A practice of (robust) statistical testing

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1. Implement a calculation of T-Test test statistic

$$t = \frac{\bar{X}_1 - \bar{X}_2}{s_p \sqrt{\frac{2}{n}}}$$

   where

$$s_p = \sqrt{\frac{s_{X_1}^2 + s_{X_2}^2}{2}},$$

   where X_1 denotes the first set of observations $\{x_i^1\}_{i=1}^n$ and X_2 denotes the first set of observations $\{x_i^2\}_{i=1}^n$, $\bar{X}_.$ denotes to sample mean and finally $s_{X.}$ denotes the sample variance.

2. Observe that the value of a test statistic under hypothesis H_0 is random variable (To do so, calculate it from two sets of numbers from Normal distribution).

3. Draw histograms of test statistics for H_0 and H_1 and observe, how they change with respect to:
   - difference in means,
   - variance of distributions,
   - number of samples.

4. Observe, how Student-t distribution with $2n-2$ degrees of freedom fits the distribution of test statistics of hypothesis H_0 independently from variance of distributions.

5. Calculate thresholds on test statistics, such that the probability of rejection hypothesis H_0 when it is true (Type I error) is $\alpha = 5\%$.
6. Empirically verify, that your thredsholds are correct. To do so, estimate the Type I error from a set of independent experiments (realizations of the test statistics under hypothesis H_0.
7. Observe, how the predicition matches the experimental results when the assumptions of T-Test is violated (both distributions have for example different variances.)
8. Observe, how the predicition matches the experimental results when number in tests comes from different distributions (e.g. Normal and Cauchy). Can you come up with a different distribution where it nicely fails?
9. Implement the test statistic U of Mann-Whitney-U statistics (see the lecture notes).
   (a) Assume we have $\{(x_i)\}_{i=1}^{n_1}$
   (b) Calculate ranks of all samples together.
   (c) Sum ranks of samples from the first population, R_1.
   (d) Sum ranks of samples from the second population, R_2.
   (e) Calculate $U_1 = R_1 - \frac{n_1(n_1+1)}{2}$ and $U_2 = R_2 - \frac{n_2(n_2+1)}{2}$.
   (f) $U = \min\{U_1, U_2\}$
10. Compare the distribution of test statistic to predicted approximation for large number of samples $U \sim \mathcal{N}\left(\frac{n_1 n_2}{2}, \frac{n_1 n_2(n_1+n_2+1)}{12}\right)$.