Lecture 4: Relational, Logical, and Set Operators, Searching and Sorting

A8B17CAS

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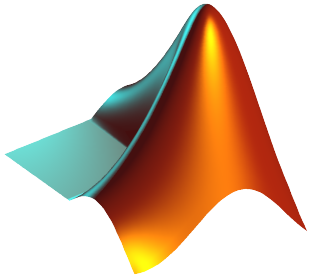
Outline

1. Relational Operators
2. Logical Operators
3. Set Operators
4. Searching and Sorting



Warm Up: Complex Power Delivered To a Circuit

There is function `calendar()`. Let us play with it! Use MATLAB Editor...

Exercise

Complete a snippet of code to get:

- A row vector of all Mondays in this month.
- How many they are?
- What day was January 1st, 1901?

Relational Operators I.

- To find out, to compare, whether "something" is greater than, less than, equal to, etc.
- The result of the comparison is always either
  - positive (`true`), logical one "1",
  - negative (`false`), logical zero "0".
- All relation operators are vector-wise.
  - It is also possible to compare vector vs. vector, matrix vs. matrix, ...
- Often in combination with logical operators (*see later*)
  - Multiple relational operators can be applied to complex expressions.

`>`	greater than
`>=`	greater than or equal to
`<`	less than
`<=`	less than or equal to
`==`	equal to
`~=`	not equal to

Relational Operators II.

- Having the vector $\mathbf{G} = \begin{pmatrix} \frac{\pi}{2} & \pi & \frac{3\pi}{2} & 2\pi \end{pmatrix}$, find elements of $\mathbf{G}$ that are
  - greater than π,
  - less than or equal to π,
  - not equal to π.
  - Try similar operations for $\mathbf{H} = \mathbf{G}^{\mathrm{T}}$.

- Find out whether $\mathbf{V} \geq \mathbf{U}$:
  - $\mathbf{V} = \begin{pmatrix} -\pi & \pi & 1 & 0 \end{pmatrix}$,
  - $\mathbf{U} = \begin{pmatrix} 1 & 1 & 1 & 1 \end{pmatrix}$.

Exercise

Relational Operators – Evaluation Goes From Left to Right!

- Find out the results of following relations.
  - Try to interpret the results.

```
2 < 1 ~= 1 % ???
```

```
r = 1/2;
0 < r < 1 % ???
```

```
(1 > A) <= true
```

Exercise

Logical Operators I.

- To to find out, whether particular condition is fulfilled.
- The result is always either
  - positive (`true`), logical one "1",
  - negative (`false`), logical zero "0".
- `all`, `any` is used to convert logical array into a scalar.
- MATLAB interprets any numerical value except 0 as `true`.
- All logical operators are vector-wise.
  - It is also possible to compare vector vs. vector, matrix vs. matrix, ...
- Function `is*` extends possibilities of logical expressions.

`&`	and
`|`	or
`~`	not
	xor
	all
	any

Logical Operators II.

Use vector $\mathbf{A} = \begin{bmatrix} \frac{\pi}{2} & \pi & \frac{3\pi}{2} & 2\pi \end{bmatrix}$ to check the following conditions.

- Which elements are equal to π or are equal to 2π.
- Which elements are greater than $\frac{\pi}{2}$ and at the same time are not equal to 2π.
- Group elements from the previous condition with vector $\mathbf{A}$.

Exercise

Logical Indexing I.

Assume a vector of 10 random numbers ranging from -10 to 10.

```
a = 20*rand(10, 1) - 10
```

- Return `true` for elements fulfilling the condition.

```
a < -5 % relation operator
```

- Return values of those elements fulfilling the condition (logical indexing).

```
a(a < -5)
```

- Put value of -5 to the position of elements fulfilling the condition.

```
a(a < -5) = -5
```

- Zero all values in the range from -5 to 5.

```
a(a > -5 & a < 5) = 0
```

- Thresholding (values below -5 set equal to -5, values above 5 set equal to 5).

```
a(a < -5 | a > 5) = ...
  sign(a(a < -5 | a > 5))*5
```

Logical Indexing II.

- Create a row vector **k** in the interval from 1 to 20 with step of 3.
- Create the vector **m** filled with elements from vector **k** that are:
  - greater than 10 and at the same time less than 16.
- Use logical operators.

Exercise

Logical Operators: Functions `all` And `any`

Create matrix **M** and answer following questions using functions `all` and `any`.

```
M = magic(3);
```

- In which of the columns are all elements greater than 2?

$$\texttt{any}\left(\begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}\right) = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix},$$

- In which of the rows is there at least one element greater than or equal to 8?

$$\texttt{all}\left(\begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}\right) = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix},$$

- Does the matrix **M** contain only positive numbers?

$$\texttt{any}\left(\texttt{all}\left(\begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}\right)\right) = \texttt{any}\left(\begin{bmatrix} 0 & 1 & 0 \end{bmatrix}\right) = 1$$

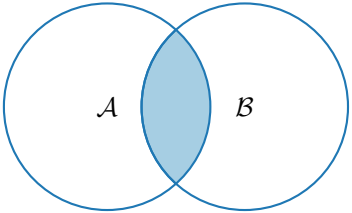
Exercise

Set Operations

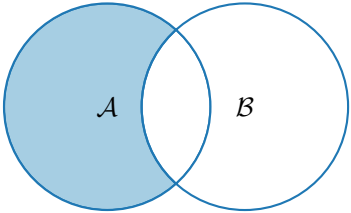
- Set operations are applicable to vectors, matrices, arrays, cells, strings, tables,...
- Mutual sizes of these structures are usually not important.

Function	Description
`intersect`	intersection of two sets
`union`	union of two sets
`setdiff`	difference of two sets
`setxor`	exclusive OR of two sets
`unique`	unique values in a set
`sort`	sorting
`sortrows`	row sorting
`ismember`	is an element member of a set?
`issorted`	is a set sorted?

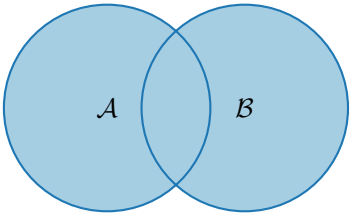
$\mathcal{A} \cap \mathcal{B}$ `intersect()`



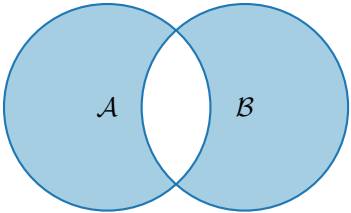
$\mathcal{A} \setminus \mathcal{B} = \mathcal{A} \cap \mathcal{B}^c$ `setdiff()`



$\mathcal{A} \cup \mathcal{B}$ `union()`



$\mathcal{A} \oplus \mathcal{B} = \overline{\mathcal{A} \cap \mathcal{B}}$ `setxor()`



Set Operations – Dirichlet's theorem

Consider three vectors **a**, **b**, **c** containing natural numbers $x \in \mathbb{N}$ so that:

- vector **a** contains all primes up to (and including) 1000,
- vector **b** contains all even numbers up to (and including) 1000,
- vector **c** is complement of **b** in the same interval.

Find vector **v** so that $\mathbf{v} = \mathbf{a} \cap (\mathbf{b} + \mathbf{c})$ and $\mathbf{b} + \mathbf{c} \equiv [b_i + c_i]$, $b_{i-1} < b_i < b_{i+1}$, $c_{i-1} < c_i < c_{i+1}$.

- What elements does **v** contain?
- How many elements are there in **v**?

Exercise

```
v =
  Columns 1 through 18
     3     7    11    19    23    31    43    47    59    67    71    79    83   103   107   127   131   139
  Columns 19 through 36
   151   163   167   179   191   199   211   223   227   239   251   263   271   283   307   311   331   347
  Columns 37 through 54
   359   367   379   383   419   431   439   443   463   467   479   487   491   499   503   523   547   563
  Columns 55 through 72
   571   587   599   607   619   631   643   647   659   683   691   719   727   739   743   751   787   811
  Columns 73 through 87
   823   827   839   859   863   883   887   907   911   919   947   967   971   983   991
ans =
    87
```

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```

Array Sorting

Sort array elements:

- column-wise, in ascending order: `>> sort(A)`
- row-wise, in ascending order: `>> sort(A, 2)`
- in descending order: `>> sort(A, 'descend')`
- in descending order, row-wise: `>> sort(A, 2, 'descend')`

Function `sortrows` sorts rows of a matrix.

- Elements of the rows are not swapped – rows are sorted as blocks.

`original matrix`

$$\begin{bmatrix} 8 & 1 & 6 \\ 3 & 5 & 7 \\ 4 & 9 & 2 \end{bmatrix}$$

`sort()`

$$\left[\begin{array}{c|c|c} 3 & 1 & 2 \\ 4 & 5 & 6 \\ 8 & 9 & 7 \end{array}\right]$$

`sortrows()`

$$\begin{bmatrix} 3 & 5 & 7 \\ 4 & 9 & 2 \\ 8 & 1 & 6 \end{bmatrix}$$

Searching in an Array

- Function `find` returns positions of non-zero (logical `true`) elements of a matrix.
- Example: Find the positions of those elements of vector $\mathbf{A} = \begin{bmatrix} \frac{\pi}{2} & \pi & \frac{3}{2}\pi & 2\pi \end{bmatrix}$ fulfilling the condition $\mathbf{A} > \pi$:

```
>> A = pi/2*(1:4)
>> find(A > pi)
```

  - Compare the above command with `>> A > pi`. What is the difference?

- To find the first/last k non-zero elements in $\mathbf{X}$:

```
>> ind = find(X, k, 'first')
>> ind = find(X, k, 'last')
```

- For more details see `>> doc find`.

Array Searching

- Sort vector $\mathbf{v} = [16 \quad 2 \quad 3 \quad 13 \quad 5 \quad 11 \quad 10 \quad 8 \quad 9 \quad 7 \quad 6 \quad 12 \quad 4 \quad 14 \quad 15 \quad 1]$ in descending order and find the elements of the vector (and their respective positions within the vector) that are divisible by three and at the same time are greater than 10.

To get the vector quickly:

```
v  = reshape(magic(4)', [1 numel(magic(4))])
```

```
v =

    16     2     3    13     5    11    10     8     9     7     6    12     4    14    15     1

v1 =

  1×16 logical array

   0   1   0   0   1   0   0   0   0   0   0   0   0   0   0   0

ans =

    15    12

ans =

     2     5
```

Exercise

Questions?

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