Relaxation heuristics

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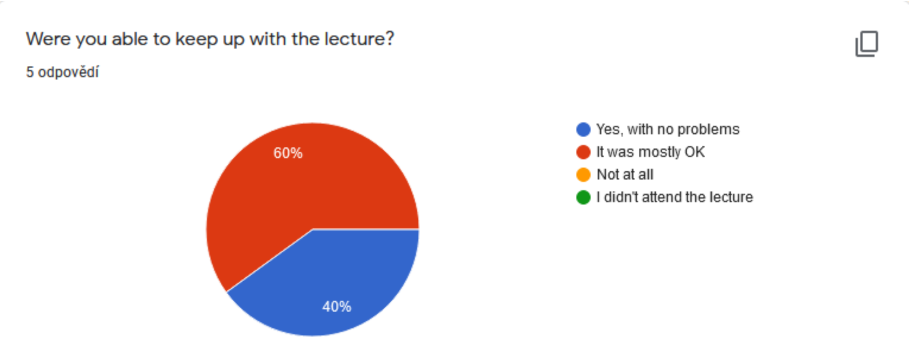
PUI Tutorial
Week 3

Lecture check

- Any questions regarding the lecture?

Feedback

- 5 reactions
- Lectures appear to get worse and worse feedback
  - Adding specific feedback might help with any issues



Obtaining heuristics

- Many different possible ways to obtain a heuristic in general
- One general idea: solve a **simplified** version of the problem

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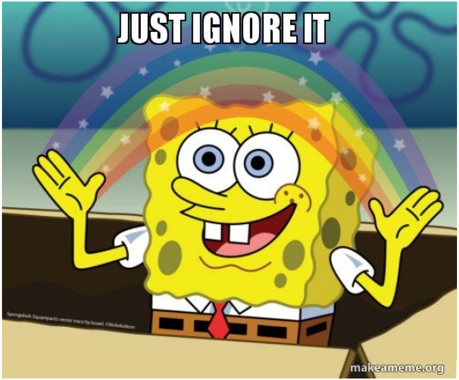
- Many different possible ways to obtain a heuristic in general
- One general idea: solve a **simplified** version of the problem
  - relaxation
  - abstraction
- This week: **relaxation**

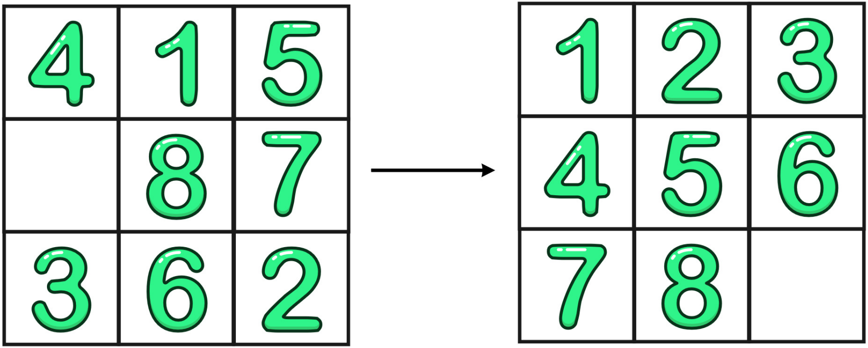
Relaxation heuristic

- Relaxation is
  - general design technique
  - usually ignoring something
  - simplifying the problem



Relaxation heuristic

- **Example:** 8-puzzle



- How would you formulate it?
- Let's try it out! http://editor.planning.domains

Relaxation heuristic

- What do we relax? Delete effects

Relaxation heuristic

- What do we relax? Delete effects
- But PDDL doesn't have any...we need STRIPS!
- **Delete relaxation**

Relaxed STRIPS planning task

Relaxation of a STRIPS planning task $\Pi = \langle F, O, s_{init}, s_{goal}, c \rangle$ is the planning task $\Pi^+ = \langle F, O^+, s_{init}, s_{goal}, c \rangle$ which contains set of relaxed operators.

Relaxation heuristic

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Relaxation of a STRIPS planning task $\Pi = \langle F, O, s_{init}, s_{goal}, c \rangle$ is the planning task $\Pi^+ = \langle F, O^+, s_{init}, s_{goal}, c \rangle$ which contains set of relaxed operators.

Relaxation of operators

Relaxation of operator $o = \langle pre(o), add(o), del(o) \rangle$ is operator $o^+ = \langle pre(o), add(o), \emptyset \rangle$.

We can modify the PDDL accordingly.

Relaxation heuristic

h^+ heuristic

The h^+ heuristic computes length of the optimal relaxed plan π^+ which is an optimal solution to the relaxed problem Π^+.

- h^* - optimal for STRIPS definition Π
- h^+ - optimal for relaxed STRIPS definition Π^+

- Computation of h^+ is still complicated.
- We can compute an **estimate** of h^+.
  - h^{max}
  - h^{add}

h^{max} heuristic

h^{max} heuristic

- STRIPS planning task $\Pi = \langle F, O, s_{init}, s_{goal}, c \rangle$
- $h^{max}(s)$ gives estimate of the distance from s to a state that satisfies s_{goal}
- $h^{max}(s) = max_{f \in s_{goal}} \Delta_1(s, f)$, where
    - $\Delta_1(s, o) = max_{f \in pre(o)} \Delta_1(s, f), \forall o \in O$
    - $\Delta_1(s, f) =$
$$\begin{cases} 0 & \text{if } f \in s, \\ \text{inf} & \text{if } \forall o \in O : f \notin add(o), \\ min\{c(o) + \Delta_1(s, o) | o \in O, f \in add(o)\} & otherwise. \end{cases}$$

h^{add} heuristic

- STRIPS planning task $\Pi = \langle F, O, s_{init}, s_{goal}, c \rangle$
- $h^{add}(s)$ gives estimate of the distance from s to a state that satisfies s_{goal}
- $h^{add}(s) = \sum_{f \in s_{goal}} \Delta_0(s, f)$, where
  - $\Delta_0(s, o) = \sum_{f \in pre(o)} \Delta_0(s, f), \forall o \in O$
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    $$\begin{cases} 0 & \text{if } f \in s, \\ \text{inf} & \text{if } \forall o \in O : f \notin add(o), \\ min\{c(o) + \Delta_0(s, o) | o \in O, f \in add(o)\} & otherwise. \end{cases}$$

Exercise h^{add}, h^{max}

Compute $h^{max}(s_{init})$ for the following problem $\Pi = \langle F, O, s_{init}, s_{goal}, c\rangle$:

$F = \{a, b, c, d, e, f, g\}$

$O =$

	pre	add	del	c
o_1	{a}	{c,d}	{a}	1
o_2	{a,b}	{e}	$\emptyset$	1
o_3	{b,e}	{d,f}	{a,e}	1
o_4	{b}	{a}	$\emptyset$	1
o_5	{d,e}	{g}	{e}	1

$s_{init} = \{a, b\}$ $s_{goal} = \{f, g\}$

h^{max} algorithm

Algorithm 1: Algorithm for computing $\mathrm{h}^{\max}(s)$.

Input: $\Pi = \langle \mathcal{F}, \mathcal{O}, s_{init}, s_{goal}, \mathrm{c} \rangle$, state s

Output: $\mathrm{h}^{\max}(s)$

for each $f \in s$ **do** $\Delta_1(s,f) \leftarrow 0$;
for each $f \in \mathcal{F} \setminus s$ **do** $\Delta_1(s,f) \leftarrow \infty$;
for each $o \in \mathcal{O}, pre(o) = \emptyset$ **do**
  for each $f \in add(o)$ **do** $\Delta_1(s,f) \leftarrow \min\{\Delta_1(s,f), \mathrm{c}(o)\}$;
end
for each $o \in \mathcal{O}$ **do** $U(o) \leftarrow |\mathrm{pre}(o)|$;
$C \leftarrow \emptyset$;
while $s_{goal} \not\subseteq C$ **do**
  $k \leftarrow \arg\min_{f \in \mathcal{F} \setminus C} \Delta_1(s,f)$;
  $C \leftarrow C \cup \{k\}$;
  for each $o \in \mathcal{O}, k \in pre(o)$ **do**
    $U(o) \leftarrow U(o) - 1$;
    if $U(o) = 0$ **then**
      for each $f \in add(o)$ **do**
        $\Delta_1(s,f) \leftarrow \min\{\Delta_1(s,f), \mathrm{c}(o) + \Delta_1(s,k)\}$;
      end
    end
  end
end
$\mathrm{h}^{\max}(s) = \max_{f \in s_{goal}} \Delta_1(s,f)$;

Exercise h^{add}, h^{max}

Compute $h^{add}(s_{init})$ for the following problem $\Pi = \langle F, O, s_{init}, s_{goal}, c \rangle$:

$F = \{a, b, c, d, e, f, g\}$

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	pre	add	del	c
o_1	{a}	{c,d}	{a}	1
o_2	{a,b}	{e}	∅	1
o_3	{b,e}	{d,f}	{a,e}	1
o_4	{b}	{a}	∅	1
o_5	{d,e}	{g}	{e}	1

$s_{init} = \{a, b\}$ $s_{goal} = \{f, g\}$

What's the difference in the algorithm?

Heuristic properties

Heuristic dominance

Admissible heuristic h_1 dominates an admissible heuristic h_2 if for every state s $h_1(s) \geq h_2(s)$

- h^+ is **admissible**, **consistent**
- h^{max} is **admissible**, **consistent**
- h^{add} is **not admissible**, **nor consistent** but can be very informative
- $h^{max} \leq h^+ \leq h^*$

Recap

- Know what is delete relaxation and which heuristics are based on it
- Know h^+, h^{max} and h^{add} heuristics
- Be capable of implementing h^{max} for your assignment

The End

Feedback form link