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## Exam test

Variant: A
Points

1. Consider the following network.

$$
\begin{equation*}
y=\sin \left(\mathbf{w}^{\top} \mathbf{x}\right)-b \tag{1}
\end{equation*}
$$

- Draw the computational graph of the forward pass of this network. Note that every operator is a node with a given arity and output. For example, the + operator is a node which has two input arguments and a single output argument, etc...
- Consider an input $\mathbf{x}=[2,1] \mathbf{w}=\left[\frac{\pi}{2}, \pi\right], b=0$ and label $l=2$.
- Compute the forward pass of the network.
- Use an $L_{2}$ loss (Mean square error) to compute the loss value between the forward prediction $y$ and label $l$. Add this loss to the computation graph.
- Use the chain rule to compute the gradient $\frac{\partial L(y, l)}{\partial \mathbf{w}}$ and estimate an update of parameters $\mathbf{w}$ with learning rate $\alpha=0.5$.

2. Consider a convolutional neural network layer $l_{1}$ which maps an $R G B$ image of size $128 \times 128$ to 16 feature maps having the same spatial dimensions as the image. The kernel size is $3 \times 3$ and uses a stride 1 .

- What is the size of padding, which ensure the same spatial resolution of the output feature map?
- How much memory (in bytes) do kernel weights in layer $l_{1}$ take up, assuming float32 weights? Ignore the bias weights.
- How many mathematical operations does this layer perform for a single forward pass. A multiplication or addition of two numbers can be considered as one operation.
- Name one way that we can regularize (prevent overfitting) a large parametric model (for example a neural network).

3. Activation function maps single input $\mathbf{x}$ on a single output value $\mathbf{y}$.

- Define a Leaky Rectified Linear Unit lrelu(x) activation function in pseudocode, with $\alpha=0.1$. The function has a single argument $\mathbf{x}$ and output $\mathbf{y}=\operatorname{lrelu}(\mathbf{x})$.
- Define the gradient of the $\operatorname{lrel} \mathbf{u}(\mathbf{x})$ activation function in pseudocode. The function has a single argument $\mathbf{x}$ and outputs $\frac{\partial l \text { relu }(x)}{\partial \mathbf{x}}$. Hint: Break up the function into two separate cases (if-else).

4. You are given batch of three one-dimensional training examples $x_{1}=5, x_{2}=2, x_{3}=1$.

- Compute output of the batch-norm layer with learnable parameters $\gamma=6, \beta=-1$.
- Compute gradient of the batch-norm layer with respect to the parameter $\beta$. Hint: output of the batch-norm layer for this batch is three-dimensional.

5. Consider MDP consisting of three states $\mathbf{x}_{0}=3, \mathbf{x}_{1}=1, \mathbf{x}_{2}=2$ and two types of actions $\mathbf{u}=1$ and $\mathbf{u}=2$, see image below. Agent selects action $\mathbf{u}$ in the state $\mathbf{x}$ according the following stochastic policy

$$
\pi_{\theta}(\mathbf{u} \mid \mathbf{x})= \begin{cases}\sigma(\theta \mathbf{x}) & \text { if } \mathbf{u}=1 \\ 1-\sigma(\theta \mathbf{x}) & \text { if } \mathbf{u}=2\end{cases}
$$

with scalar parameter $\theta=2$. This policy maps one-dimensional state $\mathbf{x}$ on the probability distribution of two possible actions $\mathbf{u}=1$ or $\mathbf{u}=2$.


Consider trajectory-reward function defined as follows:

$$
r(\tau)=\sum_{\mathbf{x}_{i} \in \tau} \frac{1}{\mathbf{x}_{i}}
$$

Given training trajectory $\tau=\left[\left(\mathbf{x}_{0}=3\right),(\mathbf{u}=1),\left(\mathbf{x}_{1}=1\right)\right]$, which consists of the single transition (outlined by red color), estimate:

- Policy gradient
$\left.\frac{\partial \log \pi_{\theta}(\mathbf{u} \mid \mathbf{x})}{\partial \theta}\right|_{\substack{\mathbf{x}=\mathbf{x}_{0} \\ \mathbf{u}=\mathbf{u}_{0}}} \cdot r(\tau)=$
- Updated weights with learning rate $\alpha=1$
$\theta^{\text {updated }}=$

