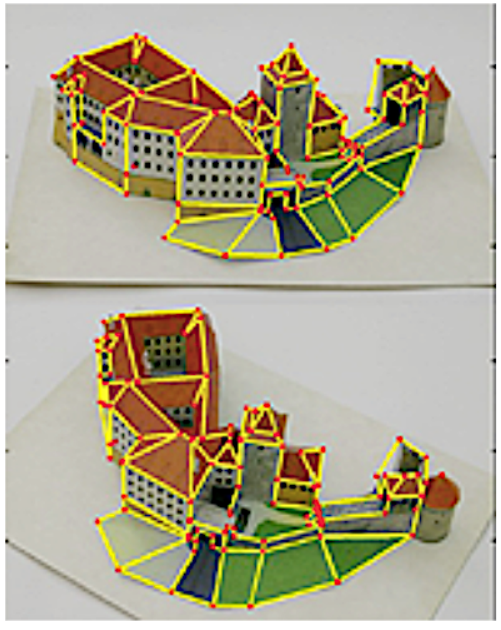
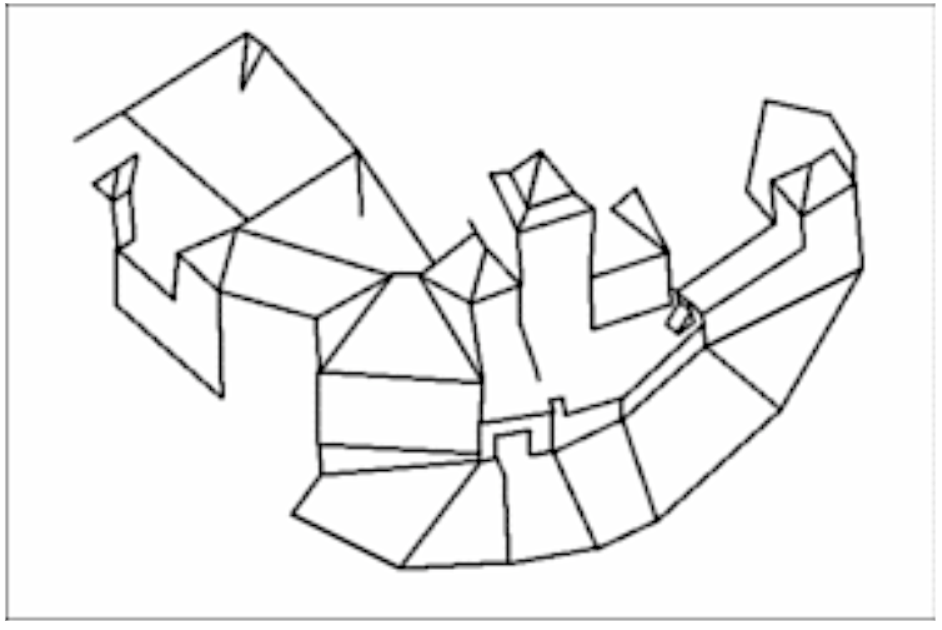
GVG – Geometry of Computer Vision & Graphics





Tomas Pajdla

2021

CIIRC – Czech Institute of Informatics Robotics and Cybernetics, CTU in Prague

NAVIGATION

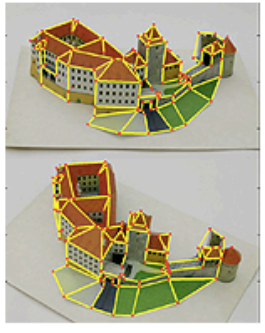
GVG

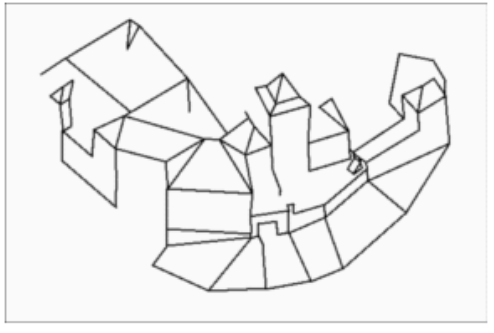
GVG Labs

ALL COURSES

Annotation BRUTE Forum Schedule Students: CZ EN | MS Teams

Geometry of Computer Vision and Graphics 2021





We will explain Euclidean, Affine, and Projective geometry basics, introduce a model of the perspective camera, and explain how images change when moving a camera. We will show how to compute camera poses and the 3D scene gemetry from images. We will demonstrate the theory in practical panorama construction tasks, finding the camera pose, adding a virtual object to a real scene, and reconstructing a 3D model of a scene from its images. We will build on our previous knowledge of linear algebra and provide fundamentals of geometry for computer vision, computer graphics, augmented reality, image processing, and object recognition.

Tomas Pajdla	Torsten Sattler	Martin Matousek	Viktor Korotynskiy	Kateryna_Zorina	Vojtech Panek
					

AAG – Applied Algebra & Geometry

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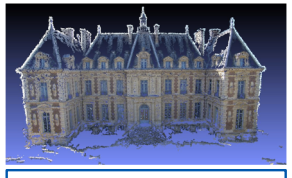
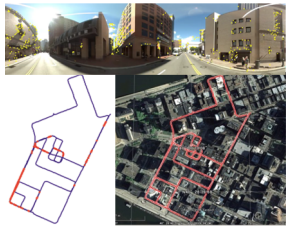
3D Reconstruction



Camera Geometry

Robotics

Visual localization



ellis
European Laboratory for Learning and Intelligent Systems

Basic & Applied Research

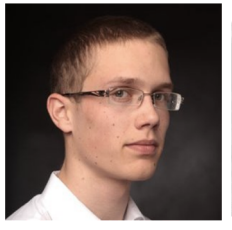
Tomas Pajdla
AAG Leader
Vision
Robotics
Mathematics

Josef Sivic
IMPACT Leader
Vision
Robotics
Machine Learning

Torsten Sattler
Researcher
3D Vision
Learning

Jiri Sedlar
Postdoc
Vision
Robotics

Michal Polic
PhD Student
3D
Vision



Pavel Trutman
PhD Student
Polynomial
Optimization

Stanislav Steidl
PhD Student
Vision
CNN

Viktor Korotynskiy
PhD Student
Algebraic
Geometry

Research

We apply elements of

- Algebra
- Geometry
- Statistics
- Optimization

in

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- Robotics
- Machine Learning

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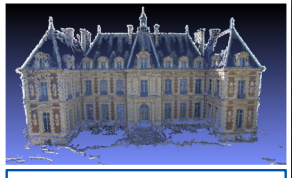


Camera Geometry

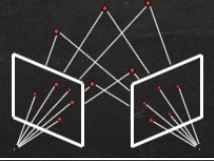
Robotics

Visual localization

Theory

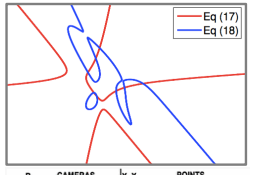
Camera Geometry

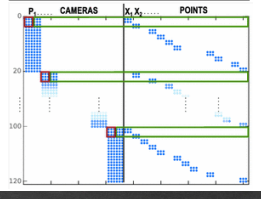
Many Minimal reconstruction Problems in RANSAC based optimization

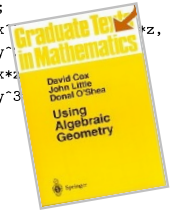


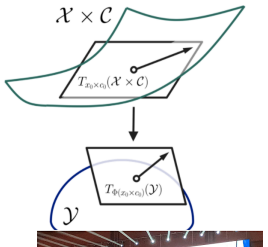
Macaulay2 program over the finite field $\mathbb{Z}/13$

```
R = ZZ/13[x,y,z, MonomialOrder=>GRevLex];
I = ideal(8*x^2*y^2 +  5*x*y^3     +  3*x^
            x^5     +  2*y^3*z^2   + 13*y^
          8*x^3     + 12*y^3       +    x*
          7*x^2*y^4 + 18*x*y^3*z^2 +    y^3
G = gens gb I
```



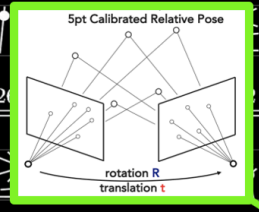


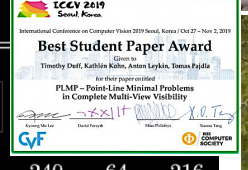




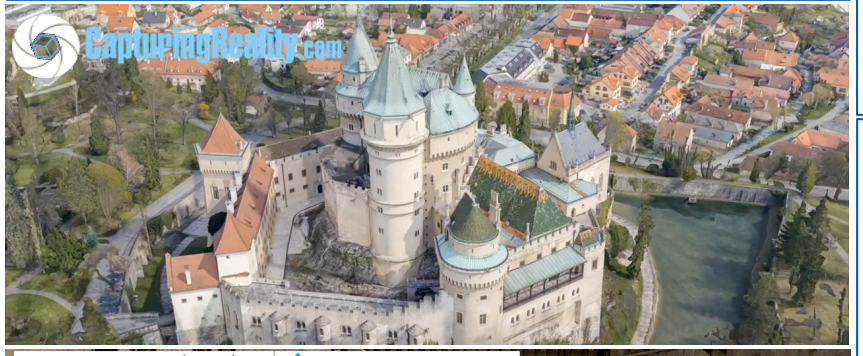
30 Minimal Problems

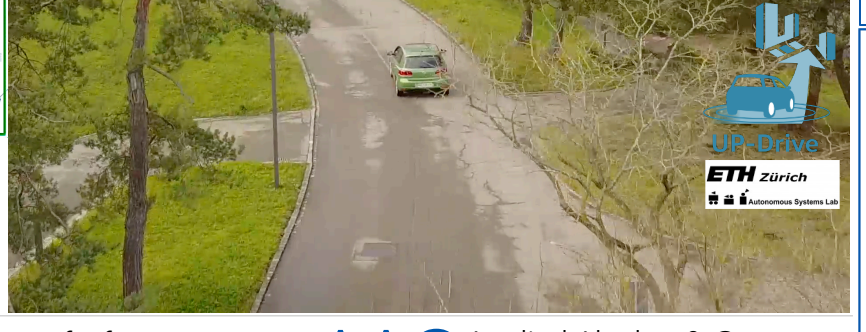
# views	6	5	5	5	4	4	4			
Configuration										
# solutions	$\approx 10^6$	11296	2(				1728			
# views	3	3					3			
Configuration										
# solutions	360	552	480	264	432	328	48(	240	64	216
# views	3	3	3	3	3	3	3	2	2	2
Configuration										
# solutions	312	224	40	144	144	144	64	20	16	12





Applications





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3D Mapping

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3D Reconstruction

Camera Geometry

Robotics

Visual localization

CapturingReality

Reconstruction by Capturing Reality (www.capturingreality.com)
Spin-off of M Bujnak and M Jancosek (PhD students of T Pajdla)
Based on 3D reconstruction techniques developed at the CTU in Prague

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3D Reconstruction

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Visual Effects

MIKROS MPC ADVERTISING MIKROS | technicolor MIKROS Animation

Open 3D Reconstruction Pipeline: alicevision.org

"See You Up There", dir. Albert Dupontel, prod. ADCB Films, Manchester Films

MIKROS

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Robotics & Machine Perception

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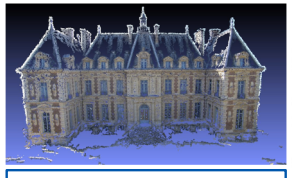

Camera Geometry

Robotics

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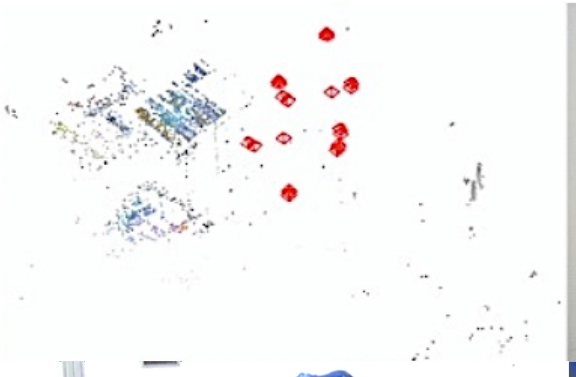

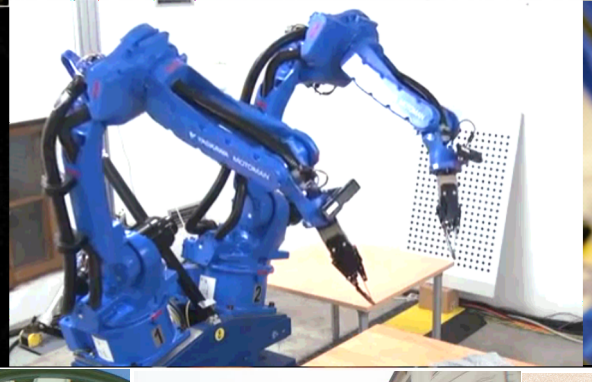

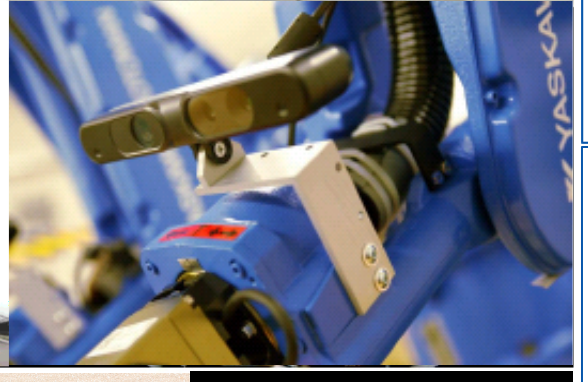

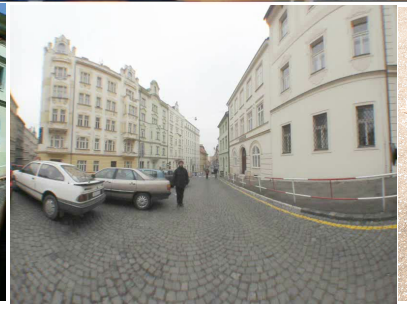

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Cameras on Cars

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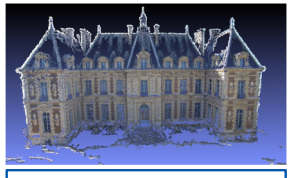
3D Reconstruction



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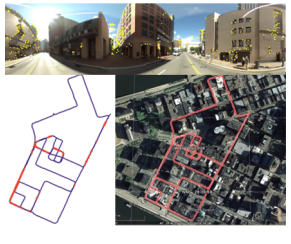
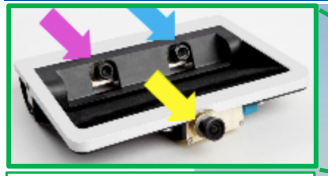
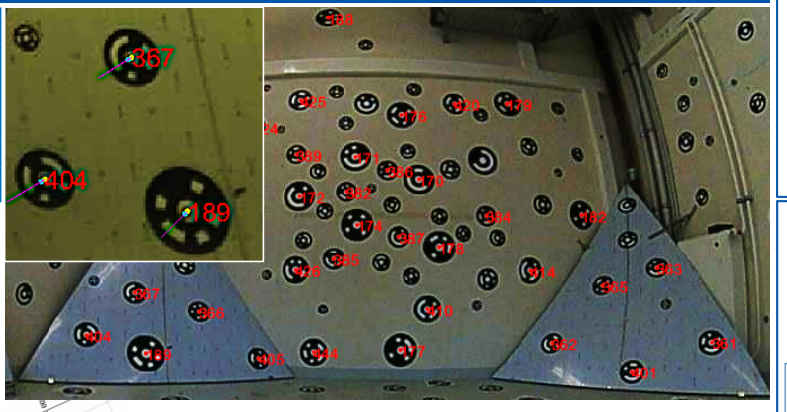
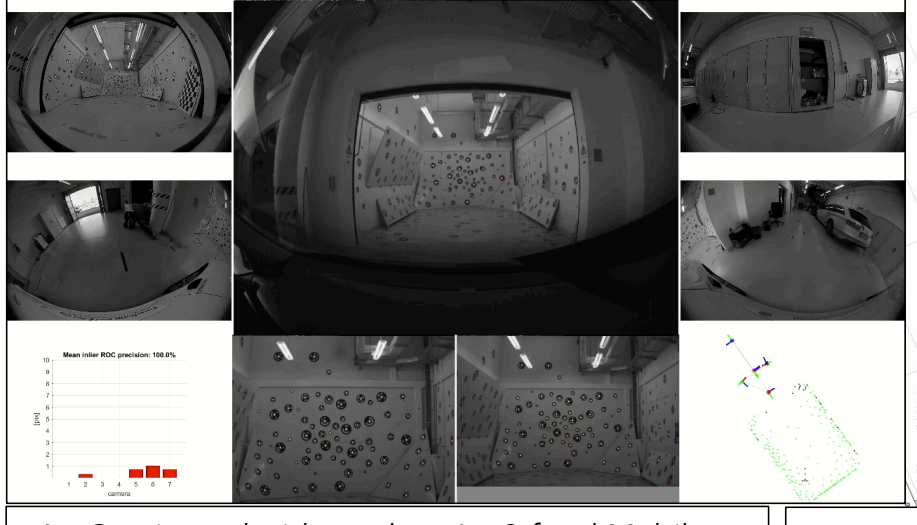




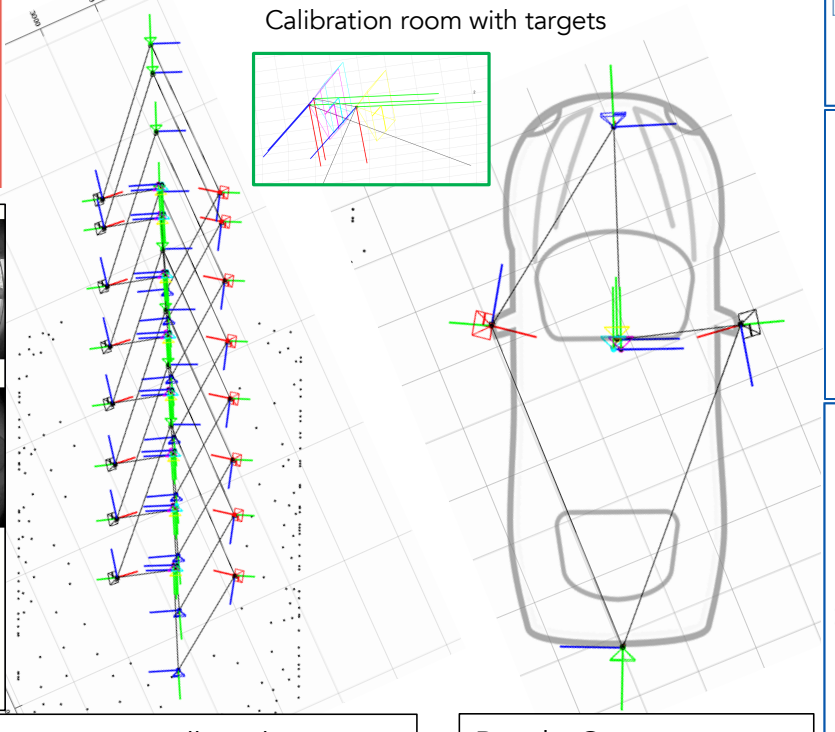
MOBILEYE 3-focal





4 x Continental wide angle + 1 x 3-focal Mobileye

Calibration room with targets



Many poses + Bundle Adjustment

Result: Cameras on car

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Autonomous Driving

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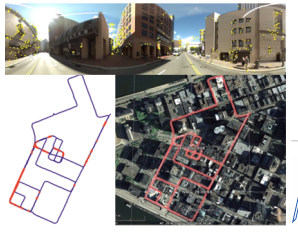

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ETH Zürich

IBM

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SLAM by ETH on cams calibrated by CTU

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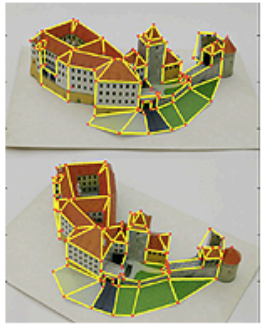

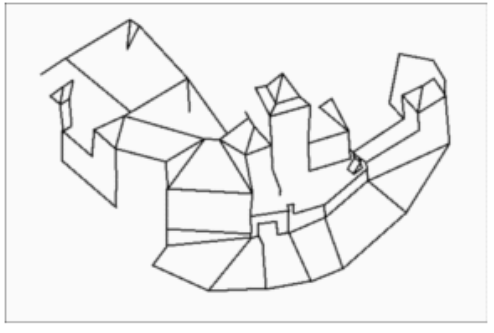

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GVG Lectures

Lectures [Monday 12:45-14:15 Online]

Tomas Pajdla, **Torsten Sattler**: Online Lectures via MS Teams

Week	Date	Lecture T Pajdla. Elements of Geometry for Computer Vision
01	15.2.	TP: Intro: Geometry of CV & CG, LA [Sec. 2.1] image coordinate system [Sec. 5] LO
02	22.2.	TP: Mathematical model of the perspective camera [Sec. 6]
03	01.3.	TP: Camera calibration and pose [Sec. 7.1]
04	08.3.	TP: Calibrated camera pose computation I & II [Sec. 7.2, 7.3]
05	15.3.	TP: Homography [Sec. 8.1-8.5]
06	22.3.	TS: Image based camera localization
07	29.3.	TS: Projective plane [9.1-9.2]
08	05.4.	*Easter Monday*
09	12.4.	TP: Vanishing points & line [Sec. 9.4, 9.5] projective space [Sec. 10] camera autocalibration [Sec. 11]
10	19.4.	TP: Vector product [Sec. 2.2, 2.3] dual space [Sec. 2.4] lines under homography [Sec. 9.3]
11	26.4.	TS: Epipolar geometry [Sec. 12.1-12.2]
12	03.5.	TP: 3D reconstruction with a calibrated camera [Sec. 12.3, 12.4]
13	10.5.	TP: Calibrated camera motion computation [Sec. 12.5], SVD
14	17.5.	TS: 3D Reconstruction pipelines

GVG Labs

Labs [Monday 14:30-16:00, 16:15-17:45 Online]

Martin Matoušek, **Viktor Korotynskiy** , **Kateryna Zorina**, **Vojěch Pánek**: Online Labs via MS Teams

- solving of algebraic problems related to vision geometry; this is without computer, i.e. 'pen-and-paper'
- solving of practical tasks (home-works) on a real data with computer

Schedule

Week	Date	Pen & Paper	Test	Assignment	Deadline
01	15.2.	Basic elements of LA	Test-α	HW-01 Image Coordinate System	
02	22.2.	Projection matrix Ex-02		HW-02 Projection Matrix	
03	01.3.	Camera internal calibration Ex-03		HW-03 Camera calibration	HW-01
04	08.3.	Polynomial solving Ex-04		HW-04a Calibrated pose I	HW-02
05	15.3.	Test 1 Example video		HW-04b Calibrated pose II	HW-03
06	22.3.	Homography Ex-06 Ex-06	Test 1	HW-05 Homography	HW-04a
07	29.3.	Projective plane Ex-07 Ex-07 video		HW-06 Panorama	HW-04b
08	05.4.	*Easter monday*			HW-05
09	12.4.	Test 2 Example (06a 06b 07a 07b)		HW-07 Autocalibration	
10	19.4.	Meet & Join Ex-10 Ex-10(04a 04b 04c)	Test 2		HW-06
11	26.4.	Epipolar geometry Ex-11		HW-08 Epipolar geometry	
12	03.5.	3D Reconstruction Ex-12		HW-09a 3D reconstruction I	HW-07
13	10.5.	Test 3 Example		HW-09b 3D reconstruction II	HW-08
14	17.5.		Test 3		HW-09a
	24.5.				HW-09b

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Assessment

1. All homework must be submitted via BRUTE and accepted.
2. At least 50% of points in total for the homework.
3. At least 50% of points in total from the tests.
4. Regular submission of homework **ends on May 24, 2021**. Later submissions are possible only by an agreement with the assistants.
5. All the above conditions have to be fulfilled, and the results have to be recorded in the Submission system before the exam.

Exam

The exam consists of a written and an oral part. It is required to achieve at least 50% of points from the written exam to be admitted to the oral exam. The grade depends on the exam (40%), tests (30%), and homework (30%).

Exam content:

1. **Linear algebra [4,5,6,7]:** linear space, basis, coordinates, linear dependence/independence, matrices, rank, determinant, eigenvalues and eigenvectors, solving systems of linear equations, Frobenius theorem and linear independence, linear function, affine function, linear mapping and its matrix, computing roots of a polynomial via eigenvalues of its companion matrix, dual space, dual basis, change of the dual basis corresponding to a change of a basis, vector product and derived linear mappings, SVD, dual space, and dual basis.
2. **Course material**

Rules

1. **Lecture:** It is very **difficult** to pass the course without attending online lectures.
2. **Labs:** It is **impossible** to pass the course without attending online labs.
3. **Homework:** Homework is assigned at a lab where it can be discussed with teaching assistants. Students work out homework **individually** (rules in Czech). The deadline for submitting homework via BRUTE is on Monday at 6:00 in the morning two weeks after the assignment. Late submissions are penalized (10% for each commenced **day** of delay but not more than 50% of points).
4. **Assessment:** see above.
5. **Tests:** Students take tests **individually**.

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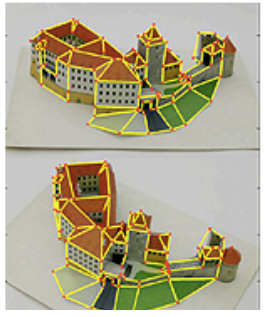
cw.fel.cvut.cz/wiki/courses/gvg/start

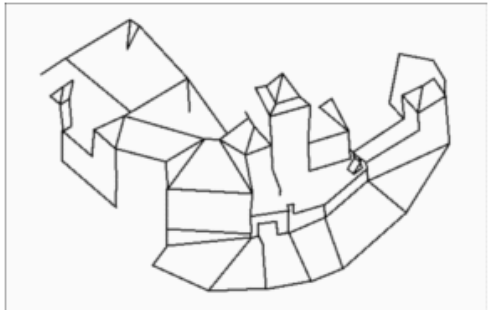
GVG

GVG Labs

ALL COURSES

Geometry of Computer Vision and Graphics 2021





MS Teams – lectures, labs, tests, the exam

BRUTE (cw.felk.cvut.cz/brute) – homework, tests, quizzes, points, feedback

Forum is the communication channel (limited emails & no messages in MS Teams)

1. Login to **BRUTE** at cw.felk.cvut.cz/brute
2. Check that you see **GVG in 2021L** in BRUTE
3. Check that you see News in the **Forum**

Questions …