PAL labs 5

19 / 10 / 2022

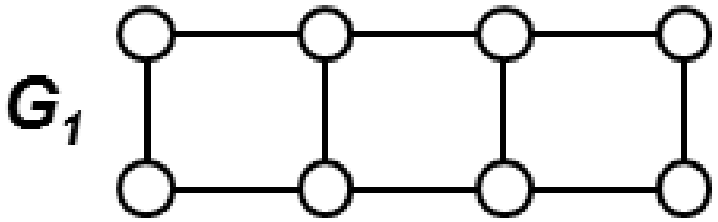
Let's start with an empty Fibonacci heap. Then, insert $2^n + 5$ unique keys ($n > 2$). After that, we apply *DeleteMin* operation including the consolidation of the heap. How many binomial trees are in the final heap?

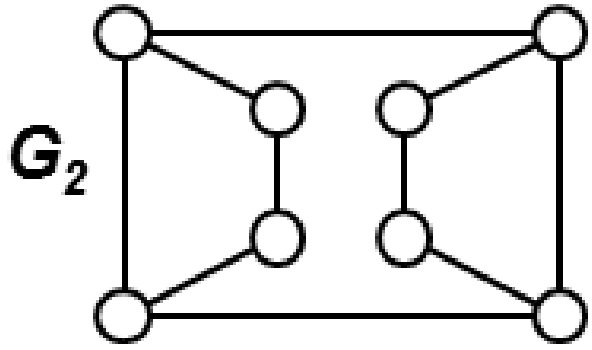
There is n ($n \geq 2$) unique keys and an empty binary heap. We add all keys one by one into the heap in a random order. What is the asymptotic complexity of this process? Is it possible that some ordering of the keys would result in a different asymptotic complexity?

Let's have $d-$ary heap with the depth of h with leaves which lie in the same depth, and which has exactly $(d^{h+1}-1)/(d-1)$ keys. What is the maximal possible and minimal number of comparisons of two keys after applying the *deleteMin* operation?

We have two undirected circuits of the same length $k \geq 2$. What is the number of isomorphisms between them?

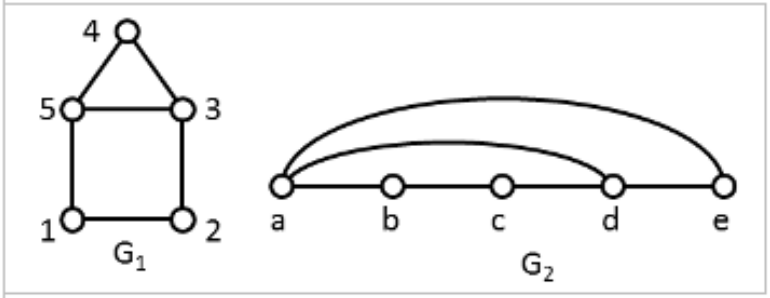
What is the number of isomorphisms between graphs G1 and G2?



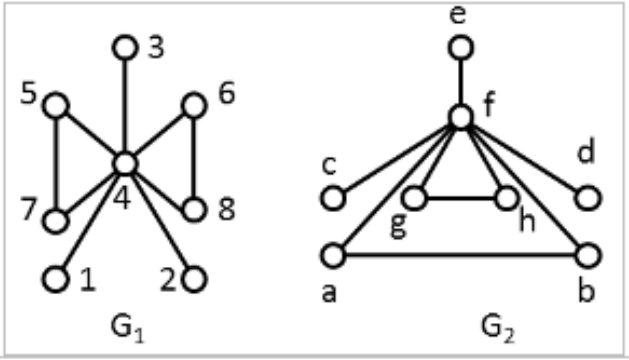


Count the number of bijections between nodes of graphs G1 and G2 in the picture which are not isomorphisms.

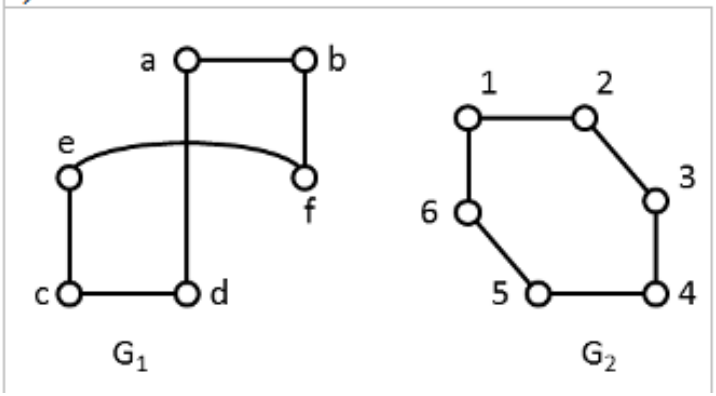
a)



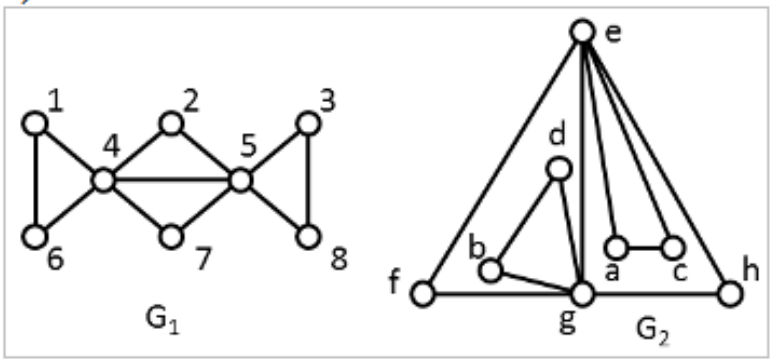
b)



c)



d)

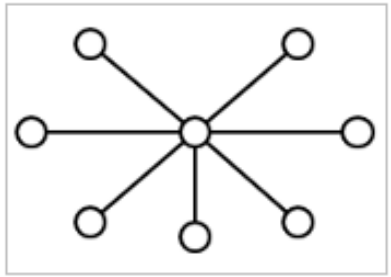


Let's have two undirected graphs, each one containing exactly n vertices and both graphs have the same score (i.e. $n-1, n-2, n-3, n-4, ..., n/2+1, n/2, n/2, n/2-1, n/2-2, ..., 3, 2, 1$), which means that almost all vertices have a unique degree with the exception of two which has the same degree of $n/2$. What is the asymptotic complexity of verifying if these two graphs are isomorphic w.r.t. n.

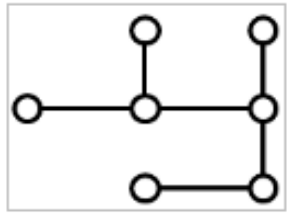
Construct a certificate for each tree and match each single node to a substring of that certificate.

a)

b)



c)



d)

Reconstruct a tree from a certificate:

a) 0101

b) 0001010110010111

c) 00010110010110010111

d) 000001011100111000101110111

Given a certificate of a tree, describe the number of leaves in the tree without reconstructing the whole tree from the certificate.

Given a certificate of a tree, describe how you can derive the maximal degree of a vertex in the tree without reconstructing the whole tree from the certificate.

Tree of type T(1,3) contains vertices of degree 1 or 3. Describe, informally, how the certificate of T(1,3)-tree looks like. Design an algorithm which verify if a certificate is of type T(1,3)-tree.