# Tutorial on LP-Based Heuristics

Daniel Fišer

Department of Computer Science,
Faculty of Electrical Engineering,
*danfis@danfis.cz*

April 8, 2020

# LP-Based Heuristics

Before you proceed with this tutorial, listed to the corresponding lecture and read Section 5 of the tutorial notes on classical planning
https://cw.fel.cvut.cz/wiki/_media/courses/be4m36pui/notes-cp.pdf

# Example Planning Task

In this tutorial, we use the following example FDR planning task $P = \langle \mathcal{V}, \mathcal{O}, s_{\text{init}}, s_{\text{goal}}, \mathrm{c} \rangle$:

$\mathcal{V} = \{A, B, C\}$,

$D_A = \{D, E\}$, $D_B = \{F, G\}$, $D_C = \{H, J, K\}$,

$s_{\text{init}} = \{A = D, B = F, C = H\}$, $s_{\text{goal}} = \{A = D, C = K\}$

$\mathcal{O} = \{o_1, o_2, o_3, o_4, o_5\}$,

| | pre | eff | c |
|---|---|---|---|
| $o_1$ | $\{A = D, C = H\}$ | $\{A = E, C = J\}$ | 2 |
| $o_2$ | $\{A = D\}$ | $\{B = G\}$ | 1 |
| $o_3$ | $\{B = G, C = J\}$ | $\{C = K\}$ | 1 |
| $o_4$ | $\{A = E\}$ | $\{A = D\}$ | 2 |
| $o_5$ | $\{C = H\}$ | $\{C = J\}$ | 5 |

# Flow Heuristic

First we construct the LP constraints for the flow heuristic.

- Each LP variable $x_o$ correspond to the operator $o \in \mathcal{O}$.
- And each variable $x_o$ counts how many times the operator $o$ appear in the optimal plan.
- Recall from the tutorial notes, that for a given variable $V$ and its value $v$, $\mathrm{prod}(\langle V, v\rangle)$ denotes a set of operators "producing" $\langle V, v\rangle$, i.e., a set of operators that set $\langle V, v\rangle$ when applied.
- Furthermore recall, that for a given variable $V$ and its value $v$, $\mathrm{cons}(\langle V, v\rangle)$ denotes a set of operators "consuming" $\langle V, v\rangle$, i.e., a set of operators that un-set $\langle V, v\rangle$ when applied.

# Flow Heuristic

So to get back to our example planning task.
We want to minimize $2x_{o_1} + x_{o_2} + x_{o_3} + 2x_{o_4} + 5x_{o_5}$
subject to
$LB_{A,D} \leq x_{o_4} - x_{o_1}$
$LB_{A,E} \leq x_{o_1} - x_{o_4}$
$LB_{B,F} \leq 0$
$LB_{B,G} \leq x_{o_2}$
$LB_{C,H} \leq -x_{o_1} - x_{o_5}$
$LB_{C,J} \leq x_{o_1} + x_{o_5} - x_{o_3}$
$LB_{C,K} \leq x_{o_3}$
where $LB_{V,v}$ is a constant for each state for which we want to compute the heuristic estimate.

# Flow Heuristic

Here is how we find out which constant to use if we compute the heuristic for the state $s$:

$$LB_{V,v} = \begin{cases} 0 & \text{if } V \in \mathrm{vars}(s_{\mathrm{goal}}) \text{ and } s_{\mathrm{goal}}[V] = v \text{ and } s[V] = v, \\ 1 & \text{if } V \in \mathrm{vars}(s_{\mathrm{goal}}) \text{ and } s_{\mathrm{goal}}[V] = v \text{ and } s[V] \neq v, \\ -1 & \text{if } (V \notin \mathrm{vars}(s_{\mathrm{goal}}) \text{ or } s_{\mathrm{goal}}[V] \neq v) \text{ and } s[V] = v, \\ 0 & \text{if } (V \notin \mathrm{vars}(s_{\mathrm{goal}}) \text{ or } s_{\mathrm{goal}}[V] \neq v) \text{ and } s[V] \neq v, \end{cases}$$

# Flow Heuristic

If $V = v$ is set in goal states and also in $s$, then $\langle V, v \rangle$ cannot be consumed more times than it is produced to reach a goal state.

$$LB_{V,v} = \begin{cases} 0 & \text{if } V \in \text{vars}(s_{\text{goal}}) \text{ and } s_{\text{goal}}[V] = v \text{ and } s[V] = v, \\ 1 & \text{if } V \in \text{vars}(s_{\text{goal}}) \text{ and } s_{\text{goal}}[V] = v \text{ and } s[V] \neq v, \\ -1 & \text{if } (V \notin \text{vars}(s_{\text{goal}}) \text{ or } s_{\text{goal}}[V] \neq v) \text{ and } s[V] = v, \\ 0 & \text{if } (V \notin \text{vars}(s_{\text{goal}}) \text{ or } s_{\text{goal}}[V] \neq v) \text{ and } s[V] \neq v, \end{cases}$$

# Flow Heuristic

If $V = v$ is set in goal states, but not in $s$, then $\langle V, v \rangle$ must be produced at least once to reach a goal state.

$$LB_{V,v} = \begin{cases} 0 & \text{if } V \in \text{vars}(s_{\text{goal}}) \text{ and } s_{\text{goal}}[V] = v \text{ and } s[V] = v, \\ 1 & \text{if } V \in \text{vars}(s_{\text{goal}}) \text{ and } s_{\text{goal}}[V] = v \text{ and } s[V] \neq v, \\ -1 & \text{if } (V \notin \text{vars}(s_{\text{goal}}) \text{ or } s_{\text{goal}}[V] \neq v) \text{ and } s[V] = v, \\ 0 & \text{if } (V \notin \text{vars}(s_{\text{goal}}) \text{ or } s_{\text{goal}}[V] \neq v) \text{ and } s[V] \neq v, \end{cases}$$

# Flow Heuristic

If $V = v$ is not necessarily set in goal states, but it is set in $s$, then we don't know how many times should be $\langle V, v \rangle$ consumed or produced. So, we set the lower bound to $-1$.

$$LB_{V,v} = \begin{cases} 0 & \text{if } V \in \mathrm{vars}(s_{\mathrm{goal}}) \text{ and } s_{\mathrm{goal}}[V] = v \text{ and } s[V] = v, \\ 1 & \text{if } V \in \mathrm{vars}(s_{\mathrm{goal}}) \text{ and } s_{\mathrm{goal}}[V] = v \text{ and } s[V] \neq v, \\ -1 & \text{if } (V \notin \mathrm{vars}(s_{\mathrm{goal}}) \text{ or } s_{\mathrm{goal}}[V] \neq v) \text{ and } s[V] = v, \\ 0 & \text{if } (V \notin \mathrm{vars}(s_{\mathrm{goal}}) \text{ or } s_{\mathrm{goal}}[V] \neq v) \text{ and } s[V] \neq v, \end{cases}$$

# Flow Heuristic

If $V = v$ is not necessarily set in goal states, and it is not set in $s$, then we can tighten the lower bound a little bit. In contrast to the previous case, $\langle V, v\rangle$ cannot be consumed before it is produced. So if we produce $\langle V, v\rangle$ $(+1)$, then we can also consume $\langle V, v\rangle$ $(-1)$, which gives us the lower bound $0$.

$$LB_{V,v} = \begin{cases} 0 & \text{if } V \in \text{vars}(s_{\text{goal}}) \text{ and } s_{\text{goal}}[V] = v \text{ and } s[V] = v, \\ 1 & \text{if } V \in \text{vars}(s_{\text{goal}}) \text{ and } s_{\text{goal}}[V] = v \text{ and } s[V] \neq v, \\ -1 & \text{if } (V \notin \text{vars}(s_{\text{goal}}) \text{ or } s_{\text{goal}}[V] \neq v) \text{ and } s[V] = v, \\ 0 & \text{if } (V \notin \text{vars}(s_{\text{goal}}) \text{ or } s_{\text{goal}}[V] \neq v) \text{ and } s[V] \neq v, \end{cases}$$

# Flow Heuristic

So, suppose we want to compute the heuristic estimate for the initial state $s_{\text{init}} = \{A = D, B = F, C = H\}$ (and recall the goal $s_{\text{goal}} = \{A = D, C = K\}$).

# Flow Heuristic

So, suppose we want to compute the heuristic estimate for the initial state $s_{\text{init}} = \{A = D, B = F, C = H\}$ (and recall the goal $s_{\text{goal}} = \{A = D, C = K\}$).
We get the following lower bounds:
$LB_{A,D} = 0$
$LB_{A,E} = 0$
$LB_{B,F} = -1$
$LB_{B,G} = 0$
$LB_{C,H} = -1$
$LB_{C,J} = 0$
$LB_{C,K} = 1$

# Flow Heuristic

So, suppose we want to compute the heuristic estimate for the initial state $s_{\text{init}} = \{A = D, B = F, C = H\}$ (and recall the goal $s_{\text{goal}} = \{A = D, C = K\}$).
Which leads to the following linear program:
minimize $2x_{o_1} + x_{o_2} + x_{o_3} + 2x_{o_4} + 5x_{o_5}$
subject to

$$
\begin{aligned}
LB_{A,D} &= 0 \leq x_{o_4} - x_{o_1} \\
LB_{A,E} &= 0 \leq x_{o_1} - x_{o_4} \\
LB_{B,F} &= -1 \leq 0 \\
LB_{B,G} &= 0 \leq x_{o_2} \\
LB_{C,H} &= -1 \leq -x_{o_1} - x_{o_5} \\
LB_{C,J} &= 0 \leq x_{o_1} + x_{o_5} - x_{o_3} \\
LB_{C,K} &= 1 \leq x_{o_3}
\end{aligned}
$$

# Potential Heuristics

Now we construct the linear program for the potential heuristic optimized for the initial state.

- In this case, each LP variable $P_{V,v}$ correspond to the fact $\langle V, v\rangle$, and each LP variable $M_V$ correspond to the variable $V$.
- The value of $P_{V,v}$ is the potential corresponding to the fact $\langle V, v\rangle$.
- $M_V$ is the upper bound on the potentials from the variable $V$.
- We use $M_V$ in situations where we don't know which value is set (in the goal or in a precondition of operator) so we prepare for the worst case.
- For example, take our goal specification $s_{\text{goal}} = \{A = D, C = K\}$: We don't know how the variable $B$ is set in goal states.
  So to ensure goal-awareness of the heuristic we could construct (in this case) two constraints: $P_{A,D} + P_{B,F} + P_{C,K} \leq 0$ and $P_{A,D} + P_{B,G} + P_{C,K} \leq 0$, which will cover all cases.

# Potential Heuristics

Now we construct the linear program for the potential heuristic optimized for the initial state.

- In this case, each LP variable $P_{V,v}$ correspond to the fact $\langle V, v\rangle$, and each LP variable $M_V$ correspond to the variable $V$.
- The value of $P_{V,v}$ is the potential corresponding to the fact $\langle V, v\rangle$.
- $M_V$ is the upper bound on the potentials from the variable $V$.
- We use $M_V$ in situations where we don't know which value is set (in the goal or in a precondition of operator) so we prepare for the worst case.
- For example, take our goal specification $s_{\text{goal}} = \{A = D, C = K\}$: We don't know how the variable $B$ is set in goal states.
  Or we can construct the constraint $P_{A,D} + M_B + P_{C,K} \leq 0$ and make sure that $M_B$ will be the maximum over $P_{B,F}$ and $P_{B,G}$ by adding auxiliary constraints $P_{B,F} \leq M_B$ and $P_{B,G} \leq M_B$.

# Potential Heuristics

Now we construct the linear program for the potential heuristic optimized for the initial state.

- In this case, each LP variable $P_{V,v}$ correspond to the fact $\langle V, v \rangle$, and each LP variable $M_V$ correspond to the variable $V$.
- The value of $P_{V,v}$ is the potential corresponding to the fact $\langle V, v \rangle$.
- $M_V$ is the upper bound on the potentials from the variable $V$.
- We use $M_V$ in situations where we don't know which value is set (in the goal or in a precondition of operator) so we prepare for the worst case.
- For example, take our goal specification $s_{\text{goal}} = \{A = D, C = K\}$: We don't know how the variable $B$ is set in goal states.
  Or we can construct the constraint $P_{A,D} + M_B + P_{C,K} \leq 0$ and make sure that $M_B$ will be the maximum over $P_{B,F}$ and $P_{B,G}$ by adding auxiliary constraints $P_{B,F} \leq M_B$ and $P_{B,G} \leq M_B$. It should be clear that the first approach can (in the worst case) generate an exponential number of constraints, whereas the second approach will always generate only a linear number constraints.

# Potential Heuristics

So, for our planning example, the linear program for the potential heuristic will maximize $P_{A,D} + P_{B,F} + P_{C,H}$
subject to

$$P_{B,F} \leq M_B$$
$$P_{B,G} \leq M_B$$
$$P_{A,D} + M_B + P_{C,K} \leq 0$$
$$P_{A,D} + P_{C,H} - P_{A,E} - P_{C,J} \leq 2$$
$$M_B - P_{B,G} \leq 1$$
$$P_{C,J} - P_{C,K} \leq 1$$
$$P_{A,E} - P_{A,D} \leq 2$$
$$P_{C,H} - P_{C,J} \leq 5$$

# Potential Heuristics

So, for our planning example, the linear program for the potential heuristic will maximize $P_{A,D} + P_{B,F} + P_{C,H}$
subject to

$P_{B,F} \leq M_B$
$P_{B,G} \leq M_B$
$P_{A,D} + M_B + P_{C,K} \leq 0$
$P_{A,D} + P_{C,H} - P_{A,E} - P_{C,J} \leq 2$
$M_B - P_{B,G} \leq 1$
$P_{C,J} - P_{C,K} \leq 1$
$P_{A,E} - P_{A,D} \leq 2$
$P_{C,H} - P_{C,J} \leq 5$

First, note that we constructed constraints only for $M_B$, but not for $M_A$ or $M_C$. The reason is that $M_A$ or $M_C$ is not used anywhere so we can simplify the linear program a little bit.

# Potential Heuristics

So, for our planning example, the linear program for the potential heuristic will maximize $P_{A,D} + P_{B,F} + P_{C,H}$
subject to
$P_{B,F} \leq M_B$
$P_{B,G} \leq M_B$
$P_{A,D} + M_B + P_{C,K} \leq 0$
$P_{A,D} + P_{C,H} - P_{A,E} - P_{C,J} \leq 2$
$M_B - P_{B,G} \leq 1$
$P_{C,J} - P_{C,K} \leq 1$
$P_{A,E} - P_{A,D} \leq 2$
$P_{C,H} - P_{C,J} \leq 5$
When we have computed the potentials $P_{V,v}$, we compute the heuristic estimate for a state $s$ as $\sum_{V \in \mathcal{V}} P_{V,v}$.

# Potential Heuristics

So, for our planning example, the linear program for the potential heuristic will maximize $P_{A,D} + P_{B,F} + P_{C,H}$
subject to
$P_{B,F} \leq M_B$
$P_{B,G} \leq M_B$
$P_{A,D} + M_B + P_{C,K} \leq 0$
$P_{A,D} + P_{C,H} - P_{A,E} - P_{C,J} \leq 2$
$M_B - P_{B,G} \leq 1$
$P_{C,J} - P_{C,K} \leq 1$
$P_{A,E} - P_{A,D} \leq 2$
$P_{C,H} - P_{C,J} \leq 5$
So, for example, for the initial state $s_{\text{init}} = \{A = D, B = F, C = H\}$, the heuristic value will be $P_{A,D} + P_{B,F} + P_{C,H}$.

# Potential Heuristics

So, for our planning example, the linear program for the potential heuristic will maximize $P_{A,D} + P_{B,F} + P_{C,H}$
subject to
$P_{B,F} \leq M_B$
$P_{B,G} \leq M_B$
$P_{A,D} + M_B + P_{C,K} \leq 0$
$P_{A,D} + P_{C,H} - P_{A,E} - P_{C,J} \leq 2$
$M_B - P_{B,G} \leq 1$
$P_{C,J} - P_{C,K} \leq 1$
$P_{A,E} - P_{A,D} \leq 2$
$P_{C,H} - P_{C,J} \leq 5$
We make sure that the sum of potentials is always admissible by making the sums goal-aware and consistent.

# Potential Heuristics

So, for our planning example, the linear program for the potential heuristic will maximize $P_{A,D} + P_{B,F} + P_{C,H}$
subject to

$P_{B,F} \leq M_B$
$P_{B,G} \leq M_B$
$P_{A,D} + M_B + P_{C,K} \leq 0$
$P_{A,D} + P_{C,H} - P_{A,E} - P_{C,J} \leq 2$
$M_B - P_{B,G} \leq 1$
$P_{C,J} - P_{C,K} \leq 1$
$P_{A,E} - P_{A,D} \leq 2$
$P_{C,H} - P_{C,J} \leq 5$

These constraints make sure that the sum will always be goal-aware.

# Potential Heuristics

So, for our planning example, the linear program for the potential heuristic will maximize $P_{A,D} + P_{B,F} + P_{C,H}$
subject to

$P_{B,F} \leq M_B$
$P_{B,G} \leq M_B$
$P_{A,D} + M_B + P_{C,K} \leq 0$
$P_{A,D} + P_{C,H} - P_{A,E} - P_{C,J} \leq 2$
$M_B - P_{B,G} \leq 1$
$P_{C,J} - P_{C,K} \leq 1$
$P_{A,E} - P_{A,D} \leq 2$
$P_{C,H} - P_{C,J} \leq 5$

This constraint make sure that the heuristic is consistent with respect to the operator $o_1$.
(Note that when $o_1$ is applied on a state $s$, we can express the heuristic estimate for $o_1[s]$ as $h(o_1[s]) = h(s) - P_{A,D} - P_{C,H} + P_{A,E} + P_{C,J}$.)

# Potential Heuristics

So, for our planning example, the linear program for the potential heuristic will maximize $P_{A,D} + P_{B,F} + P_{C,H}$
subject to
$P_{B,F} \leq M_B$
$P_{B,G} \leq M_B$
$P_{A,D} + M_B + P_{C,K} \leq 0$
$P_{A,D} + P_{C,H} - P_{A,E} - P_{C,J} \leq 2$
$M_B - P_{B,G} \leq 1$
$P_{C,J} - P_{C,K} \leq 1$
$P_{A,E} - P_{A,D} \leq 2$
$P_{C,H} - P_{C,J} \leq 5$
These constraints ensure consistency with respect to $o_2$.

# Potential Heuristics

So, for our planning example, the linear program for the potential heuristic will maximize $P_{A,D} + P_{B,F} + P_{C,H}$
subject to
$P_{B,F} \leq M_B$
$P_{B,G} \leq M_B$
$P_{A,D} + M_B + P_{C,K} \leq 0$
$P_{A,D} + P_{C,H} - P_{A,E} - P_{C,J} \leq 2$
$M_B - P_{B,G} \leq 1$
$P_{C,J} - P_{C,K} \leq 1$
$P_{A,E} - P_{A,D} \leq 2$
$P_{C,H} - P_{C,J} \leq 5$
And so on, for $o_3$, $o_4$, and $o_5$.

# Potential Heuristics

So, for our planning example, the linear program for the potential heuristic will maximize $P_{A,D} + P_{B,F} + P_{C,H}$
subject to
$P_{B,F} \leq M_B$
$P_{B,G} \leq M_B$
$P_{A,D} + M_B + P_{C,K} \leq 0$
$P_{A,D} + P_{C,H} - P_{A,E} - P_{C,J} \leq 2$
$M_B - P_{B,G} \leq 1$
$P_{C,J} - P_{C,K} \leq 1$
$P_{A,E} - P_{A,D} \leq 2$
$P_{C,H} - P_{C,J} \leq 5$
So the sum of potentials for every state will be an admissable estimate.