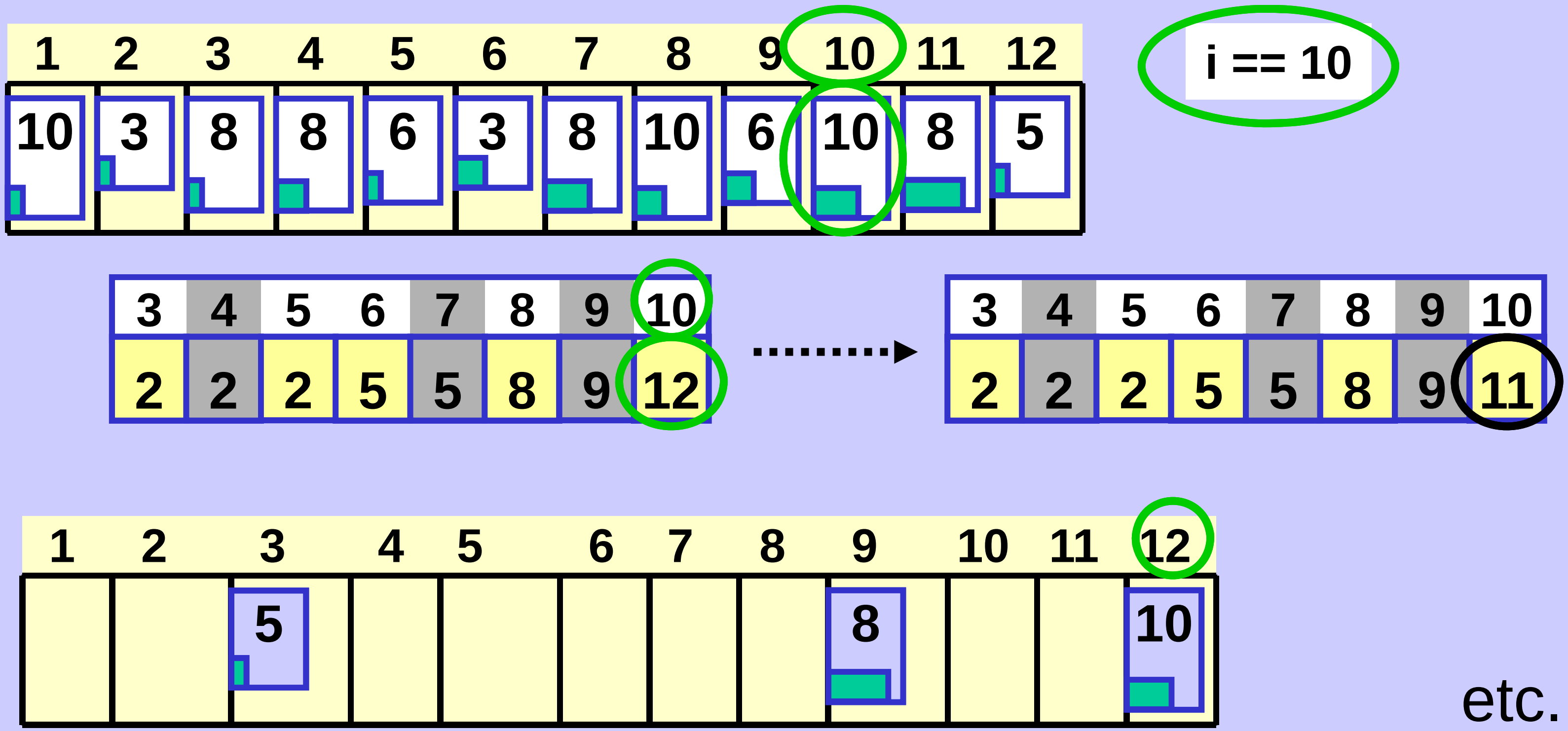


Counting sort

Step 3

`i == N-2`

```
for i in range(N, 0, -1):
    output[freq[input[i]]] = input[i]
    freq[input[i]] -= 1
```



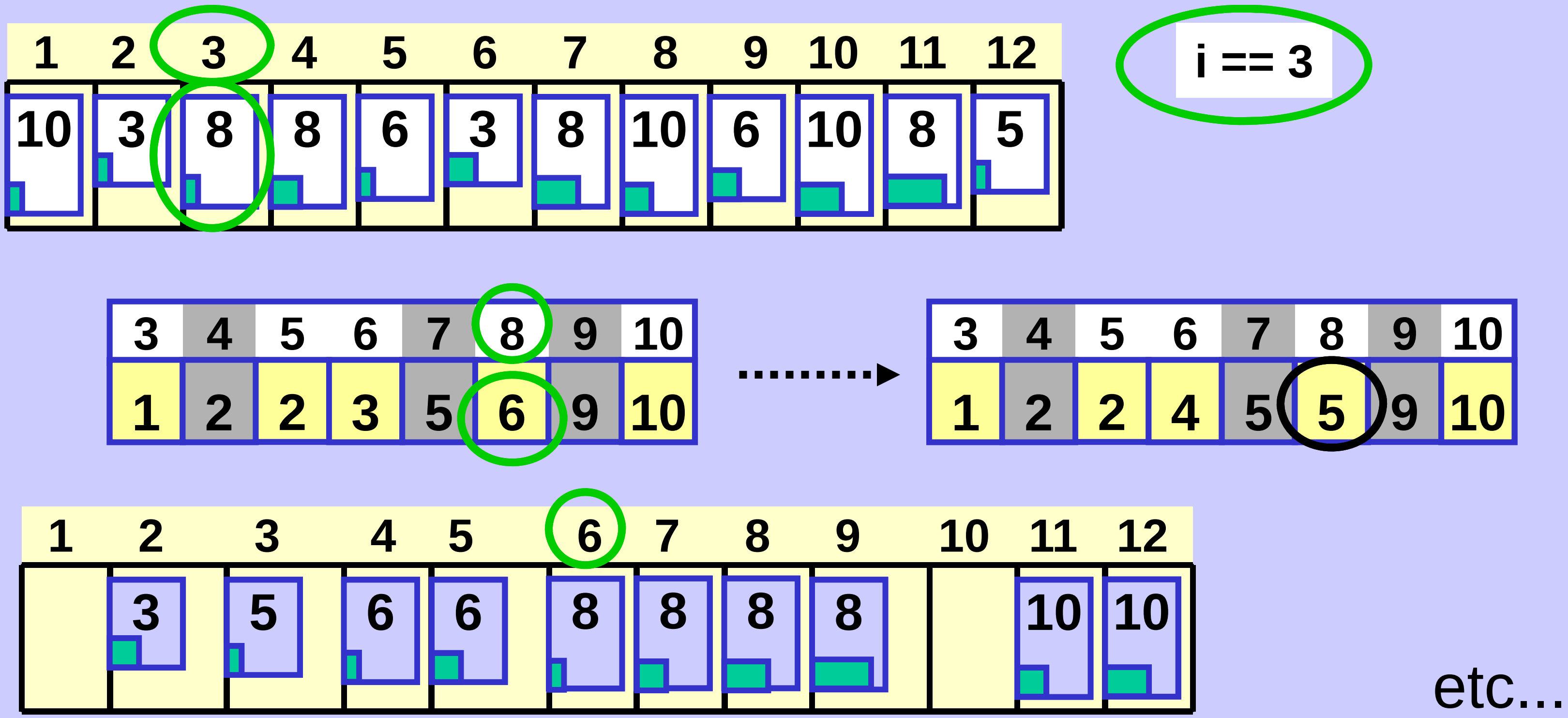
etc...

Counting sort

Step 3

i == 3

```
for i in range(N, 0, -1):
    output[freq[input[i]]] = input[i]
    freq[input[i]] -= 1
```



Sorts complexities overview

Array size n	Worst case	Best case	Average, "expected" case	Stable
Selection sort	$\Theta(n^2)$	$\Theta(n^2)$	$\Theta(n^2)$	No
Insertion sort	$\Theta(n^2)$	$\Theta(n)$	$\Theta(n^2)$	Yes
Bubble sort *)	$\Theta(n^2)$	$\Theta(n^2)$	$\Theta(n^2)$	Yes
Quick sort	$\Theta(n^2)$	$\Theta(n \cdot \log(n))$	$\Theta(n \cdot \log(n))$	No **)
Merge sort	$\Theta(n \cdot \log(n))$	$O(n \cdot \log(n))$	$\Theta(n \cdot \log(n))$	Yes
Heap sort	$\Theta(n \cdot \log(n))$	$\Theta(n \cdot \log(n))$	$\Theta(n \cdot \log(n))$	No
Radix sort	$\Theta(n)$	$\Theta(n)$	$\Theta(n)$	Yes
Counting sort	$\Theta(n)$	$\Theta(n)$	$\Theta(n)$	Yes

*) Not recommended to use

**) Stable slow versions exist

Small sorting experiment

Environment

Intel(R) 1.8 GHz, Microsoft Windows XP SP3, jdk 1.6.0_16.

Organization

Explored the sorts which compare the elements value (double).
Each datasets of particular datasizes used in all sorts.
Arrays values generated by the pseudorandom generator.
The results are the averages of repeated runs.

Conclusion

There is no particular sort method which would be optimal in all circumstances.
The performance is influenced by the data size and by the degree of the original organisation (partial order) of the data.

Small sorting experiment

Array size	% sorted	Time in milliseconds if not indicated otherwise					
		Sort					
		Select	Insert	Bubble	Quick	Merge	Heap
10	0%	0.0005	★ 0.0002	0.0005	0.0004	0.0009	0.0005
10	90%	0.0004	★ 0.0001	0.0004	0.0004	0.0007	0.0005
100	0%	0.028	0.016	0.043	0.081	0.014	★ 0.011
100	90%	0.026	★ 0.003	0.030	0.010	0.011	0.011
1 000	0%	2.36	1.30	4.45	★ 0.12	0.19	0.17
1 000	90%	2.31	0.18	2.86	0.16	★ 0.15	0.16
10 000	0%	228	130	450	★ 1.57	2.40	2.31
10 000	90%	229	17.5	285	1.93	★ 1.68	2.11
100 000	0%	22 900	12 800	45 000	★ 18.7	31.4	31.4
100 000	90%	22 900	1 760	28 500	27.4	★ 24.6	25.5
1 000 000	0%	38 min	22 min	75 min	★ 237	385	570
1 000 000	90%	38 min	2.9 min	47.5 min	336	★ 301	381

Degree of order. 100% –x% of sorted data are randomly chosen and their values are randomly changed.

Small sorting experiment

Array size	% sorted	Ratio of slowdown (>1) compared to Quick sort					
		Sort					
		Select	Insert ◻	Bubble ◻	Quick	Merge ◻	Heap
10	0%	1.3	★ 0.7	1.4	1	✗ 2.5	1.4
10	90%	1	★ 0.26	0.96	1	✗ 1.8	1.3
100	0%	3.4	✗ 1.8	5.4	★ 1	1.75	1.35
100	90%	2.46	★ 0.28	2.9	1	✗ 1.07	1.07
1 000	0%	20	✗ 11	37.5	★ 1	1.65	1.4
1 000	90%	15	✗ 1.2	18.5	1	★ 0.95	1.03
10 000	0%	146	✗ 83	287	★ 1	1.53	1.48
10 000	90%	118	✗ 9.1	148	1	★ 0.87	1.09
100 000	0%	1 220	✗ 686	2 410	★ 1	1.7	1.7
100 000	90%	837	✗ 64.1	1 040	1	★ 0.9	0.93
1 000 000	0%	9 960	✗ 5 400	19 000	★ 1	1.6	2.41
1 000 000	90%	6 820	521	8 480	1	★ 0.9	1.14

Fastest ★ Slowest ✗ Stable ◻

Selection and Bubble sort do not compete.

Small sorting experiment

Array size	% sorted	Ratio of slowdown (> 1) when comparing the sort speeds of unsorted and partially sorted data.					
		Sort					
		Select	Insert	Bubble	Quick	Merge	Heap
10	0%	1	1	1	1	1	1
10	90%	0.8	0.5	0.8	1	0.8	1
100	0%	1	1	1	1	1	1
100	90%	0.9	0.2	0.68	1.27	0.78	1
1 000	0%	1	1	1	1	1	1
1 000	90%	0.98	0.14	0.64	1.31	0.75	0.95
10 000	0%	1	1	1	1	1	1
10 000	90%	1.0	0.14	0.63	1.23	0.7	0.91
100 000	0%	1	1	1	1	1	1
100 000	90%	1.0	0.14	0.63	1.46	0.78	0.81
1 000 000	0%	1	1	1	1	1	1
1 000 000	90%	1.0	0.14	0.63	1.42	0.78	0.67

Stable