LM-Cut Heuristic

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Before you proceed with this tutorial, read Section 3 of the tutorial notes on classical planning
https://cw.fel.cvut.cz/wiki/_media/courses/be4m36pui/notes-cp.pdf

Disjunctive Operator Landmarks

Disjunctive Operator Landmark

A **disjunctive operator landmark** $L \subseteq \mathcal{O}$ is a set of operators such that every plan contains at least one operator from L.

Suppose we have a problem with two plans:

1. $\pi_1 = (o_1, o_2, o_3, o_4)$
2. $\pi_2 = (o_1, o_5, o_2, o_6)$

Which of the following sets are disjunctive operator landmarks?

1. $\{o_1\}$
2. $\{o_1, o_3\}$
3. $\{o_2, o_3\}$
4. $\{o_1, o_2, o_3, o_4\}$
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6. $\{o_4, o_6\}$

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3. $\{o_2, o_3\}$ ✓
4. $\{o_1, o_2, o_3, o_4\}$ ✓
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5. $\{o_3, o_4\}$ ✗
6. $\{o_4, o_6\}$

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6. $\{o_4, o_6\}$ ✓

LM-Cut using Algorithm 2

Algorithm 2

- Before getting into the example on the next slide, read the definitions of **supporter**, **justification graph**, and **s-t-cut**.
- On line 5, Algorithm 2 constructs a planning tasks Π_1 that is equivalent to the input planning task Π except:
  - The goal is set to the singleton $\{G\}$ (G is a new fact) and the auxiliary zero-cost operator o_{goal} leads from the original goal to $\{G\}$.
  - The initial state is set to the singleton $\{I\}$ (I is a new fact) and the auxiliary zero-cost operator o_{init} leads from $\{I\}$ to the state s for which we want to compute the heuristic estimate.
  - This way, we can compute the heuristic estimate for the initial state $\{I\}$ consisting of a single fact and we can use $\{G\}$ as the goal consisting of a single fact.
  - In our examples, we will skip this step, because all examples will have a single fact in the goal and in the state for which we will compute the heuristic estimate.

Example 1

Example 1

Compute $h^{\text{lm-cut}}(s_{\text{init}})$ using Algorithm 2 for the planning task $\Pi = \langle \mathcal{F}, \mathcal{O}, s_{\text{init}}, s_{\text{goal}}, c \rangle$:

$\mathcal{F} = \{s, t, q_1, q_2, q_3\}$, $s_{\text{init}} = \{s\}$, $s_{\text{goal}} = \{t\}$

$\mathcal{O} =$

	pre	add	del	c
o_1	$\{s\}$	$\{q_1, q_2\}$	$\emptyset$	1
o_2	$\{s\}$	$\{q_1, q_3\}$	$\emptyset$	1
o_3	$\{s\}$	$\{q_2, q_3\}$	$\emptyset$	1
fin	$\{q_1, q_2, q_3\}$	$\{t\}$	$\emptyset$	0

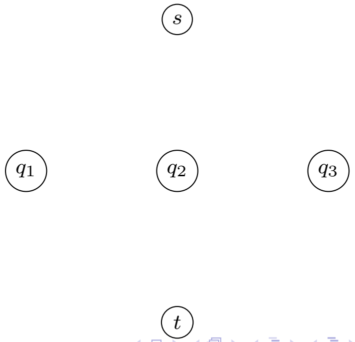
Example 1

For each operator, we will keep track of their costs and supporters.
For each fact, we will keep track of their Δ_1-values.
During the algorithm, we will maintain the justification graph where each node correponds to a fact.
And we keep track of the heuristic estimate (initialized to zero).

	pre	add	c	supp
o_1	$\{s\}$	$\{q_1, q_2\}$	1	?
o_2	$\{s\}$	$\{q_1, q_3\}$	1	?
o_3	$\{s\}$	$\{q_2, q_3\}$	1	?
fin	$\{q_1, q_2, q_3\}$	$\{t\}$	0	?

	s	q_1	q_2	q_3	t
Δ_1	?	?	?	?	?

$\mathsf{h}^{\text{lm-cut}}(s) = 0$



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	pre	add	c	supp
o_1	$\{s\}$	$\{q_1, q_2\}$	1	?
o_2	$\{s\}$	$\{q_1, q_3\}$	1	?
o_3	$\{s\}$	$\{q_2, q_3\}$	1	?
fin	$\{q_1, q_2, q_3\}$	$\{t\}$	0	?

	s	q_1	q_2	q_3	t
Δ_1	?	?	?	?	?

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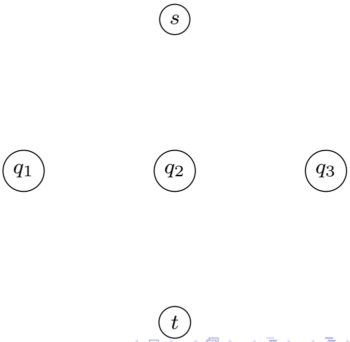

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	pre	add	c	supp
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o_2	$\{s\}$	$\{q_1, q_3\}$	1	?
o_3	$\{s\}$	$\{q_2, q_3\}$	1	?
fin	$\{q_1, q_2, q_3\}$	$\{t\}$	0	?

	s	q_1	q_2	q_3	t
Δ_1	?	?	?	?	?

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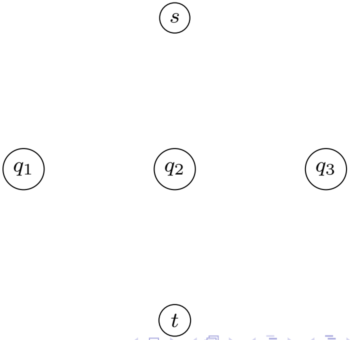

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For each fact, we will keep track of their Δ_1-values.

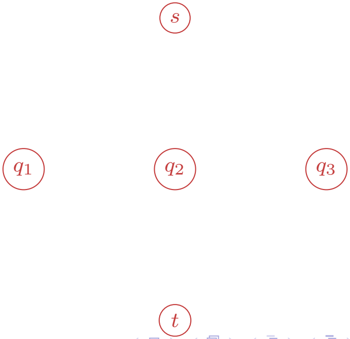
During the algorithm, we will maintain the justification graph where each node correponds to a fact.

And we keep track of the heuristic estimate (initialized to zero).

	pre	add	c	supp
o_1	$\{s\}$	$\{q_1, q_2\}$	1	?
o_2	$\{s\}$	$\{q_1, q_3\}$	1	?
o_3	$\{s\}$	$\{q_2, q_3\}$	1	?
fin	$\{q_1, q_2, q_3\}$	$\{t\}$	0	?

	s	q_1	q_2	q_3	t
Δ_1	?	?	?	?	?

$$\mathsf{h}^{\text{lm-cut}}(s) = 0$$



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During the algorithm, we will maintain the justification graph where each node correponds to a fact.
And we keep track of the heuristic estimate (initialized to zero).

	pre	add	c	supp
o_1	$\{s\}$	$\{q_1, q_2\}$	1	?
o_2	$\{s\}$	$\{q_1, q_3\}$	1	?
o_3	$\{s\}$	$\{q_2, q_3\}$	1	?
fin	$\{q_1, q_2, q_3\}$	$\{t\}$	0	?

	s	q_1	q_2	q_3	t
Δ_1	?	?	?	?	?

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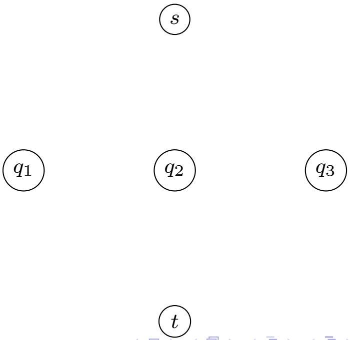


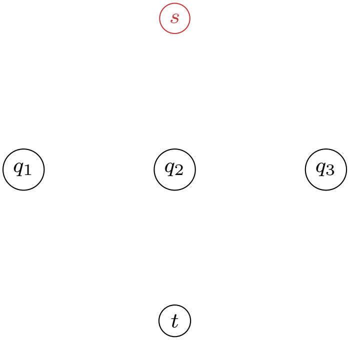
Example 1

Remember that we want to compute the heuristic estimate for the state that corresponds to the fact s and the goal consists of a single fact t.

	pre	add	c	supp
o_1	$\{s\}$	$\{q_1, q_2\}$	1	?
o_2	$\{s\}$	$\{q_1, q_3\}$	1	?
o_3	$\{s\}$	$\{q_2, q_3\}$	1	?
fin	$\{q_1, q_2, q_3\}$	$\{t\}$	0	?

	s	q_1	q_2	q_3	t
Δ_1	?	?	?	?	?

$\mathsf{h}^{\text{lm-cut}}(s) = 0$



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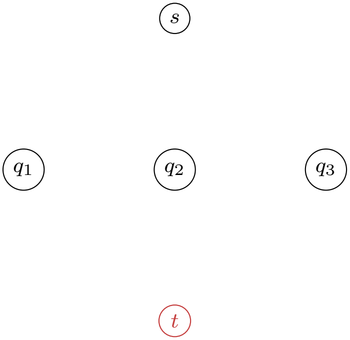
	pre	add	c	supp
o_1	$\{s\}$	$\{q_1, q_2\}$	1	?
o_2	$\{s\}$	$\{q_1, q_3\}$	1	?
o_3	$\{s\}$	$\{q_2, q_3\}$	1	?
fin	$\{q_1, q_2, q_3\}$	$\{t\}$	0	?

	s	q_1	q_2	q_3	t
Δ_1	?	?	?	?	?

$h^{\text{lm-cut}}(s) = 0$



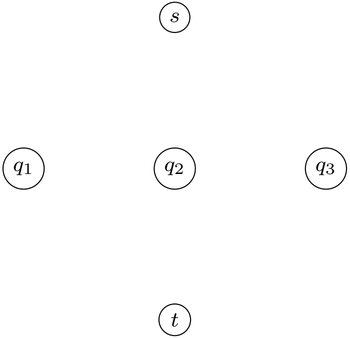
Example 1

The first step is to compute h^{max} for the state $\{s\}$ (test on line 7) and we keep Δ_1-values for the construction of justification graph (line 8).

	pre	add	c	supp
o_1	$\{s\}$	$\{q_1, q_2\}$	1	?
o_2	$\{s\}$	$\{q_1, q_3\}$	1	?
o_3	$\{s\}$	$\{q_2, q_3\}$	1	?
fin	$\{q_1, q_2, q_3\}$	$\{t\}$	0	?

	s	q_1	q_2	q_3	t
Δ_1	0	1	1	1	1

$h^{\text{lm-cut}}(s) = 0$



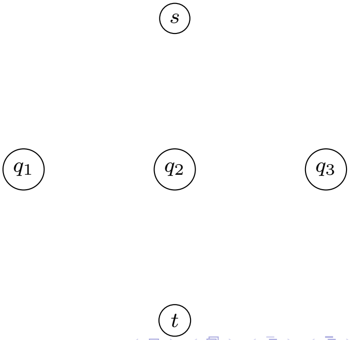
Example 1

Next, we need to find a supporter for each operator.
The supporter is the fact from the operator's precondition with the highest Δ_1-value, breaking ties arbitrarily, i.e., if there are two or more facts all with the same (highest) Δ_1-value, we can choose freely.
For the operator fin, we choose q_1.

	pre	add	c	supp
o_1	$\{s\}$	$\{q_1, q_2\}$	1	s
o_2	$\{s\}$	$\{q_1, q_3\}$	1	s
o_3	$\{s\}$	$\{q_2, q_3\}$	1	s
fin	$\{q_1, q_2, q_3\}$	$\{t\}$	0	q_1

	s	q_1	q_2	q_3	t
Δ_1	0	1	1	1	1

$h^{\text{lm-cut}}(s) = 0$



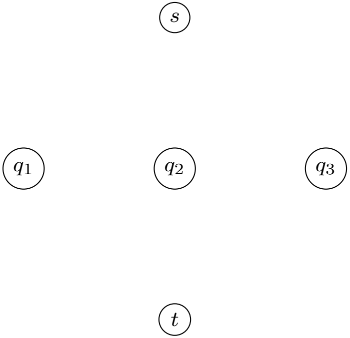
Example 1

Now we have found supporters, so we can construct the justification graph (line 8): For every operator and every fact from its add effect, we create an edge leading from the operator's supporter to that fact and we label the edge with the operator.

	pre	add	c	supp
o_1	$\{s\}$	$\{q_1, q_2\}$	1	s
o_2	$\{s\}$	$\{q_1, q_3\}$	1	s
o_3	$\{s\}$	$\{q_2, q_3\}$	1	s
fin	$\{q_1, q_2, q_3\}$	$\{t\}$	0	q_1

	s	q_1	q_2	q_3	t
Δ_1	0	1	1	1	1

$h^{\text{lm-cut}}(s) = 0$



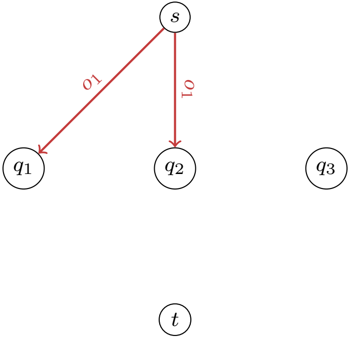
Example 1

For operator o_1, we have an edge from s to q_1 and another edge from s to q_2.

	pre	add	c	supp
o_1	$\{s\}$	$\{q_1, q_2\}$	1	s
o_2	$\{s\}$	$\{q_1, q_3\}$	1	s
o_3	$\{s\}$	$\{q_2, q_3\}$	1	s
fin	$\{q_1, q_2, q_3\}$	$\{t\}$	0	q_1

	s	q_1	q_2	q_3	t
Δ_1	0	1	1	1	1

$\mathsf{h}^{\text{lm-cut}}(s) = 0$



Example 1

For operator o_1, we have an edge from s to q_1 and another edge from s to q_2.
For operator o_2, we have an edge from s to q_1 and from s to q_3.

	pre	add	c	supp
o_1	$\{s\}$	$\{q_1, q_2\}$	1	s
o_2	$\{s\}$	$\{q_1, q_3\}$	1	s
o_3	$\{s\}$	$\{q_2, q_3\}$	1	s
fin	$\{q_1, q_2, q_3\}$	$\{t\}$	0	q_1

	s	q_1	q_2	q_3	t
Δ_1	0	1	1	1	1

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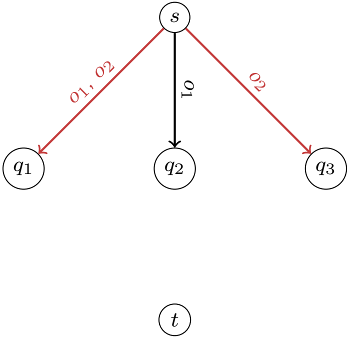

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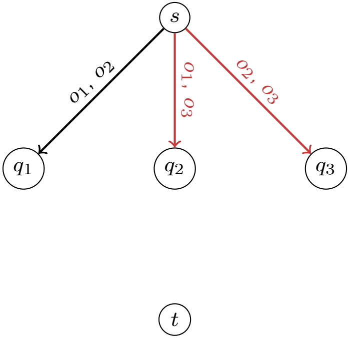
For operator o_1, we have an edge from s to q_1 and another edge from s to q_2.
For operator o_2, we have an edge from s to q_1 and from s to q_3.
For operator o_3, we have an edge from s to q_2 and from s to q_3.

	pre	add	c	supp
o_1	$\{s\}$	$\{q_1, q_2\}$	1	s
o_2	$\{s\}$	$\{q_1, q_3\}$	1	s
o_3	$\{s\}$	$\{q_2, q_3\}$	1	s
fin	$\{q_1, q_2, q_3\}$	$\{t\}$	0	q_1

	s	q_1	q_2	q_3	t
Δ_1	0	1	1	1	1

$h^{\text{lm-cut}}(s) = 0$



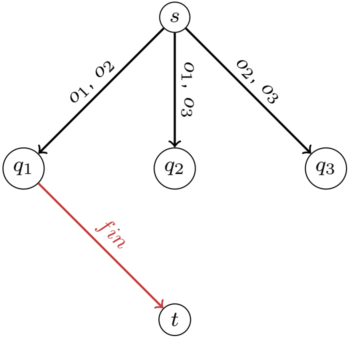
Example 1

For operator o_1, we have an edge from s to q_1 and another edge from s to q_2.
For operator o_2, we have an edge from s to q_1 and from s to q_3.
For operator o_3, we have an edge from s to q_2 and from s to q_3.
For operator fin, we have an edge from q_1 to t.

	pre	add	c	supp
o_1	$\{s\}$	$\{q_1, q_2\}$	1	s
o_2	$\{s\}$	$\{q_1, q_3\}$	1	s
o_3	$\{s\}$	$\{q_2, q_3\}$	1	s
fin	$\{q_1, q_2, q_3\}$	$\{t\}$	0	q_1

	s	q_1	q_2	q_3	t
Δ_1	0	1	1	1	1

$h^{\text{lm-cut}}(s) = 0$



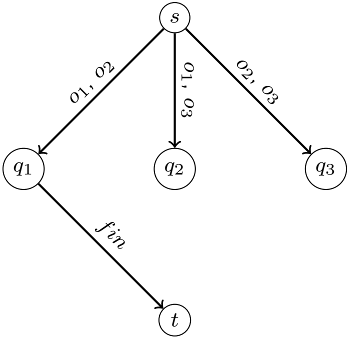
Example 1

Next, we find the s-t-cut (line 9), i.e., we need to find N^0, $N^\star$, and N^b.

	pre	add	c	supp
o_1	$\{s\}$	$\{q_1, q_2\}$	1	s
o_2	$\{s\}$	$\{q_1, q_3\}$	1	s
o_3	$\{s\}$	$\{q_2, q_3\}$	1	s
fin	$\{q_1, q_2, q_3\}$	$\{t\}$	0	q_1

	s	q_1	q_2	q_3	t
Δ_1	0	1	1	1	1

$\mathsf{h}^{\text{lm-cut}}(s) = 0$



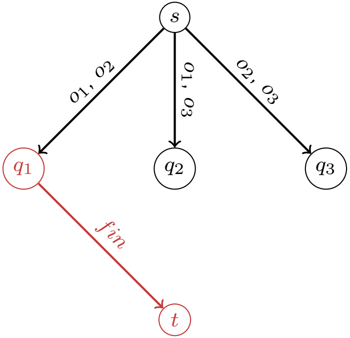
Example 1

$N^\star$ contains all nodes from which t is reachable with a zero-cost path.
Since only fin has a zero cost, $N^\star = \{q_1, t\}$.

	pre	add	c	supp
o_1	$\{s\}$	$\{q_1, q_2\}$	1	s
o_2	$\{s\}$	$\{q_1, q_3\}$	1	s
o_3	$\{s\}$	$\{q_2, q_3\}$	1	s
fin	$\{q_1, q_2, q_3\}$	$\{t\}$	0	q_1

	s	q_1	q_2	q_3	t
Δ_1	0	1	1	1	1

$h^{\text{lm-cut}}(s) = 0$



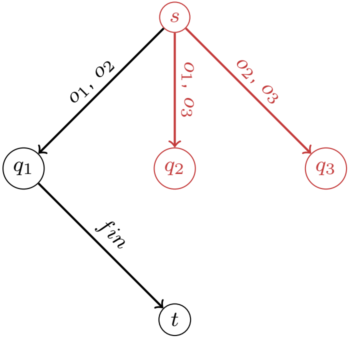
Example 1

$N^\star = \{q_1, t\}$
N^0 contains all nodes reachable from s without passing through any node from $N^\star$.
$N^0 = \{s, q_2, q_3\}$
N^b is empty in this case, but we don't care about it anyway.

	pre	add	c	supp
o_1	$\{s\}$	$\{q_1, q_2\}$	1	s
o_2	$\{s\}$	$\{q_1, q_3\}$	1	s
o_3	$\{s\}$	$\{q_2, q_3\}$	1	s
fin	$\{q_1, q_2, q_3\}$	$\{t\}$	0	q_1

	s	q_1	q_2	q_3	t
Δ_1	0	1	1	1	1

$h^{\text{lm-cut}}(s) = 0$

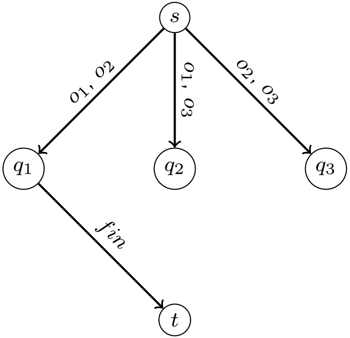


Now, we can determine our first landmark as labels of edges going from N^0 to $N^\star$ (line 11).

	pre	add	c	supp
o_1	$\{s\}$	$\{q_1, q_2\}$	1	s
o_2	$\{s\}$	$\{q_1, q_3\}$	1	s
o_3	$\{s\}$	$\{q_2, q_3\}$	1	s
fin	$\{q_1, q_2, q_3\}$	$\{t\}$	0	q_1

	s	q_1	q_2	q_3	t
Δ_1	0	1	1	1	1

$\mathsf{h}^{\text{lm-cut}}(s) = 0$



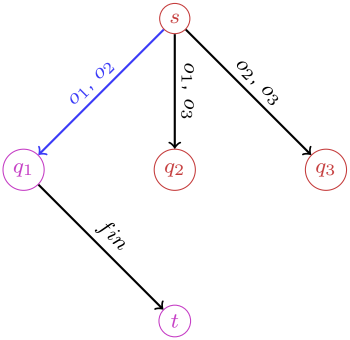
Example 1

Now, we can determine our first landmark as labels of edges going from N^0 to $N^\star$ (line 11).
$N^0 = \{s, q_2, q_3\}$, $N^\star = \{q_1, t\}$
Landmark $L = \{o_1, o_2\}$.

	pre	add	c	supp
o_1	$\{s\}$	$\{q_1, q_2\}$	1	s
o_2	$\{s\}$	$\{q_1, q_3\}$	1	s
o_3	$\{s\}$	$\{q_2, q_3\}$	1	s
fin	$\{q_1, q_2, q_3\}$	$\{t\}$	0	q_1

	s	q_1	q_2	q_3	t
Δ_1	0	1	1	1	1

$\mathsf{h}^{\text{lm-cut}}(s) = 0$



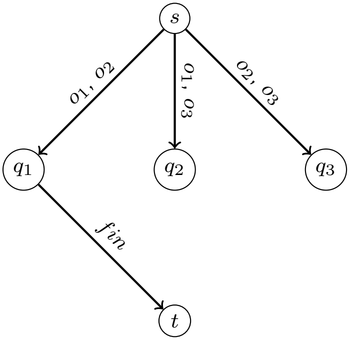
Example 1

Landmark $L = \{o_1, o_2\}$.
So, now we know that every plan for our problem must contain either o_1 or o_2.

	pre	add	c	supp
o_1	$\{s\}$	$\{q_1, q_2\}$	1	s
o_2	$\{s\}$	$\{q_1, q_3\}$	1	s
o_3	$\{s\}$	$\{q_2, q_3\}$	1	s
fin	$\{q_1, q_2, q_3\}$	$\{t\}$	0	q_1

	s	q_1	q_2	q_3	t
Δ_1	0	1	1	1	1

$h^{\text{lm-cut}}(s) = 0$



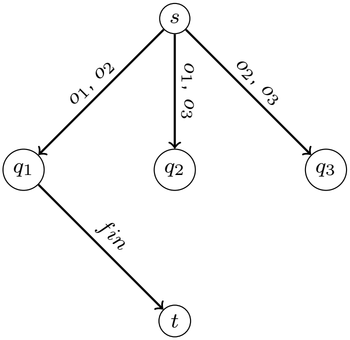
Example 1

The cost of the landmark $L = \{o_1, o_2\}$ is the minimum over the costs of operators from L, i.e., the cost is 1 (line 11).
And we can update the estimate to $\mathsf{h}^{\text{lm-cut}}(s) = 1$ (line 12).

	pre	add	c	supp
o_1	$\{s\}$	$\{q_1, q_2\}$	1	s
o_2	$\{s\}$	$\{q_1, q_3\}$	1	s
o_3	$\{s\}$	$\{q_2, q_3\}$	1	s
fin	$\{q_1, q_2, q_3\}$	$\{t\}$	0	q_1

	s	q_1	q_2	q_3	t
Δ_1	0	1	1	1	1

$\mathsf{h}^{\text{lm-cut}}(s) = 1$



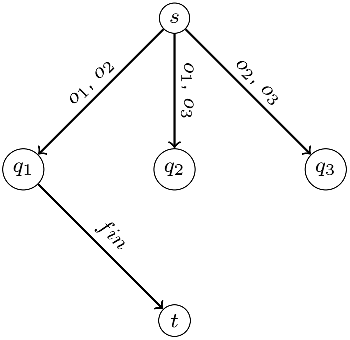
Example 1

As the last step of the cycle, we update the cost of the operators from $L = \{o_1, o_2\}$ by substracting the cost of the landmark.

	pre	add	c	supp
o_1	$\{s\}$	$\{q_1, q_2\}$	0	s
o_2	$\{s\}$	$\{q_1, q_3\}$	0	s
o_3	$\{s\}$	$\{q_2, q_3\}$	1	s
fin	$\{q_1, q_2, q_3\}$	$\{t\}$	0	q_1

	s	q_1	q_2	q_3	t
Δ_1	0	1	1	1	1

$$h^{\text{lm-cut}}(s) = 1$$



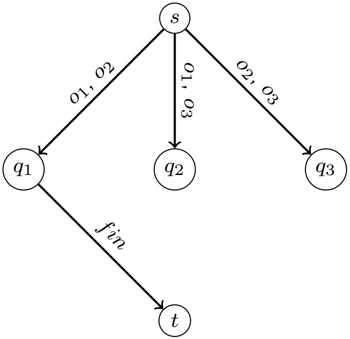
Example 1

And we repeat the cycle...

	pre	add	c	supp
o_1	$\{s\}$	$\{q_1, q_2\}$	0	s
o_2	$\{s\}$	$\{q_1, q_3\}$	0	s
o_3	$\{s\}$	$\{q_2, q_3\}$	1	s
fin	$\{q_1, q_2, q_3\}$	$\{t\}$	0	q_1

	s	q_1	q_2	q_3	t
Δ_1	0	1	1	1	1

$h^{\text{lm-cut}}(s) = 1$



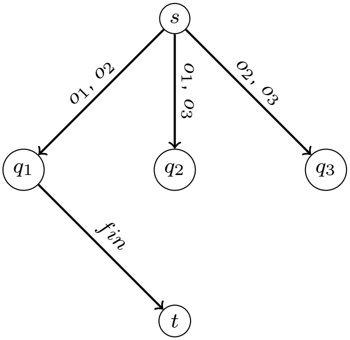
Example 1

We compute $\mathsf{h}^{\max}$ and keep the Δ_1-values (with the updated costs of operators). Since $\mathsf{h}^{\max}(s) = 0$, we terminate the algorithm and return the heuristic estimate $\mathsf{h}^{\text{lm-cut}}(s) = 1$.

	pre	add	c	supp
o_1	$\{s\}$	$\{q_1, q_2\}$	0	s
o_2	$\{s\}$	$\{q_1, q_3\}$	0	s
o_3	$\{s\}$	$\{q_2, q_3\}$	1	s
fin	$\{q_1, q_2, q_3\}$	$\{t\}$	0	q_1

	s	q_1	q_2	q_3	t
Δ_1	0	0	0	0	0

$\mathsf{h}^{\text{lm-cut}}(s) = 1$



Example 1

Now let's think about what we found out...

Before you proceed, try to work out yourself the following:

- Find optimal plans for this example planning task.
- Find optimal relaxed plans for this task.
- Compute $h^\star(s_{\text{init}})$ and $h^+(s_{\text{init}})$.
- Go back to the selection of supporters. We have selected q_1 as a supporter of the operator fin, but we could have selected q_2 or q_3. What would change if we selected q_2 or q_3 instead of q_1? Would we have a different heuristic estimate? How would the landmarks differ?

Example 1

The optimal plans for our example planning task are: (o_1, o_2, fin), (o_1, o_3, fin), (o_2, o_3, fin), (o_2, o_1, fin), ... And these plans are also optimal relaxed plans.
Therefore, $h^\star(s_{\text{init}}) = h^+(s_{\text{init}}) = 2$.

Tie-breaking in the selection of supporters

Changing the supporter of the operator fin does not change the outcome in terms of the heuristic estimate.
However, we obtain different disjunctive operator landmarks:

- With q_1 as the supporter of fin we got $L = \{o_1, o_2\}$,
- with q_2 we get $L = \{o_1, o_3\}$,
- and with q_3 we get $L = \{o_2, o_3\}$.

All all of these sets are, indeed, disjunctive operator landmarks. Therefore, a hitting set over all landmarks will tell us that every plan must contain at least two of the operators from $\{o_1, o_2, o_3\}$, yielding the perfect heuristic estimate 2.

Example 1

Tie-breaking in the selection of supporters

Changing the supporter of the operator fin does not change the outcome in terms of the heuristic estimate.

However, we obtain different disjunctive operator landmarks:

- With q_1 as the supporter of fin we got $L = \{o_1, o_2\}$,
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All all of these sets are, indeed, disjunctive operator landmarks. Therefore, a hitting set over all landmarks will tell us that every plan must contain at least two of the operators from $\{o_1, o_2, o_3\}$, yielding the perfect heuristic estimate 2.

Note however, that in order to get the perfect estimate, we needed to run Algorithm 2 for all possible selections of supporters (which can be exponential in the worst case) and then solve a hitting set problem which is an NP-Complete problem.

Note also, that LM-Cut is a relaxation heuristic so the best we can hope is to get perfect h$^+$.

Example 2

Compute $h^{\text{lm-cut}}(s_{\text{init}})$ using Algorithm 2 for the planning task $\Pi = \langle \mathcal{F}, \mathcal{O}, s_{\text{init}}, s_{\text{goal}}, c \rangle$:

$\mathcal{F} = \{a, b, c, d, e, i, g\}$

$s_{\text{init}} = \{i\}$, $s_{\text{goal}} = \{g\}$

$\mathcal{O} =$

	pre	add	del	c
o_1	$\{i\}$	$\{a, b\}$	$\emptyset$	2
o_2	$\{i\}$	$\{b, c\}$	$\emptyset$	3
o_3	$\{a, c\}$	$\{d\}$	$\{c\}$	1
o_4	$\{b, d\}$	$\{e\}$	$\{b\}$	3
o_5	$\{a, c, e\}$	$\{g\}$	$\{c, d\}$	1
o_6	$\{a\}$	$\{e\}$	$\{a, c\}$	5

(Note again, that both s_{init} and s_{goal} are singletons, so we skip line 5 of Algorithm 2.)

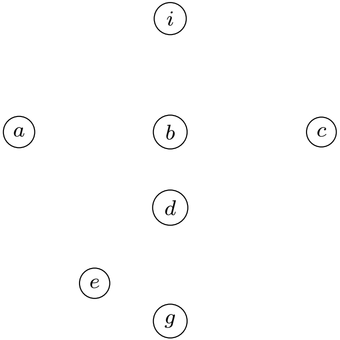
Example 2

We, again, keep track of operators' costs and supporters, Δ_1-values of facts, the $\mathsf{h}^{\text{lm-cut}}$ estimate, and the justification graph.
The fact i corresponds to the state for which we want to compute the estimate, and the fact g corresponds to the goal.

	pre	add	c	supp
o_1	$\{i\}$	$\{a, b\}$	2	?
o_2	$\{i\}$	$\{b, c\}$	3	?
o_3	$\{a, c\}$	$\{d\}$	1	?
o_4	$\{b, d\}$	$\{e\}$	3	?
o_5	$\{a, c, e\}$	$\{g\}$	1	?
o_6	$\{a\}$	$\{e\}$	5	?

	i	a	b	c	d	e	g
Δ_1	?	?	?	?	?	?	?

$\mathsf{h}^{\text{lm-cut}}(s) = 0$



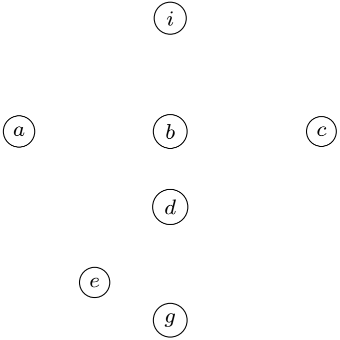
Example 2

Compute $h^{\max}(\{i\})$ and keep all Δ_1-values.
Compute supporters of operators.

	pre	add	c	supp
o_1	$\{i\}$	$\{a, b\}$	2	i
o_2	$\{i\}$	$\{b, c\}$	3	i
o_3	$\{a, c\}$	$\{d\}$	1	c
o_4	$\{b, d\}$	$\{e\}$	3	d
o_5	$\{a, c, e\}$	$\{g\}$	1	e
o_6	$\{a\}$	$\{e\}$	5	a

	i	a	b	c	d	e	g
Δ_1	0	2	2	3	4	7	8

$h^{\text{lm-cut}}(s) = 0$



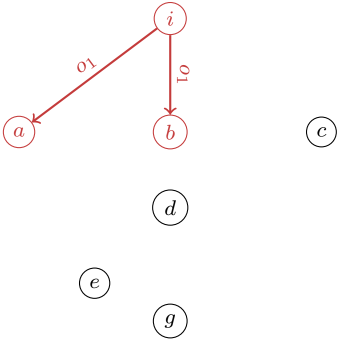
Example 2

Construct the justification graph.

	pre	add	c	supp
o_1	$\{i\}$	$\{a, b\}$	2	i
o_2	$\{i\}$	$\{b, c\}$	3	i
o_3	$\{a, c\}$	$\{d\}$	1	c
o_4	$\{b, d\}$	$\{e\}$	3	d
o_5	$\{a, c, e\}$	$\{g\}$	1	e
o_6	$\{a\}$	$\{e\}$	5	a

	i	a	b	c	d	e	g
Δ_1	0	2	2	3	4	7	8

$h^{\text{lm-cut}}(s) = 0$



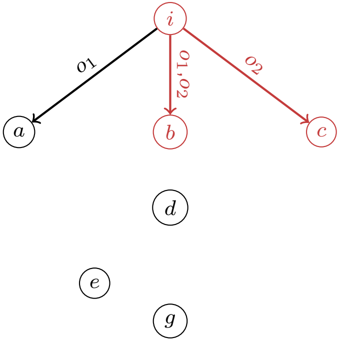
Example 2

Construct the justification graph.

	pre	add	c	supp
o_1	$\{i\}$	$\{a, b\}$	2	i
o_2	$\{i\}$	$\{b, c\}$	3	i
o_3	$\{a, c\}$	$\{d\}$	1	c
o_4	$\{b, d\}$	$\{e\}$	3	d
o_5	$\{a, c, e\}$	$\{g\}$	1	e
o_6	$\{a\}$	$\{e\}$	5	a

	i	a	b	c	d	e	g
Δ_1	0	2	2	3	4	7	8

$h^{\text{lm-cut}}(s) = 0$



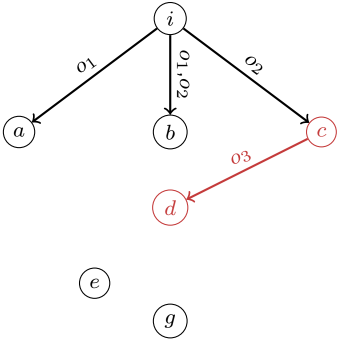
Example 2

Construct the justification graph.

	pre	add	c	supp
o_1	$\{i\}$	$\{a, b\}$	2	i
o_2	$\{i\}$	$\{b, c\}$	3	i
o_3	$\{a, c\}$	$\{d\}$	1	c
o_4	$\{b, d\}$	$\{e\}$	3	d
o_5	$\{a, c, e\}$	$\{g\}$	1	e
o_6	$\{a\}$	$\{e\}$	5	a

	i	a	b	c	d	e	g
Δ_1	0	2	2	3	4	7	8

$$h^{\text{lm-cut}}(s) = 0$$



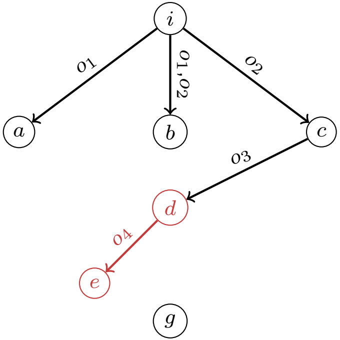
Example 2

Construct the justification graph.

	pre	add	c	supp
o_1	$\{i\}$	$\{a, b\}$	2	i
o_2	$\{i\}$	$\{b, c\}$	3	i
o_3	$\{a, c\}$	$\{d\}$	1	c
o_4	$\{b, d\}$	$\{e\}$	3	d
o_5	$\{a, c, e\}$	$\{g\}$	1	e
o_6	$\{a\}$	$\{e\}$	5	a

	i	a	b	c	d	e	g
Δ_1	0	2	2	3	4	7	8

$$h^{\text{lm-cut}}(s) = 0$$



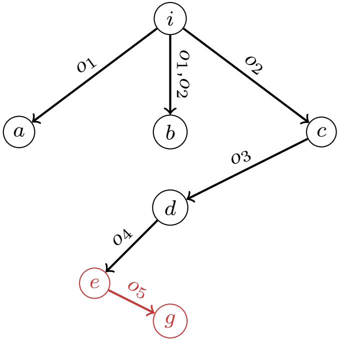
Example 2

Construct the justification graph.

	pre	add	c	supp
o_1	$\{i\}$	$\{a, b\}$	2	i
o_2	$\{i\}$	$\{b, c\}$	3	i
o_3	$\{a, c\}$	$\{d\}$	1	c
o_4	$\{b, d\}$	$\{e\}$	3	d
o_5	$\{a, c, e\}$	$\{g\}$	1	e
o_6	$\{a\}$	$\{e\}$	5	a

	i	a	b	c	d	e	g
Δ_1	0	2	2	3	4	7	8

$$h^{\text{lm-cut}}(s) = 0$$



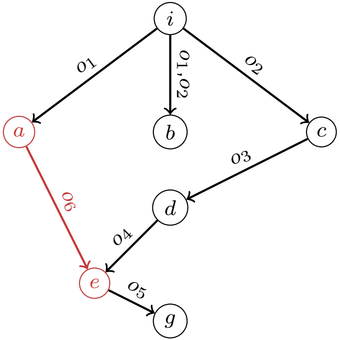
Example 2

Construct the justification graph.

	pre	add	c	supp
o_1	$\{i\}$	$\{a, b\}$	2	i
o_2	$\{i\}$	$\{b, c\}$	3	i
o_3	$\{a, c\}$	$\{d\}$	1	c
o_4	$\{b, d\}$	$\{e\}$	3	d
o_5	$\{a, c, e\}$	$\{g\}$	1	e
o_6	$\{a\}$	$\{e\}$	5	a

	i	a	b	c	d	e	g
Δ_1	0	2	2	3	4	7	8

$h^{\text{lm-cut}}(s) = 0$



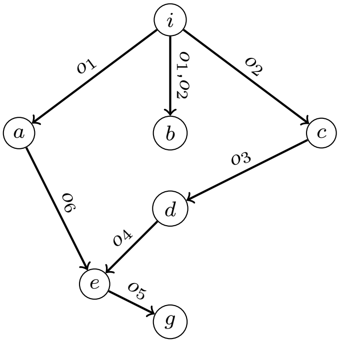
Example 2

Find N^0, $N^\star$, and N^b.

	pre	add	c	supp
o_1	$\{i\}$	$\{a, b\}$	2	i
o_2	$\{i\}$	$\{b, c\}$	3	i
o_3	$\{a, c\}$	$\{d\}$	1	c
o_4	$\{b, d\}$	$\{e\}$	3	d
o_5	$\{a, c, e\}$	$\{g\}$	1	e
o_6	$\{a\}$	$\{e\}$	5	a

	i	a	b	c	d	e	g
Δ_1	0	2	2	3	4	7	8

$\mathsf{h}^{\text{lm-cut}}(s) = 0$



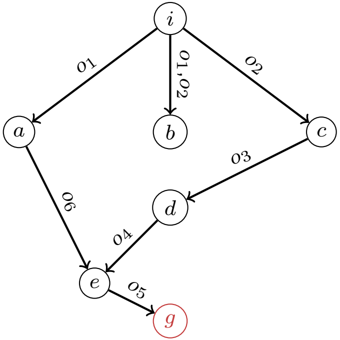
Example 2

Find N^0, $N^\star$, and N^b.
$N^\star = \{g\}$, because all operators are non-zero, i.e., g is reachable only from itself.

	pre	add	c	supp
o_1	$\{i\}$	$\{a, b\}$	2	i
o_2	$\{i\}$	$\{b, c\}$	3	i
o_3	$\{a, c\}$	$\{d\}$	1	c
o_4	$\{b, d\}$	$\{e\}$	3	d
o_5	$\{a, c, e\}$	$\{g\}$	1	e
o_6	$\{a\}$	$\{e\}$	5	a

	i	a	b	c	d	e	g
Δ_1	0	2	2	3	4	7	8

$h^{\text{lm-cut}}(s) = 0$



Example 2

Find N^0, $N^\star$, and N^b.

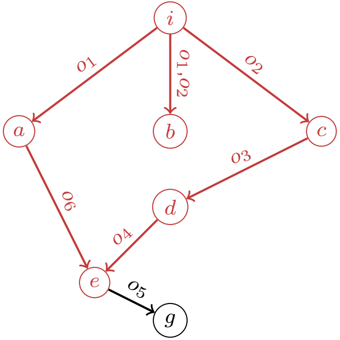
$N^\star = \{g\}$, because all operators are non-zero, i.e., g is reachable only from itself.

$N^0 = \{i, a, b, c, d, e\}$.

	pre	add	c	supp
o_1	$\{i\}$	$\{a, b\}$	2	i
o_2	$\{i\}$	$\{b, c\}$	3	i
o_3	$\{a, c\}$	$\{d\}$	1	c
o_4	$\{b, d\}$	$\{e\}$	3	d
o_5	$\{a, c, e\}$	$\{g\}$	1	e
o_6	$\{a\}$	$\{e\}$	5	a

	i	a	b	c	d	e	g
Δ_1	0	2	2	3	4	7	8

$h^{\text{lm-cut}}(s) = 0$



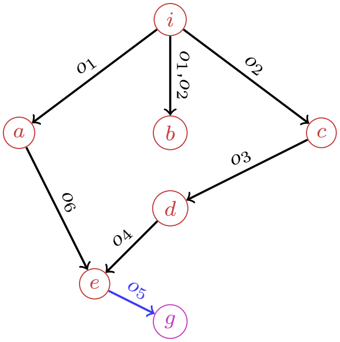
Example 2

$N^0 = \{i, a, b, c, d, e\}$
$N^\star = \{g\}$
This makes the (disjunctive operator) landmark L_1:
$L_1 = \{o_5\}$

	pre	add	c	supp
o_1	$\{i\}$	$\{a, b\}$	2	i
o_2	$\{i\}$	$\{b, c\}$	3	i
o_3	$\{a, c\}$	$\{d\}$	1	c
o_4	$\{b, d\}$	$\{e\}$	3	d
o_5	$\{a, c, e\}$	$\{g\}$	1	e
o_6	$\{a\}$	$\{e\}$	5	a

	i	a	b	c	d	e	g
Δ_1	0	2	2	3	4	7	8

$h^{\text{lm-cut}}(s) = 0$



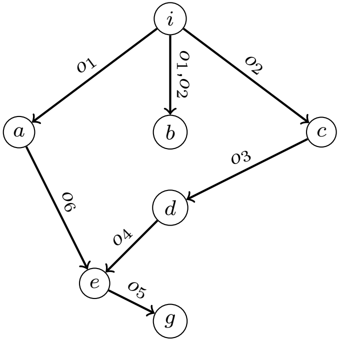
Example 2

$L_1 = \{o_5\}$
The cost of L_1 is 1 (the cost of o_5), therefore we update the estimate

	pre	add	c	supp
o_1	$\{i\}$	$\{a, b\}$	2	i
o_2	$\{i\}$	$\{b, c\}$	3	i
o_3	$\{a, c\}$	$\{d\}$	1	c
o_4	$\{b, d\}$	$\{e\}$	3	d
o_5	$\{a, c, e\}$	$\{g\}$	1	e
o_6	$\{a\}$	$\{e\}$	5	a

	i	a	b	c	d	e	g
Δ_1	0	2	2	3	4	7	8

$h^{\text{lm-cut}}(s) = 1$



Example 2

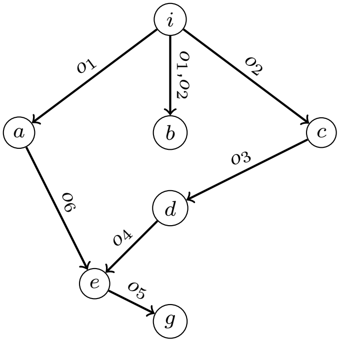
$L_1 = \{o_5\}$

The cost of L_1 is 1 (the cost of o_5), therefore we update the estimate and substract the cost of the landmark from the cost of the operators from L_1

	pre	add	c	supp
o_1	$\{i\}$	$\{a, b\}$	2	i
o_2	$\{i\}$	$\{b, c\}$	3	i
o_3	$\{a, c\}$	$\{d\}$	1	c
o_4	$\{b, d\}$	$\{e\}$	3	d
o_5	$\{a, c, e\}$	$\{g\}$	0	e
o_6	$\{a\}$	$\{e\}$	5	a

	i	a	b	c	d	e	g
Δ_1	0	2	2	3	4	7	8

$h^{\text{lm-cut}}(s) = 1$



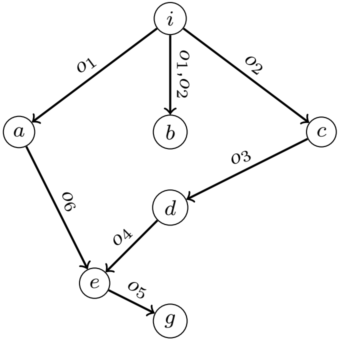
Example 2

And we continue with the next cycle...

	pre	add	c	supp
o_1	$\{i\}$	$\{a, b\}$	2	i
o_2	$\{i\}$	$\{b, c\}$	3	i
o_3	$\{a, c\}$	$\{d\}$	1	c
o_4	$\{b, d\}$	$\{e\}$	3	d
o_5	$\{a, c, e\}$	$\{g\}$	0	e
o_6	$\{a\}$	$\{e\}$	5	a

	i	a	b	c	d	e	g
Δ_1	0	2	2	3	4	7	8

$h^{\text{lm-cut}}(s) = 1$

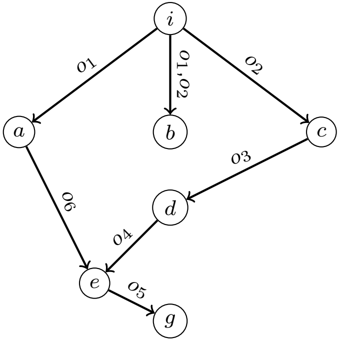


Compute $h^{\max}$, keep Δ_1-values, and update supporters.

	pre	add	c	supp
o_1	$\{i\}$	$\{a, b\}$	2	i
o_2	$\{i\}$	$\{b, c\}$	3	i
o_3	$\{a, c\}$	$\{d\}$	1	c
o_4	$\{b, d\}$	$\{e\}$	3	d
o_5	$\{a, c, e\}$	$\{g\}$	0	e
o_6	$\{a\}$	$\{e\}$	5	a

	i	a	b	c	d	e	g
Δ_1	0	2	2	3	4	7	7

$h^{\text{lm-cut}}(s) = 1$



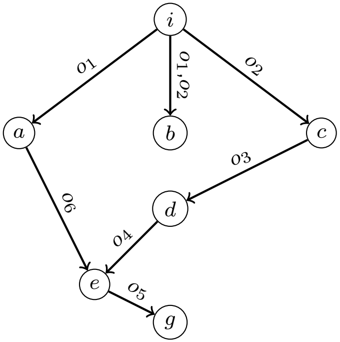
Example 2

Supporters did not change, so the justification graph also remains the same.

	pre	add	c	supp
o_1	$\{i\}$	$\{a, b\}$	2	i
o_2	$\{i\}$	$\{b, c\}$	3	i
o_3	$\{a, c\}$	$\{d\}$	1	c
o_4	$\{b, d\}$	$\{e\}$	3	d
o_5	$\{a, c, e\}$	$\{g\}$	0	e
o_6	$\{a\}$	$\{e\}$	5	a

	i	a	b	c	d	e	g
Δ_1	0	2	2	3	4	7	7

$$h^{\text{lm-cut}}(s) = 1$$



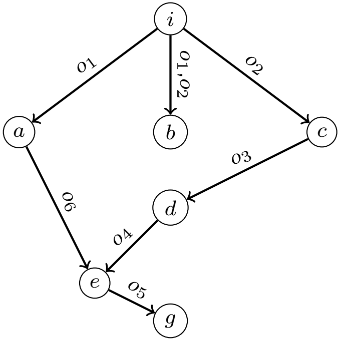
Example 2

Find N^0, $N^\star$, and N^b.

	pre	add	c	supp
o_1	$\{i\}$	$\{a, b\}$	2	i
o_2	$\{i\}$	$\{b, c\}$	3	i
o_3	$\{a, c\}$	$\{d\}$	1	c
o_4	$\{b, d\}$	$\{e\}$	3	d
o_5	$\{a, c, e\}$	$\{g\}$	0	e
o_6	$\{a\}$	$\{e\}$	5	a

	i	a	b	c	d	e	g
Δ_1	0	2	2	3	4	7	7

$h^{\text{lm-cut}}(s) = 1$



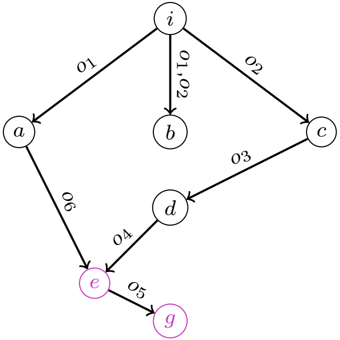
Find N^0, $N^\star$, and N^b.

$N^\star = \{g, e\}$, because now g is reachable from e with the zero-cost operator o_5.

	pre	add	c	supp
o_1	$\{i\}$	$\{a, b\}$	2	i
o_2	$\{i\}$	$\{b, c\}$	3	i
o_3	$\{a, c\}$	$\{d\}$	1	c
o_4	$\{b, d\}$	$\{e\}$	3	d
o_5	$\{a, c, e\}$	$\{g\}$	0	e
o_6	$\{a\}$	$\{e\}$	5	a

	i	a	b	c	d	e	g
Δ_1	0	2	2	3	4	7	7

$h^{\text{lm-cut}}(s) = 1$



Example 2

Find N^0, $N^\star$, and N^b.

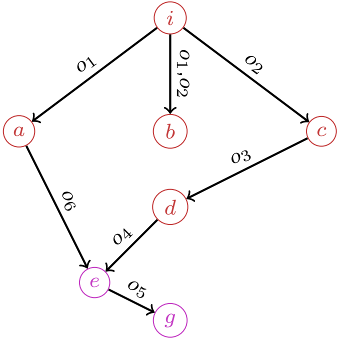
$N^\star = \{g, e\}$, because now g is reachable from e with the zero-cost operator o_5.

$N^0 = \{i, a, b, c, d\}$.

	pre	add	c	supp
o_1	$\{i\}$	$\{a, b\}$	2	i
o_2	$\{i\}$	$\{b, c\}$	3	i
o_3	$\{a, c\}$	$\{d\}$	1	c
o_4	$\{b, d\}$	$\{e\}$	3	d
o_5	$\{a, c, e\}$	$\{g\}$	0	e
o_6	$\{a\}$	$\{e\}$	5	a

	i	a	b	c	d	e	g
Δ_1	0	2	2	3	4	7	7

$h^{\text{lm-cut}}(s) = 1$



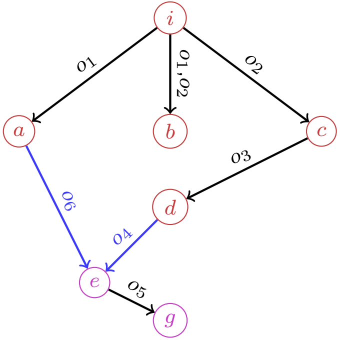
Example 2

$N^0 = \{i, a, b, c, d\}$
$N^\star = \{g, e\}$
This makes the next landmark $L_2 = \{o_4, o_6\}$

	pre	add	c	supp
o_1	$\{i\}$	$\{a, b\}$	2	i
o_2	$\{i\}$	$\{b, c\}$	3	i
o_3	$\{a, c\}$	$\{d\}$	1	c
o_4	$\{b, d\}$	$\{e\}$	3	d
o_5	$\{a, c, e\}$	$\{g\}$	0	e
o_6	$\{a\}$	$\{e\}$	5	a

	i	a	b	c	d	e	g
Δ_1	0	2	2	3	4	7	7

$\mathrm{h}^{\mathrm{lm\text{-}cut}}(s) = 1$



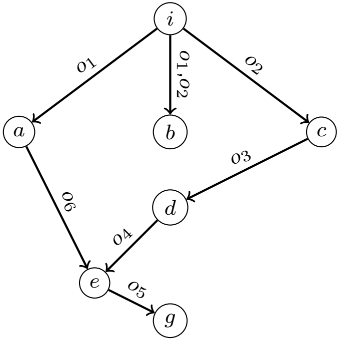
Example 2

The cost of $L_2 = \{o_4, o_6\}$ is 3 $(\min(c(o_4), c(o_6)) = \min(3, 5) = 3)$
So we increase our estimate, and substract the cost 3 from $c(o_4)$ and $c(o_6)$.

	pre	add	c	supp
o_1	$\{i\}$	$\{a, b\}$	2	i
o_2	$\{i\}$	$\{b, c\}$	3	i
o_3	$\{a, c\}$	$\{d\}$	1	c
o_4	$\{b, d\}$	$\{e\}$	0	d
o_5	$\{a, c, e\}$	$\{g\}$	0	e
o_6	$\{a\}$	$\{e\}$	2	a

	i	a	b	c	d	e	g
Δ_1	0	2	2	3	4	7	7

$h^{\text{lm-cut}}(s) = 4$



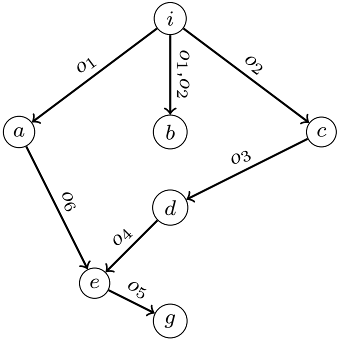
Example 2

And next cycle...

	pre	add	c	supp
o_1	$\{i\}$	$\{a, b\}$	2	i
o_2	$\{i\}$	$\{b, c\}$	3	i
o_3	$\{a, c\}$	$\{d\}$	1	c
o_4	$\{b, d\}$	$\{e\}$	0	d
o_5	$\{a, c, e\}$	$\{g\}$	0	e
o_6	$\{a\}$	$\{e\}$	2	a

	i	a	b	c	d	e	g
Δ_1	0	2	2	3	4	7	7

$h^{\text{lm-cut}}(s) = 4$



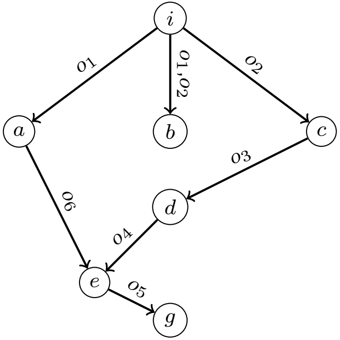
Example 2

Compute Δ_1-values, update supporters if necessary...
Supporters remain the same, so the justification graph remains the same.

	pre	add	c	supp
o_1	$\{i\}$	$\{a, b\}$	2	i
o_2	$\{i\}$	$\{b, c\}$	3	i
o_3	$\{a, c\}$	$\{d\}$	1	c
o_4	$\{b, d\}$	$\{e\}$	0	d
o_5	$\{a, c, e\}$	$\{g\}$	0	e
o_6	$\{a\}$	$\{e\}$	2	a

	i	a	b	c	d	e	g
Δ_1	0	2	2	3	4	4	4

$h^{\text{lm-cut}}(s) = 4$



Example 2

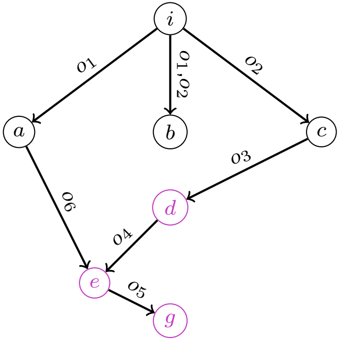
Find N^0, $N^\star$, and N^b...

$N^\star = \{d, e, g\}$, because now g is reachable from e via o_5 and from d via (o_4, o_5).

	pre	add	c	supp
o_1	$\{i\}$	$\{a, b\}$	2	i
o_2	$\{i\}$	$\{b, c\}$	3	i
o_3	$\{a, c\}$	$\{d\}$	1	c
o_4	$\{b, d\}$	$\{e\}$	0	d
o_5	$\{a, c, e\}$	$\{g\}$	0	e
o_6	$\{a\}$	$\{e\}$	2	a

	i	a	b	c	d	e	g
Δ_1	0	2	2	3	4	4	4

$h^{\text{lm-cut}}(s) = 4$



Example 2

Find N^0, $N^\star$, and N^b...

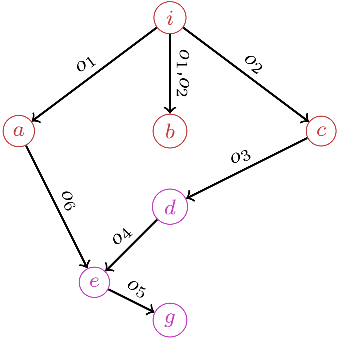
$N^\star = \{d, e, g\}$, because now g is reachable from e via o_5 and from d via (o_4, o_5).

$N^0 = \{i, a, b, c\}$.

	pre	add	c	supp
o_1	$\{i\}$	$\{a, b\}$	2	i
o_2	$\{i\}$	$\{b, c\}$	3	i
o_3	$\{a, c\}$	$\{d\}$	1	c
o_4	$\{b, d\}$	$\{e\}$	0	d
o_5	$\{a, c, e\}$	$\{g\}$	0	e
o_6	$\{a\}$	$\{e\}$	2	a

	i	a	b	c	d	e	g
Δ_1	0	2	2	3	4	4	4

$h^{\text{lm-cut}}(s) = 4$



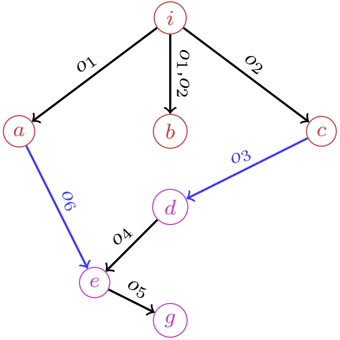
Example 2

$N^0 = \{i, a, b, c\}$
$N^\star = \{d, e, g\}$
This makes the next landmark $L_3 = \{o_3, o_6\}$.

	pre	add	c	supp
o_1	$\{i\}$	$\{a, b\}$	2	i
o_2	$\{i\}$	$\{b, c\}$	3	i
o_3	$\{a, c\}$	$\{d\}$	1	c
o_4	$\{b, d\}$	$\{e\}$	0	d
o_5	$\{a, c, e\}$	$\{g\}$	0	e
o_6	$\{a\}$	$\{e\}$	2	a

	i	a	b	c	d	e	g
Δ_1	0	2	2	3	4	4	4

$h^{\text{lm-cut}}(s) = 4$



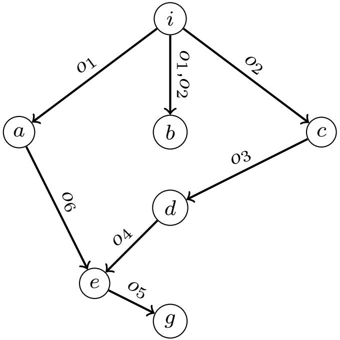
Example 2

The cost of $L_3 = \{o_3, o_6\}$ is 1 $(\min(c(o_3), c(o_6)) = \min(1, 2) = 1)$
So we increase our estimate, and substract the cost 1 from $c(o_3)$ and $c(o_6)$.

	pre	add	c	supp
o_1	$\{i\}$	$\{a, b\}$	2	i
o_2	$\{i\}$	$\{b, c\}$	3	i
o_3	$\{a, c\}$	$\{d\}$	0	c
o_4	$\{b, d\}$	$\{e\}$	0	d
o_5	$\{a, c, e\}$	$\{g\}$	0	e
o_6	$\{a\}$	$\{e\}$	1	a

	i	a	b	c	d	e	g
Δ_1	0	2	2	3	4	4	4

$h^{\text{lm-cut}}(s) = 5$



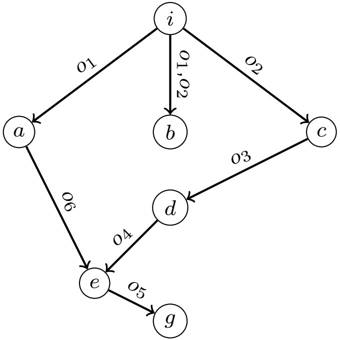
Example 2

And next cycle...

	pre	add	c	supp
o_1	$\{i\}$	$\{a, b\}$	2	i
o_2	$\{i\}$	$\{b, c\}$	3	i
o_3	$\{a, c\}$	$\{d\}$	0	c
o_4	$\{b, d\}$	$\{e\}$	0	d
o_5	$\{a, c, e\}$	$\{g\}$	0	e
o_6	$\{a\}$	$\{e\}$	1	a

	i	a	b	c	d	e	g
Δ_1	0	2	2	3	4	4	4

$h^{\text{lm-cut}}(s) = 5$



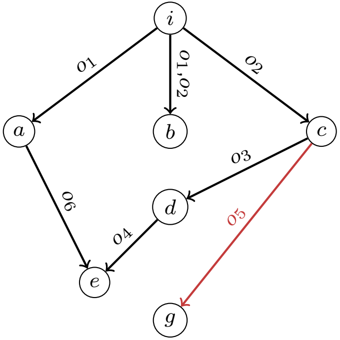
Example 2

Compute Δ_1-values, update supporters if necessary...
Now, we can choose a different supporter for o_5, because both c and e has the Δ_1-value 3.
So, let's try to change the supporter from e to c and therefore modify the justification graph.

	pre	add	c	supp
o_1	$\{i\}$	$\{a, b\}$	2	i
o_2	$\{i\}$	$\{b, c\}$	3	i
o_3	$\{a, c\}$	$\{d\}$	0	c
o_4	$\{b, d\}$	$\{e\}$	0	d
o_5	$\{a, c, e\}$	$\{g\}$	0	c
o_6	$\{a\}$	$\{e\}$	1	a

	i	a	b	c	d	e	g
Δ_1	0	2	2	3	3	3	3

$\mathrm{h}^{\mathrm{lm\text{-}cut}}(s) = 5$



Example 2

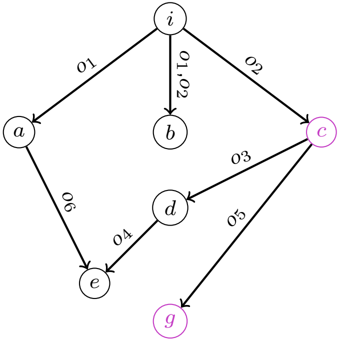
Find N^0, $N^\star$, and N^b...

$N^\star = \{c, g\}$, because now g is reachable only from c via the zero-cost operator o_5.

	pre	add	c	supp
o_1	$\{i\}$	$\{a, b\}$	2	i
o_2	$\{i\}$	$\{b, c\}$	3	i
o_3	$\{a, c\}$	$\{d\}$	0	c
o_4	$\{b, d\}$	$\{e\}$	0	d
o_5	$\{a, c, e\}$	$\{g\}$	0	c
o_6	$\{a\}$	$\{e\}$	1	a

	i	a	b	c	d	e	g
Δ_1	0	2	2	3	3	3	3

$h^{\text{lm-cut}}(s) = 5$



Example 2

Find N^0, $N^\star$, and N^b...

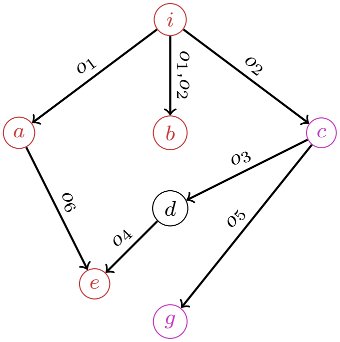
$N^\star = \{c, g\}$, because now g is reachable only from c via the zero-cost operator o_5.

$N^0 = \{i, a, b, e\}$ (Note that $N^b = \{d\}$, because d is not reachable from i without crossing c which belongs to $N^\star$.)

	pre	add	c	supp
o_1	$\{i\}$	$\{a, b\}$	2	i
o_2	$\{i\}$	$\{b, c\}$	3	i
o_3	$\{a, c\}$	$\{d\}$	0	c
o_4	$\{b, d\}$	$\{e\}$	0	d
o_5	$\{a, c, e\}$	$\{g\}$	0	c
o_6	$\{a\}$	$\{e\}$	1	a

	i	a	b	c	d	e	g
Δ_1	0	2	2	3	3	3	3

$h^{\text{lm-cut}}(s) = 5$



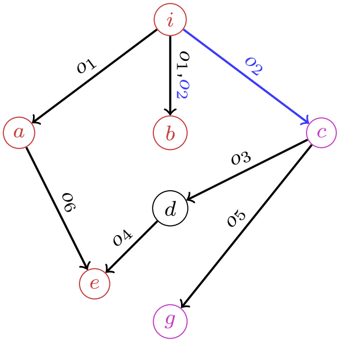
Example 2

$N^0 = \{i, a, b, e\}$
$N^\star = \{c, g\}$
So now the landmark is $L_4 = \{o_2\}$ (don't get confused by the fact that the label o_2 is also on an edge that does not cross from N^0 to $N^\star$—it doesn't matter).

	pre	add	c	supp
o_1	$\{i\}$	$\{a, b\}$	2	i
o_2	$\{i\}$	$\{b, c\}$	3	i
o_3	$\{a, c\}$	$\{d\}$	0	c
o_4	$\{b, d\}$	$\{e\}$	0	d
o_5	$\{a, c, e\}$	$\{g\}$	0	c
o_6	$\{a\}$	$\{e\}$	1	a

	i	a	b	c	d	e	g
Δ_1	0	2	2	3	3	3	3

$h^{\text{lm-cut}}(s) = 5$



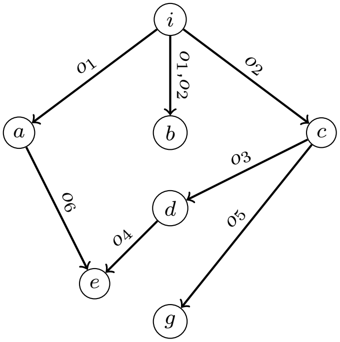
Example 2

The cost of $L_4 = \{o_2\}$ is 3 because $c(o_2) = 3$, so we increase our estimate, and descrese the cost of o_2.

	pre	add	c	supp
o_1	$\{i\}$	$\{a, b\}$	2	i
o_2	$\{i\}$	$\{b, c\}$	0	i
o_3	$\{a, c\}$	$\{d\}$	0	c
o_4	$\{b, d\}$	$\{e\}$	0	d
o_5	$\{a, c, e\}$	$\{g\}$	0	c
o_6	$\{a\}$	$\{e\}$	1	a

	i	a	b	c	d	e	g
Δ_1	0	2	2	3	3	3	3

$h^{\text{lm-cut}}(s) = 8$



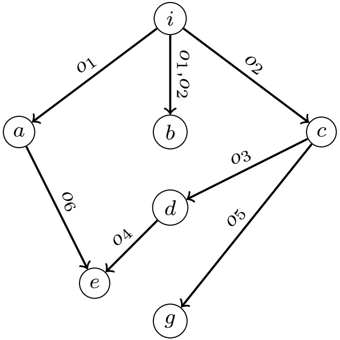
Example 2

Next cycle...

	pre	add	c	supp
o_1	$\{i\}$	$\{a, b\}$	2	i
o_2	$\{i\}$	$\{b, c\}$	0	i
o_3	$\{a, c\}$	$\{d\}$	0	c
o_4	$\{b, d\}$	$\{e\}$	0	d
o_5	$\{a, c, e\}$	$\{g\}$	0	c
o_6	$\{a\}$	$\{e\}$	1	a

	i	a	b	c	d	e	g
Δ_1	0	2	2	3	3	3	3

$$h^{\text{lm-cut}}(s) = 8$$



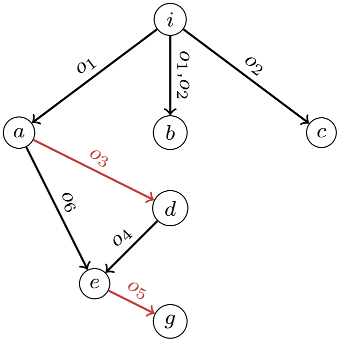
Example 2

Compute Δ_1-values, update supporters if necessary...
Now, we have to change the supporter of o_3, and o_5. For o_5, we can choose between a and e, so let's choose e. Therefore, we also need to modify the justification graph.

	pre	add	c	supp
o_1	$\{i\}$	$\{a, b\}$	2	i
o_2	$\{i\}$	$\{b, c\}$	0	i
o_3	$\{a, c\}$	$\{d\}$	0	a
o_4	$\{b, d\}$	$\{e\}$	0	d
o_5	$\{a, c, e\}$	$\{g\}$	0	e
o_6	$\{a\}$	$\{e\}$	1	a

	i	a	b	c	d	e	g
Δ_1	0	2	0	0	2	2	2

$h^{\text{lm-cut}}(s) = 8$



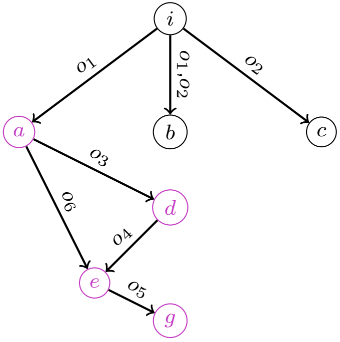
Example 2

Find N^0, $N^\star$, and N^b...
$N^\star = \{a, d, e, g\}$ (consider the zero-cost path (o_3, o_4, o_5)).

	pre	add	c	supp
o_1	$\{i\}$	$\{a, b\}$	2	i
o_2	$\{i\}$	$\{b, c\}$	0	i
o_3	$\{a, c\}$	$\{d\}$	0	a
o_4	$\{b, d\}$	$\{e\}$	0	d
o_5	$\{a, c, e\}$	$\{g\}$	0	e
o_6	$\{a\}$	$\{e\}$	1	a

	i	a	b	c	d	e	g
Δ_1	0	2	0	0	2	2	2

$h^{\text{lm-cut}}(s) = 8$



Example 2

Find N^0, $N^\star$, and N^b...

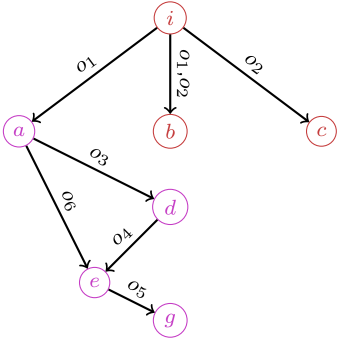
$N^\star = \{a, d, e, g\}$ (consider the zero-cost path (o_3, o_4, o_5)).

$N^0 = \{i, b, c\}$

	pre	add	c	supp
o_1	$\{i\}$	$\{a, b\}$	2	i
o_2	$\{i\}$	$\{b, c\}$	0	i
o_3	$\{a, c\}$	$\{d\}$	0	a
o_4	$\{b, d\}$	$\{e\}$	0	d
o_5	$\{a, c, e\}$	$\{g\}$	0	e
o_6	$\{a\}$	$\{e\}$	1	a

	i	a	b	c	d	e	g
Δ_1	0	2	0	0	2	2	2

$h^{\text{lm-cut}}(s) = 8$



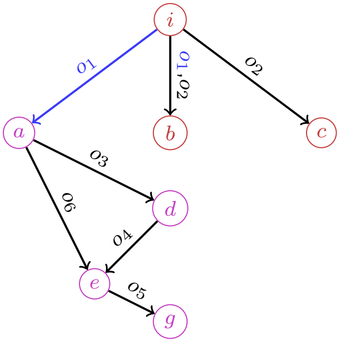
Example 2

Find N^0, $N^\star$, and N^b...
$N^\star = \{a, d, e, g\}$ (consider the zero-cost path (o_3, o_4, o_5)).
$N^0 = \{i, b, c\}$
So the next landmark is $L_5 = \{o_1\}$.

	pre	add	c	supp
o_1	$\{i\}$	$\{a, b\}$	2	i
o_2	$\{i\}$	$\{b, c\}$	0	i
o_3	$\{a, c\}$	$\{d\}$	0	a
o_4	$\{b, d\}$	$\{e\}$	0	d
o_5	$\{a, c, e\}$	$\{g\}$	0	e
o_6	$\{a\}$	$\{e\}$	1	a

	i	a	b	c	d	e	g
Δ_1	0	2	0	0	2	2	2

$h^{\text{lm-cut}}(s) = 8$



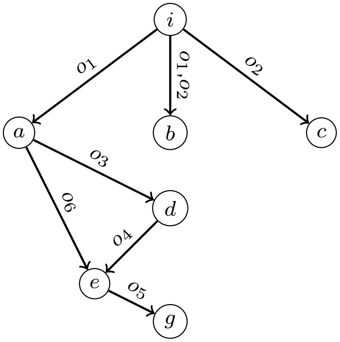
Example 2

The cost of the landmark $L_5 = \{o_1\}$ is 2, so we increase our estimate and descrease the cost of o_1.

	pre	add	c	supp
o_1	$\{i\}$	$\{a, b\}$	0	i
o_2	$\{i\}$	$\{b, c\}$	0	i
o_3	$\{a, c\}$	$\{d\}$	0	a
o_4	$\{b, d\}$	$\{e\}$	0	d
o_5	$\{a, c, e\}$	$\{g\}$	0	e
o_6	$\{a\}$	$\{e\}$	1	a

	i	a	b	c	d	e	g
Δ_1	0	2	0	0	2	2	2

$h^{\text{lm-cut}}(s) = 10$



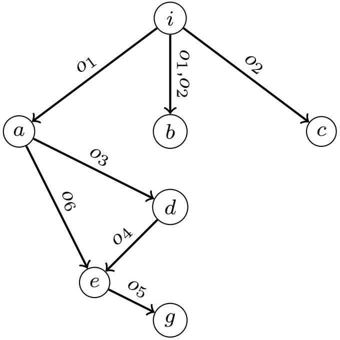
Example 2

In the next cycle we compute Δ_1-values, find out that the $h^{\max}$ is zero and therefore terminate the algorithm with the heuristic estimate 10.

	pre	add	c	supp
o_1	$\{i\}$	$\{a, b\}$	0	i
o_2	$\{i\}$	$\{b, c\}$	0	i
o_3	$\{a, c\}$	$\{d\}$	0	a
o_4	$\{b, d\}$	$\{e\}$	0	d
o_5	$\{a, c, e\}$	$\{g\}$	0	e
o_6	$\{a\}$	$\{e\}$	1	a

	i	a	b	c	d	e	g
Δ_1	0	0	0	0	0	0	0

$h^{\text{lm-cut}}(s) = 10$



Example 2

- Think about the way we used $h^{\max}$ as a subprocedure for LM-Cut. Can we say something about the relation between $h^{\max}$ and $h^{\text{lm-cut}}$?
- Now, try to re-compute the heuristic by yourself, but with different supporters.
- For example, you can go back and set the supporter of o_5 to e (instead of c as we did).
- You'll see that depending on your choice of supporters in the following steps, you can get either 10 or 8 as the resulting heuristic estimate.
- Bare in mind that both of these values are correct! This is the reason why we could not give you the one "correct" LM-Cut heuristic estimate for the planning tasks in your first assignment.
- The selection of the best supporter is still an open question in classical planning. If you are interested in trying to tackle this problem (as your semestral project/diploma thesis) let us know.
- Nevertheless, LM-Cut is still one of the best-performing, state-of-the-art heuristics in classical planning.