

# Deep Learning (BEV033DLE)

## Lecture 1.

Czech Technical University in Prague

- ◆ Biological neurons
- ◆ Visual cortex and pathway
- ◆ Artificial neuron models
- ◆ Neural network architectures
- ◆ Application examples
- ◆ Stochastic neurons

# Organisational Matters

**Teachers:** Alexander Shekhovtsov, Jan Šochman and Boris Flach

**Format:** 1 lecture & 1 lab per week (6 credits), labs of two types (alternating)

- ◆ practical labs: implementation of selected methods (Python/PyTorch), submission of results: upload system BRUTE, 4 weeks p.a., hard deadline
- ◆ theoretical labs: solving theoretical assignments  
assignments are published in advance, you are expected to present/discuss solutions

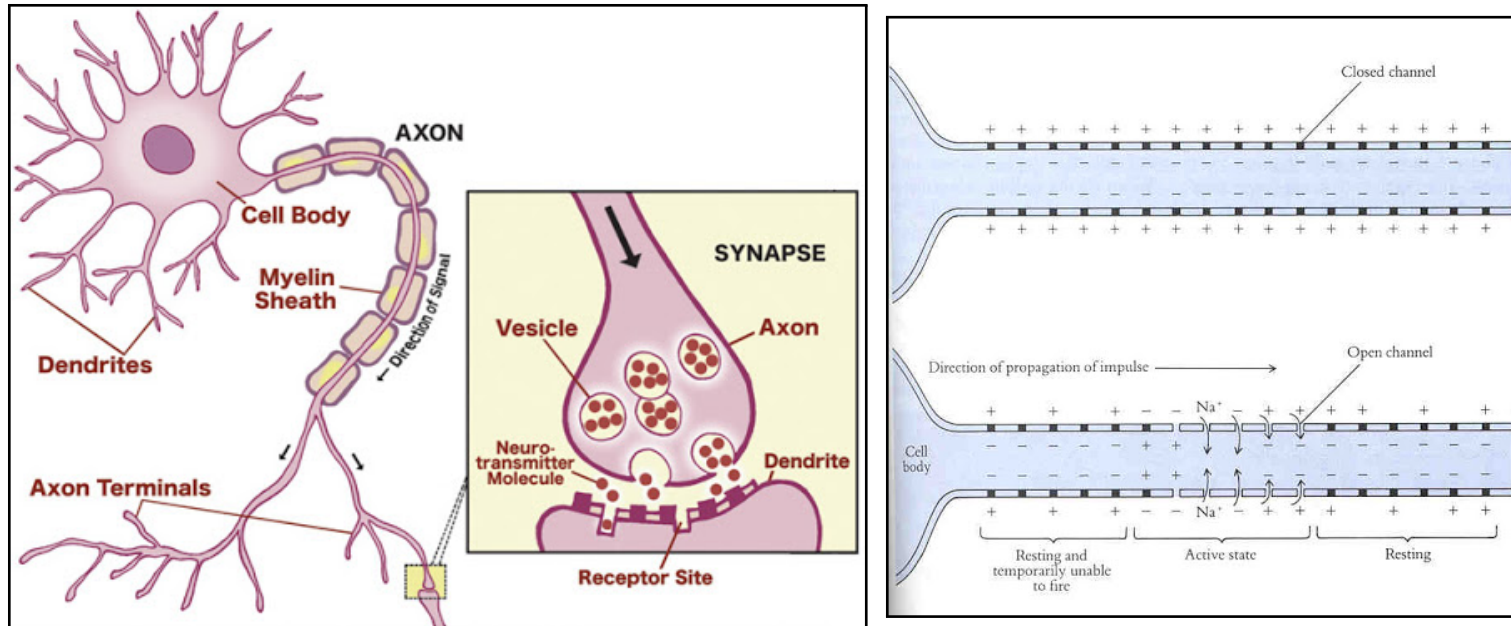
**Grading:** 50% practical labs + 50% written exam = 100% (+ bonus points)

## Prerequisites:

- ◆ calculus, linear algebra and optimisation
- ◆ basics of graph theory and related algorithms
- ◆ pattern recognition and machine learning (AE4B33RPZ)

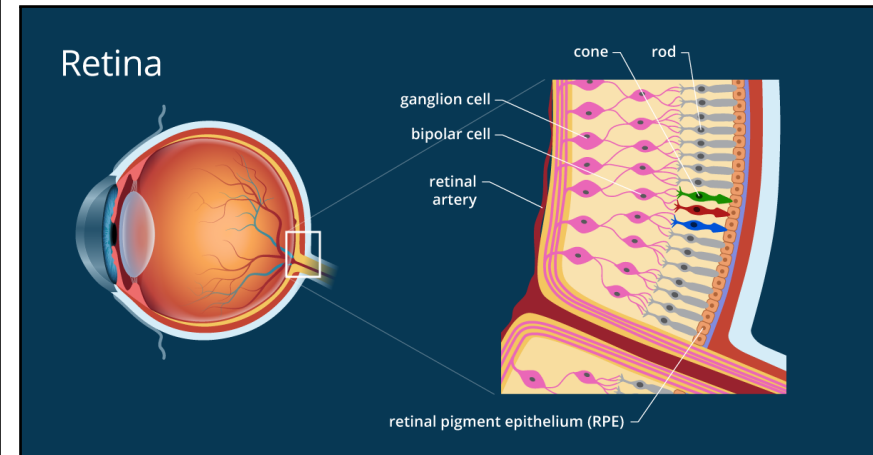
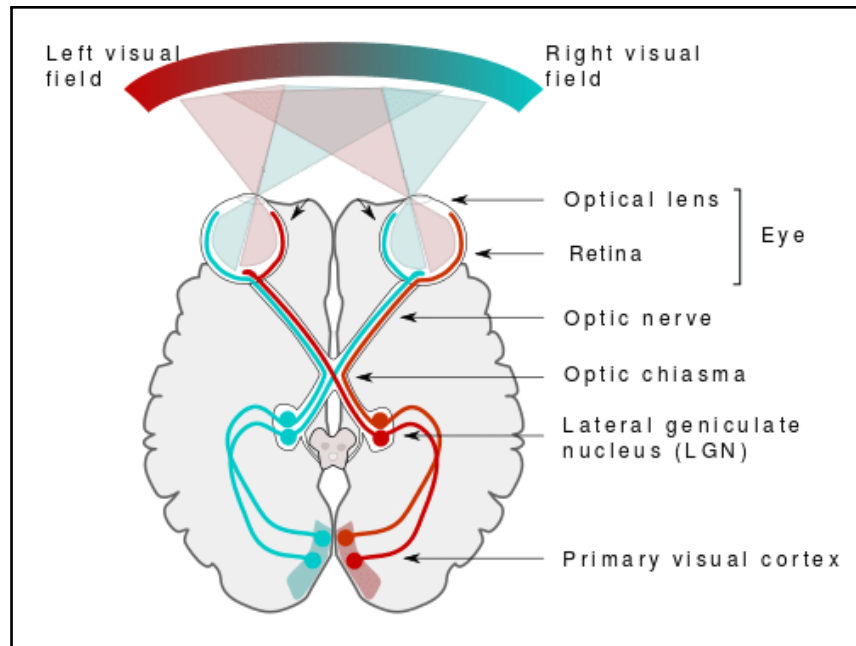
More details: <https://cw.fel.cvut.cz/wiki/courses/bev033dle/start>

## Biological neurons



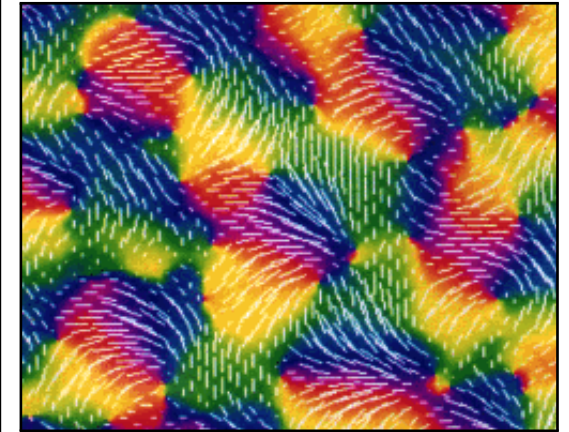
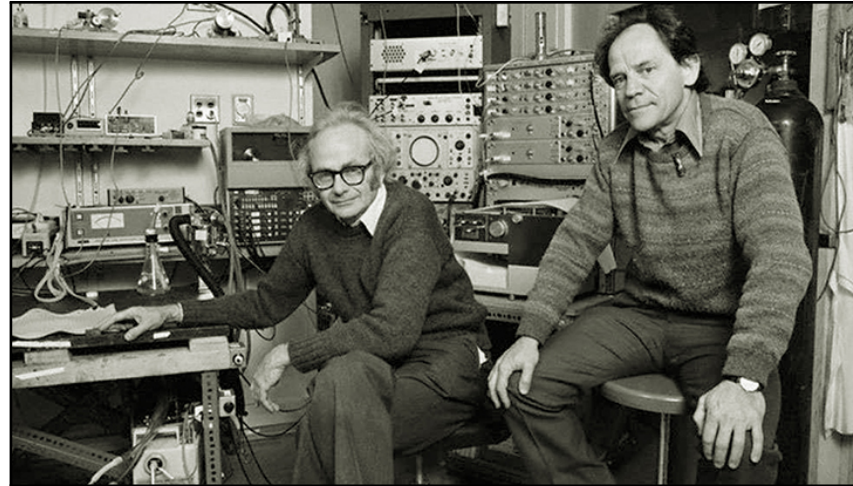
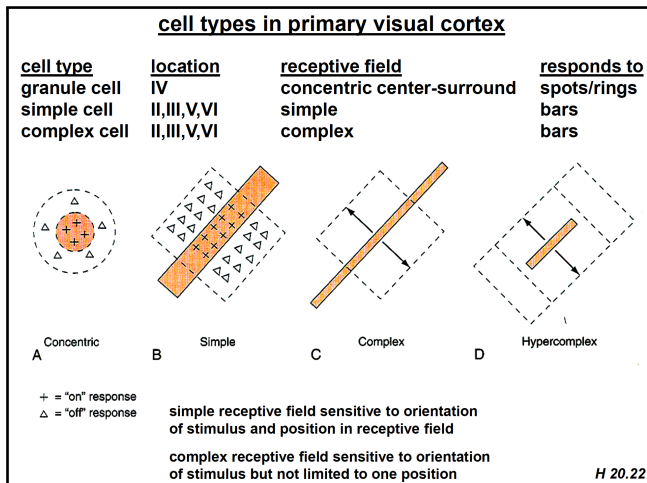
- ◆ physiology of neuron cells: dendrite, soma, axon, axon branches, synaptic connections
- ◆ resting state: interplay of selective voltage gated ion channels and pumps leads to negative potential
- ◆ activation: strong simultaneous excitation by incoming signals (dendrite)- triggers depolarisation and action potential at axon hillock - travelling electric signal (spike) along the axon
- ◆ spike arriving at synaptic endings - release of neuro-transmitters into the synaptic gap - excitation or inhibition of the postsynaptic neurons

# Biological neural networks: visual cortex



- ◆ Retina photoreceptors:  $\sim 10^7$  cones and  $\sim 10^8$  rods, several processing layers, last layer:  $\sim 10^6$  ganglion cells (“center on” and “center off”)
- ◆ Optical nerve - lateral geniculate nucleus - V1 first layer of visual cortex
- ◆ First layers in visual cortex resemble convolutional networks (with feedback)
- ◆ simple and complex cells in V1 have circular shaped receptive fields

# Biological neural networks: visual cortex



## Cell types in V1

David Hubel and Torsten Wiesel  
(Nobelprize, 1981)

Orientation columns  
in V1

- ◆ recording specific patterns that stimulate individual neurons, simple and complex cells in V1,
- ◆ retinotopy, ocular dominance columns, orientation columns,
- ◆ emergence of architecture/connectivity: blocking visual input from an eye input in critical development phases dramatically alters the connectivity pattern,
- ◆ absence of visual stimuli during critical development periods leads to deterioration of connectivity and suppresses emergency of cells responding to specific visual stimuli.

## Artificial neurons

Artificial neuron as elementary computational unit (McCulloch-Pitts 1943)

$$y = f\left(\sum_i w_i x_i + b\right) = f(w^T x + b),$$

with  $x = (x_1, \dots, x_n)$  - inputs,  $w = (w_1, \dots, w_n)$  - weights,  $b$  - bias and  $f: \mathbb{R} \rightarrow \mathbb{R}$  - a nonlinear activation function.

**binary output:**  $y = 0, 1$ ,  $f = H$  - Heaviside function or  $y = \pm 1$ ,  $f = \text{sign}$  - sign function

- ◆ represents a binary classifier
- ◆ Notice, that the mapping is invariant to parameter scaling  $(w, b) \rightarrow (\lambda w, \lambda b)$
- ◆ networks with binary outputs & binary valued weights/biases are highly relevant for mobile applications
- ◆ the mapping is piece-wise constant, networks can not be learned by gradient descent

**graded response:**  $y \in [0, 1]$ ,  $f = S$  - sigmoid function or  $y \in [-1, 1]$ ,  $f = \tanh$

- ◆ squashed output (spike frequency)
- ◆ the mapping is differentiable, but can have vanishing gradients

## Artificial neurons

**Rectified Linear Units:**  $y \in \mathbb{R}_+$ ,  $f(x) = \max(0, x)$  - ReLU or  $f(x) = \max(\alpha x, x)$  - Leaky ReLU

- ◆ the mapping is differentiable (a.e.),
- ◆ ReLU neurons can have vanishing gradients,
- ◆ notice, that the mapping scales linearly with  $(w, b) \rightarrow (\lambda w, \lambda b)$
- ◆ LReLU neurons can be generalised to **maxout neurons** with tunable slopes  
 $f(x) = \max(\alpha_1 x + b_1, \alpha_2 x + b_2)$

**Softmax unit/layer:** suppose the network outputs should represent class probabilities (conditioned on the input), i.e. they must be non-negative and sum up to one.

- ◆  $x = (x_1, \dots, x_K)$  are **scores** for the  $K$  classes
- ◆  $y = (y_1, \dots, y_K)$  are interpreted as class probabilities, where

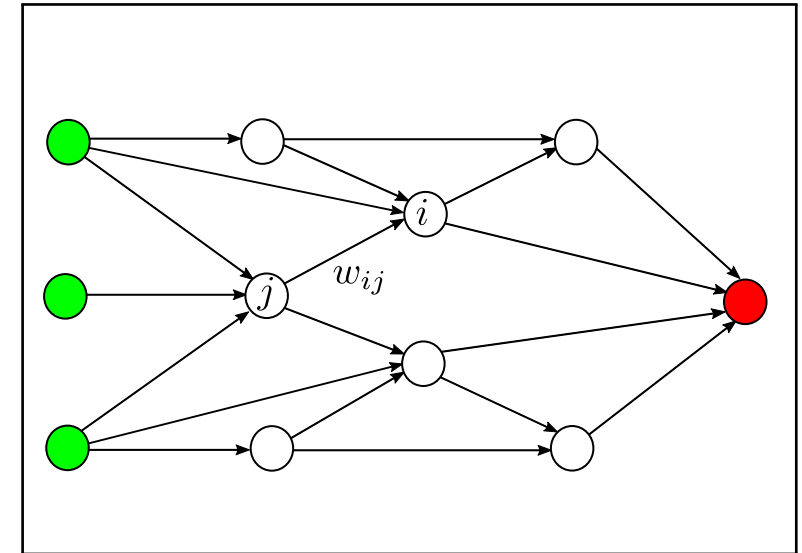
$$y_k = \frac{e^{x_k}}{\sum_{m=1}^K e^{x_m}}$$



## Network architectures

A general **feed forward network** is given by

- ◆  $(V, E, w)$  a directed acyclic weighted graph
- ◆  $\{x_i \in \mathbb{R} \mid i \in V\}$  real valued variables associated with the nodes of the graph
- ◆  $\{f_i \mid i \in V\}$  activation functions associated with the nodes.



The network defines a mapping  $x_I \mapsto x_T$ , where  $I \subset V$  are the input nodes and  $T \subset V$  are the output nodes of the graph. It is defined recursively by

$$x_i = f_i(a_i) = f_i\left(\sum_{j \in pa(i)} w_{ij}x_j\right)$$

The value (output) of each node  $i \in V$  becomes a function  $x_i = F_i(x_I, w)$  of the inputs  $x_I$  and the network parameters  $w$ .



## Network architectures

A **layered feed forward network** is organised in **layers**  $V = \bigcup_{k=0}^L V^k$  with edges between nodes in consecutive layers only. We call  $V^0$  the input layer,  $V^L$  the output layer and the remaining layers hidden.

The input-output mapping  $F: x^0 \rightarrow x^L$  can be represented as composition of mappings

$$x^L = F(x^0) = f^L \circ A^L \circ \dots \circ f^1 \circ A^1 x^0,$$

where

- ◆  $x^k$  denotes the output vector of neurons in layer  $k$ ,
- ◆  $A^k$  denotes the linear (affine) mapping  $A^k x^{k-1} = W^k x^{k-1} + b^k$ ,
- ◆  $f^k$  denotes the elementwise application of the activation function of layer  $k$ .

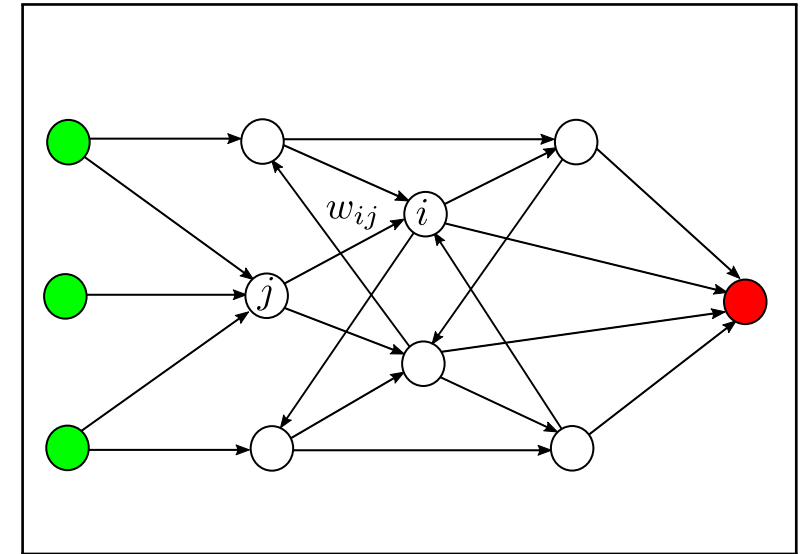
This representation allows to “modularise” the forward computation and the computation of the derivatives of the outputs w.r.t. to the network parameters.

An important special case of FFNs are **convolutional neural networks**, which will be discussed later in the course.

## Network architectures

A general **recurrent** network is given by

- ◆  $(V, E, w)$  a directed weighted graph (with cycles)
- ◆  $\{x_i \in \mathbb{R} \mid i \in V\}$  real valued variables associated with the nodes of the graph
- ◆  $\{f_i \mid i \in V\}$  activation functions associated with the nodes.



Notice, that this leads to a **dynamical system** even if we clamp the states of the input neurons.

$$x_i(t) = f_i(a_i) = f_i\left(\sum_{j \in \mathcal{N}_i} w_{ij}x_j(t-1)\right)$$

There are different possible **updating schedules**

- ◆ **parallel:** compute new outputs for all neurons and update them synchronously,
- ◆ **sequential:** fix an ordering of neurons and update them one at a time.

## Network architectures

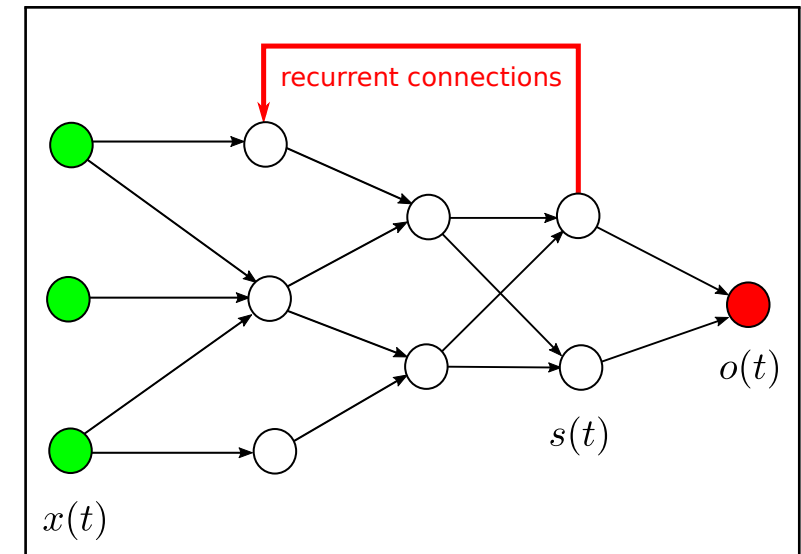
Recurrent networks for time sequence processing:

- ◆ consider layered networks with additional feedback connections from penultimate layer to first layer,
- ◆ this leads to the dynamical system

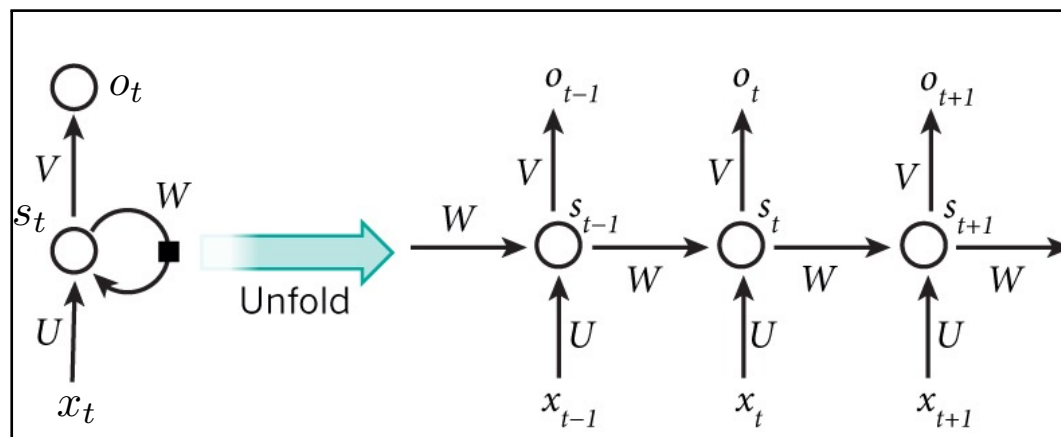
$$s(t) = F(x(t), s(t-1))$$

$$o(t) = G(s(t)),$$

where  $x(t)$  denotes inputs,  $s(t)$  denotes outputs of the penultimate layer and  $o(t)$  denotes outputs

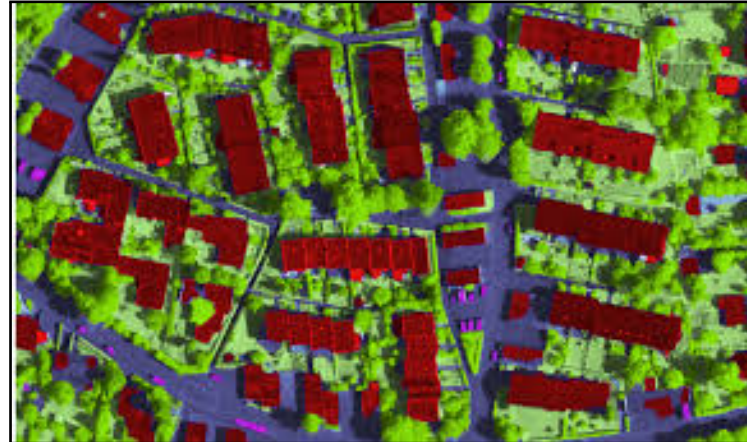


It can be “unfolded” in time

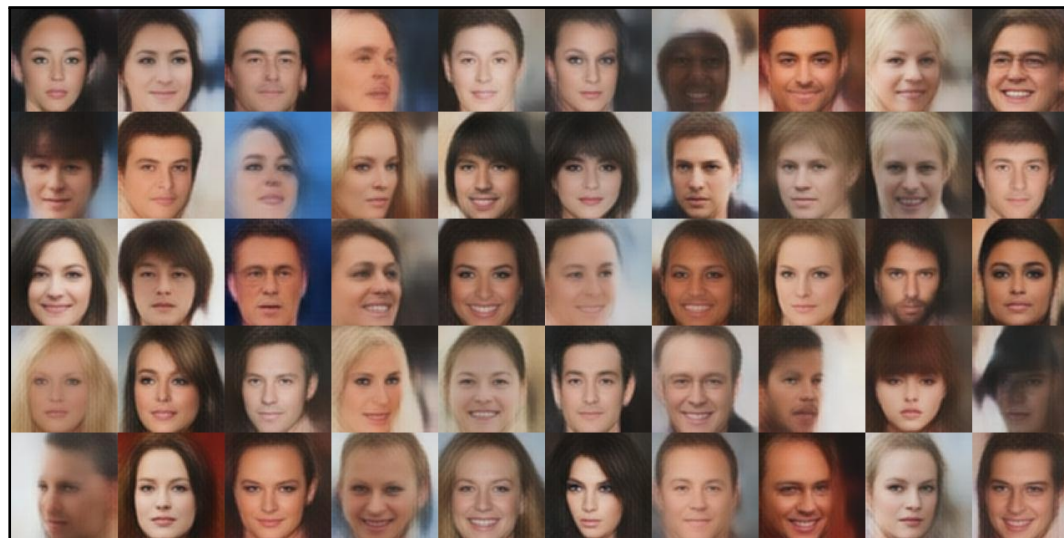


## Application examples

**Semantic segmentation:** input image  $\rightarrow$  CNN  $\rightarrow$  output segment class probabilities for each pixel

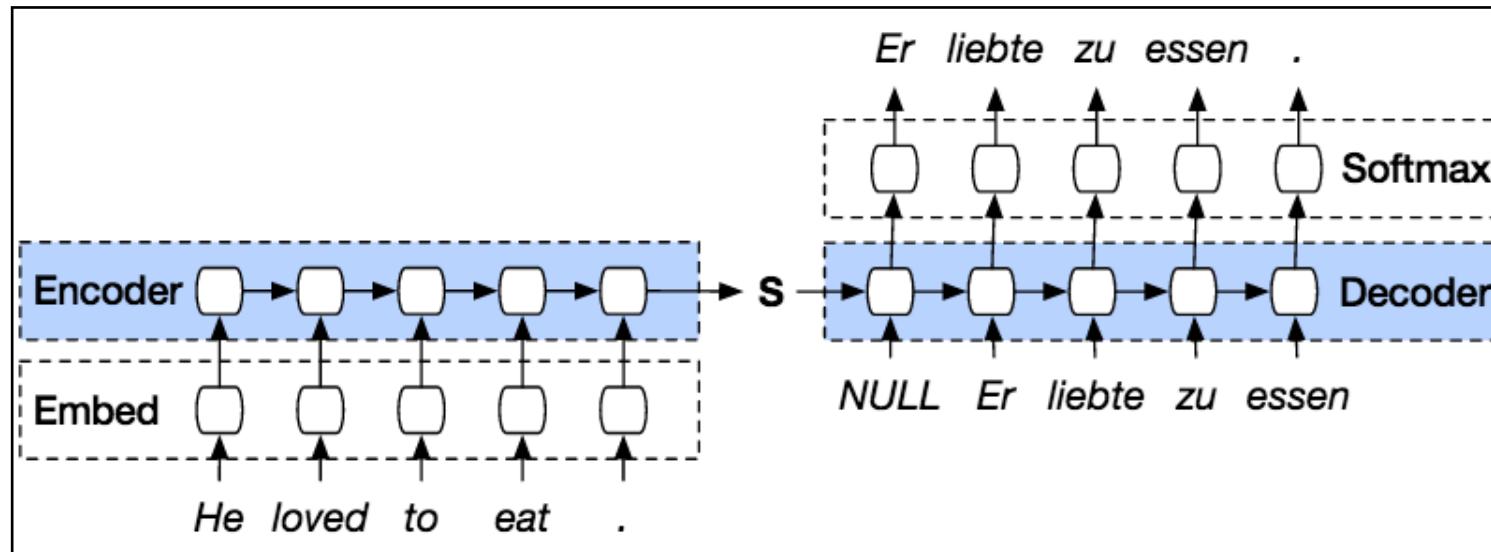


**Generative networks:** sample latent noise  $Z \sim \mathcal{N}(0, C) \rightarrow$  DNN  $\rightarrow$  output image



## Application examples

**Natural Language Processing:** machine translation by “sequence to sequence” RNNs



## Sidestep: stochastic neurons

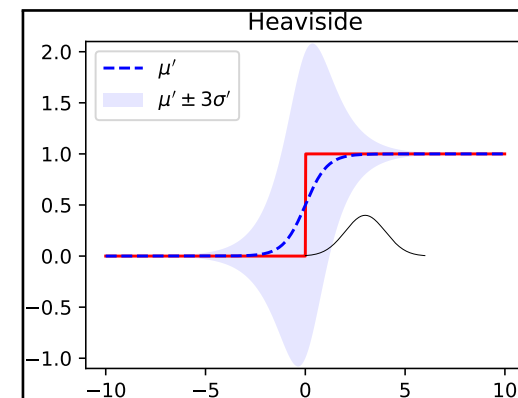
Why stochastic neurons?

- ◆ recognition: randomised predictors might be easier to learn (regularisation)
- ◆ networks with binary output neurons: make them accessible for learning

**Stochastic neuron** with additive injected noise

$$y = f\left(\sum_i w_i x_i + b + z\right)$$

$z \sim$  standard logistic distribution



The expected output conditioned on input becomes smooth, even if the activation function is not. E.g. for  $f = H$ , we obtain

$$\mathbb{E}_z[Y = 1 \mid X = x] = S\left(\sum_i w_i x_i + b\right)$$

because the sigmoid function  $S$  is the c.d.f.  $F$  of the standard logistic distribution.

$$\mathbb{P}(Y = 1 \mid x) = \mathbb{P}(a(x) + z \geq 0) = \mathbb{P}(z \geq -a(x)) = 1 - F(-a(x)) = F(a(x)) = S(a(x)).$$