Elements of Geometry for Computer Vision and Computer Graphics

Translation of Euclid's Elements by Adelardus Bathensis (1080–1152)

2021 Lecture 10

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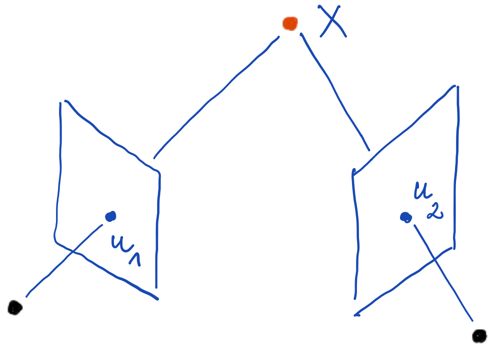
3 Two-view scene reconstruction

3D reconstruction by triangulation

Imagine two cameras giving two images of the space from two different view points. We will next investigate how to (re-)construct camera projection matrices and meaningful coordinates of points in the space such that the reconstructed cameras and the reconstructed points generate the images.



3.1 Epipolar geometry

Figure 3.1 shows two cameras with different centers C_1, C_2 and image planes π_1, π_2, observing a general point X as u_1, u_2. Baseline b connecting

$$u_1, u_2 \longrightarrow X$$

$$\begin{bmatrix} \vec{u}_{1\alpha_1} \\ \vec{u}_{2\alpha_2} \end{bmatrix} \longrightarrow \vec{X}_\delta$$

$$\mathbb{R}^4 \longrightarrow \mathbb{R}^3$$

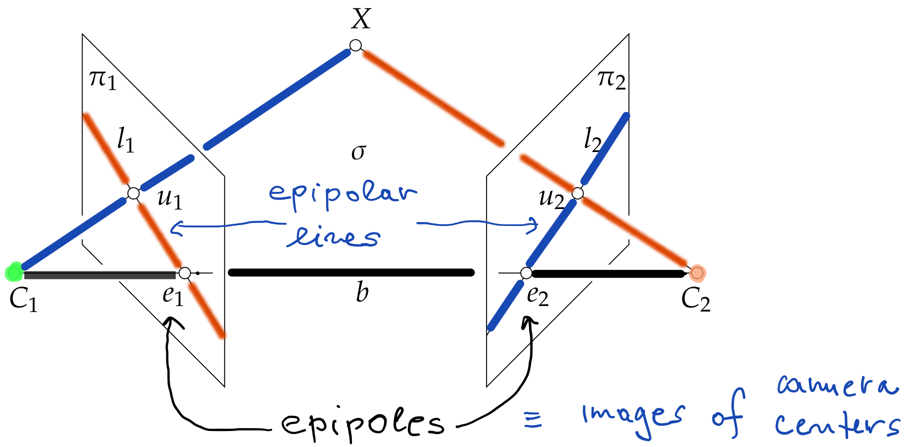


Figure 3.1: Epipolar geometry of two cameras.

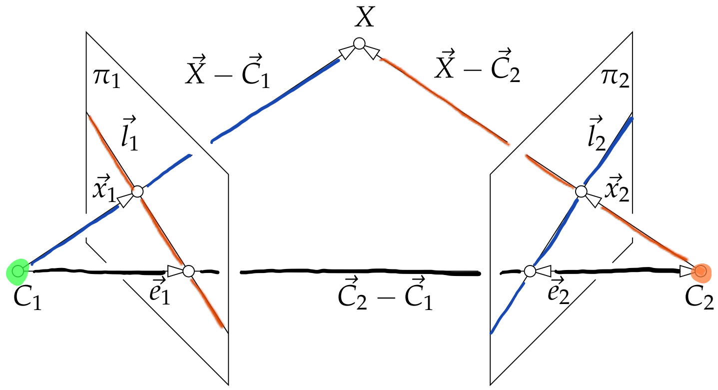


Figure 3.2: Vectors of the epipolar geometry.

image centers C_1, C_2 intersects π_1, π_2 in *epipoles* e_1, e_2. Points C_1, C_2 and X form *epipolar plane* σ, which intersects π_1 in *epipolar line* l_1 and π_2 in epipolar line l_2. Epipolar line l_1 passes through epipole e_1 and through image point u_1. Epipolar line l_2 passes through epipole e_2 and through image point u_2.

Let us next find the relationship between image points, epipoles, epipolar lines as a function of camera parameters, Figure 3.2. Assume a world coordinate system (O, δ) and cameras C_1, C_2 with camera projection matrices

$$\boxed{\mathtt{P}_1 = \left[\mathtt{K}_1\mathtt{R}_1 \,|\, -\mathtt{K}_1\mathtt{R}_1\vec{C}_{1\delta}\right]} \quad \text{and} \quad \boxed{\mathtt{P}_2 = \left[\mathtt{K}_2\mathtt{R}_2 \,|\, -\mathtt{K}_2\mathtt{R}_2\vec{C}_{2\delta}\right]} \tag{3.1}$$

Point X is projected to image planes π_1, π_2, with respective coordinate systems (o_1, β_1), (o_2, β_2), as

$$\zeta_1\, \vec{x}_{1\beta_1} = \mathtt{P}_1 \begin{bmatrix} \vec{X}_\delta \\ 1 \end{bmatrix} \quad \text{and} \quad \zeta_2\, \vec{x}_{2\beta_2} = \mathtt{P}_2 \begin{bmatrix} \vec{X}_\delta \\ 1 \end{bmatrix} \tag{3.2}$$

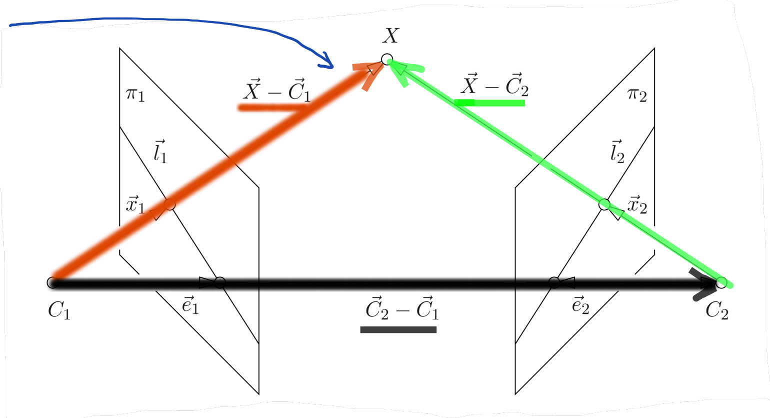
for some $\zeta_1 > 0$ and $\zeta_2 > 0$, which then leads to

$$\zeta_1\, \vec{x}_{1\beta_1} = \mathtt{P}_1 \begin{bmatrix} \vec{X}_\delta \\ 1 \end{bmatrix} \quad \text{and} \quad \zeta_2\, \vec{x}_{2\beta_2} = \mathtt{P}_2 \begin{bmatrix} \vec{X}_\delta \\ 1 \end{bmatrix}$$

$$\zeta_1\, \vec{x}_{1\beta_1} = \mathtt{K}_1\mathtt{R}_1(\vec{X}_\delta - \vec{C}_{1\delta}) \quad \text{and} \quad \zeta_2\, \vec{x}_{2\beta_2} = \mathtt{K}_2\mathtt{R}_2(\vec{X}_\delta - \vec{C}_{2\delta}) \tag{3.3}$$

$$\zeta_1\, \mathtt{R}_1^\top \mathtt{K}_1^{-1} \vec{x}_{1\beta_1} = \vec{X}_\delta - \vec{C}_{1\delta} \qquad \zeta_2\, \mathtt{R}_2^\top \mathtt{K}_2^{-1} \vec{x}_{2\beta_2} = \vec{X}_\delta - \vec{C}_{2\delta} \tag{3.4}$$

Consider now that vectors $\vec{X}_\delta - \vec{C}_{1\delta}$, $\vec{X}_\delta - \vec{C}_{2\delta}$ and $\vec{C}_{2\delta} - \vec{C}_{1\delta}$ form a triangle and hence



$$\vec{C}_{2\delta} - \vec{C}_{1\delta} = (\vec{X}_\delta - \vec{C}_{1\delta}) - (\vec{X}_\delta - \vec{C}_{2\delta}) \tag{3.5}$$

$$\vec{C}_{2\delta} - \vec{C}_{1\delta} = \zeta_1\, \mathtt{R}_1^\top \mathtt{K}_1^{-1} \vec{x}_{1\beta_1} - \zeta_2\, \mathtt{R}_2^\top \mathtt{K}_2^{-1} \vec{x}_{2\beta_2} \tag{3.6}$$

with $\zeta_1 > 0$ and $\zeta_2 > 0$ for the standard choice of camera coordinate systems.

We shall next eliminate depths ζ_1, ζ_2 by exploiting the vector product identities, see Paragraph 1.3.

$$\vec{0} = \vec{x} \times \vec{x} = [\vec{x}]_\times\, \vec{x} \tag{3.7}$$

$$\vec{0} = \vec{y}^\top (\vec{x} \times \vec{y}) = \vec{y}^\top\, [\vec{x}]_\times\, \vec{y} \tag{3.8}$$

for all $\vec{x}, \vec{y} \in \mathbb{R}^3$.

3 → 2 terms

We first vector-multiply Equation 3.6 by $\vec{C}_{2\delta} - \vec{C}_{1\delta}$ from the left to get

$$\mathbf{0} = \left[\vec{C}_{2\delta} - \vec{C}_{1\delta}\right]_\times \zeta_1\, \mathtt{R}_1^\top \mathtt{K}_1^{-1} \vec{x}_{1\beta_1} - \left[\vec{C}_{2\delta} - \vec{C}_{1\delta}\right]_\times \zeta_2\, \mathtt{R}_2^\top \mathtt{K}_2^{-1} \vec{x}_{2\beta_2} \tag{3.9}$$

and then multiply Equation 3.9 by $\zeta_2\, \vec{x}_{2\beta_2}^\top \mathtt{K}_2^{-\top} \mathtt{R}_2$ from the left to get

2 → 1 term

$$0 = \zeta_2\, \vec{x}_{2\beta_2}^\top \mathtt{K}_2^{-\top} \mathtt{R}_2 \left[\vec{C}_{2\delta} - \vec{C}_{1\delta}\right]_\times \zeta_1\, \mathtt{R}_1^\top \mathtt{K}_1^{-1} \vec{x}_{1\beta_1} \tag{3.10}$$

which, since $\zeta_1 \neq 0$ and $\zeta_2 \neq 0$, is equivalent with

elimination of ζ_1, ζ_2

$$0 = \vec{x}_{2\beta_2}^\top \mathtt{K}_2^{-\top} \mathtt{R}_2 \left[\vec{C}_{2\delta} - \vec{C}_{1\delta}\right]_\times \mathtt{R}_1^\top \mathtt{K}_1^{-1} \vec{x}_{1\beta_1} \tag{3.11}$$

$$0 = \vec{x}_{2\beta_2}^\top \mathtt{K}_2^{-\top} \mathtt{E}\, \mathtt{K}_1^{-1} \vec{x}_{1\beta_1} \tag{3.12}$$

$$0 = \vec{x}_{2\beta_2}^\top \mathtt{F}\, \vec{x}_{1\beta_1} \tag{3.13}$$

Essential matrix

$$E = R_2 [\vec{C}_{2\delta} - \vec{C}_{1\delta}]_\times R_1^T$$

Fundamental matrix

$$F = K_2^{-T} E K_1^{-1}$$

where we introduced the *essential matrix* $\mathtt{E} \in \mathbb{R}^{3\times 3}$ as

$$\mathtt{E} = \mathtt{R}_2 \left[\vec{C}_{2\delta} - \vec{C}_{1\delta}\right]_\times \mathtt{R}_1^\top \tag{3.14}$$

and the *fundamental matrix* $\mathtt{F} \in \mathbb{R}^{3\times 3}$ as

$$\mathtt{F} = \mathtt{K}_2^{-\top}\mathtt{R}_2 \left[\vec{C}_{2\delta} - \vec{C}_{1\delta}\right]_\times \mathtt{R}_1^\top \mathtt{K}_1^{-1} \tag{3.15}$$

Let us next introduce epipoles to pass from vectors in δ to vectors in β_1, β_2, which are measurable in images.

The projection e_1 of the the camera center $\vec{C}_2$ to the first image as well as the projection e_2 of the the camera center $\vec{C}_1$ to the second image are obtained as

$$\zeta_1\, \vec{e}_{1\beta_1} = \mathtt{P}_1 \begin{bmatrix} \vec{C}_{2\delta} \\ 1 \end{bmatrix} = \mathtt{K}_1\mathtt{R}_1(\vec{C}_{2\delta} - \vec{C}_{1\delta}) \tag{3.16}$$

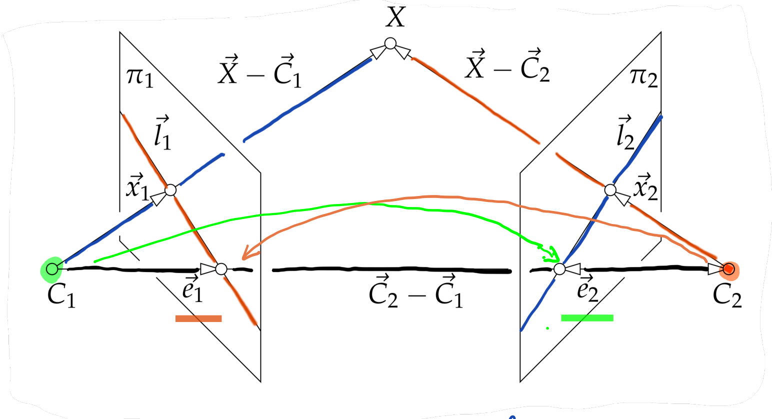
$$\zeta_2\, \vec{e}_{2\beta_2} = \mathtt{P}_2 \begin{bmatrix} \vec{C}_{1\delta} \\ 1 \end{bmatrix} = \mathtt{K}_2\mathtt{R}_2(\vec{C}_{1\delta} - \vec{C}_{2\delta}) \tag{3.17}$$

for some $\zeta_1 > 0$ and $\zeta_2 > 0$.

We can now substitute Equation 3.16 into Equation 3.15 to get

$$\begin{aligned} \mathtt{F} &= \mathtt{K}_2^{-\top}\mathtt{R}_2 \left[\vec{C}_{2\delta} - \vec{C}_{1\delta}\right]_\times \mathtt{R}_1^\top \mathtt{K}_1^{-1} && (3.18)\\ &= \mathtt{K}_2^{-\top}\mathtt{R}_2 \left[\zeta_1\, \mathtt{R}_1^\top \mathtt{K}_1^{-1} \vec{e}_{1\beta_1}\right]_\times \mathtt{R}_1^\top \mathtt{K}_1^{-1} && (3.19)\\ &= \zeta_1 \mathtt{K}_2^{-\top}\mathtt{R}_2 \frac{(\mathtt{R}_1^\top \mathtt{K}_1^{-1})^{-\top}}{\left|(\mathtt{R}_1^\top \mathtt{K}_1^{-1})^{-\top}\right|} \left[\vec{e}_{1\beta_1}\right]_\times && (3.20)\\ &= \frac{\zeta_1}{|\mathtt{K}_1|} \mathtt{K}_2^{-\top}\mathtt{R}_2\mathtt{R}_1^\top \mathtt{K}_1^\top \left[\vec{e}_{1\beta_1}\right]_\times && (3.21) \end{aligned}$$

We used the result from § 2, which shows how the vector product behaves under the change of a basis.



Epipoles ≡ images of camera centers

$C_2 \xrightarrow{P_1} e_1$ $\quad$ $C_1 \xrightarrow{P_2} e_2$

$\zeta_1 \vec{e}_{1\beta_1} = K_1 R_1 (\vec{C}_{2\delta} - \vec{C}_{1\delta})$

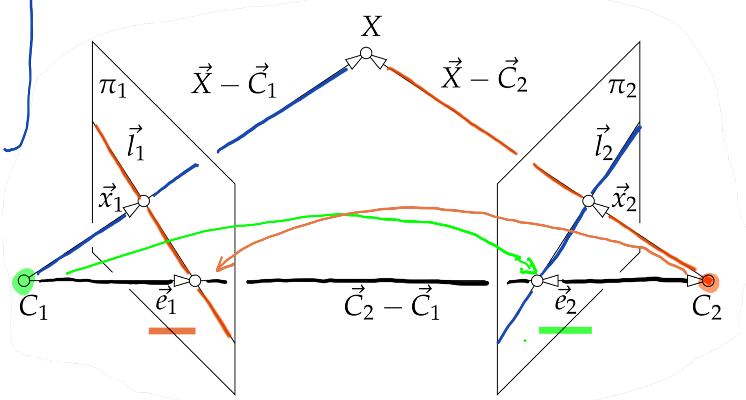
$\zeta R_1^\top K_1^{-1} \vec{e}_{1\beta_1} = \vec{C}_{2\delta} - \vec{C}_{1\delta}$

$[A\vec{x}]_\times A = \dfrac{A^{-\top}}{|A^{-\top}|} [\vec{x}]_\times$

Analogically, we substitute Equation 3.17 into Equation 3.15 to get

$$
\begin{aligned}
\mathtt{F} &= \mathtt{K}_2^{-\top}\mathtt{R}_2\left[\vec{C}_{2\delta}-\vec{C}_{1\delta}\right]_\times \mathtt{R}_1^\top\mathtt{K}_1^{-1} && (3.22)\\
&= \mathtt{K}_2^{-\top}\mathtt{R}_2\left[-\zeta_2\,\mathtt{R}_2^\top\mathtt{K}_2^{-1}\vec{e}_{2\beta_2}\right]_\times \mathtt{R}_1^\top\mathtt{K}_1^{-1} && (3.23)\\
&= \left(\left[\zeta_2\,\mathtt{R}_2^\top\mathtt{K}_2^{-1}\vec{e}_{2\beta_2}\right]_\times \mathtt{R}_2^\top\mathtt{K}_2^{-1}\right)^\top \mathtt{R}_1^\top\mathtt{K}_1^{-1} && (3.24)\\
&= \left(\frac{\zeta_2}{|\mathtt{K}_2|}\mathtt{R}_2^\top\mathtt{K}_2^\top\left[\vec{e}_{2\beta_2}\right]_\times\right)^\top \mathtt{R}_1^\top\mathtt{K}_1^{-1} && (3.25)\\
&= -\frac{\zeta_2}{|\mathtt{K}_2|}\left[\vec{e}_{2\beta_2}\right]_\times \mathtt{K}_2\mathtt{R}_2\mathtt{R}_1^\top\mathtt{K}_1^{-1} && (3.26)
\end{aligned}
$$

Analogically as above



$$\mathtt{F} = \frac{\zeta_1}{|\mathtt{K}_1|}\mathtt{K}_2^{-\top}\mathtt{R}_2\mathtt{R}_1^\top\mathtt{K}_1^\top\left[\vec{e}_{1\beta_1}\right]_\times$$

$$\mathtt{F}\,\vec{e}_{1\beta_1} = \frac{\zeta_1}{|\mathtt{K}_1|}\mathtt{K}_2^{-\top}\mathtt{R}_2\mathtt{R}_1^\top\mathtt{K}_1^\top\underbrace{\left[\vec{e}_{1\beta_1}\right]_\times \vec{e}_{1\beta_1}}_{0} = 0$$

We used additional properties of the linear representation of the vector product from § 3.

We see from Equations 3.21 and 3.26 that it is possible to recover homogeneous coordinates of the epipoles from $\mathtt{F}$ by solving equations

$$\mathtt{F}\,\vec{e}_{1\beta_1} = 0 \quad \text{and} \quad \mathtt{F}^\top\vec{e}_{2\beta_2} = 0 \tag{3.27}$$

for a non-zero multiples of $\vec{e}_{1\beta_1}$, $\vec{e}_{2\beta_2}$. We also see that matrix $\mathtt{F}$ has rank smaller than three since it has a non-zero null space $\vec{e}_{1\beta_1}$. Since, rank of $\left[\vec{C}_{2\delta}-\vec{C}_{1\delta}\right]_\times$ is two for non-zero $\vec{C}_{2\delta}-\vec{C}_{1\delta}$, $\mathtt{F}$ has rank two when camera centers do not coincide.

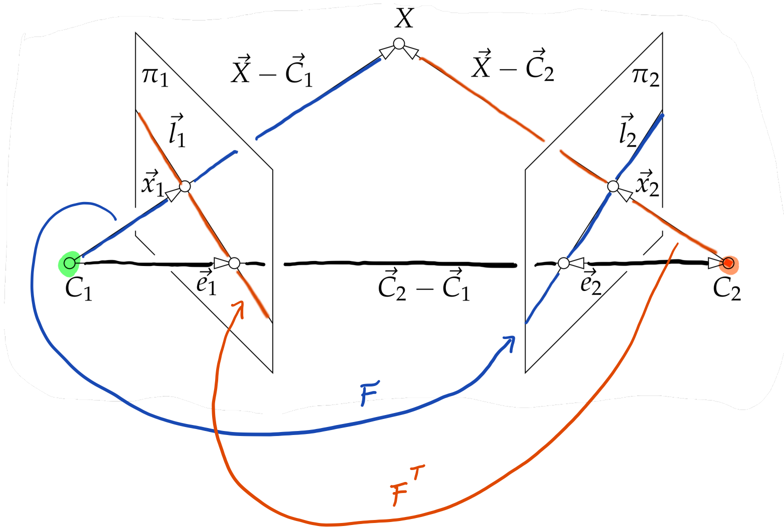
Let us look at the epipolar lines. Epipolar lines pass through the corresponding points in images and the epipoles, i.e. $l_1 = x_1 \vee e_1$ and $l_2x = x_2 \vee e_2$. Consider that there holds

$$
\begin{aligned}
\vec{x}_{2\beta_2}^\top\mathtt{F}\,\vec{e}_{1\beta_1} &= 0 \quad \text{and} \quad \vec{x}_{1\beta_1}^\top\mathtt{F}^\top\vec{e}_{2\beta_2} = 0 && (3.28)\\
\vec{x}_{2\beta_2}^\top\mathtt{F}\,\vec{x}_{1\beta_1} &= 0 \qquad\qquad \vec{x}_{1\beta_1}^\top\mathtt{F}^\top\vec{x}_{2\beta_2} = 0 && (3.29)\\
& && (3.30)
\end{aligned}
$$

and therefore homogeneous coordinates $\vec{l}_{1\bar{\beta}_1}$ $\vec{l}_{2\bar{\beta}_2}$ of epipolar lines generated by $\vec{x}_{2\beta_2}$ and $\vec{x}_{1\beta_1}$, respectively, are obtained as

$$\vec{l}_{1\bar{\beta}_1} = \mathtt{F}^\top \vec{x}_{2\beta_2} \quad \text{and} \quad \vec{l}_{2\bar{\beta}_2} = \mathtt{F}\,\vec{x}_{1\beta_1} \tag{3.31}$$

for $\vec{x}_{2\beta_2} \neq \vec{e}_{2\beta_2}$ and $\vec{x}_{1\beta_1} \neq \vec{e}_{1\beta_1}$.



3.2 Computing epipolar geometry from image matches

Let us look at how to compute the epipolar geometry between images from image matches. Our goal is to find matrix $\mathtt{G} = \tau\mathtt{F}$ for some real non-zero τ using Equation 3.13. Let us introduce

$$\mathtt{G} = \begin{bmatrix} g_{11} & g_{12} & g_{13} \\ g_{21} & g_{22} & g_{23} \\ g_{31} & g_{32} & g_{33} \end{bmatrix} \tag{3.32}$$

and write Equation 3.13 as

$w_{1i} = w_{2i} = 1$
for affine points

$$0 = \vec{x}_{2i\beta_2}^\top \mathtt{G}\, \vec{x}_{1i\beta_1} = \begin{bmatrix} u_{2i} & v_{2i} & w_{2i} \end{bmatrix} \begin{bmatrix} g_{11} & g_{12} & g_{13} \\ g_{21} & g_{22} & g_{23} \\ g_{31} & g_{32} & g_{33} \end{bmatrix} \begin{bmatrix} u_{1i} \\ v_{1i} \\ w_{1i} \end{bmatrix} \tag{3.33}$$

$$0 = \begin{bmatrix} u_{2i}\,u_{1i} & u_{2i}\,v_{1i} & u_{2i}\,w_{1i} & v_{2i}\,u_{1i} & v_{2i}\,v_{1i} & v_{2i}\,w_{1i} & w_{2i}\,u_{1i} & w_{2i}\,v_{1i} & w_{2i}\,w_{1i} \end{bmatrix} \begin{bmatrix} g_{11} \\ g_{12} \\ \vdots \\ g_{33} \end{bmatrix} = [u_{1i}\ v_{1i}\ w_{1i}] \otimes [u_{2i}\ v_{2i}\ w_{2i}]\, v(G)$$

for the i-th pair of the corresponding points $\vec{x}_{1i\beta_1}$, $\vec{x}_{2i\beta_2}$ in the two images. Notice that we can work even with ideal points when $w_{1i} = 0$ or $w_{2i} = 0$.

We can solve this way for a non-zero multiple of $\mathtt{F}$ from eight correspondences in a general position, i.e. not all on a plane or on some special quadrics passing through camera centers [11]. If there is noise in image coordinates, we in general get a rank three matrix.

To avoid this problem, we can use only seven point correspondences to compute a two dimensional space of solutions

$$\mathsf{G} = \mathsf{G}_1 + \alpha\,\mathsf{G}_2 \tag{3.34}$$

generated form its basis G_1, G_2 by α. Then we use the constraint

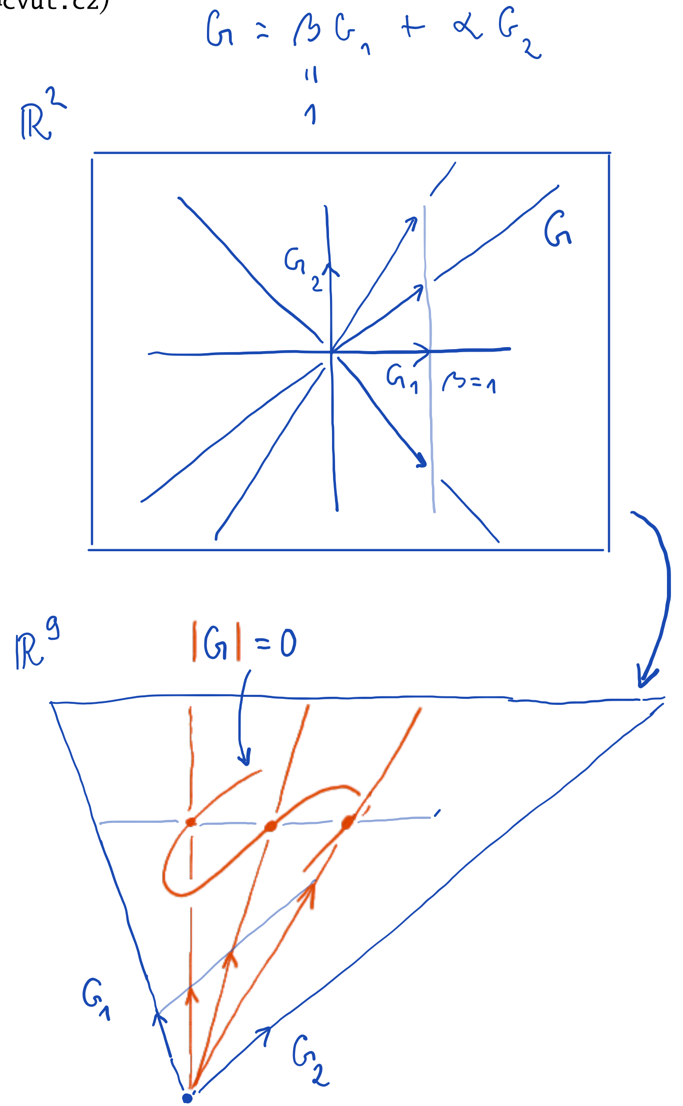
$$0 = |\mathsf{G}| = |\mathsf{G}_1 + \alpha\,\mathsf{G}_2| = \left|\begin{bmatrix} g_{111} & g_{112} & g_{113} \\ g_{121} & g_{122} & g_{123} \\ g_{131} & g_{132} & g_{133} \end{bmatrix} + \alpha \begin{bmatrix} g_{211} & g_{212} & g_{213} \\ g_{221} & g_{222} & g_{223} \\ g_{231} & g_{232} & g_{233} \end{bmatrix}\right| \tag{3.35}$$

to find α by solving a third order polynomial

$$\begin{aligned}
0 &= a_3\,\alpha^3 + a_2\,\alpha^2 + a_1\,\alpha + a_0 && (3.36)\\
a_3 &= |\mathsf{G}_2| \\
a_2 &= g_{221}\,g_{232}\,g_{113} - g_{221}\,g_{212}\,g_{133} + g_{211}\,g_{222}\,g_{133} + g_{231}\,g_{112}\,g_{223} \\
& + g_{231}\,g_{212}\,g_{123} - g_{211}\,g_{223}\,g_{132} - g_{231}\,g_{122}\,g_{213} - g_{231}\,g_{222}\,g_{113} \\
& - g_{211}\,g_{123}\,g_{232} + g_{121}\,g_{232}\,g_{213} + g_{221}\,g_{132}\,g_{213} + g_{131}\,g_{212}\,g_{223} \\
& - g_{121}\,g_{212}\,g_{233} - g_{111}\,g_{223}\,g_{232} - g_{221}\,g_{112}\,g_{233} + g_{211}\,g_{122}\,g_{233} \\
& + g_{111}\,g_{222}\,g_{233} - g_{131}\,g_{222}\,g_{213} \\
a_1 &= g_{111}\,g_{122}\,g_{233} + g_{111}\,g_{222}\,g_{133} + g_{231}\,g_{112}\,g_{123} - g_{121}\,g_{112}\,g_{233} \\
& - g_{211}\,g_{123}\,g_{132} - g_{221}\,g_{112}\,g_{133} - g_{231}\,g_{122}\,g_{113} + g_{211}\,g_{122}\,g_{133} \\
& + g_{121}\,g_{132}\,g_{213} + g_{121}\,g_{232}\,g_{113} + g_{131}\,g_{212}\,g_{123} - g_{121}\,g_{212}\,g_{133} \\
& - g_{131}\,g_{222}\,g_{113} + g_{221}\,g_{132}\,g_{113} - g_{111}\,g_{123}\,g_{232} - g_{131}\,g_{122}\,g_{213} \\
& + g_{131}\,g_{112}\,g_{223} - g_{111}\,g_{223}\,g_{132} \\
a_0 &= |\mathsf{G}_1|
\end{aligned}$$

That will give us up to three rank two matrices G.

Notice that we assumed that G was constructed with a non-zero coefficient at G_1. We therefore also need to check $\mathsf{G} = \mathsf{G}_2$ for a solution.



3.3 Ambiguity in two-view reconstruction

The goal of scene reconstruction from its two views is to find camera projection matrices $\mathtt{P}_1$, $\mathtt{P}_2$, and coordinates of points in the scene $\vec{X}_\delta$ such that the points $\vec{X}_\delta$ are projected by cameras $\mathtt{P}_1, \mathtt{P}_2$ to observed image points $\vec{x}_{1\beta_1}, \vec{x}_{2\beta_2}$

$$\zeta_1\, \vec{x}_{1\beta_1} = \mathtt{P}_1 \begin{bmatrix} \vec{X}_\delta \\ 1 \end{bmatrix} \quad \text{and} \quad \zeta_2\, \vec{x}_{2\beta_2} = \mathtt{P}_2 \begin{bmatrix} \vec{X}_\delta \\ 1 \end{bmatrix} \tag{3.37}$$

for some positive real ζ_1, ζ_2.

Assume that there are some cameras $\mathtt{P}_1, \mathtt{P}_2$, and coordinates of points in the scene $\vec{X}_\delta$ such that Equation 3.37 holds true. Then, for every 4×4 real regular matrix $\mathtt{H}$ we can get new camera matrices $\mathtt{P}'_1$, $\mathtt{P}'_2$ and new point coordinates $\vec{X}'_\delta$ as

$$\mathtt{P}'_1 = \mathtt{P}_1\, \mathtt{H}^{-1} \quad \mathtt{P}'_2 = \mathtt{P}_2\, \mathtt{H}^{-1} \quad \begin{bmatrix} \vec{X}'_\delta \\ 1 \end{bmatrix} = \mathtt{H} \begin{bmatrix} \vec{X}_\delta \\ 1 \end{bmatrix} \tag{3.38}$$

which also project to the same image points

Does not change

Ambiguity = choice of coordinate system in $\mathbb{P}^3$

$$\zeta_1\, \vec{x}_{1\beta_1} = \mathtt{P}_1 \begin{bmatrix} \vec{X}_\delta \\ 1 \end{bmatrix} = \mathtt{P}_1\, \mathtt{H}^{-1}\mathtt{H} \begin{bmatrix} \vec{X}_\delta \\ 1 \end{bmatrix} = \mathtt{P}'_1 \begin{bmatrix} \vec{X}'_\delta \\ 1 \end{bmatrix} \tag{3.39}$$

$$\zeta_2\, \vec{x}_{2\beta_2} = \mathtt{P}_2 \begin{bmatrix} \vec{X}_\delta \\ 1 \end{bmatrix} = \mathtt{P}_2\, \mathtt{H}^{-1}\mathtt{H} \begin{bmatrix} \vec{X}_\delta \\ 1 \end{bmatrix} = \mathtt{P}'_2 \begin{bmatrix} \vec{X}'_\delta \\ 1 \end{bmatrix} \tag{3.40}$$

We see that in general we can reconstruct the cameras and the scene points only up to some unknown transformation of the space. We also see that the transformation is more general than just changing a basis in $\mathbb{R}^3$ where we represent affine points $\vec{X}_\delta$. Matrix $\mathtt{H}$ acts in the three-dimensional affine space exactly as homography on two-dimensional affine space.

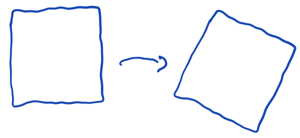
Let us next look at a somewhat simpler situation when camera calibration matrices $\mathtt{K}_1$, $\mathtt{K}_2$ are known. In such a case we can make sure that $\mathtt{H}$

H = general linear transform in $\mathbb{P}^3$

Interesting subgroups

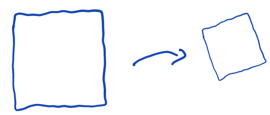
Euclidean motion

$$\begin{bmatrix} R & t \\ 0^\top & 1 \end{bmatrix} \qquad R^\top R = I \qquad |R| = 1$$



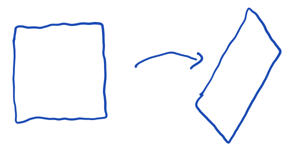
Similarity

$$\begin{bmatrix} sR & t \\ 0^\top & 1 \end{bmatrix} \qquad R^\top R = I \qquad |R| = 1$$



Affine transform

$$\begin{bmatrix} A & t \\ 0^\top & 1 \end{bmatrix} \qquad |A| \neq 0$$



Projective transform

$$\begin{bmatrix} A & t \\ p^\top & h_{33} \end{bmatrix} \qquad p^\top \neq 0^\top$$