Mean Shift Tracker

Lecturer:

Jiří Matas

Authors:

Ondřej Drbohlav and Jiří Matas

Center for Machine Perception

Czech Technical University in Prague

http://cmp.felk.cvut.cz

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Mean Shift Tracking

- Tracked region described by a histogram
- Let $\mathbf{q}$ and $\mathbf{p}$ denote the normalized histograms of template and candidate regions, respectively, with B bins.

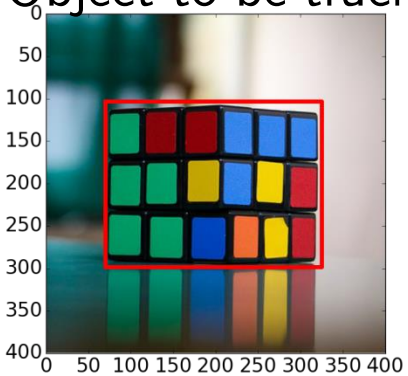
$$\sum_{i=1}^{B} p_i = 1, \quad \sum_{i=1}^{B} q_i = 1\,.$$

- The similarity of histograms is measured by Bhatacharyya coefficient:

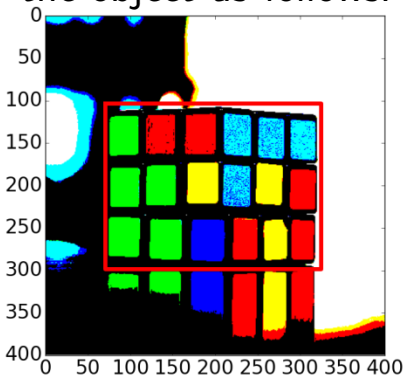
$$\rho(\mathbf{p}, \mathbf{q}) = \sum_{i=1}^{B} \sqrt{p_i q_i}$$

- Tracking = searching for a region with histogram which has highest similarity to the template histogram

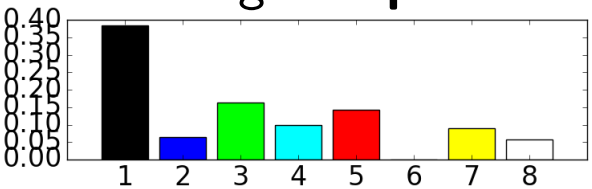
Object to be tracked:



This example uses color histogram with 8 bins (2x2x2 RGB.) MS 'sees' the object as follows:

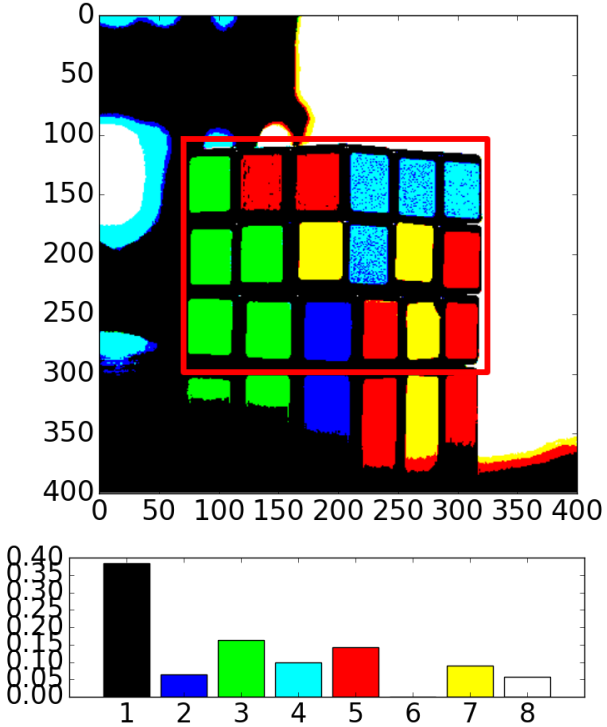


Histogram $\mathbf{q}$:



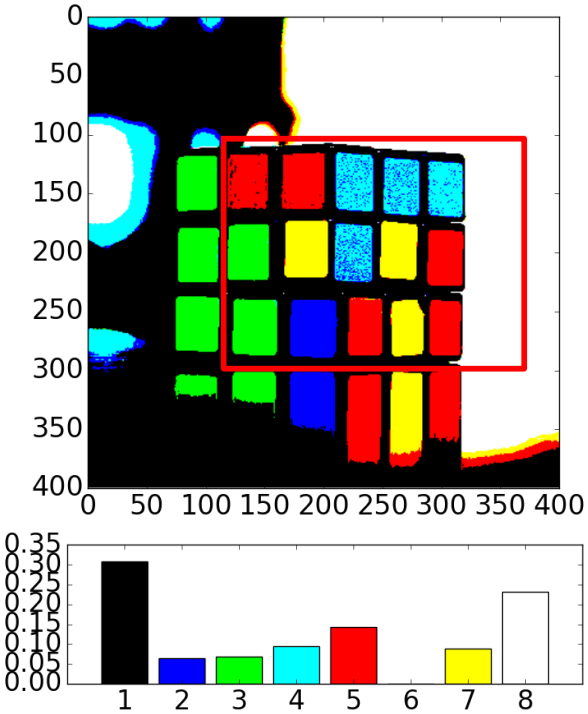
Histogram representation, Bhatacharyya coef.

Template region



Template histogram **q**

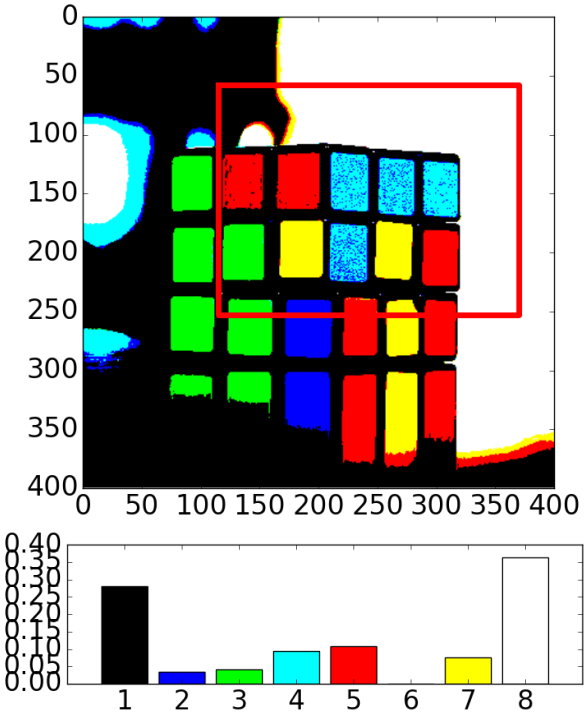
Candidate region 1



Candidate histogram **p**
Bhatacharyya coef:

0.958

Candidate region 2

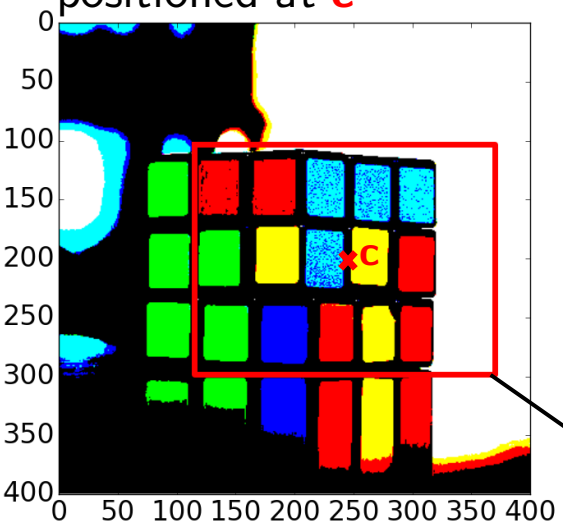


Candidate histogram **p**
Bhatacharyya coef:

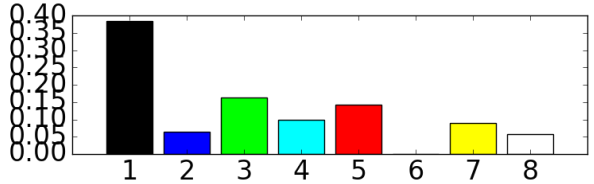
0.905

Iteration of Mean Shift

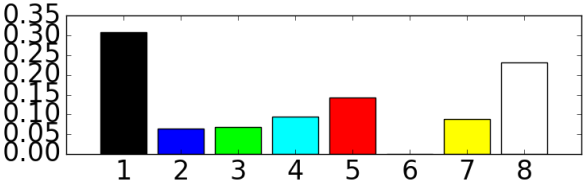
Candidate region $R(\mathbf{c})$, positioned at $\mathbf{c}$



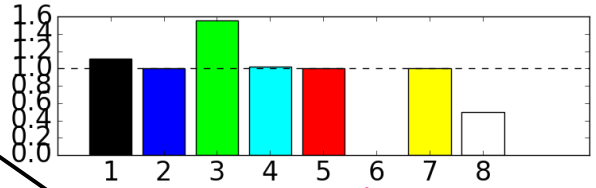
Template histogram $\mathbf{q}$



Candidate histogram $\mathbf{p}$

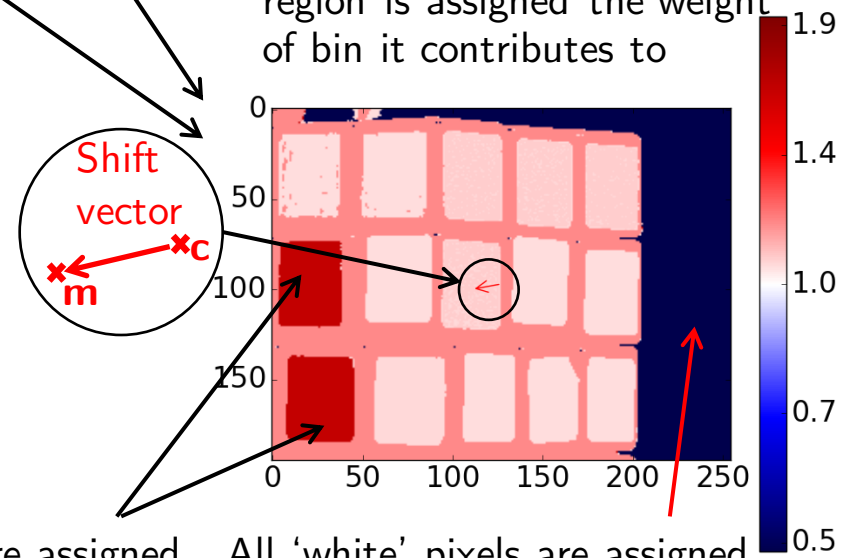


1) Bin weights $\mathbf{w}$ are computed



$$w_i = \sqrt{\frac{q_i}{p_i}}$$

2) Each pixel in the candidate region is assigned the weight of bin it contributes to



3) Mean $\mathbf{m}$ is computed over the candidate region $R(\mathbf{c})$:

$$\mathbf{m} = \frac{\sum_{\mathbf{x}\in R(\mathbf{c})} w_{i(\mathbf{x})}\mathbf{x}}{\sum_{\mathbf{x}\in R(\mathbf{c})} w_{i(\mathbf{x})}}$$

[$i(\mathbf{x})$ maps pixel to its bin]

4) Center of the candidate region R is shifted to $\mathbf{m}$ (if this improves the B. coef. If not, $\mathbf{m} \leftarrow \frac{1}{2}(\mathbf{m}+\mathbf{c})$, and repeat.)

All 'green' pixels are assigned high weight because they are less frequent in $\mathbf{p}$ than in $\mathbf{q}$

All 'white' pixels are assigned low weight because they are more frequent in $\mathbf{p}$ than in $\mathbf{q}$

Mean Shift Tracking Algorithm

Given initial candidate region position $\mathbf{c}$ and template histogram $\mathbf{q}$:

1. Compute the histogram $\mathbf{p}(\mathbf{c})$ of candidate region $R(\mathbf{c})$. Compute weights w_i for all bins.

2. Compute the 'center of gravity' $\mathbf{m}$ of the region, with pixels weighted by respective weights:

$$\mathbf{m} = \frac{\sum_{\mathbf{x} \in R(\mathbf{c})} w_{i(\mathbf{x})} \, \mathbf{x}}{\sum_{\mathbf{x} \in R(\mathbf{c})} w_{i(\mathbf{x})}} \tag{1}$$

   Compute histogram $\mathbf{p}(\mathbf{m})$ of the candidate region at the new position $\mathbf{m}$ and the Bhatacharyya coefficient $\rho(\mathbf{p}(\mathbf{m}), \mathbf{q})$

3. While $\rho(\mathbf{p}(\mathbf{m}), \mathbf{q}) < \rho(\mathbf{p}(\mathbf{c}), \mathbf{q})$ do: $\mathbf{m} \leftarrow \frac{1}{2}(\mathbf{m} + \mathbf{c})$

4. If $\|\mathbf{m} - \mathbf{c}\| <$ threshold, stop. Otherwise, $\mathbf{c} \leftarrow \mathbf{m}$ and goto step 1.

Example

- Histogram description: 16x16x16 RGB
- No histogram adaptation (template histogram is the same over the whole sequence)
- Frame 1 (initial frame) and template region:

Example (video)

frame 2

Converged box (6 iters)
$\rho = 0.932$

Initial box
$\rho = 0.791$

Notes

- Mean Shift Tracker is based on Kernel Density Estimation (KDE) theory.
- The form of an MS iteration can be interpreted as a gradient ascent.
- Computing the mean as it's been described is just one possible option. It corresponds to assuming Epanechnikov kernel for the KDE. Another widely used possibility is the Normal (Gaussian) kernel.

Math Background

Formulations largely compatible with the seminal paper:

Comaniciu, D., Ramesh, V., Meer, P.: *Real-time tracking of non-rigid objects using mean shift*. pp. 142–149 (2000)

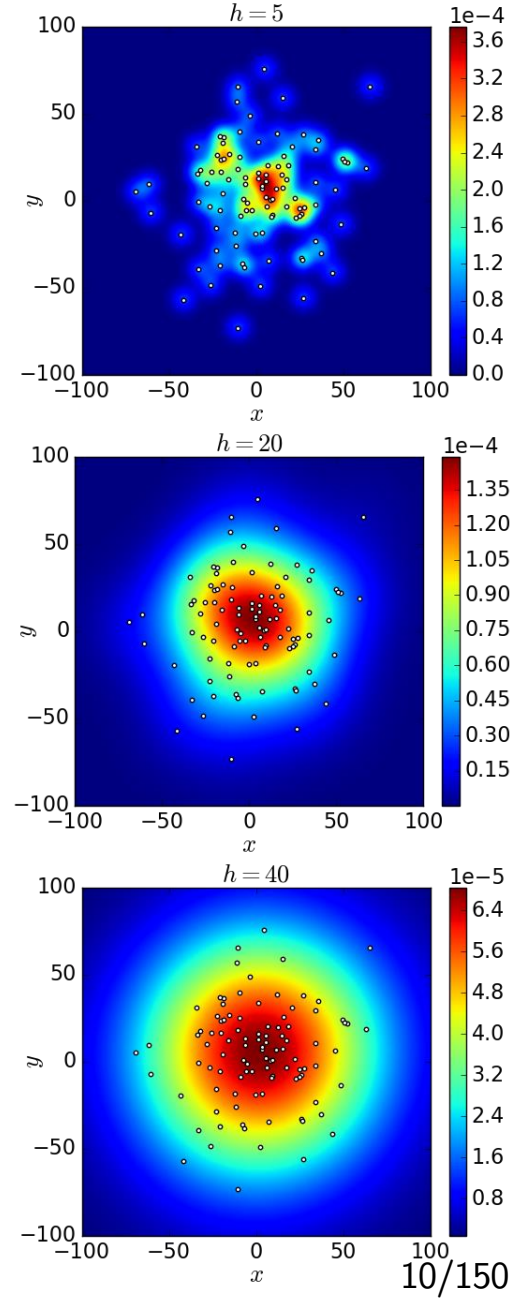
Mean Shift: Kernel Density Estimation

Given $\{\mathbf{x}_i,\, i = 1, ..., N\}$ the set of points in $\mathbb{R}^d$, the *multivariate kernel density estimator* with kernel $K(\mathbf{x})$ and bandwith h is computed as

$$p(\mathbf{x}) = \frac{1}{Nh^d} \sum_{i=1}^{N} K\left(\frac{\mathbf{x} - \mathbf{x}_i}{h}\right) . \qquad (1)$$

Example: Normal kernel K_N:

$$K_N(\mathbf{x}) = (2\pi)^{-d/2} \exp\left(-\frac{1}{2}\|\mathbf{x}\|^2\right) \qquad (2)$$



Kernel density estimation for different bandwiths *h*.

Data points (*N*=100) are sampled from normal distribution with std=30.

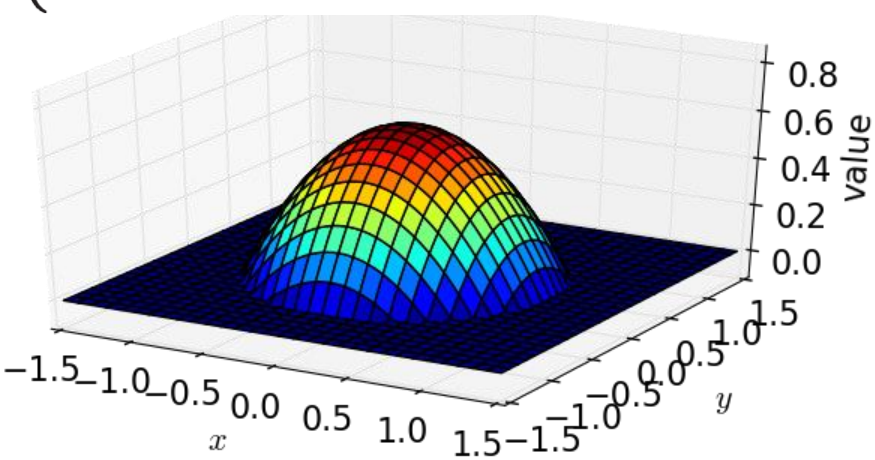
Mean Shift: Kernel Density Estimation

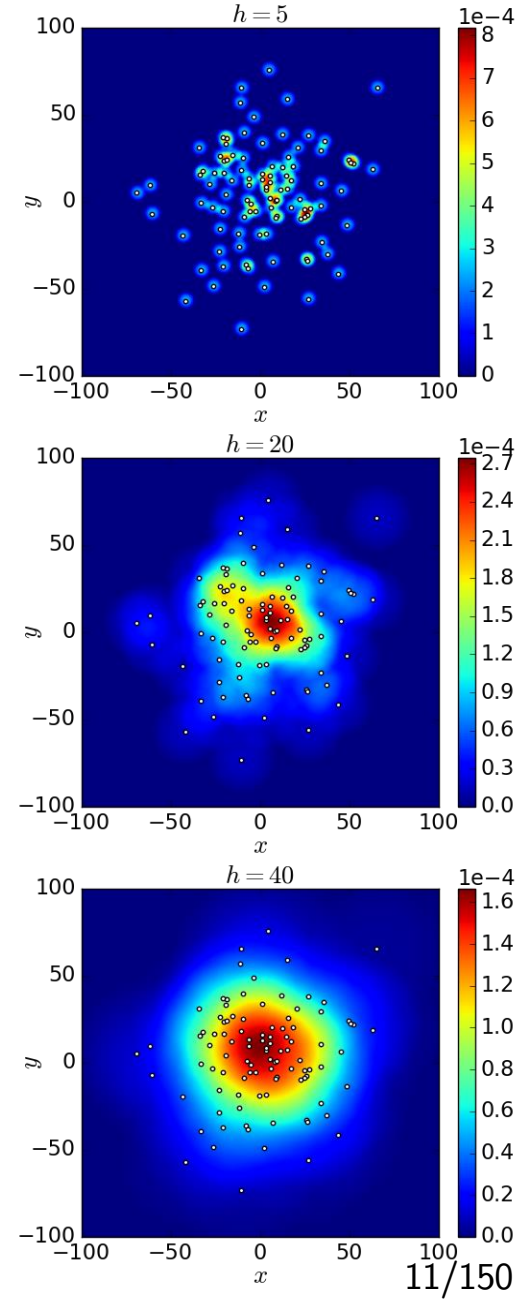
Given $\{\mathbf{x}_i,\, i = 1, ..., N\}$ the set of points in $\mathbb{R}^d$, the *multivariate kernel density estimator* with kernel $K(\mathbf{x})$ and bandwith h is computed as

$$p(\mathbf{x}) = \frac{1}{Nh^d} \sum_{i=1}^{N} K\left(\frac{\mathbf{x} - \mathbf{x}_i}{h}\right) . \qquad (1)$$

Example: *Epanechnikov* kernel K_E (c_d: volume of d-dimensional sphere):

$$K_E(\mathbf{x}) = \begin{cases} \frac{1}{2} c_d^{-1}(d+2)(1 - \|\mathbf{x}\|^2) & \text{if } \|x\| < 1 \\ 0 & \text{otherwise} \end{cases} \qquad (2)$$





Mean Shift: Kernel Profile (isotropic kernels)

Let us rewrite the computation of KDE

$$p(\mathbf{x}) = \frac{1}{Nh^d} \sum_{i=1}^{N} K\left(\frac{\mathbf{x} - \mathbf{x}_i}{h}\right) \tag{1}$$

using the **kernel profile** $k(z) = k(\|\mathbf{x}\|^2) = K(\mathbf{x})$:

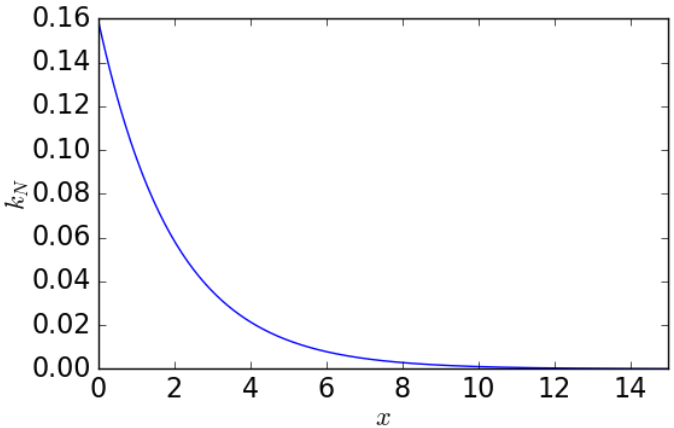
$$p(\mathbf{x}) = \frac{1}{Nh^d} \sum_{i=1}^{N} k\left(\left\|\frac{\mathbf{x} - \mathbf{x}_i}{h}\right\|^2\right), \tag{2}$$

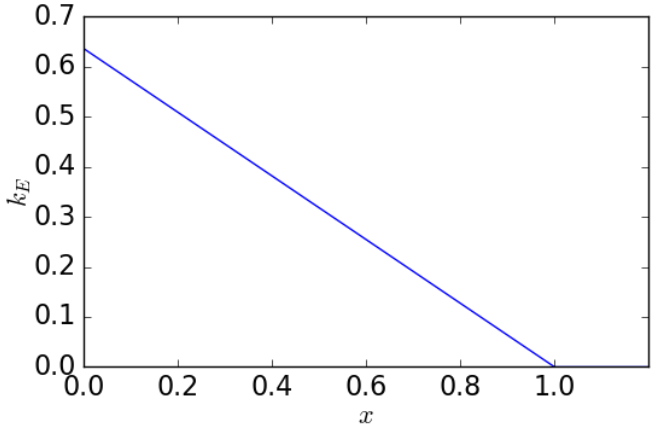
as this will simplify further derivations. The normal and Epanechnikov profiles are then:

$$k_N(x) = (2\pi)^{-d/2} \exp\left(-\frac{1}{2}x\right), \tag{3}$$

$$k_E(x) = \begin{cases} \frac{1}{2}c_d^{-1}(d+2)(1-x) & \text{if } x < 1 \\ 0 & \text{otherwise} \end{cases} \tag{4}$$

Normal and Epanechnikov kernel profiles:





Mean Shift: Mode Seeking

Task: Given $\{\mathbf{x}_i,\, i = 1, ..., N\}$ the set of points in $\mathbb{R}^d$, a kernel with profile $k(x)$ and bandwith parameter h, find the local mode of distribution $p(\mathbf{x})$ estimated using KDE, starting from initial mode estimate $\mathbf{y}_0$. This will employ grad. ascent.

$$p(\mathbf{x}) = \frac{1}{Nh^d} \sum_{i=1}^{N} k\left(\left\|\frac{\mathbf{x}-\mathbf{x}_i}{h}\right\|^2\right), \tag{1}$$

$$\nabla p(\mathbf{x}) = \frac{2}{Nh^{d+2}} \sum_{i=1}^{N} k'\left(\left\|\frac{\mathbf{x}-\mathbf{x}_i}{h}\right\|^2\right)(\mathbf{x}-\mathbf{x}_i) = \frac{2}{Nh^{d+2}} \times$$

$$\sum_{i=1}^{N} k'\left(\left\|\frac{\mathbf{x}-\mathbf{x}_i}{h}\right\|^2\right)\left(\mathbf{x} - \frac{\sum_{m=1}^{N} \mathbf{x}_m k'\left(\left\|\frac{\mathbf{x}-\mathbf{x}_m}{h}\right\|^2\right)}{\sum_{m=1}^{N} k'\left(\left\|\frac{\mathbf{x}-\mathbf{x}_m}{h}\right\|^2\right)}\right). \tag{2}$$

For discussed kernel profiles, $k'(x)$ is negative (or zero); define $g(x) = -k'(x)$. Then the last equation is rewritten as

$$\nabla p(\mathbf{x}) = \frac{2}{Nh^{d+2}} \sum_{i-1}^{N} g\left(\left\|\frac{\mathbf{x}-\mathbf{x}_i}{h}\right\|^2\right)\left(\frac{\sum_{m=1}^{N} \mathbf{x}_m g\left(\left\|\frac{\mathbf{x}-\mathbf{x}_m}{h}\right\|^2\right)}{\sum_{m=1}^{N} g\left(\left\|\frac{\mathbf{x}-\mathbf{x}_m}{h}\right\|^2\right)} - \mathbf{x}\right). \tag{3}$$

Mean Shift: Mode Seeking

$$\nabla p(\mathbf{x}) = \overbrace{\frac{2}{Nh^{d+2}} \sum_{i=1}^{N} g\left(\left\|\frac{\mathbf{x}-\mathbf{x}_i}{h}\right\|^2\right)}^{\text{const} > 0} \underbrace{\left(\overbrace{\frac{\sum_{m=1}^{N} \mathbf{x}_m g\left(\left\|\frac{\mathbf{x}-\mathbf{x}_m}{h}\right\|^2\right)}{\sum_{m=1}^{N} g\left(\left\|\frac{\mathbf{x}-\mathbf{x}_m}{h}\right\|^2\right)}}^{\text{(kernel-weighted) sample } \textbf{mean}} - \mathbf{x}\right)}_{\textbf{mean shift} \text{ vector } M_{h,G}(\mathbf{x})} . \quad (1)$$

Note that $g(x) = -k'(x)$ defines a kernel $G(\mathbf{x}) = Cg(\|\mathbf{x}\|^2)$
(C is a norm. constant)

- For Epanechnikov profile, $g(x)$ profile is a constant and $G(\mathbf{x})$ is thus a uniform kernel.
- For normal profile, $g(x)$ is again a normal profile and $G(\mathbf{x})$ is thus equal to $K(\mathbf{x})$.

Mean Shift: Mode Seeking

$$\nabla p(\mathbf{x}) = \overbrace{\frac{2}{Nh^{d+2}} \sum_{i=1}^{N} g\left(\left\|\frac{\mathbf{x}-\mathbf{x}_i}{h}\right\|^2\right)}^{\text{const} > 0} \underbrace{\left(\overbrace{\frac{\sum_{m=1}^{N} \mathbf{x}_m g\left(\left\|\frac{\mathbf{x}-\mathbf{x}_m}{h}\right\|^2\right)}{\sum_{m=1}^{N} g\left(\left\|\frac{\mathbf{x}-\mathbf{x}_m}{h}\right\|^2\right)}}^{\text{(kernel-weighted) sample } \textbf{mean}} - \mathbf{x}\right)}_{\textbf{mean shift} \text{ vector } M_{h,G}(\mathbf{x})} . \quad (1)$$

The **Mean Shift** procedure for mode seeking:

- Given $\mathbf{y}_1$,
- Compute

$$\mathbf{y}_{j+1} = \frac{\sum_{i=1}^{N} \mathbf{x}_i g\left(\left\|\frac{\mathbf{y_j}-\mathbf{x}_i}{h}\right\|^2\right)}{\sum_{i=1}^{N} g\left(\left\|\frac{\mathbf{y_j}-\mathbf{x}_i}{h}\right\|^2\right)}, \quad j = 1, 2, \ldots \quad (1)$$

- Proof of convergence: [CRM] Comaniciu, Ramesh and Meer: *Real-Time Tracking of Non-Rigid Objects using Mean Shift*. In CVPR 2000.

Mean Shift Tracking

- Let $\mathbf{x}_i$ $(i = 1, 2, ..., N)$ be pixel locations and $b(\mathbf{x}_i)$ be a function which assigns the histogram bin index to pixel location $\mathbf{x}_i$.
- Let the center of template be at $\mathbf{c}$ and its pixel set be S_q.
- The histogram description q_u of the template is computed as:

$$q_u = C_q \sum_{\substack{\mathbf{x}_i \in S_q \\ b(\mathbf{x}_i)=u}} k\left(\left\|\frac{\mathbf{c} - \mathbf{x}_i}{h_q}\right\|^2\right), \quad u = 1, 2, ..., B \tag{1}$$

  thus votes are weighted by a kernel profile k with bandwith h_q (C_q is a norm. constant. and B is the number of bins.)
- The histogram description p_u of target candidate centered at $\mathbf{y}$ and with extent comprising pixels S_p is computed in the same way:

$$p_u = C_p \sum_{\substack{\mathbf{x}_i \in S_p \\ b(\mathbf{x}_i)=u}} k\left(\left\|\frac{\mathbf{y} - \mathbf{x}_i}{h_p}\right\|^2\right), \quad u = 1, 2, ..., B \tag{2}$$

Mean Shift Tracking

The similarity between template and candidate histograms $\mathbf{q} = (q_1, q_2, ..., q_B)$ and $\mathbf{p} = (p_1, p_2, ..., p_B)$ is measured using the Bhattacharyya coefficient:

$$\rho(\mathbf{p}(\mathbf{y}), \mathbf{q}) = \sum_{u=1}^{B} \sqrt{p_u(\mathbf{y}) q_u} \,. \tag{1}$$

Note that if $\mathbf{p} = \mathbf{q}$ then $\rho = 1$.

Let us make Taylor expansion around the current histogram values $\mathbf{p}(\mathbf{y}_0)$ computed at position $\mathbf{y}_0$, thus at $\mathbf{p}(\mathbf{y}) = \mathbf{p}(\mathbf{y}_0) + \Delta\mathbf{p}$:

$$\begin{aligned}
\rho(\mathbf{p}(\mathbf{y}_0) + \Delta\mathbf{p}, \mathbf{q}) &= \sum_{u=1}^{B} \sqrt{(p_u(\mathbf{y}_0) + \Delta p_u) q_u} \\
&\approx \sum_{u=1}^{B} \sqrt{p_u(\mathbf{y}_0) q_u} + \frac{1}{2} \sum_{u=1}^{B} \frac{1}{\sqrt{p_u(\mathbf{y}_0) q_u}} q_u \Delta p_u \qquad (2) \\
&= \frac{1}{2} \sum_{u=1}^{B} \sqrt{p_u(\mathbf{y}_0) q_u} + \frac{1}{2} \sum_{u=1}^{B} \sqrt{\frac{q_u}{p_u(\mathbf{y_0})}} p_u(\mathbf{y}) \qquad (3)
\end{aligned}$$

Mean Shift Tracking

For $\mathbf{p}(\mathbf{y}) = \mathbf{p}(\mathbf{y}_0) + \Delta\mathbf{p}$:

$$\rho(\mathbf{p}(\mathbf{y}), \mathbf{q}) = \frac{1}{2}\sum_{u=1}^{B}\sqrt{p_u(\mathbf{y}_0)q_u} + \frac{1}{2}\sum_{u=1}^{B}\sqrt{\frac{q_u}{p_u(\mathbf{y_0})}}p_u(\mathbf{y}) \tag{1}$$

Recall that

$$p_u = C_p \sum_{\substack{\mathbf{x}_i \in S_p \\ b(\mathbf{x}_i)=u}} k\left(\left\|\frac{\mathbf{y}-\mathbf{x}_i}{h_p}\right\|^2\right), \quad u = 1, 2, ..., B\,. \tag{2}$$

Inserting (2) to (1) we get

$$\rho(\mathbf{p}(\mathbf{y}), \mathbf{q}) = \frac{1}{2}\sum_{u=1}^{B}\sqrt{p_u(\mathbf{y}_0)q_u} + \frac{C_p}{2}\sum_{u=1}^{B}\sum_{\substack{\mathbf{x}_i \in S_p \\ b(\mathbf{x}_i)=u}}\sqrt{\frac{q_u}{p_u(\mathbf{y_0})}}k\left(\left\|\frac{\mathbf{y}-\mathbf{x}_i}{h_p}\right\|^2\right)$$

$$= \underbrace{\frac{1}{2}\sum_{u=1}^{B}\sqrt{p_u(\mathbf{y}_0)q_u}}_{\text{const}} + \boxed{\frac{C_p}{2}\sum_{\mathbf{x}_i \in S_p} w_i k\left(\left\|\frac{\mathbf{y}-\mathbf{x}_i}{h_p}\right\|^2\right)}, \quad w_i = \sqrt{\frac{q_{b(\mathbf{x}_i)}}{p_{b(\mathbf{x}_i)}(\mathbf{y_0})}} \tag{3}$$

KDE estimation!

Mean Shift Tracking

In order to maximize $\rho(\mathbf{p}(\mathbf{y}), \mathbf{q})$

$$\rho(\mathbf{p}(\mathbf{y}), \mathbf{q}) = \text{const}(\mathbf{y}_0) + \frac{C_p}{2} \sum_{\mathbf{x}_i \in S_p} w_i k \left(\left\| \frac{\mathbf{y} - \mathbf{x}_i}{h_p} \right\|^2 \right) , \tag{1}$$

$$\text{where } w_i = \sqrt{\frac{q_{b(\mathbf{x}_i)}}{p_{b(\mathbf{x}_i)}(\mathbf{y_0})}} , \tag{2}$$

we need to find the mode of prob. density estimated using KDE with kernel profile k and bandwith h_p, where the individual pixels of target candidate are weighted by weights w_i. This can be done using the Mean Shift procedure, with kernel profile $g(x) = -k'(x)$.

Mean Shift Tracking

This leads to the Mean Shift tracking algorithm:

1. Given initial estimate for candidate position $\mathbf{y}_0$, compute $\rho(\mathbf{p}(\mathbf{y}_0), \mathbf{q})$.
2. Compute weights w_i according to (2) on the previous slide.
3. Update the new location $\mathbf{y}_1$ of target candidate:

$$\mathbf{y}_1 = \frac{\sum_{\mathbf{x}_i \in S_p} \mathbf{x}_i w_i g\left(\left\|\frac{\mathbf{y}_0 - \mathbf{x}_i}{h_p}\right\|^2\right)}{\sum_{\mathbf{x}_i \in S_p} w_i g\left(\left\|\frac{\mathbf{y}_0 - \mathbf{x}_i}{h_p}\right\|^2\right)} \tag{1}$$

   Compute $\mathbf{p}(\mathbf{y}_1)$ and $\rho(\mathbf{p}(\mathbf{y}_1), \mathbf{q})$

4. While $\rho(\mathbf{p}(\mathbf{y}_1), \mathbf{q}) < \rho(\mathbf{p}(\mathbf{y}_0), \mathbf{q})$ do: $\mathbf{y}_1 \leftarrow \frac{1}{2}(\mathbf{y}_0 + \mathbf{y}_1)$
5. If $\|\mathbf{y_1} - \mathbf{y_0}\|$, stop. Otherwise, $\mathbf{y}_0 \leftarrow \mathbf{y}_1$ and goto step 1.

Mean Shift tracking example

Feature space: 16×16×16 quantized RGB
Target: manually selected on 1st frame
Average mean-shift iterations: 4

Mean Shift tracking example

D. Comaniciu, V. Ramesh, P. Meer: *Kernel-Based Object Tracking* TPAMI, 2003

Pros and cons

- Low computational cost (easily real-time)
- Surprisingly robust
  - Invariant to pose and viewpoint
  - Often no need to update reference color model

- Invariance comes at a price
  - Position estimate prone to fluctuation
  - Scale and orientation not well captured
  - Sensitive to color clutter (e.g., teammates in team sports)
- Local search by gradient descent

- Problems:
  - abrupt moves
  - occlusions

slide credit:
Patrick Perez