Peaking Power Plants

March 8, 2022

1 The Peaking Power-Plants

1.1 Motivation

You are the boss of an electricity distribution company. Based on the contracts with the end-users, you know the demand on the amount of the electrical energy between hour t and $(t+1)$ next day; i.e., you have vector $\bar{d} = (d_0, d_1, \ldots, d_{23})$ of demands.

Your company owns several power-plants and wants to cover the demands by switching these plants on and off. There are two types of power-plants: `base` and `peak` plants (there are n_{base} and n_{peak} of these plants).

- **Base power-plants** (zdroje základního zatížení) are cheap, but it takes a long time to start them and turn them off; therefore, they need to be either turned on or turned off the whole day. These plants produce e_{base} energy every hour and their running cost is c_{base} every hour.
- **Peak power-plants** (špičkové zdroje) are fast, and can be turned on/off every hour, but typically the price is high. These plants produce e_{peak} energy every hour and their running cost c_{peak} (every hour).

The surplus energy is being stored in the batteries (storage) and can be used later. However, there is a loss modeled by parameter $\gamma \in [0, 1]$. If k units of energy leave the storage, only $\gamma \cdot k$ units can be used to cover demands (the rest, i.e., $(1 - \gamma) \cdot k$, is lost).

The capacity of the storage is limited to s_{max} units, and due to technological restrictions, it is impossible to take the energy from the storage and store it inside simultaneously. The produced energy needs to cover the demands or be stored in the batteries (energy cannot just vanish). The storage is empty at the beginning.

You want to minimize to total cost (turning the power-plants on/off) while covering all the demands.

1.2 Input

You are given the following:

- $\mathbf{d} = (d_0, d_1, \ldots, d_{23})$ – vector of demands
- n_{base}, n_{peak} – number of the respective power-plants
- c_{base}, c_{peak} – cost needed for the running of the respective type of the plant (per hour)
- e_{base}, e_{peak} – amount of energy generated by the respective type (per hour)
- s_{max} – storage capacity
- $(1-\gamma)$ – energy loss of the storage (only $\gamma \cdot k$ energy units can be used out of k units)

For the testing purposes, you can experiment with the following instance:

[1]:
```
d = [5, 5, 5, 5, 5, 10, 10, 15, 20, 20, 30, 30, 40, 50, 60, 60, 60, 50, 40, 30,␣
 ↪30, 20, 10, 5]

n_base = 3
e_base = 7
c_base = 2.0 / 24.0

n_peak = 40
e_peak = 2
c_peak = 12

s_max = 100

gamma = 0.75
```

1.3 Output

Your goal is to find the number of `base` power-plants that should be active throughout the whole day, as well as the number of `peak` power-plants that should be active every hour such that the cost is minimized.

The **optimal solution** for the given instance is:

- 2 active base power-plants
- $(0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 5, 8, 13, 7, 23, 23, 23, 18, 13, 8, 8, 3, 0, 0)$ active peak power-plants (every hour)

The corresponding cost is 1840.

1.4 Exercise

Implement the ILP modelfor the peaking power-plants problem, solve it and examine the solution.

[2]:
```
import gurobipy as gb

# MODEL
m = gb.Model()

# - ADD VARIABLES
# TODO

# - ADD CONSTRAINTS
# TODO

# - SET OBJECTIVE
# TODO
```

```
m.optimize()
```

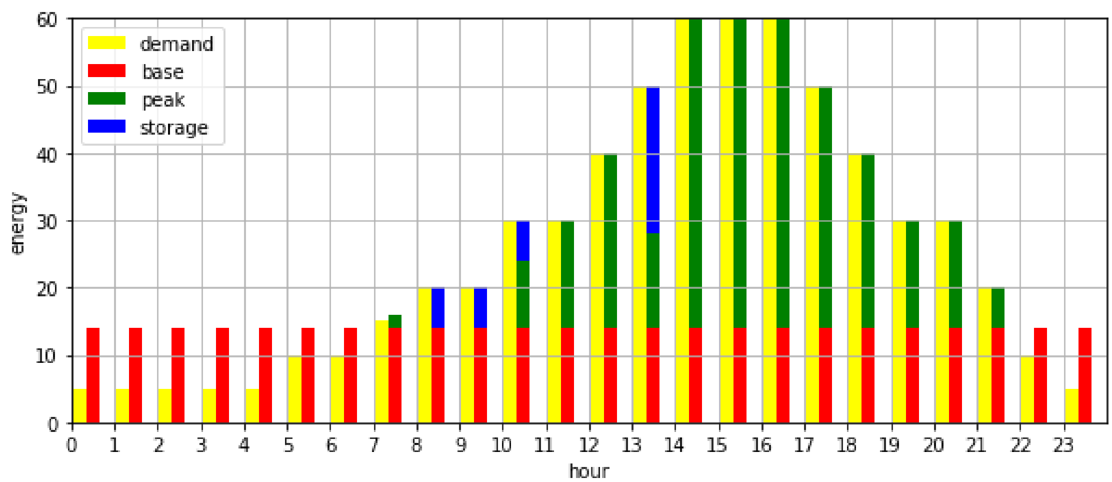
```
Academic license - for non-commercial use only
Optimize a model with 0 rows, 0 columns and 0 nonzeros
Coefficient statistics:
  Matrix range     [0e+00, 0e+00]
  Objective range  [0e+00, 0e+00]
  Bounds range     [0e+00, 0e+00]
  RHS range        [0e+00, 0e+00]
Presolve time: 0.01s
Presolve: All rows and columns removed
Iteration    Objective       Primal Inf.    Dual Inf.      Time
       0    0.0000000e+00   0.000000e+00   0.000000e+00      0s

Solved in 0 iterations and 0.01 seconds
Optimal objective  0.000000000e+00
```

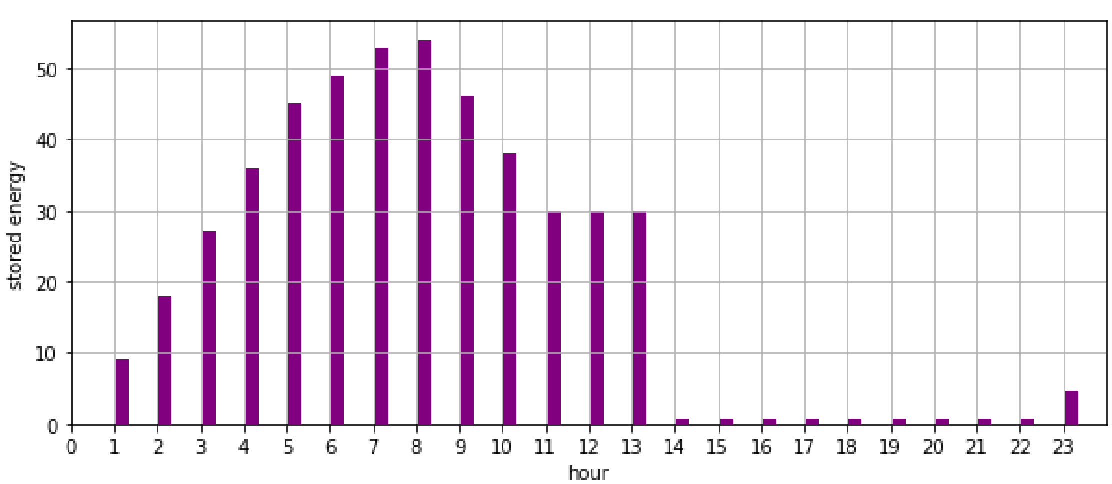
1.5 Solution visualization

[4]:
```
import matplotlib.pyplot as plt
import numpy as np

def plot_demands(n_base, n_peak_every_hour, storage_take_every_hour,␣
 ↪storage_state_every_hour):
    """
    n_base: number of active base power-plants
    n_peak_every_hour: a list containing the number of active peak power-plants␣
 ↪(every hour)
    storage_take_every_hour: a list containing the amount of energy, which is␣
 ↪taken from the storage (every hour)
    storage_state_every_hour: a list containing the amount of energy, which is␣
 ↪stored in the storage (every hour)
    """
    # Demand plot
    T = 24
    margin = 0.2
    width = 0.3
    plt.figure(figsize=(10, 4))
    plt.bar([t + margin for t in range(T)], d, width=width, color='yellow')

    # - base power-plants
    bottom = np.zeros(T)
    g_base = np.array([e_base * n_base for t in range(T)])
    plt.bar([t + margin + width for t in range(T)],
            g_base,
            width=width,
            bottom=bottom,
```

```
            color='red')
    bottom += g_base

    # - peak power-plants
    g_peak = np.array([e_peak * n_peak_every_hour[t] for t in range(T)])
    plt.bar([t + margin + width for t in range(T)],
            g_peak,
            width=width,
            bottom=bottom,
            color='green')
    bottom += g_peak

    # - storage
    g_a_take = np.array([gamma * storage_take_every_hour[t] for t in range(T)])
    plt.bar([t + margin + width for t in range(T)],
            g_a_take,
            width=width,
            bottom=bottom,
            color='blue')
    bottom += g_a_take

    plt.xlabel("hour")
    plt.ylabel("energy")
    plt.legend(['demand', 'base', 'peak', 'storage'], ncol=1, loc=2)
    plt.xlim(0, 24)
    plt.xticks(range(24), [i % 24 for i in range(24)])
    plt.grid()

    # Storage plot.
    plt.figure(figsize=(10, 4))
    plt.xlim(0, 24)
    plt.xticks(range(24), [i % 24 for i in range(24)])
    plt.xlabel("hour")
    plt.ylabel("stored energy")
    plt.bar([t + margin for t in range(T)], [storage_state_every_hour[t] for t␣
 ↪in range(T)], width=width, color='purple')
    plt.grid()

    plt.show()

# The optimal solution
n_base_opt = 2
n_peak_opt = [0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 5, 8, 13, 7, 23, 23, 23, 18, 13, 8,␣
 ↪8, 3, 0, 0]
storage_take_opt = [0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 8.0, 8.0, 7.
 ↪999999999999973, 0.0, 0.0, 29.333333333333275, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0,␣
 ↪0.0, 0.0, 0.0, 0.0]
```

```
storage_state_opt = [0.0, 9.0, 17.999999999999773, 26.99999999999942, 35.999999999999, 44.99999999999851, 48.999999999997975, 52.99999999999757, 53.999999999997, 45.999999999997, 37.999999999997, 29.99999999999703, 29.999999999997026, 29.999999999997026, 0.6666666666637511, 0.6666666666637511, 0.6666666666637511, 0.6666666666637511, 0.6666666666637511, 0.6666666666637511, 0.6666666666637511, 0.6666666666637511, 0.6666666666637511, 4.666666666663695]

# TODO: visualize your solution
# Call this function with
# - the number of base power-plants to be activated,
# - list of the numbers of the peak power-plants to be activated (every hour),
# - list containing the amount of power taken from the storage (every hour),
# - list containing the amount of energy in the storage (every hour)
plot_demands(n_base_opt, n_peak_opt, storage_take_opt, storage_state_opt)
```





```
[ ]:
```