Wavelet texture descriptors

Šonka&Hlaváč&Boyle book, Unser 1995, Leung&Malik 2001, Reyes-Aldasoro 2007

Jan Kybic

2020

Texture

- texture information = image information - shape & color
- perceptually shift invariant (at a given scale)
- medical imaging - stochastic (weak) texture
- random, described by statistical descriptors

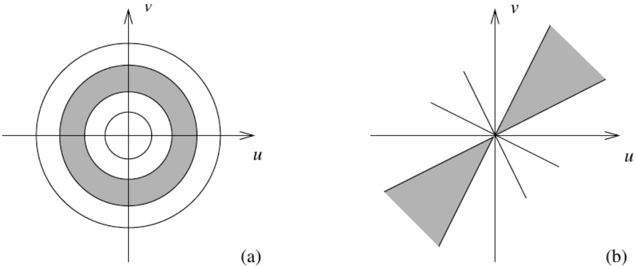
Statistical texture description

- mean, variance,

Evaluate autocorrelation coefficients for several different values of parameters p, q:

$$C_{ff}(p,q) = \frac{MN}{(M-p)(N-q)} \frac{\sum_{i=1}^{M-p} \sum_{j=1}^{N-q} f(i,j) f(i+p, j+q)}{\sum_{i=1}^{M} \sum_{j=1}^{N} f^2(i,j)} , \qquad (15.1)$$

-
- image power spectrum $\left|\hat{F}\right|$



- *co-occurrence matrices, fractal dimensions...*

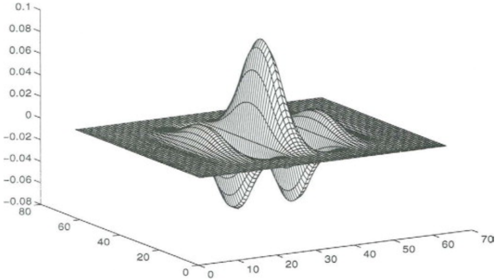
Filterbank descriptors

- Filterbank g_i,energy descriptors $d_i = \|f * g_i\|^2$

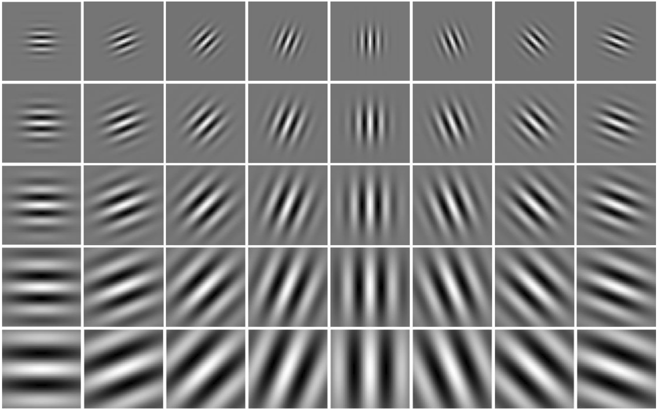
Filterbank descriptors

- Filterbank g_i,energy descriptors $d_i = \|f * g_i\|^2$
- Gabor filters

$$G_c[i,j] = Be^{-\frac{(i^2+j^2)}{2\sigma^2}} \cos(2\pi f(i\cos\theta + j\sin\theta))$$
$$G_s[i,j] = Ce^{-\frac{(i^2+j^2)}{2\sigma^2}} \sin(2\pi f(i\cos\theta + j\sin\theta))$$

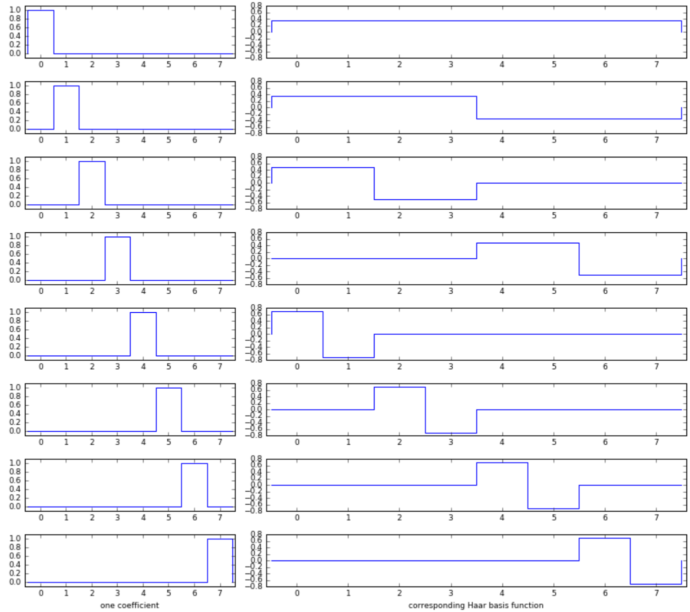


2D Gabor filters



by Mohammad Haghighat

Haar wavelets



from unix4lyfe.org

Two-scale relationship

- Two-scale relationship

$$h_{i+1}(k) = [h]_{\uparrow 2^i} * h_i(k)$$
$$g_{i+1}(k) = [g]_{\uparrow 2^i} * h_i(k)$$

Nested subspaces

- Orthogonal basis functions

$$\varphi_{i,l}(k) = 2^{i/2}\, h_i(k - 2^i l)\ ,$$
$$\psi_{i,l}(k) = 2^{i/2}\, g_i(k - 2^i l)\ ,$$

- Nested subspaces $V_i = \mathrm{span}\{\varphi_{i,l}\}_{l\in Z}$ $\quad l_2 \supset V_0 \supset V_1 \supset \ldots \supset V_I$
- Complement space $V_{i-1} = V_i + W_i$, $V_i \perp W_i$, $W_i = span\left\{\varphi_{i,l}\right\}_{l\in Z}$

Wavelet representation

- Orthogonal projection to V_i

$$x_{(i)}(k) = \sum_{l \in Z} s_{(i)}(l)\, \varphi_{i,l}\ ,$$
$$s_{(i)}(l) = \left\langle x(k), \varphi_{i,l}(k) \right\rangle_{l_2}$$

- Residual (high frequency)

$$x_{(i-1)}(k) - x_{(i)}(k) = \sum_{l \in Z} d_{(i)}(l)\, \psi_{i,l}\ ,$$
$$d_{(i)}(l) = \left\langle x(k), \psi_{i,l}(k) \right\rangle_{l_2}$$

- Complete wavelet expansion

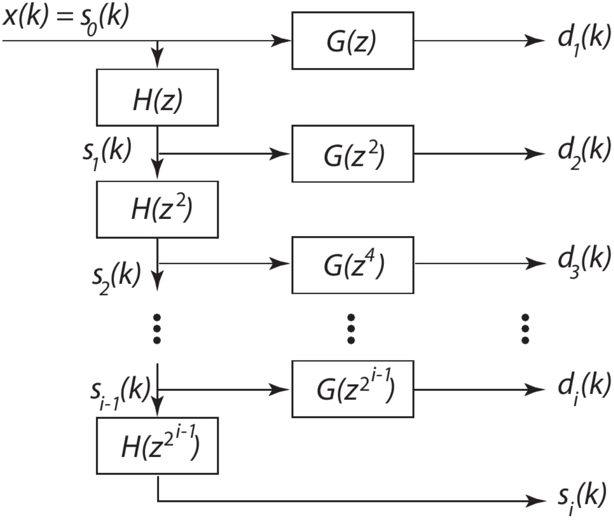
$$x(k) = \sum_{l \in Z} s_{(I)}(l)\, \varphi_{I,l} + \sum_{i=1}^{I} \sum_{l \in Z} d_{(i)}(l)\, \psi_{i,l}$$

Undecimated WT

- Standard (decimated) WT is sensitive to shift
- Low pass $s_l = h_l * \tilde{x}(k)$
- High pass $d_i = g_i * \tilde{x}(k)$
- Recursive decomposition

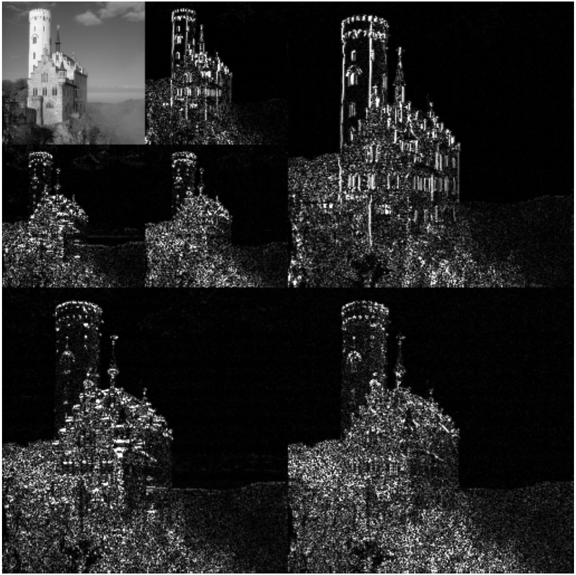
$$s_{i+1}(k) = [h]_{\uparrow 2^i} * s_i(k)$$
$$d_{i+1}(k) = [g]_{\uparrow 2^i} * s_i(k)$$

Recursive decomposition



2D Wavelets

Separability: $h_l(x)h_l(y)$, $h_l(x)g_l(y)$, $g_l(x)h_l(y)$, $g_l(x)g_l(y)$



by Alessio Damato, Wikipedia

Wavelet features

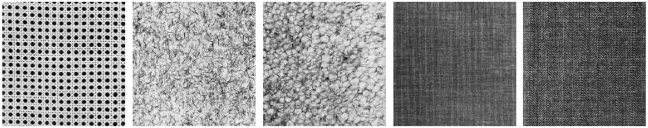
- Wavelet coefficients

$$\mathbf{y}(k,l) = \left(y_i(k,l)\right)_{i=1,\ldots,N} = \left[s_I(k,l);\ d_I(k,l);\ \ldots;\ d_1(k,l)\right]^T$$

- In 2D, $3I+1$ channels
- Texture characterized by $p(y_i)$
- Descriptors

$$v_i = \frac{1}{N_R} \sum_{(k,l)\in R} y_i^2(k,l)$$

Brodatz textures



(a) 1 (b) 2 (c) 3 (d) 4 (e) 5

Texture classification example

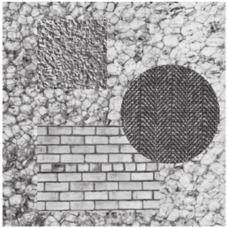
	1	2	3	4	5	6	7	8	9	10
1	20	0	0	0	0	0	0	0	1	0
2	0	17	0	0	0	0	0	0	0	0
3	0	0	15	0	0	0	0	0	0	0
4	0	0	0	18	0	0	0	0	0	0
5	0	0	0	0	12	0	0	0	0	0
6	0	0	0	0	0	15	0	0	0	0
7	0	0	0	0	0	0	22	0	0	0
8	0	0	0	0	0	0	0	22	3	0
9	1	0	0	0	0	0	0	0	15	0
10	0	0	0	0	0	0	0	0	0	19

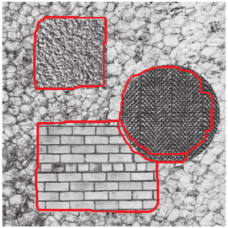
Table 15.2: Wavelet confusion matrix.

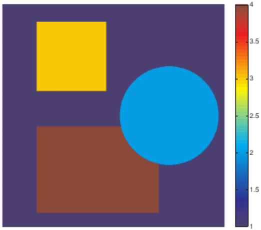
Svoboda, Kybic, Hlaváč book

Texture segmentation

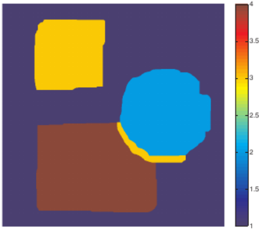
- For segmentation - average variance over a window
- $p(y_i)$ assumed normal, GMM parameters found by EM
- GraphCut regularization


(b)


(c)



(e)



(f)