Question 1.

Let h, h' be propositional conjunctions. Is $h' \models h$ equivalent to $h \subseteq h'$? Justify your answer.

Question 2.

Consider basis expansion introducing additional n attributes holding the inverted values contained in the original observation x. Such transformation turns general conjunctions (disjunctions) into *monotone* ones, thus into linearly separable ones, i.e., learnable by Winnow algorithm. How will the transformation change the algorithm's mistake bound?

Question 3.

Consider the halving algorithm with hypothesis class (initial hypothesis) $\mathcal{H}_1$ of all non-contradictory conjunctions on 3 propositional variables.

1. Determine $|\mathcal{H}_1|$.
2. Give an upper bound on $|\mathcal{H}_2|$ given that first prediction was incorrect.

Question 4.

Consider halving algorithm with the initial version space $\mathcal{H}$ consisting

(a) of all conjunctions of exactly 3 different non-negative literals, i.e.,

$$\mathcal{H} = \{ p_i \wedge p_j \wedge p_k \mid 1 \leq i < j < k \leq n \}$$

(b) of all conjunctions that use some of the given variables (and the empty conjunction).

(c) of all n-CNFs.

1. For each scenario, determine if the learner learns $\mathcal{H}$ online (in the mistake-bound model) and justify your answer.
2. For each scenario where the learner successfully learns in the mistake bound model, decide if they learn efficiently as well. Assume that checking the consistency of a single hypothesis with an observation takes a unit of time.
3. For the **first** case, assume the first example is $(0, 1, 1, 1, \ldots 1)$ and it is a negative instance. What will be the learner's prediction for the second example, which is $(0, 1, 0, 1, \ldots)$? Justify your answer.