Data with Semantics – RDF(S)

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October 3, 2022

Outline

- Core RDF
- RDF features
- Metamodeling in RDFS
- RDF Syntaxes
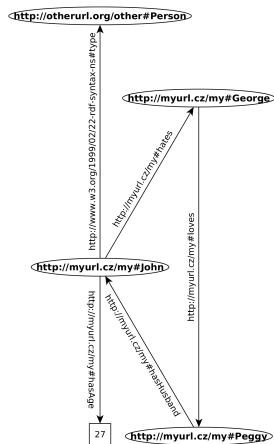- RDF Datasets
- Semantics of RDF(S)

1. RDF Vocabularies
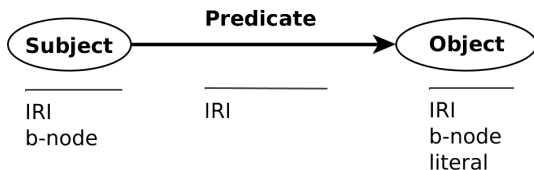
2. Advanced: Formal Semantics of RDF(S)

Core RDF

RDF

- RDF = **R**esource **D**escription **F**ramework
- RDF 1.0 – W3C Recommendation in 2004,
- RDF Graph is a graph, where each
    - node is either an IRI (ellipse), a literal (rectangle), or a blank node (blank ellipse)
    - edge is labeled with IRI

RDF Triple

is an ordered triple of the form (*Subject*, *Predicate*, *Object*):

Definitions

RDF Graph is a set of RDF triples (in fact edges)

RDF Term is either an *IRI*, a *blank node*, or a *literal*

IRIs

- IRI = International Resource Identifier
- denotes a *document*, or a real *thing*

    ```
    <http://myurl.cz/my#Peggy>
    <http://myurl.cz/my/document-about-peggy>
    ```

- using hash (#) or slash (/) for delimiting particular entities in a namespace
- mapped to URIs = backward-compatibility

Note

- Two IRIs are equal iff their string representations are equal.
- No IRI is equal to any blank node, or literal.

Namespaces

can be abbreviated using prefixes to improve readability

> `rdf:type` (can be also abbreviated as `a`) instead of
> `http://www.w3.org/1999/02/22-rdf-syntax-ns#type`.

- rdf: `http://www.w3.org/1999/02/22-rdf-syntax-ns#`. This namespace defines basic resources, like `rdf:type`, `rdf:Property`.
- rdfs: `http://www.w3.org/2000/01/rdf-schema#`. This namespace is used for metamodeling, like `rdfs:Class`, or `rdfs:subPropertyOf`.
- xsd: `http://www.w3.org/2001/XMLSchema#`, for referencing XML Schema datatypes reused by RDF, like `xsd:integer`, or `xsd:string`.

Note

Often, a shortened IRI with empty prefix (e.g. `:x`) is used in examples. In such cases, the namespace is fixed, but unimportant for the example, if not stated otherwise.

Literals

- denote basic data values, like strings, integers, or calendar data.

Definition

A literal consists of:

a lexical form , being a Unicode string,

a datatype IRI , being an IRI identifying a datatype,

a language tag , iff the datatype IRI is `http://www.w3.org/1999/02/22-rdf-syntax-ns#langString`.

Two literals are equal iff their 1) lexical forms, 2) datatypes, 3) language tags equal.

- $\underbrace{\text{"dolphin"}}_{\text{lex. form}} \underbrace{\text{@en}}_{\text{lang. tag}}$

- $\underbrace{\text{"dolphin"}}_{\text{lex. form}} \text{^^} \underbrace{\text{xsd:string}}_{\text{datatype IRI}}$

- `"128"^^xsd:integer`

- `"2010-01-19T16:00:00Z"`
  `^^xsd:dateTime`

Datatypes

- reused from XML Schema (e.g. `xsd:string`) plus `rdf:HTML` and `rdf:XMLLiteral`

Definition

A datatype consists of:

lexical space , e.g. a set $\{"0", "01", \ldots\}$ of strings made of numbers 0-9.

value space , e.g. a set of integers $\{0, 1, \ldots, \infty\}$,

lexical-to-value mapping $L2V$, e.g.
$$L2V(\text{datatype for } \texttt{xsd:integer}) = \{\langle "01", 1\rangle, \ldots\}.$$

- most XML Schema built-in datatypes:
  - `xsd:string`, `xsd:boolean`, `xsd:integer`, `xsd:decimal`, `xsd:dateTimeStamp`, `xsd:base64Binary`, ...
- `rdf:HTML` – for embedding HTML as literals
- `rdf:XMLLiteral` – for embedding XML as literals
- custom datatypes can be defined on different levels – XML Schema, OWL 2, ...

RDF Example (Turtle syntax)

```
@prefix qb:             <http://purl.org/linked-data/cube#> .
@prefix cssz-measure:   <https://data.cssz.cz/ontology/measure/> .
@prefix rdf:            <http://www.w3.org/1999/02/22-rdf-syntax-ns#> .
@prefix y-onto:         <https://data.cssz.cz/ontology/years/> .
@prefix xsd:            <http://www.w3.org/2001/XMLSchema#> .
@prefix skos:           <http://www.w3.org/2004/02/skos/core#> .
@prefix cssz:           <https://data.cssz.cz/ontology/> .
@prefix rdfs:           <http://www.w3.org/2000/01/rdf-schema#> .
@prefix cssz-dimension: <https://data.cssz.cz/ontology/dimension/> .
@prefix d-2011-t:       <https://data.cssz.cz/resource/observation/invalidita/2011/pk_id/t/> .
@prefix d-invalidita:   <https://data.cssz.cz/resource/dataset/invalidita> .

d-2011-t:60-64
  rdf:type qb:Observation ;
  qb:dataSet d-invalidita: ;
  cssz-dimension:druh-duchodu
    <https://data.cssz.cz/resource/pension-kind/PK_ID_2010> ;
  cssz-dimension:pohlavi
    <https://data.cssz.cz/ontology/sdmx/code/sex-T> ;
  cssz-dimension:refPeriod
    <https://data.cssz.cz/resource/reference.data.gov.uk/id/gregorian-year/2011> ;
  cssz-dimension:skupina-diagnoz-dle-who
    <https://data.cssz.cz/resource/icd-10/chapter/C_T> ;
  cssz-dimension:vekova-kategorie
    <https://data.cssz.cz/generated/resource/age/Y60T64> ;
  cssz-measure:pocet-nove-priznanych-duchodu "234"^^xsd:integer .
```
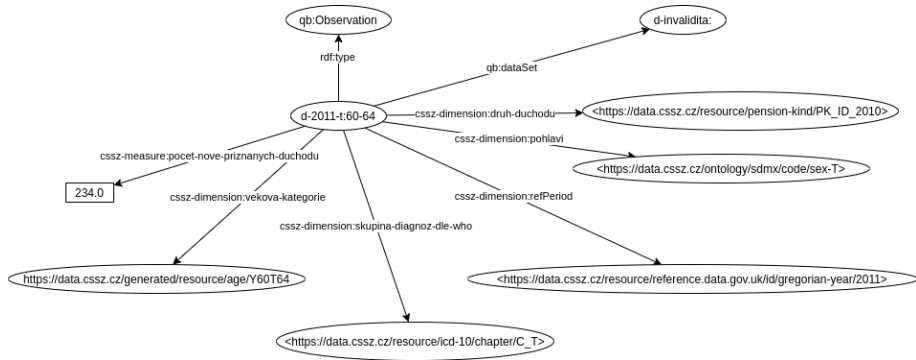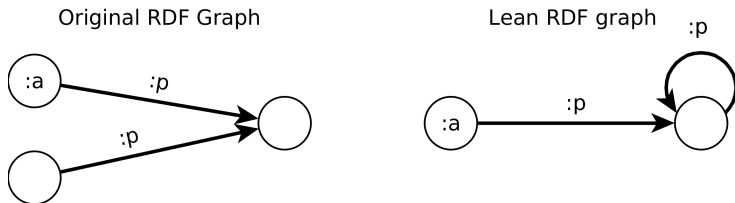
RDF Example (graph)

Blank Nodes (b-nodes)

- denote existential variables,

Definition

Ground RDF Graph is an RDF Graph containing no b-nodes.

  Instance of RDF Graph G_1 is an RDF Graph in which some b-nodes are be replaced by an arbitrary RDF Term.

Lean RDF Graph G_1 has no instance G_2 which is a proper subgraph of G_1.

Original RDF Graph   Lean RDF graph
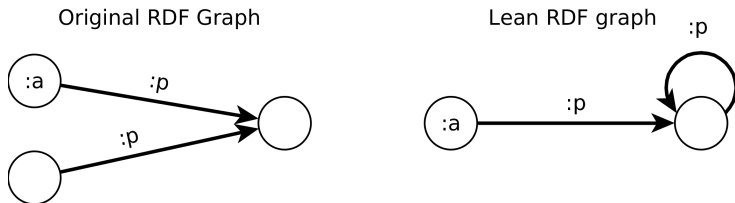
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Original RDF Graph

Lean RDF graph

Blank Nodes (b-nodes)

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- are local to the RDF document (cannot be reused outside),
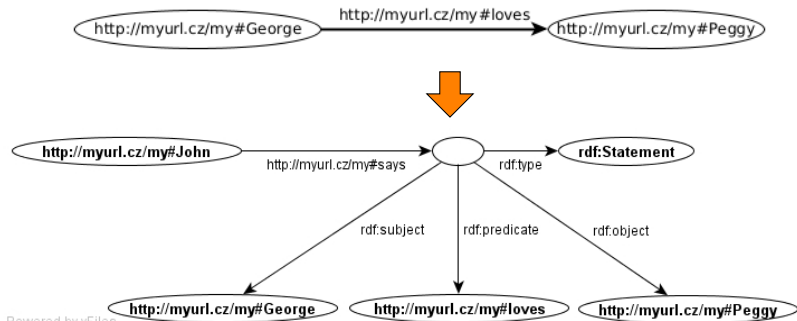- in Turtle/N-TRIPLES/SPARQL have _: prefix, e.g. _:x,

---

Definition

Ground RDF Graph is an RDF Graph containing no b-nodes.

Instance of RDF Graph G_1 is an RDF Graph in which some b-nodes are be replaced by an arbitrary RDF Term.

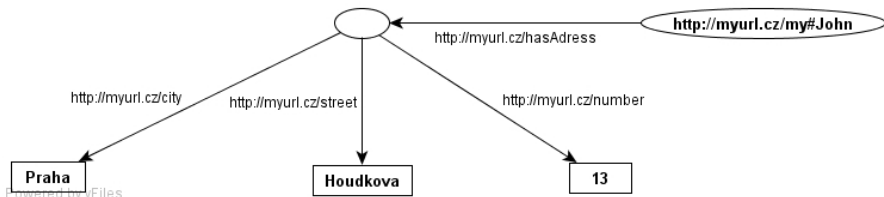Lean RDF Graph G_1 has no instance G_2 which is a proper subgraph of G_1.

Original RDF Graph

Lean RDF graph
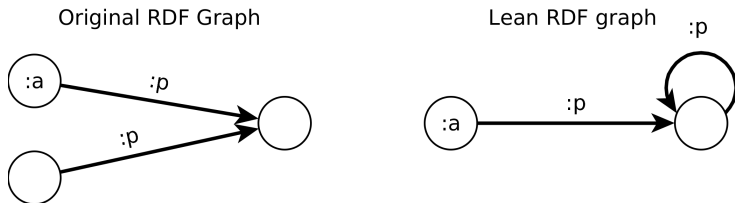
Blank Nodes for statement reification

Blank Nodes for expressing complex values
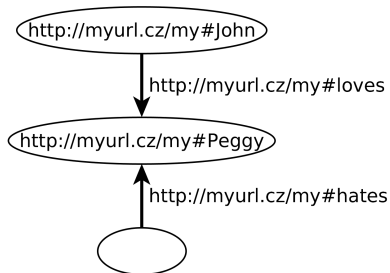
Blank Nodes for other use-cases

- container description – multisets, sequences, alternatives
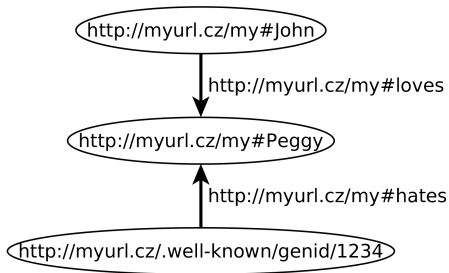- modeling n-ary relations (e.g. `birth`)

Blank Node Skolemization

- replacing the blank nodes with fresh IRIs (*Skolem IRI*) to allow stronger identification of those resources
- the meaning of the RDF graph remains the same as before skolemization
- skolemized IRIs `http://.../.well-known/genid/xxx`, where `xxx` is a placeholder for a generated identifier.

Original RDF Graph

Skolemized RDF Graph

RDF features
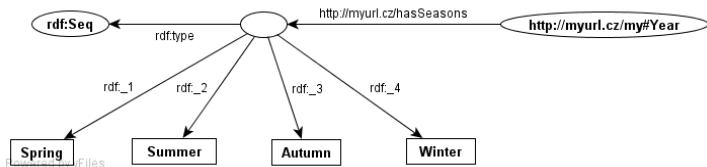
RDF containers

- `rdf:Bag` denotes an unordered sets of possibly repeating elements (multiset),
- `rdf:Seq` denotes an ordered seequence,
- `rdf:Alt` denotes an alternative choice from given resources/literals

- Container elements can be addressed by means of the `rdf:_x` property, where 'x' is a positive number,

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- Containers are **not closed** – someone else can assert statements adding elements to our container,
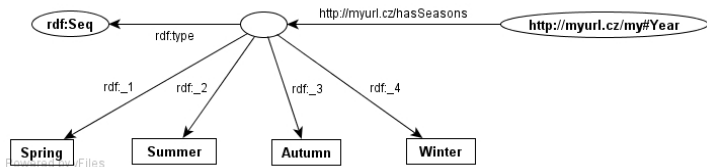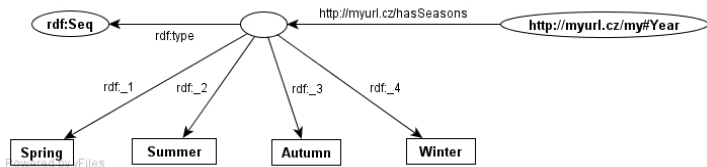
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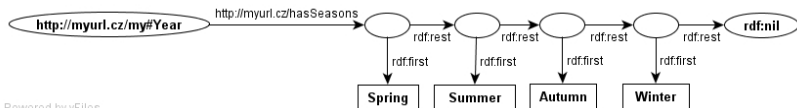- Containers are **not closed** – someone else can assert statements adding elements to our container,
- Containers can be modeled by means of blank nodes.

RDF collections

Powered by yFiles

- represent **closable** containers, similarly as LISP/Prolog lists

RDF collections
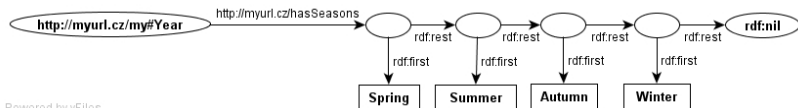
Powered by yFiles

- represent **closable** containers, similarly as LISP/Prolog lists
- `rdf:List` represents a list; the list head is available through `rdf:first` and the property is available through `rdf:rest`. The list can be closed by means of an empty list `rdf:nil`.

RDF Model – Axiomatic Triples
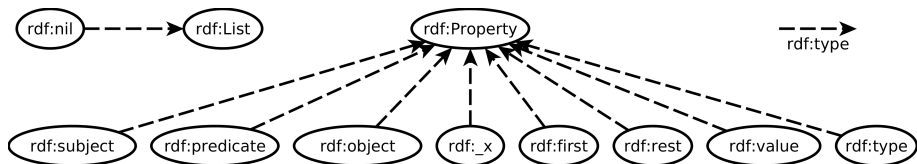
Figure: Visualization of axiomatic triples of RDF. Precise definition can be found in [**Patel-Schneider:14:RS**]

RDF 1.1

- RDF 1.1 Primer – W3C Working Group Note [**Schreiber:13:RP**]

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  - identification of resources by IRIs
  - all literals are *typed*, new datatypes introduced:

    ```
    rdf:langString
    rdf:HTML
    rdf:XMLLiteral
    ```

  The last two are non-normative in RDF 1.1

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    ```
    rdf:langString
    rdf:HTML
    rdf:XMLLiteral
    ```

    The last two are non-normative in RDF 1.1
  - additional XSD datatypes

    ```
    xsd:duration,
    xsd:dayTimeDuration,
    xsd:yearMonthDuration,
    xsd:dateTimeStamp
    ```

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    - additional XSD datatypes

    ```
    xsd:duration,
    xsd:dayTimeDuration,
    xsd:yearMonthDuration,
    xsd:dateTimeStamp
    ```

- additional serialization – JSON-LD, Turtle, TriG, N-Quads

RDF*

- extending RDF 1.1 to support more efficient reification
- https://arxiv.org/pdf/1406.3399.pdf
- statement `:man :hasSpouse :woman.`
- reified statement `<<:man :hasSpouse :woman>>`
  `:startDate "2020-02-11" .`

Metamodeling in RDFS

RDFS Basics

- RDFS = RDF Schema
- simple metamodeling language
- rdfs being shortcut for
  http://www.w3.org/2000/01/rdf-schema#
- rdf being shortcut for
  http://www.w3.org/1999/02/22-rdf-syntax-ns#
- RDF Schema 1.0 – W3C Recommendation in 2004
  [**Brickley:04:RVD**]
- basic metamodeling vocabulary:

```
rdf:type,
rdfs:Class,
rdfs:subClassOf,
rdf:Property,
rdfs:subPropertyOf,
rdfs:domain,
rdfs:range
```

Classes

- define instances :

```
ex:John rdf:type ex:Person .
```

Classes

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  ```
  ex:John rdf:type ex:Person .
  ```

- define classes (class `rdfs:Class`) :
  ```
  ex:Person rdf:type rdfs:Class .
  ```

Classes

- define instances :

  ```
  ex:John rdf:type ex:Person .
  ```

- define classes (class `rdfs:Class`) :

  ```
  ex:Person rdf:type rdfs:Class .
  ```

- create class hierarchies (property `rdfs:subClassOf`) :

  ```
  ex:Woman rdfs:subClassOf ex:Person .
  ```

Classes

- define instances :

    ```
    ex:John rdf:type ex:Person .
    ```

- define classes (class rdfs:Class) :

    ```
    ex:Person rdf:type rdfs:Class .
    ```

- create class hierarchies (property rdfs:subClassOf) :

    ```
    ex:Woman rdfs:subClassOf ex:Person .
    ```

- multiple inheritance :

    ```
    ex:Woman rdfs:subClassOf ex:Person .
    ex:Woman rdfs:subClassOf ex:Female.
    ```

Properties

- property definitions (resource `rdf:Property`) :

  ```
  ex:hasParent rdf:type rdf:Property .
  ```

- creation of property hierarchies (property `rdfs:subPropertyOf`) :

  ```
  ex:hasMother rdfs:subPropertyOf ex:hasParent.
  ```

- multiple inheritance

- domain and range definition :

  ```
  ex:hasMother rdfs:domain ex:Person .
  ex:hasMother rdfs:range ex:Woman
  ```

- domains/ranges considered as conjunction :

  ```
  ex:hasMother rdfs:range ex:Person .
  ex:hasMother rdfs:range ex:Female .
  ```

RDFS Model – Axiomatic Triples

Figure: Visualization of axiomatic triples of RDFS. Precise definition can be found in [**Patel-Schneider:14:RS**]

RDF Syntaxes

Syntaxes

Turtle family

N-TRIPLES , simple triples, for batch processing

TURTLE , well-readable, compact

TriG , extension of TURTLE for multiple graphs (RDF datasets)

N-QUADS , extension of N-TRIPLES for multiple graphs (RDF datasets)

RDF/XML , a frame-based syntax

JSON-LD , JSON syntax for RDF 1.1

RDF-A , syntax for embedding RDF 1.1 into HTML

N-TRIPLES

suitable for loading large data volumes

```
<http://www.myurl.cz/my#George> <http://www.myurl.cz/my#loves> <http://www.myurl.cz/my#Peggy> .
<http://www.myurl.cz/my#Peggy> <http://www.myurl.cz/my#hasHusband> <http://www.myurl.cz/my#John>
<http://www.myurl.cz/my#John> <http://www.myurl.cz/my#hates> <http://www.myurl.cz/my#George> .
<http://www.myurl.cz/my#John> <http://www.w3.org/1999/02/22-rdf-syntax-ns#type>
    <http://www.otherurl.org/other#Person> .
<http://www.myurl.cz/my#John> <http://www.myurl.cz#hasAge>
    "27"^^<http://www.w3.org/2001/XMLSchema#integer> .
```

TURTLE

extension of N-TRIPLES, allowing shortcuts

```
@prefix rdf: <http://www.w3.org/1999/02/22-rdf-syntax-ns#> .
@prefix my: <http://www.myurl.cz/my#> .
@prefix xsd: <http://www.w3.org/2001/XMLSchema#> .
my:George my:loves my:Peggy .
my:Peggy my:hasHusband my:John .
my:John rdf:type <http://www.otherurl.org/other#Person> ;
        my:hates my:George ;
        my:hasAge "27"^^xsd:integer.
```

```
:a :p1 :o1 ;
   :p2 :o2 .
```

```
:a :p1 :o1 .
:a :p2 :o2 .
```

```
:a :p :o1, :o2 .
```

```
:a :p :o1 .
:a :p :o2 .
```

TURTLE

extension of N-TRIPLES, allowing shortcuts

```
:a :p1 [
    :p2 :o2 ;
    :p3 :o3 .
]
```

```
:a :p1 _:x .
_:x :p2 :o2 .
_:x :p3 :o3 .
```

```
:a :p (:o1 :o2 :o3).
```

```
:a :p _:a .
_:a rdf:first :o1 .
_:a rdf:rest _:b .
_:b rdf:first :o2 .
_:b rdf:rest _:c .
_:c rdf:first :o3 .
_:c rdf:rest rdf:nil .
```

RDF/XML
readable, expressive, plenty of syntactic sugar

```
<rdf:RDF xmlns:rdf="http://www.w3.org/1999/02/22-rdf-syntax-ns#"
         xmlns:base="http://myurl.cz/my#"
         xmlns:my="http://myurl.cz/my#"
         xmlns:other="http://otherurl.org/other#">
<rdf:Description rdf:ID="George">
  <my:loves rdf:about="http://myurl.cz/my#Peggy"/>
</rdf:Description>
<rdf:Description rdf:ID="Peggy">
  <my:hasHusband rdf:about="http://myurl.cz/my#John"/>
</rdf:Description>
<other:Person rdf:ID="John">
  <my:hates rdf:about="http://myurl.cz/my#George"/>
  <my:hasAge rdf:datatype="http://www.w3.org/2001/XMLSchema#integer">
    27
  </my:hasAge>
</other:Person>
</rdf:RDF>
```

RDF Datasets

Definition

RDF dataset is a collection of RDF graphs:

$$DS = \{DG, (i_1, G_1), \ldots, (i_n, G_n)\}$$

consisting of a **default (unnamed) RDF graph** DG and zero or more **named RDF graphs** G_k identified by their IRI/blank node i_k.

- Default graphs might be independent on named graphs (in RDF4J they are not – default graph contains union of all named graphs).
- Blank nodes can be reused between different graphs in a single RDF dataset.
- For SPARQL 1.1, RDF dataset cannot use blank nodes as graph names.

RDF Merge

- **Merge** of RDF graphs G_1 and G_2 is an RDF graph created as follows:
  - rename b-nodes in G_1, so that no b-node label occur in both G_1 and G_2.
  - union G_1 and G_2.
- Example:
  - G_1 :

    ```
    @prefix : <http://www.myurl.cz/my#> .
    :a :p _:b .
    :a :q _:c .
    ```

  - G_2 :

    ```
    @prefix : <http://www.myurl.cz/my#> .
    :a :s _:c .
    :a :t _:d .
    ```

  - merge of G_1 and G_2 :

    ```
    @prefix : <http://www.myurl.cz/my#> .
    :a :p _:b .
    :a :q _:c .
    :a :s _:e .
    :a :t _:d .
    ```

Semantics of RDF(S)

1 RDF Vocabularies

2 Advanced: Formal Semantics of RDF(S)

Entailment Regimes and Semantic Extension

Precise definition of RDF semantics can be found in
[**Patel-Schneider:14:RS**]

Definition

Semantic Extension is a set of semantic constraints on an RDF graph.

Entailment Regime is a set of entailments defined by the corresponding
*semantic extension*.

- Four entailment regimes are predefined in RDF specs:

  Simple entailment provides only structural matching of graphs with
  possible b-node renaming

  D entailment additionally interprets datatypes

  RDF entailment interprets RDF vocabulary

  RDFS entailment interprets RDF and RDFS vocabularies

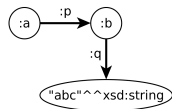- All entailment regimes must be *monotonic* extensions of simple
  entailment
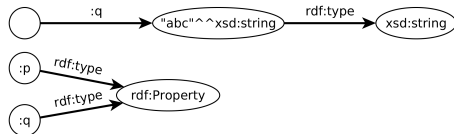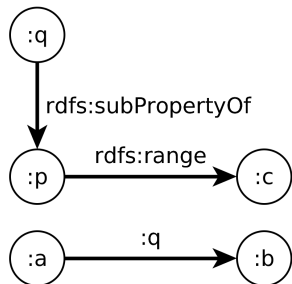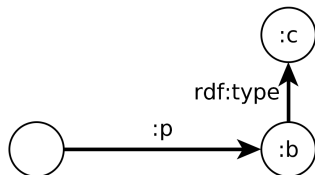
Simple Entailment Example
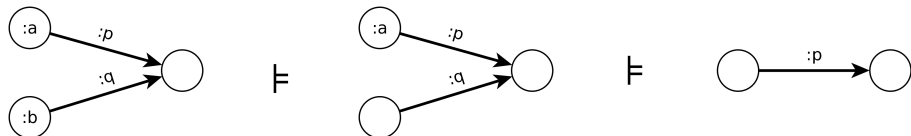
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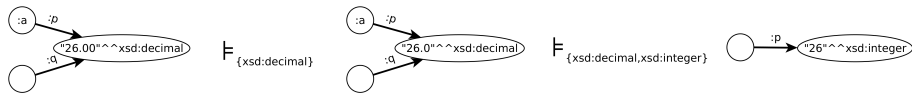
RDF Entailment Example

RDFS Entailment Example

1 RDF Vocabularies

2 Advanced: Formal Semantics of RDF(S)

RDF Vocabularies

Vocabularies

Various predefinied vocabularies can be reused in your data, e.g.:

- RDF – `https://www.w3.org/TR/rdf11-primer`
- RDFS – `https://www.w3.org/TR/rdf-schema/`
- OWL – `https://www.w3.org/TR/owl2-overview/`
- schema.org – `http://schema.org/docs/schemas.html`
- Dublin Core –
  `http://dublincore.org/documents/dc-rdf/.`
  `https://www.dublincore.org/specifications/`
  `dublin-core/dcmi-terms/`
- FOAF – `http://www.foaf-project.org/`
- DCAT – `https://www.w3.org/TR/vocab-dcat-2/`
- VOID – `http://www.w3.org/TR/void/`
- . . . and many others

schema.org

Source:
https://wiki.eclipse.org/File:Schema-main-2.0.101D.png
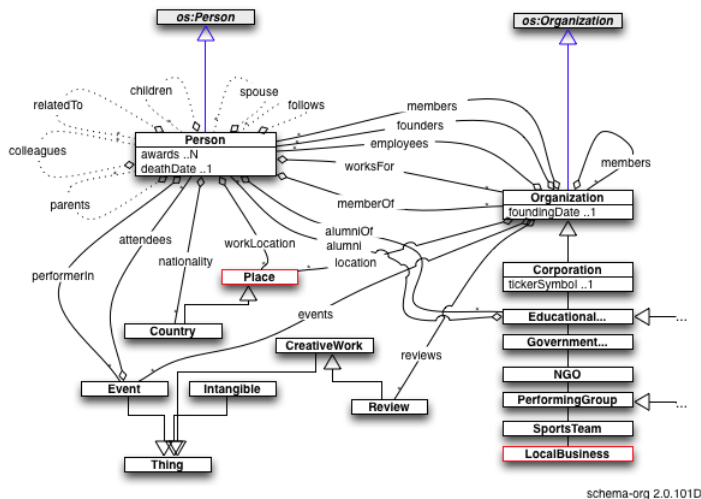
Dublin Core

Table 1. The Fifteen Elements of "Simple Dublin Core"

Identifier	Definition
Title	A name given to the resource.
Creator	An entity primarily responsible for making the content of the resource.
Subject	The topic of the content of the resource.
Description	An account of the content of the resource.
Publisher	An entity responsible for making the resource available.
Contributor	An entity responsible for making contributions to the content of the resource.
Date	A date associated with an event in the life cycle of the resource.
Type	The nature or genre of the content of the resource.
Format	The physical or digital manifestation of the resource.
Identifier	An unambiguous reference to the resource within a given context.
Source	A reference to a resource from which the present resource is derived.
Language	A language of the intellectual content of the resource.
Relation	A reference to a related resource.
Coverage	The extent or scope of the content of the resource.
Rights	Information about rights held in and over the resource.

Source: Sugimoto, Shigeo, Thomas Baker and Stuart L. Weibel. "Dublin Core: Process and Principles." ICADL (2002).

1 RDF Vocabularies

2 Advanced: Formal Semantics of RDF(S)

Advanced: Formal Semantics of RDF(S)

Simple Interpretation

Definition

A finite interpretation $I = (IR, IP, IEXT, IS, IL)$ w.r.t. vocabulary $N = (N_{IRI}, N_{lit})$ is defined as follows:

- IR is a set of *resources*
- IP is a set of *properties* (often $IP \subseteq IR$)
- $IEXT$ is a mapping $IEXT : IP \to IR \times IR$
- IS is a mapping $IS : N_{IRI} \to IR \cup IP$
- IL is a partial mapping $IL : N_{lit} \to IR$

Simple Interpretation Example

```
@prefix : <http://www.myurl.cz/my#> .
:John :loves :Mary .
:John :childcount 2 .
```

- $IR = \{John, Mary, 2\}$ (real resources)
- $IP = \{loves, childcount\}$ (real properties)
- $IEXT = \{(loves, \langle John, Mary \rangle),$
  $(childcount, \langle John, 2 \rangle)\}$
- $IS = \{\langle \texttt{http} : //\texttt{www.myurl.cz/my\#John}, John \rangle,$
  $\langle \texttt{http} : //\texttt{www.myurl.cz/my\#Mary}, Mary \rangle,$
  $\langle \texttt{http} : //\texttt{www.myurl.cz/my\#loves}, loves \rangle,$
  $\langle \texttt{http} : //\texttt{www.myurl.cz/my\#childcount}, childcount \rangle\}$
- $IL =$
  $\{\langle \texttt{"2"^^http} : //\texttt{www.w3.org/2001/XMLSchema\#integer}, 2 \rangle\}$

Simple Entailment

Simple entailment is just a "structural matching with b-node rewriting."

Semantic Conditions on Simple Entailment

- if E is a literal, then $I(E) = IL(E)$
- if E is an IRI, then $I(E) = IS(E)$
- if E is a ground triple (s, p, o), then $I(E) = true$ iff $I(p) \in IP$ and $\langle I(s), I(o) \rangle \in IEXT(I(p))$
- if E is a ground RDF graph, then $I(E) = true$ iff $I(E') = true$ for each triple $E' \in E$
- if E is an RDF graph, then $I(E) = true$ iff there exists a mapping $A : N_{bnode} \to IR$, such that $I(A(E)) = true$, where $A(E)$ is E, where each blank node B is replaced by $A(B)$.
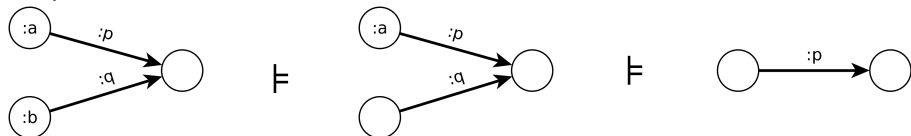
Simple Entailment

- graph G_1 (simply) entails graph G_2 (denoted $G_1 \models G_2$) if $I(G_2) = true$ whenever $I(G_1) = true$.
- if $G_1 \models G_2$ and $G_2 \models G_1$ then they are *logically equivalent*.

How to Check Simple Entailment ?

Interpolation lemma

Graph G_1 simply entails graph G_2 iff a subgraph of G_1 is an instance of G_2.

Simple entailment is NP in the size of G_1 and G_2.

D-Entailment

In addition to blank nodes, *D*-entailment ($\models_D$) interprets datatypes in the set *D* of recognized datatypes. Literals with non-recognized datatypes are treated as uninterpreted.

Semantic Conditions on D-Entailment

- if `rdf:langString` $\in D$, then for each literal *lex@lang*:,
  $IL(lex@lang) = \langle lex, lowercase(lang) \rangle$
- if *dIRI* $\in D$, then for each literal *lex^^dIRI*:
  $IL(lex\text{^^}dIRI) = L2V(I(dIRI))(lex)$, where
    - $I(dIRI)$ is a datatype identified by *dIRI*
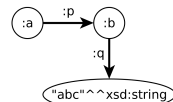   - $L2V(d)$ transforms a lexical value to the value space of *d*.
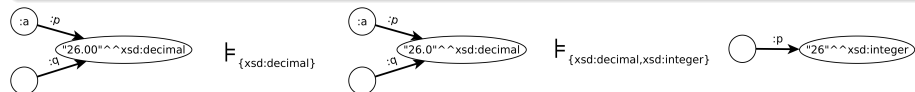
RDF-Entailment

In addition to D-entailment, RDF-entailment w.r.t D interprets properties in the RDF vocabulary.

Entailment rules

rule	G contains	t_i, s.t. $G \models_{RDF-D} t_i$
GrdfD1	$(s, p, lex\text{\textasciicircum}\text{\textasciicircum}d)$ $d \in D$	$(lex\text{\textasciicircum}\text{\textasciicircum}d, \mathrm{rdf:type}, d)$
rdfD2	(s, p, o)	$(p, \mathrm{rdf:type}, \mathrm{rdf:Property})$

For example:
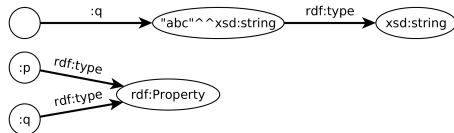
RDFS-Entailment
RDFS-entailment w.r.t D interprets most RDF and RDFS vocabulary.

Entailment rules

| rule	G contains	t_i, s.t. $G \models_{RDFS-D} t_i$
rdfs1	any IRI $dIRI \in D$ in G	$(dIRI, \mathrm{rdf:type}, \mathrm{rdfs:Datatype})$
rdfs2	(s, p, o), $(p, \mathrm{rdfs:domain}, w)$	$(s, \mathrm{rdf:type}, w)$
rdfs3	(s, p, o), $(p, \mathrm{rdfs:range}, w)$	$(o, \mathrm{rdf:type}, w)$
rdfs4	(s, p, o)	$(s, \mathrm{rdf:type}, \mathrm{rdfs:Resource})$ $(o, \mathrm{rdf:type}, \mathrm{rdfs:Resource})$
rdfs5	$(p_1, \mathrm{rdfs:subPropertyOf}, p_2)$ $(p_2, \mathrm{rdfs:subPropertyOf}, p_3)$	$(p_1, \mathrm{rdfs:subPropertyOf}, p_3)$
rdfs6	$(p, \mathrm{rdf:type}, \mathrm{rdf:Property})$	$(p, \mathrm{rdfs:subPropertyOf}, p)$
rdfs7	$(p_1, \mathrm{rdfs:subPropertyOf}, p_2)$ (s, p_1, o)	(s, p_2, o)
rdfs8	$(s, \mathrm{rdf:type}, \mathrm{rdfs:Class})$	$(s, \mathrm{rdfs:subClassOf}, \mathrm{rdfs:Resource})$
rdfs9	$(c_1, \mathrm{rdfs:subClassOf}, c_2)$ $(s, \mathrm{rdf:type}, c_1)$	$(s, \mathrm{rdf:type}, c_2)$
rdfs10	$(c, \mathrm{rdf:type}, \mathrm{rdfs:Class})$	$(c, \mathrm{rdfs:subClassOf}, c)$
rdfs11	$(c_1, \mathrm{rdfs:subClassOf}, c_2)$ $(c_2, \mathrm{rdfs:subClassOf}, c_3)$	$(c_1, \mathrm{rdfs:subClassOf}, c_3)$
rdfs12	$(p, \mathrm{rdf:type},$ $\mathrm{rdfs:ContainerMembershipProperty})$	$(p, \mathrm{rdfs:subPropertyOf},$ $\mathrm{rdfs:member})$
rdfs13	$(d, \mathrm{rdf:type}, \mathrm{rdfs:Datatype})$	$(d, \mathrm{rdfs:subClassOf}, \mathrm{rdfs:Literal})$

RDFS Entailment Example
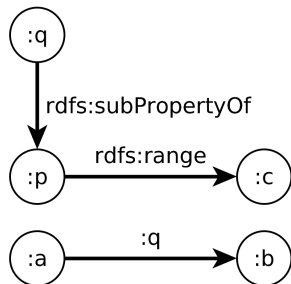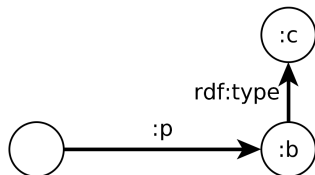
Entailment Checking

All discussed entailments can be checked by applying the entailment rules on *generalized RDF graphs*, i.e. **graphs that allow all RDF Terms in all positions – subject, predicate, object**.

Entailment checking procedure

$G_1 \models_X G_2$, iff $Clos_X(G_1)$ simply entails G_2, where $Clos_X(G_1)$ is constructed as follows:

1. Add to G_1 all axiomatic triples for $X \in \{\text{RDF-D,RDFS-D}\}$ (visualized in Figure 1, resp. Figure 2)

2. For each container membership property IRI p occuring in G_1, add to G_1 corresponding axiomatic triples for X containing p.

3. If no triples were added in the previous step, add axiomatic triples for X containing `rdf:_1`.

4. Apply rules for X (i.e. $\{GrdfD1, rdfD2\}$ for $X = RDF$, or $\{Grdf1, rdfD2, rdfs1, \ldots, rdfs13\}$ for $X = RDFS$) with $D = \{rdf : langString, xsd : string\}$, until exhaustion.

Entailment Checking Complexity

- the previous procedure is finite and polynomial
- simple entailment checking itself is NP
- the less blank nodes, the more efficient