Problem solving by search

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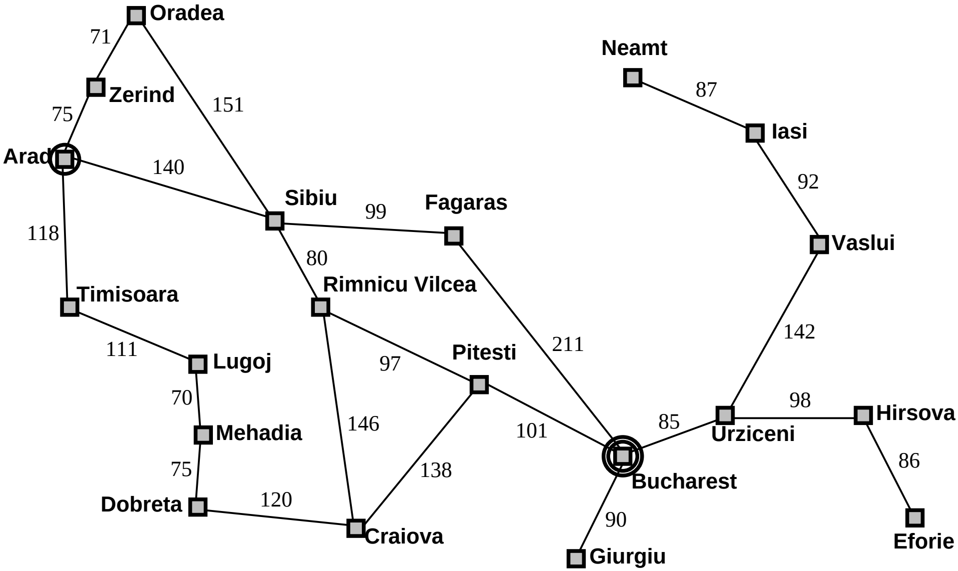
Notes

Outline

- Search problem.
- State space graphs.
- Search trees.
- Strategies: which tree branches to choose?
- Strategy/Algorithm properties.
- Programming infrastructure.

Notes

Example: Traveling in Romania



Notes

Ok, start with a simple one, almost everybody knows about the navigation - path planning problem. Waze, Garmin, . . .
Can you think about more problems?
For example:

- Touring problems. Special case: Traveling salesperson problem – each city must be visited exactly once.
- Planning robot movements – mobile robot or manipulator.
- VLSI (chip) layout.
- ...

Example: Map of Romania

Goal:

be in Bucharest

Problem formulation:

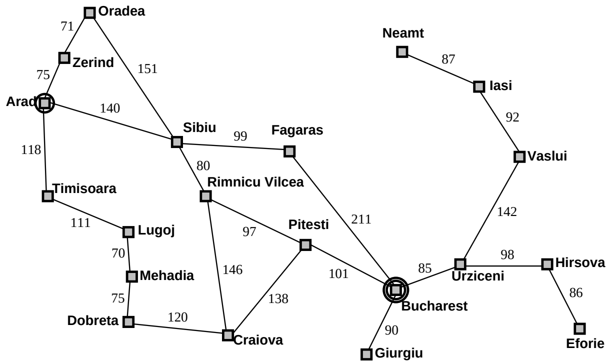
states: position in a city (cities)

actions: drive between cities

Solution:

Sequence of cities (path)

(action sequence [2])

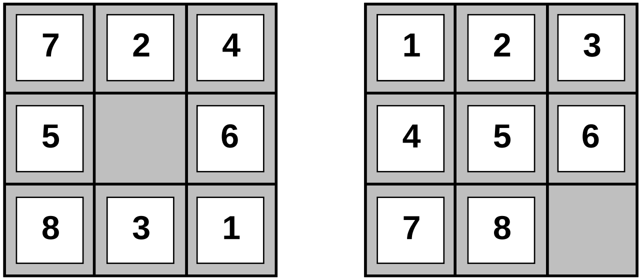


Notes

Classical problem from the Book [2], we use it, too.

states and actions will be frequently discussed in several lectures and algorithms. It is important to fully understand them.

Example: The 8-puzzle



Start State **Goal State**

states?
actions?
solution?
cost?

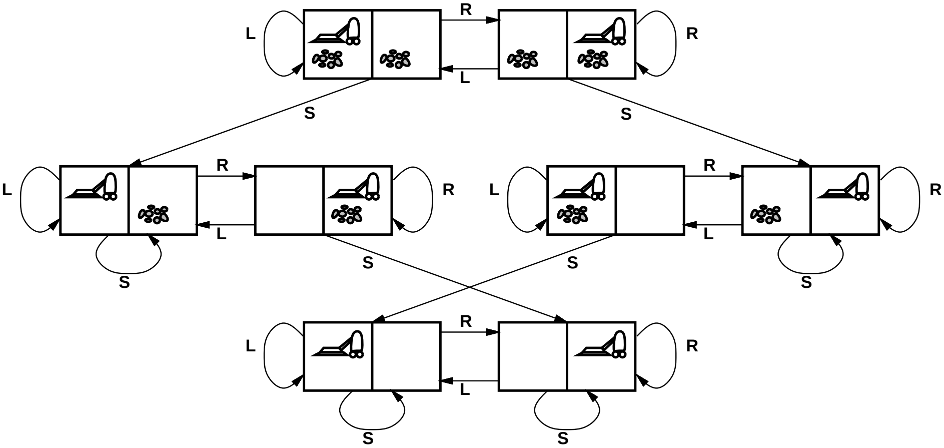
Notes

Also known as $n-1$ puzzle.

- States: Location of each of the 8 tiles and the blank.
- Number of states: 9!
- Initial state: any state. (Note that any given goal state can be reached from exactly half of the initial states.)
- Actions: Movements of the blank space: Left, Right, Up, Down (or a subset of these)
- Solution / goal test: Check whether state matches the goal configuration.
- Path cost: nr. steps in the path (each step costs 1)

Toy problem (3.2.1) from [2].

Example: Vacuum cleaner



states?
actions?
solution?
cost?

Notes

- States: Determined by agent location and dirt location. The agent is in one of two locations, each of which may or may not contain dirt.
- Number of states: 2×2^2 (two possible choices for agent location; for every location, choice dirt vs. no dirt). For n locations: $n \times 2^n$
- Initial state: any state
- Actions: Left, Right, Suck (larger envs. can have also Up and Down)
- Solution / goal test: Are all squares clean?
- Path cost: nr. steps in the path (each step costs 1)

Toy problem (3.2.1) from [2].

A Search Problem

- State space (including Start/Initial state): position, board configuration,
- Actions : drive to, Up, Down, Left ...
- Transition model : Given state and action return state (and cost)
- Goal test : Are we done?

Notes

We will use the terminology throught the next 5-6 lectures; also for Markov (Sequential) Decision Processes, Reinforcement Learning.

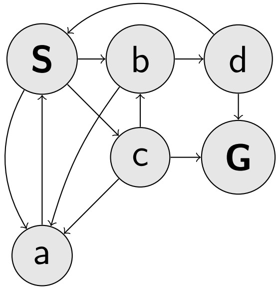
Make a mental test: You are a robot, going from home to school. What would be states, actions, transition model, goal test?

State Space Graphs

State space graph: a representation of a search problem

- Graph Nodes – states – are abstracted world configurations
- Arcs represent action results
- Goal test – a set of goal nodes

Each state occurs only *once* in a state (search) space.

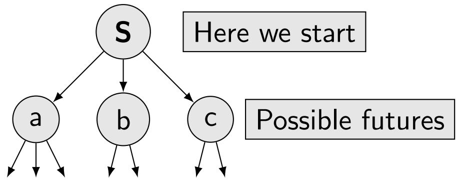


Notes

Formalizing a real world problem – (creating) a state space graph – could be a problem in itself. I put creating into brackets as it may be also infinite.
Close connection to graph algorithms like Dijkstra, Floyd-Warshall.

- Graph algorithms assume complete info about the graphs - the main input.
- For many real-world problems, the graph is not known in advance.
- The state space graph is *revealed during the search*. The graph serves as an abstraction - mental model - rather than as an actual data representation.
- Many real world problems have too many vertices, think about $n-1$ puzzle or chess, number of possible configurations is enormous.
- A solution can be actually quite shallow.

Search Trees

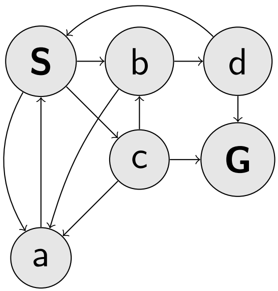


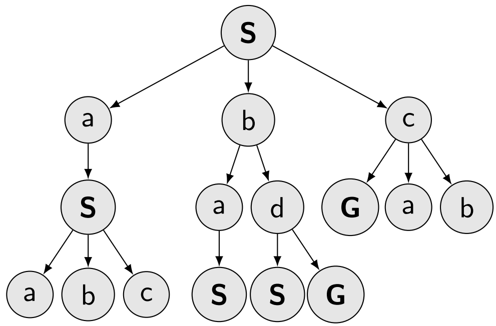
- A "what if" tree of plans and their outcomes
- Start node is the root
- Children are successors
- Nodes show/contains states, but correspond to *plans* that achieve those states

Notes

- What if decision about an action, repeats ...
- Nodes in the search tree are not the same as the nodes in the state space graph.

State Space Graphs vs. Search Trees



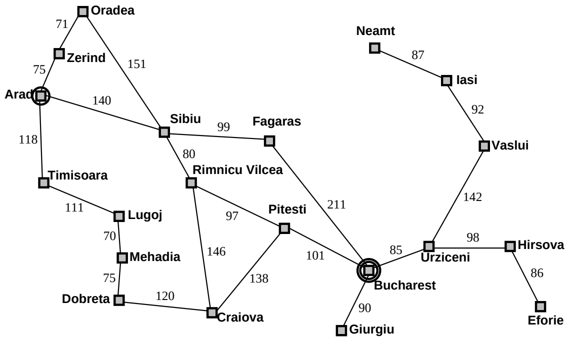


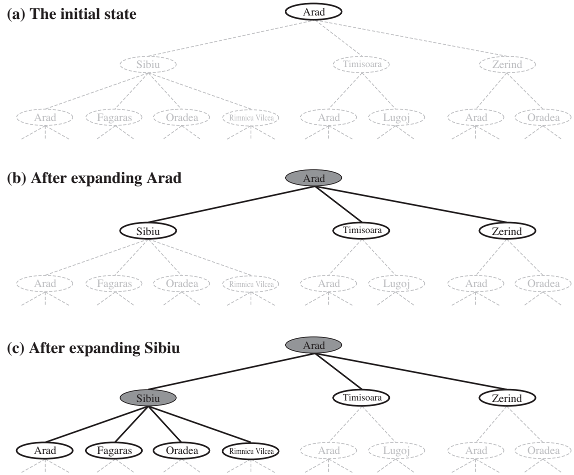
How big is the search tree?

Notes

- 'S' denotes Start; 'G' denotes Goal.
- There could be *multiple* search trees above *one* state space, depending on the algorithm.
- When going through the unfolding of the *search tree* (on the right), one may already introduce that there are *leaf nodes* at the *frontier*; one of them always gets *expanded*.
- A search tree can be *much bigger* than the state space. (E.g., states 'S', 'a',... appear more than once in the search tree...)
- Note also that search does not have stop when 'G' is reached for the first time. We may need the shortest path...
- These properties will be discussed next.

From problem/transition graph to search tree (Romania)



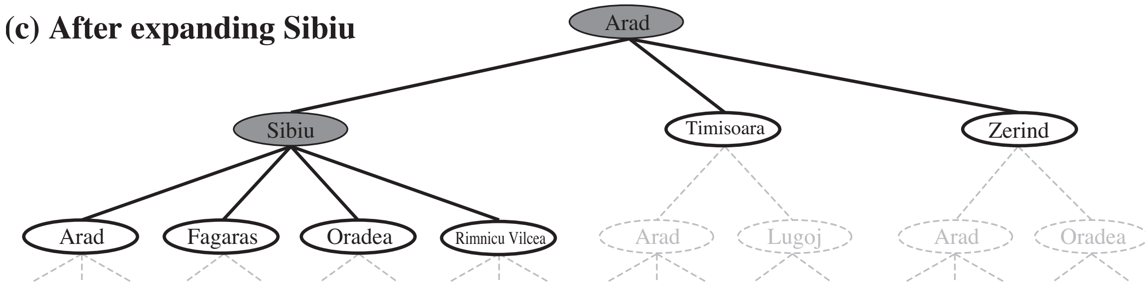


Problem/transition graph is revealed incrementally.
The revealing strategy can be visualized as a search tree.

Notes

Images from [2].

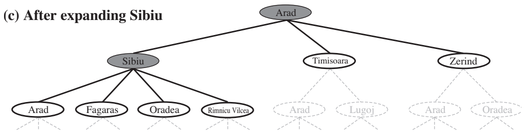
Search elements - unvisited, dead, alive states



- Expand plans - possible ways (tree nodes).
- Manage/Maintain fringe (or frontier) of plans under consideration.
- Expand new nodes *wisely(?).*

Notes

Tree search algorithm

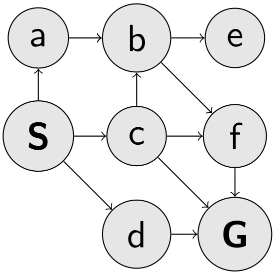


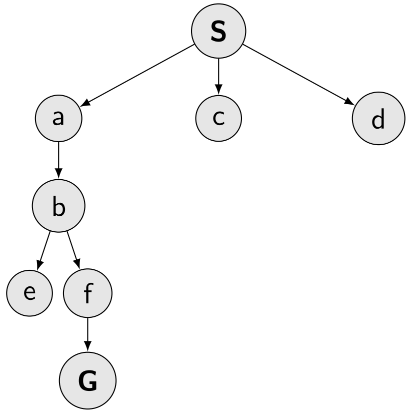
function TREE_SEARCH(problem) **return** a solution or failure
    initialize by using the initial state of the problem
    loop
        if no candidates for expansion **then return** failure
        else choose a leaf node for expansion
        end if
        if the node contains a goal state **then return** the solution
        end if
        Expand the node and add the resulting nodes to the tree
    end loop
end function

Notes

A *general* tree search algorithm. Individual search algorithms vary primarily in how they choose which state to expand next – the "search strategy".

Example of a tree search





Which nodes to *explore*?

What are the properties of a strategy/algorithm?

Notes

Before going to the next slide, think about algorithms. What properties of an algorithm would you want?

Search (algorithm) properties

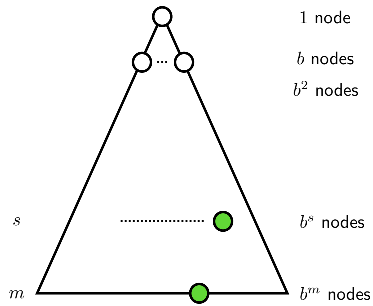
- Guaranteed to find a solution (if exists)? Complete?
- Guaranteed to find the least cost path? Optimal?
- How many steps - an operation with a node? Time complexity?
- How many nodes to remember? Space/Memory complexity?

How many nodes in a (search) tree? What are tree parameters?

Notes

Draw a (symbolic–think about a triangle) sketch of a (search) tree. It may grow upwards or downwards. How would you characterize/parametrize *size* of a tree.

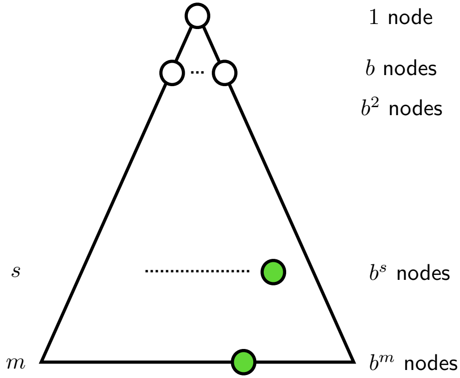
- Depth of the tree d.
- Max-Depth of the tree m. Can be ∞.
- Branching factor b.
- s denotes the shallowest Goal.
- How many nodes in the whole tree?



Strategies

How to traverse/build a search tree?

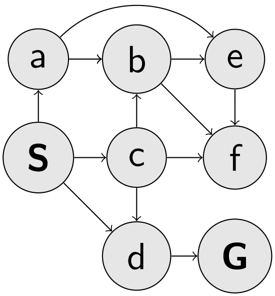
- Depth of the tree d.
- Max-Depth of the tree m. Can be ∞.
- Branching factor b.
- s denotes the shallowest Goal .
- How many nodes in the whole tree?

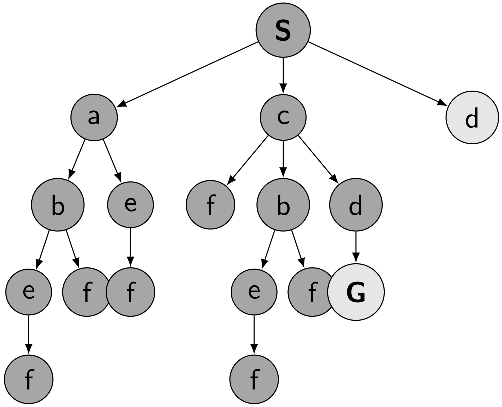


Notes

It is perhaps worth to remember that the search tree is built as the algorithm goes. Or better said, the tree is a human friendly representation of the machine run.

Depth-First Search (DFS)





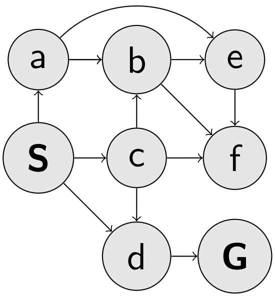
What are the DFS properties (complete, optimal, time, space)?

Notes

- In animation, we will do the expansion step at once.
- Expanded (explored) nodes become darker gray.
- *frontier* - set of nodes are light gray.
- When to stop the search?
- Thinking about optimality, what is the best solution we seek?

DFS properties

- Time complexity?
- Space complexity?
- Complete?
- Optimal?

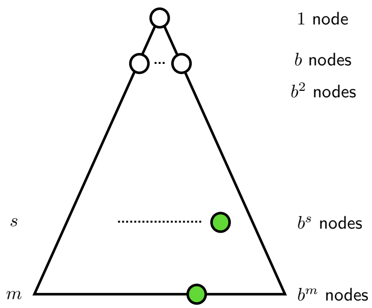


b, m, s, Time complexity?

A $\mathcal{O}(bm)$
B $\mathcal{O}(b^m)$
C $\mathcal{O}(m^b)$
D ∞

Notes

- Time, can process the whole tree: b^m
- Space, only the path so far: bm (a path from root to leaf (m), plus siblings on the path are also on the frontier ($b \times m$))
- Completness: m may be ∞ hence, not in general
- Optimality: No! It just takes the first solution found.



DFS properties

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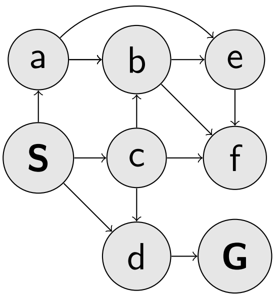

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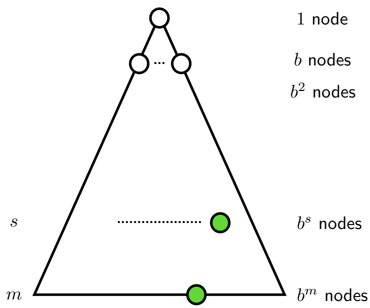
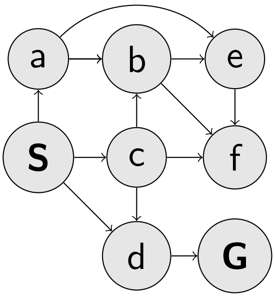
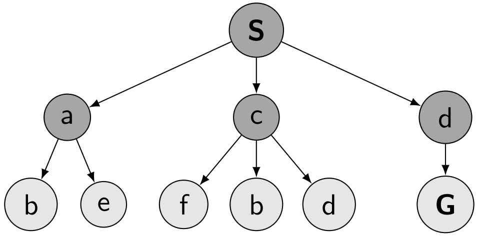


Breadth-First Search (BFS)



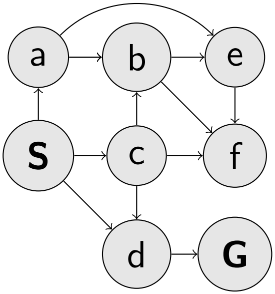


What are the BFS properties?

Notes

BFS properties

- Time complexity?
- Space complexity?
- Complete?
- Optimal?

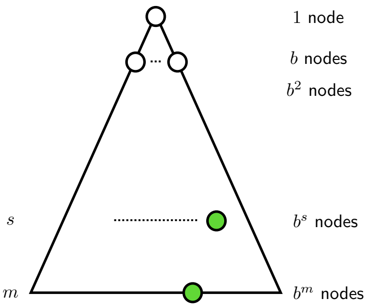


b, m, s, Time complexity?

A $\mathcal{O}(bm)$
B $\mathcal{O}(b^m)$
C $\mathcal{O}(m^b)$
D $\mathcal{O}(b^s)$

Notes

- Time, can process the whole tree until s: b^s, well actually $b + b^2 + b^3 + \cdots + b^s$ but the last layer vastly dominates. Try some calculations for various b.
- Space, all the frontier: b^s
- Completness: Yes!
- Optimality, it does not miss the shallowest solution, hence if all the transition costs are 1: Yes!



BFS properties

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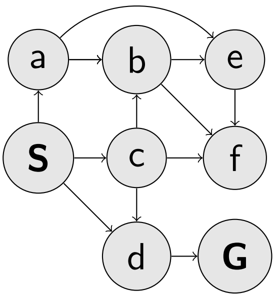

b, m, s, Space complexity?

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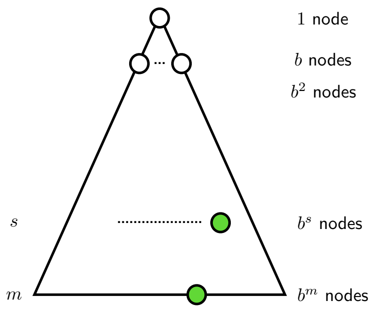
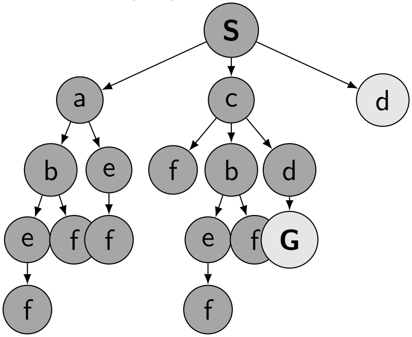
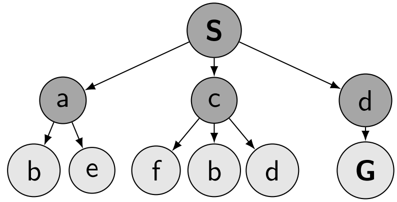


DFS vs BFS

What are (dis)advantages of the individual strategies?





Notes

What is the impression from the animation? BFS seems better.

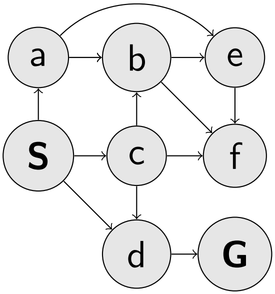
- However, let's not jump to conclusions!
- Draw for yourself a different graph and contruct appropriate trees.
- Not everything is visible from the animations.
- Draw a comparison table.

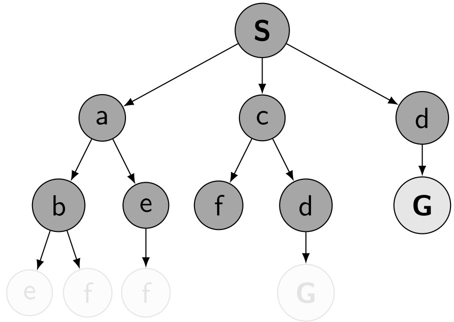
	Complete	Optimal	Time	Space
DFS	N (Y if no cycles)	N	$O(b^m)$	$O(mb)$
BFS	Y	Y	$O(b^m)$	$O(b^m)$

- Exponential complexity is scary.
- Practically, space complexity is even more critical. It is not about "waiting longer" but "memory overflow"... (and freezing the machine)
- This motivates the algorithm modification we look at next.

DFS with limited depth, `maxdepth=2`

Do not follow nodes with `depth` > `maxdepth`





Notes

- Remedy to DFS failing in infinite state spaces.
- Supplying a predetermined max depth – nodes at this depth are treated as if they had no successors.
- However, an additional source of algorithm *incompleteness*. Solution can obviously be deeper than *maxdepth* – unless we know something about the problem. Think about our map of Romania. There are 20 cities. Hence, $maxdepth = 19$ is a possible choice. Taking a closer look, any city can be reached from any other city in max. 9 steps (*state space diameter*), giving a more strict and hence better limit.

Iterative deepening DFS (ID-DFS)

- Start with `maxdepth = 1`
- Perform DFS with limited depth. Report success or failure.
- If failure, forget everything, increase `maxdepth` and repeat DFS

Is it not a terrible waste to forget everything between steps?

Notes

Really, how much do we repeat/waste? The "upper levels", close to the root, are repeated many times. However, in a tree, most nodes are the bottom levels and nr. nodes traversed is what counts. More specifically, for a solution at depth s, the nodes on the bottm level are generated only once, those on the next-to-bottom level 2x … children of the root are generated $s\times$. Compare the number of nodes generated ID-DFS vs. BFS:

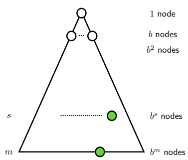
$$N(\text{ID-DFS}) = (s)b + (s-1)b^2 + (s-2)b^3 + \cdots + (1)b^s$$

$$N(\text{BFS}) = b + b^2 + b^3 + \cdots + b^s$$

Try some calculations for various s and b. For $b = 10$ and $d = 5$:

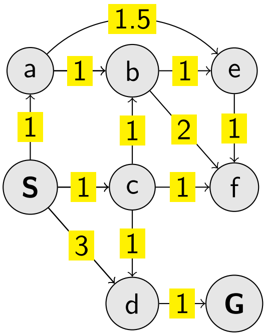
$$N(\text{ID-DFS}) = 50 + 400 + 3000 + 20000 + 100000 = 123450$$

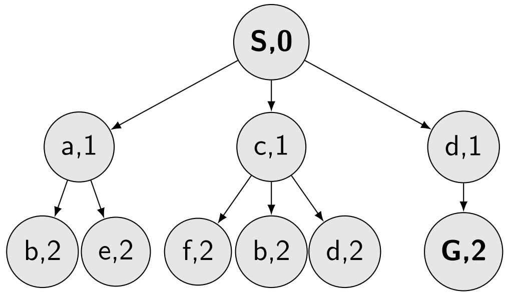
$$N(\text{BFS}) = 10 + 100 + 1000 + 10000 + 100000 = 111110$$



(Example from [2].)

Cost sensitive search

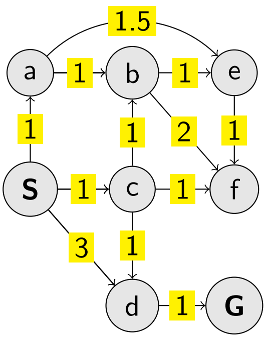


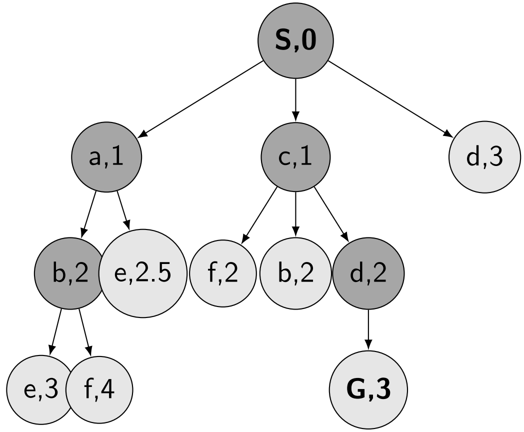


- In BFS, DFS, node $\pm$depth was the node-value.
- How was the depth actually computed?
- How to evaluate nodes with path cost?

Notes

Uniform Cost Search (UCS)





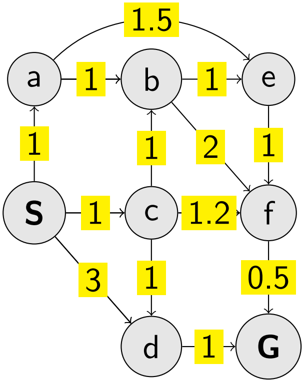
When to check the goal (and stop) the search? When visiting or expanding the node?

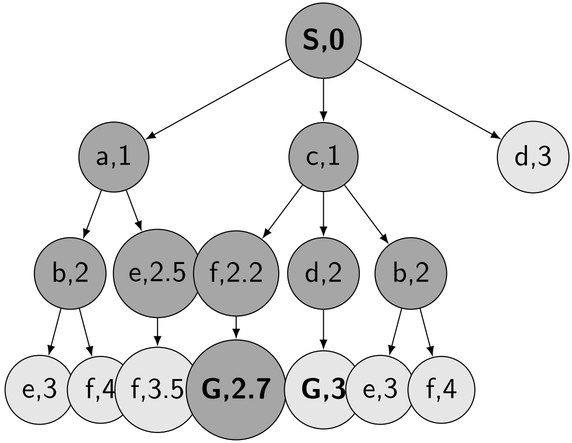
Notes

Simple extension of BFS. Instead of expanding shallowest node, the node with smallest path cost so far is expanded.
Two differences:

- Goal test applied to a node when *selected for expansion* – not when first generated. (First goal generated may be on a suboptimal path.)
- Test is added in case a better path is found to a node currently on the frontier.

When to stop, when visiting or expanding?

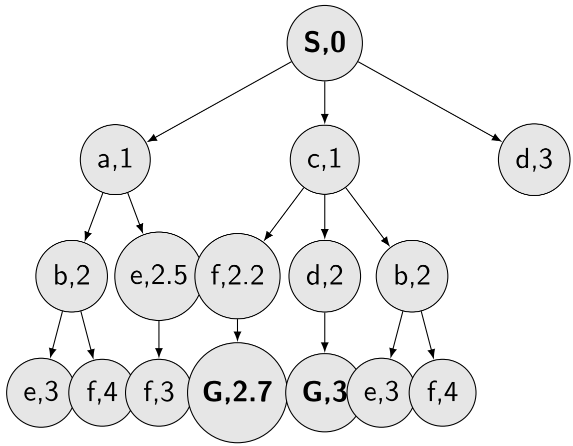




Notes

UCS properties

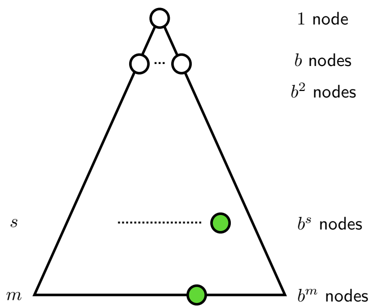
- Time complexity?
- Space complexity?
- Complete?
- Optimal?



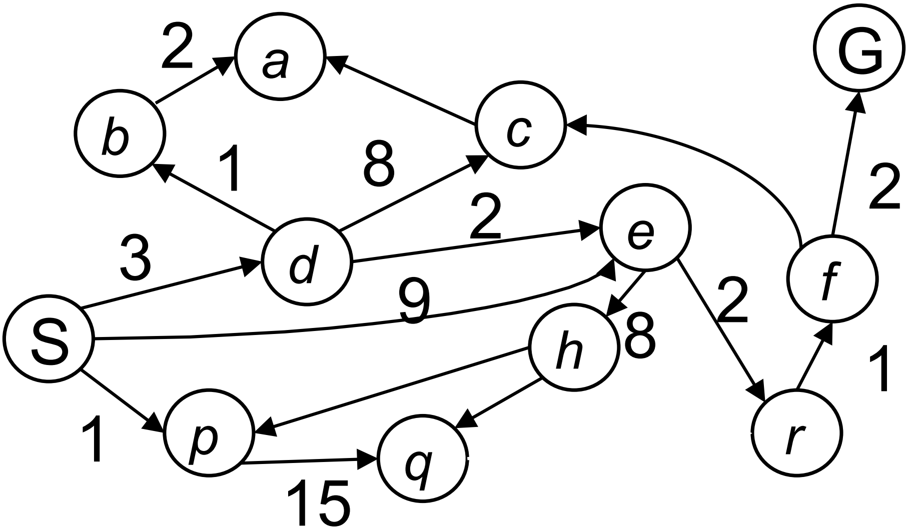
Notes

Solution cost C^*, transition cost at least ϵ. Effective depth, roughly C^*/ϵ.

- Time: $b^{C^*/\epsilon}$
- Space: $b^{C^*/\epsilon}$
- Completness: Yes!
- Optimality: Yes! Why?



Example: Graph with costs



Notes

Try it on paper, mark which nodes are in frontier, mark lines of equal cost.

Infrastructure for (tree) search algorithms

What should a tree node `n` know?

- `n.state`
- `n.parent`
- `n.pathcost`

Perhaps we may add something later, if needed . . .

Notes

How to organize nodes?

The Python examples are just suggestions, . . .

- A dynamically linked structure (`list()`).
- Add a node (`list.insert(node)`).
- Take a node and remove from the structure (`node=list.pop()`).
- Check the Python modules `heapq`[1] and `queue`[2] for inspiration.

[1] https://docs.python.org/3.5/library/heapq.html
[2] https://docs.python.org/3.5/library/queue.html

Notes

Very likely, you discussed `heapq` and `queue` in some programming, algorithms or data structures related courses.

What is the solution?

- We stop when Goal is reached.
- How do we construct the path?

Notes

References, further reading

Some figures if from [2]. Chapter 2 in [1] provides a compact/dense intro into search algorithms.

[1] Steven M. LaValle.
Planning Algorithms.
Cambridge, 1st edition, 2006.
Online version available at: http://planning.cs.uiuc.edu.

[2] Stuart Russell and Peter Norvig.
Artificial Intelligence: A Modern Approach.
Prentice Hall, 3rd edition, 2010.
http://aima.cs.berkeley.edu/.

Notes