

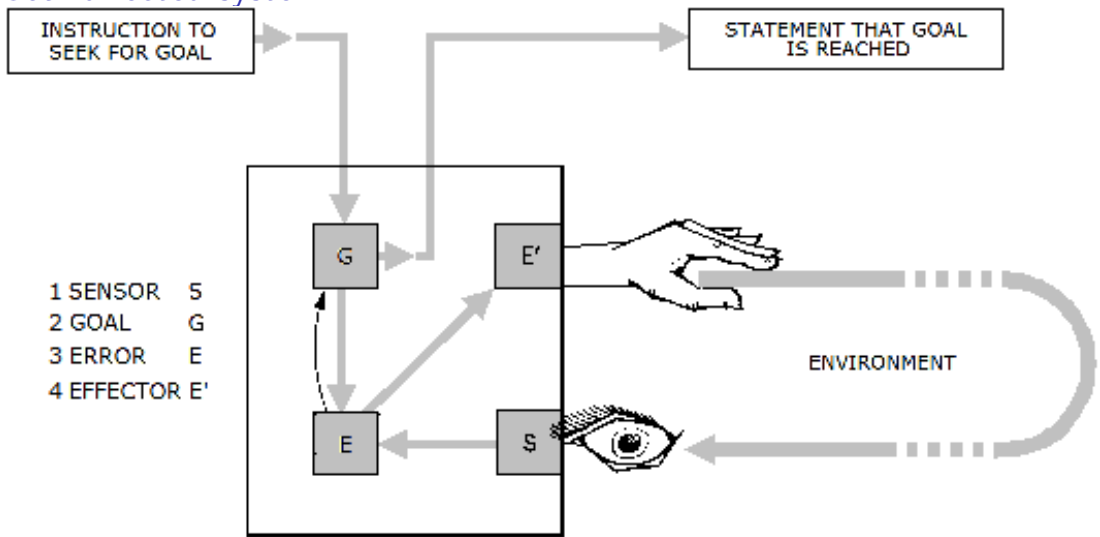
# Reinforcement learning

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# Goal-directed system



**A SIMPLE GOAL-DIRECTED SYSTEM**

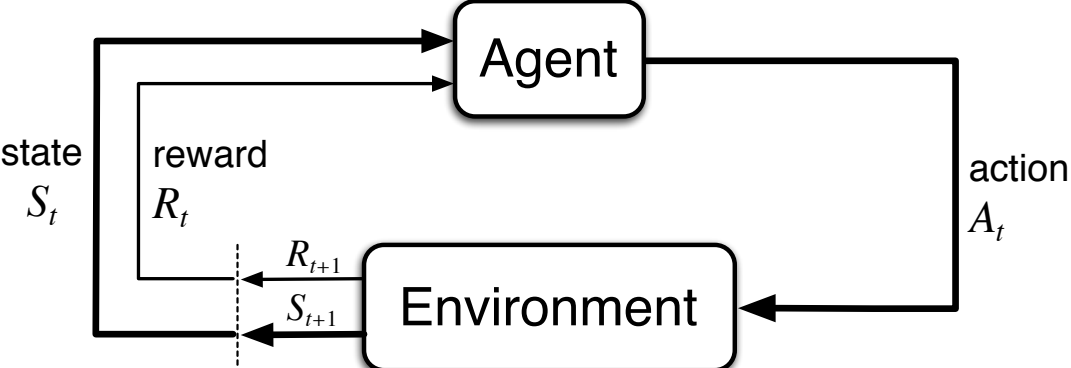
1

<sup>1</sup>Figure from <http://www.cybsoc.org/gcyb.htm>

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Notes

# Reinforcement Learning



2

- ▶ Feedback in form of Rewards
- ▶ Learn to act so as to maximize expected rewards.

<sup>2</sup>Scheme from [3]

### Autonomous Flipper Control with Safety Constraints

Martin Pecka, Vojtěch Šalanský,  
Karel Zimmermann, Tomáš Svoboda

experiments utilizing  
Constrained Relative Entropy Policy Search

**Video: Learning safe policies<sup>3</sup>**

<sup>3</sup>M. Pecka, V. Salansky, K. Zimmermann, T. Svoboda. Autonomous flipper control with safety constraints. In Intelligent Robots and Systems (IROS), 2016, [https://youtu.be/\\_oUMbBtoRcs](https://youtu.be/_oUMbBtoRcs)

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#### Notes

Policy search is a more advanced topic, only touched by this course. Later in master programme.

# From off-line (MDPs) to on-line (RL)

Markov decision process – MDPs. Off-line search, we know:

- ▶ A set of states  $s \in \mathcal{S}$  (map)
- ▶ A set of actions per state.  $a \in \mathcal{A}$
- ▶ A transition model  $T(s, a, s')$  or  $p(s'|s, a)$  (robot)
- ▶ A reward function  $r(s, a, s')$  (map, robot)

Looking for the optimal policy  $\pi(s)$ . We can plan/search before the robot enters the environment.

---

## Notes

For MDPs, we know  $p, r$  for all possible states and actions.

# From off-line (MDPs) to on-line (RL)

Markov decision process – MDPs. Off-line search, we know:

- ▶ A set of states  $s \in \mathcal{S}$  (map)
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- ▶ A reward function  $r(s, a, s')$  (map, robot)

Looking for the optimal policy  $\pi(s)$ . We can plan/search before the robot enters the environment.

On-line problem:

- ▶ Transition model  $p$  and reward function  $r$  not known.
- ▶ Agent/robot must act and learn from experience.

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## Notes

For MDPs, we know  $p, r$  for all possible states and actions.

# (Transition) Model-based learning

The main idea: Do something and:

- ▶ Learn an approximate model from experiences.
- ▶ Solve as if the model was correct.

---

## Notes

- Where to start?
- When does it end?
- How long does it take?
- When to stop (the learning phase)?

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The main idea: Do something and:

- ▶ Learn an approximate model from experiences.
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Learning MDP model:

- ▶ In  $s$  try  $a$ , observe  $s'$ , count  $(s, a, s')$ .
- ▶ Normalize to get an estimate of  $p(s' | s, a)$ .
- ▶ Discover (by observation) each  $r(s, a, s')$  when experienced.

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Solve the learned MDP.

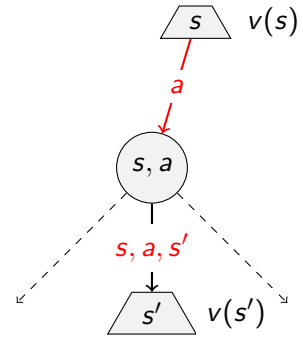
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## Notes

- Where to start?
- When does it end?
- How long does it take?
- When to stop (the learning phase)?

## Reward function $r(s, a, s')$

- ▶  $r(s, a, s')$  - reward for taking  $a$  in  $s$  and landing in  $s'$ .
- ▶ In Grid world, we assumed  $r(s, a, s')$  to be the same everywhere.
- ▶ In the real world, it is different (going up, down, ...)



In ai-gym `env.step(action)` returns  $s', r(s, \text{action}, s')$ .

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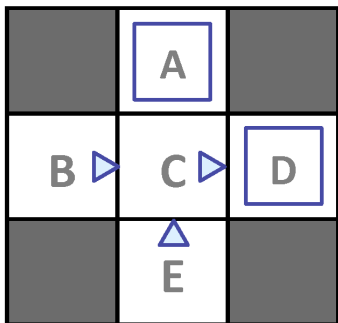
### Notes

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In ai-gym `env.step(action)` returns  $s', r(s, \text{action}, s'), \dots$ . It is defined by the environment (robot simulator, system, ...) not by the (algorithms)

# Model-based learning: Grid example

## Input Policy $\pi$



Assume:  $\gamma = 1$

## Observed Episodes (Training)

### Episode 1

B, east, C, -1  
C, east, D, -1  
D, exit, x, +10

### Episode 2

B, east, C, -1  
C, east, D, -1  
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### Episode 3

E, north, C, -1  
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D, exit, x, +10

### Episode 4

E, north, C, -1  
C, east, A, -1  
A, exit, x, -10

4

<sup>4</sup>Figure from [1]

# Learning transition model

$$p(D \mid C, \text{east}) = ?$$

## Episode 1

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D, exit, x, +10

## Episode 2

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## Episode 4

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C, east, A, -1  
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# Learning reward function

$r(C, \text{east}, D) = ?$

## Episode 1

B, east, C, -1  
C, east, D, -1  
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## Episode 2

B, east, C, -1  
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Notes

# Model based vs model-free: Expected age $E[A]$

Random variable age  $A$ .

$$E[A] = \sum_a P(A = a)a$$

We do not know  $P(A = a)$ . Instead, we collect  $N$  samples  $[a_1, a_2, \dots, a_N]$ .

---

## Notes

Just to avoid confusion. There are many more samples than possible ages (positive integer). Think about  $N \gg 100$ .

- Model based – eventually, we learn the correct model.
- Model free – no need for weighting; this is achieved through the frequencies of different ages within the samples (most frequent and hence most probable ages simply come up many times).

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## Model based

$$\hat{P}(a) = \frac{\text{num}(a)}{N}$$

$$E[A] \approx \sum_a \hat{P}(a)a$$

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Model based

$$\hat{P}(a) = \frac{\text{num}(a)}{N}$$

$$E[A] \approx \sum_a \hat{P}(a)a$$

Model free

$$E[A] \approx \frac{1}{N} \sum_i a_i$$

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# Model-free learning

# Passive learning (evaluating given policy)

- ▶ **Input:** a fixed policy  $\pi(s)$
- ▶ We want to know how good it is.
- ▶  $r, p$  not known.
- ▶ Execute policy ...
- ▶ and learn on the way.
- ▶ **Goal:** learn the state values  $v^\pi(s)$

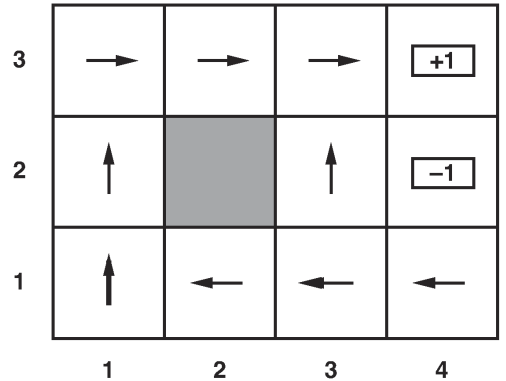


Image from [2]

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## Notes

Executing policies - training, then learning from the observations. We want to do the policy evaluation but the necessary model is not known.

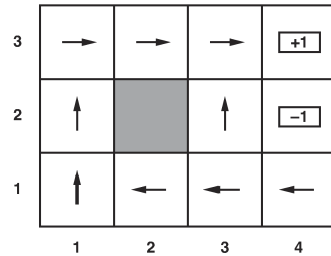
The word passive means we just follow a prescribed policy  $\pi(s)$ .

# Direct evaluation from episodes

Value of  $s$  for  $\pi$  – expected sum of discounted rewards – expected return

$$v^\pi(S_t) = E \left[ \sum_{k=0}^{\infty} \gamma^k R_{t+k+1} \right]$$

$$v^\pi(S_t) = E[G_t]$$



---

## Notes

- Act according to the policy.
- When visiting a state, remember what the sum of discounted rewards (returns) turned out to be.
- Compute average of the returns.
- Each trial episode provides a sample of  $v^\pi$ .

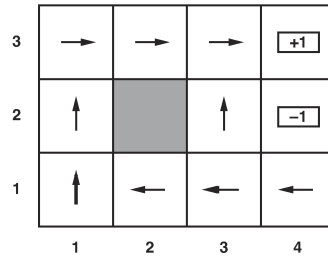
What is  $v(3, 2)$  after these episodes?

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## Notes

- Act according to the policy.
- When visiting a state, remember what the sum of discounted rewards (returns) turned out to be.
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- Each trial episode provides a sample of  $v^\pi$ .

What is  $v(3, 2)$  after these episodes?

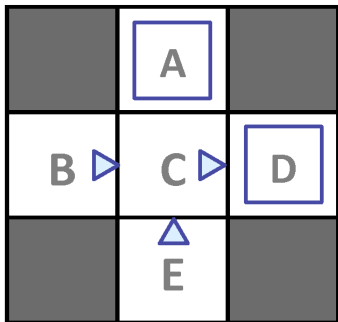
## Direct evaluation from episodes, $v^\pi(S_t) = E[G_t]$ , $\gamma = 1$

$(1, 1) \xrightarrow{-0.04} (1, 2) \xrightarrow{-0.04} (1, 3) \xrightarrow{-0.04} (1, 2) \xrightarrow{-0.04} (1, 3) \xrightarrow{-0.04} (2, 3) \xrightarrow{-0.04} (3, 3) \xrightarrow{-0.04} (4, 3) +1$   
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What is  $v(3, 2)$  after these episodes?

# Direct evaluation: Grid example

## Input Policy $\pi$



Assume:  $\gamma = 1$

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### Episode 1

B, east, C, -1  
C, east, D, -1  
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### Episode 3

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### Episode 4

E, north, C, -1  
C, east, A, -1  
A, exit, x, -10

## Notes

	-10 A	
+8 B	+4 C	+10 D
	-2 E	

# Direct evaluation: Grid example, $\gamma = 1$

What is  $v(C)$  after the 4 episodes?

## Episode 1

B, east, C, -1  
C, east, D, -1  
D, exit, x, +10

## Episode 2

B, east, C, -1  
C, east, D, -1  
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C, east, D, -1  
D, exit, x, +10

## Episode 4

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C, east, A, -1  
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## Direct evaluation: Grid example, $\gamma = 1$

What is  $v(C)$  after the 4 episodes?

Let  $M$  be the number of recorded episodes.

Let  $N$  be the number of samples used to compute the averages.

What is the relation of  $M$  and  $N$ ?

- A**  $N = M$
- B**  $N \leq M$
- C**  $N \geq M$
- D**  $N$  has no relation to  $M$

### Episode 1

B, east, C, -1  
C, east, D, -1  
D, exit, x, +10

### Episode 2

B, east, C, -1  
C, east, D, -1  
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E, north, C, -1  
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D, exit, x, +10

### Episode 4

E, north, C, -1  
C, east, A, -1  
A, exit, x, -10



## Direct evaluation algorithm (every-visit version)

$(1, 1) \cdot 0.4 \rightsquigarrow (1, 2) \cdot 0.4 \rightsquigarrow (1, 3) \cdot 0.4 \rightsquigarrow (1, 2) \cdot 0.4 \rightsquigarrow (1, 3) \cdot 0.4 \rightsquigarrow (2, 3) \cdot 0.4 \rightsquigarrow (3, 3) \cdot 0.4 \rightsquigarrow (4, 3) + 1$   
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Input: a policy  $\pi$  to be evaluated

Initialize:

$V(s) \in \mathbb{R}$ , arbitrarily, for all  $s \in \mathcal{S}$

$Returns(s) \leftarrow$  an empty list, for all  $s \in \mathcal{S}$

Loop forever (for each episode):

Generate an episode following  $\pi$ :  $S_0, A_0, R_1, S_1, A_1, R_2, \dots, S_{T-1}, A_{T-1}, R_T$

$G \leftarrow 0$

Loop backwards for each step of episode,  $t = T - 1, T - 2, \dots, 0$ :

$G \leftarrow R_{t+1} + \gamma G$

Append  $G$  to  $Returns(S_t)$

$V(S_t) \leftarrow \text{average}(Returns(S_t))$

---

### Notes

The algorithm can be easily expanded to  $Q(S_t, A_t)$ . Instead of visiting  $S_t$  we consider visiting of a pair  $S_t, A_t$ .

## Direct evaluation algorithm (first-visit version)

$(1, 1) \xrightarrow{.04} (1, 2) \xrightarrow{.04} (1, 3) \xrightarrow{.04} (1, 2) \xrightarrow{.04} (1, 3) \xrightarrow{.04} (2, 3) \xrightarrow{.04} (3, 3) \xrightarrow{.04} (4, 3) +1$   
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 $(1, 1) \xrightarrow{.04} (2, 1) \xrightarrow{.04} (3, 1) \xrightarrow{.04} (3, 2) \xrightarrow{.04} (4, 2) -1 .$

Input: a policy  $\pi$  to be evaluated

Initialize:

$V(s) \in \mathbb{R}$ , arbitrarily, for all  $s \in \mathcal{S}$

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Loop forever (for each episode):

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Loop backwards for each step of episode,  $t = T - 1, T - 2, \dots, 0$ :

$G \leftarrow R_{t+1} + \gamma G$

If  $S_t$  does not appear in  $S_0, S_1, \dots, S_{t-1}$ : // Use the return for the first visit only

Append  $G$  to  $Returns(S_t)$

$V(S_t) \leftarrow \text{average}(Returns(S_t))$

---

### Notes

The algorithm can be easily expanded to  $Q(S_t, A_t)$ . Instead of visiting  $S_t$  we consider visiting of a pair  $S_t, A_t$ .

# Direct evaluation: analysis

The good:

- ▶ Simple, easy to understand and implement.
- ▶ Does not need  $p, r$  and eventually it computes the true  $v^\pi$ .

---

## Notes

In second trial, we visit  $(3, 2)$  for the first time. We already know that the successor  $(3, 3)$  has probably a high value but the method does not use until the end of the trial episode.

Before updating  $V(s)$  we have to wait until the training episode ends.

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- ▶ Each state value learned in isolation.
- ▶ State values are not independent
- ▶  $v^\pi(s) = \sum_{s'} p(s' | s, \pi(s)) [r(s, \pi(s), s') + \gamma v^\pi(s')]$

---

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Before updating  $V(s)$  we have to wait until the training episode ends.

## (on-line) Policy evaluation?

In each round, replace  $V$  with a one-step-look-ahead

$$V_0^\pi(s) = 0$$

$$V_{k+1}^\pi(s) \leftarrow \sum_{s'} p(s' | s, \pi(s)) [r(s, \pi(s), s') + \gamma V_k^\pi(s')]$$

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Problem: both  $p(s' | s, \pi(s))$  and  $r(s, \pi(s), s')$  unknown!

## Use samples for evaluating policy?

MDP ( $p, r$  known) : Update  $V$  estimate by a weighted average:

$$V_{k+1}^{\pi}(s) \leftarrow \sum_{s'} p(s' | s, \pi(s)) [r(s, \pi(s), s') + \gamma V_k^{\pi}(s')]$$

---

### Notes

It looks promising. Unfortunately, we cannot do it that way. After an action, the robot is in a next state and cannot go back to the very same state where it was before. Energy was consumed and some actions may be irreversible; think about falling into a hole. We have to utilize the  $s, a, s'$  experience anytime when performed/visited.



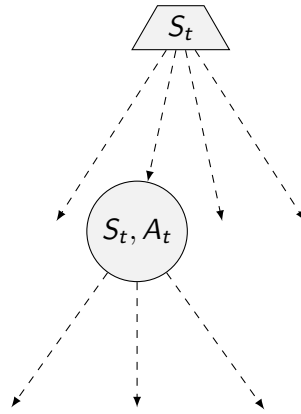
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What about stop, try, try,  $\dots$ , and average?

Trials at time  $t$ .  $\pi(S_t) \rightarrow A_t$ , repeat  $A_t$ .



---

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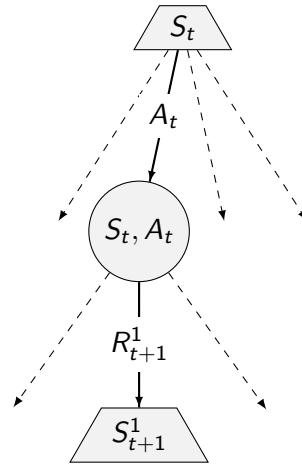
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What about stop, try, try, ..., and average?

Trials at time  $t$ .  $\pi(S_t) \rightarrow A_t$ , repeat  $A_t$ .

$$\text{trial}^1 = R_{t+1}^1 + \gamma V(S_{t+1}^1)$$



## Notes

It looks promising. Unfortunately, we cannot do it that way. After an action, the robot is in a next state and cannot go back to the very same state where it was before. Energy was consumed and some actions may be irreversible; think about falling into a hole. We have to utilize the  $s, a, s'$  experience anytime when performed/visited.

# Use samples for evaluating policy?

MDP ( $p, r$  known) : Update  $V$  estimate by a weighted average:

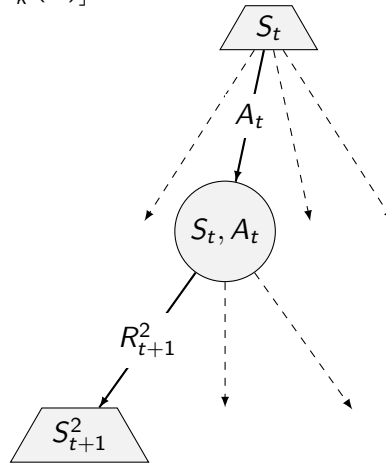
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---

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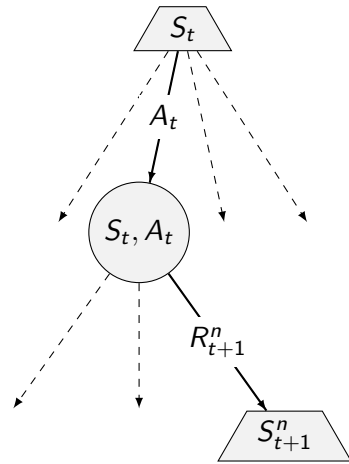
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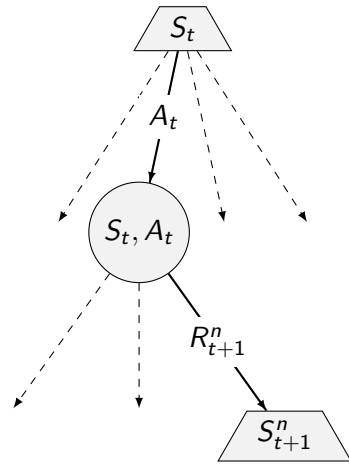
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$$V(S_t) \leftarrow \frac{1}{n} \sum_i \text{trial}^i$$



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# Temporal-difference value learning

$(1, 1) \xrightarrow{-0.04} (1, 2) \xrightarrow{-0.04} (1, 3) \xrightarrow{-0.04} (1, 2) \xrightarrow{-0.04} (1, 3) \xrightarrow{-0.04} (2, 3) \xrightarrow{-0.04} (3, 3) \xrightarrow{-0.04} (4, 3) +1$   
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---

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Note the main difference. In *Direct evaluation*, we had to wait until the end of the episode, compute  $G_t$  for each  $t$  on the way, and then we update  $V(S_t)$ . We can do it  $\alpha$  incrementally

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$$V(1, 3) = R_{t+1} + V(2, 3) = -0.04 + 0.92 = 0.88$$

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- ▶  $V(S_t) \leftarrow (1 - \alpha)V(S_t) + \alpha(\text{new sample})$

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# Exponential moving average

$$\bar{x}_n = (1 - \alpha)\bar{x}_{n-1} + \alpha x_n$$

What does it remember about the past? Try to derive:

$$\bar{x}_n = f(\alpha, x_n, x_{n-1}, x_{n-2}, x_{n-3}, \dots)$$

---

## Notes

Recursively inserting we end up with

$$\bar{x}_n = \alpha \left[ x_n + (1 - \alpha)x_{n-1} + (1 - \alpha)^2 x_{n-2} + \dots \right]$$

We already know the sum of geometric series for  $r < 1$

$$1 + r + r^2 + r^3 + \dots = \frac{1}{1 - r}$$

Putting  $r = 1 - \alpha$ , we see that

$$\frac{1}{\alpha} = 1 + (1 - \alpha) + (1 - \alpha)^2 + \dots$$

And hence:

$$\bar{x}_n = \frac{x_n + (1 - \alpha)x_{n-1} + (1 - \alpha)^2 x_{n-2} + \dots}{1 + (1 - \alpha) + (1 - \alpha)^2 + (1 - \alpha)^3 + \dots}$$

a weighted average that exponentially forgets about the past.

# Example: TD Value learning

$$V(S_t) \leftarrow V(S_t) + \alpha(R_{t+1} + \gamma V(S_{t+1}) - V(S_t))$$

	<span style="color: blue;">A</span> 0	
<span style="color: blue;">B</span> 0	<span style="color: blue;">C</span> 0	<span style="color: blue;">D</span> 8
	<span style="color: blue;">E</span> 0	

- ▶ Values represent initial  $V(s)$
- ▶ Assume:  $\gamma = 1, \alpha = 0.5, \pi(s) \Rightarrow$

## Notes

### States

	A	
B	C	D
	E	

Assume:  $\gamma = 1, \alpha = 1/2$

### Observed Transitions

B, east, C, -2

	0	
0	0	8
	0	

C, east, D, -2

	0	
-1	0	8
	0	

	0	
-1	3	8
	0	

$$V^\pi(s) \leftarrow (1 - \alpha)V^\pi(s) + \alpha [R(s, \pi(s), s') + \gamma V^\pi(s')]$$

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## Notes

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- ▶  $(C, \rightarrow, D), -2, \Rightarrow V(C)?$

## Notes

### States

	A	
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B, east, C, -2

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0	0	8
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## Temporal difference value learning: algorithm

Input: the policy  $\pi$  to be evaluated

Algorithm parameter: step size  $\alpha \in (0, 1]$

Initialize  $V(s)$ , for all  $s \in \mathcal{S}^+$ , arbitrarily except that  $V(\text{terminal}) = 0$

Loop for each episode:

    Initialize  $S$

    Loop for each step of episode:

$A \leftarrow$  action given by  $\pi$  for  $S$

        Take action  $A$ , observe  $R, S'$

$V(S) \leftarrow V(S) + \alpha [R + \gamma V(S') - V(S)]$

$S \leftarrow S'$

    until  $S$  is terminal



# What is wrong with the temporal difference Value learning?

The Good: Model-free value learning by mimicking Bellman updates.

---

## Notes

Learn Q-values, not V-values, and make the action selection model-free too!

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The Good: Model-free value learning by mimicking Bellman updates.

The Bad: How to turn values into a (new) policy?

$$\blacktriangleright \pi(s) = \arg \max_a \sum_{s'} p(s' | s, a) [r(s, a, s') + \gamma V(s')]$$

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## Notes

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# Active reinforcement learning

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## Notes

So far we walked as prescribed by a  $\pi(s)$  because we did not know how to act better.

## Reminder: $V$ , $Q$ -value iteration for MDPs

Value/Utility iteration (depth limited evaluation):

- ▶ Start:  $V_0(s) = 0$
- ▶ In each step update  $V$  by looking one step ahead:  
$$V_{k+1}(s) \leftarrow \max_a \sum_{s'} p(s' | s, a) [r(s, a, s') + \gamma V_k(s')]$$

$Q$  values more useful (think about updating  $\pi$ )

- ▶ Start:  $Q_0(s, a) = 0$
- ▶ In each step update  $Q$  by looking one step ahead:  
$$Q_{k+1}(s, a) \leftarrow \sum_{s'} p(s' | s, a) \left[ r(s, a, s') + \gamma \max_{a'} Q_k(s', a') \right]$$

---

### Notes

Draw the  $(s)$ - $(s,a)$ - $(s')$ - $(s',a')$  tree. It will be also handy when discussing exploration vs. exploitation – where to drive next.

## Q-learning

$$\text{MDP update: } Q_{k+1}(s, a) \leftarrow \sum_{s'} p(s' | s, a) \left[ r(s, a, s') + \gamma \max_{a'} Q_k(s', a') \right]$$

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There are alternatives how to compute the trial value. SARSA method takes  $Q(S_{t+1}, A_{t+1})$  directly, not the max. More next week.

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Learn Q values as the robot/agent goes (temporal difference)

- ▶ Drive the robot and fetch rewards  $(s, a, s', R)$

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$$\text{trial} = R_{t+1} + \gamma \max_a Q(S_{t+1}, a)$$

---

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$$\text{trial} = R_{t+1} + \gamma \max_a Q(S_{t+1}, a)$$

- ▶  $\alpha$  update  
 $Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha(\text{trial} - Q(S_t, A_t))$   
or (the same)  
 $Q(S_t, A_t) \leftarrow (1 - \alpha)Q(S_t, A_t) + \alpha \text{trial}$

---

## Notes

There are alternatives how to compute the trial value. SARSA method takes  $Q(S_{t+1}, A_{t+1})$  directly, not the max. More next week.

# Q-learning

MDP update:  $Q_{k+1}(s, a) \leftarrow \sum_{s'} p(s' | s, a) \left[ r(s, a, s') + \gamma \max_{a'} Q_k(s', a') \right]$

Learn Q values as the robot/agent goes (temporal difference)

- ▶ Drive the robot and fetch rewards  $(s, a, s', R)$
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In each step  $Q$  approximates the optimal  $q^*$  function.

---

## Notes

There are alternatives how to compute the trial value. SARSA method takes  $Q(S_{t+1}, A_{t+1})$  directly, not the max. More next week.

# Q-learning: algorithm

step size  $0 < \alpha \leq 1$

initialize  $Q(s, a)$  for all  $s \in \mathcal{S}, a \in \mathcal{S}(s)$

**repeat** episodes:

    initialize  $S$

**for** for each step of episode: **do**

        choose  $A$  from  $S$

        take action  $A$ , observe  $R, S'$

$Q(S, A) \leftarrow Q(S, A) + \alpha [R + \gamma \max_a Q(S', a) - Q(S, A)]$

$S \leftarrow S'$

**end for** until  $S$  is terminal

**until** Time is up, ...

# From Q-learning to Q-learning agent

- ▶ Drive the robot and fetch rewards.  $(s, a, s', R)$
- ▶ We know old estimates  $Q(s, a)$  (and  $Q(s', a')$ ), if not, initialize.
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## Notes

Q-function for a discrete, finite problem? But what about continuous space or discrete but a very large one?

Use the  $(s)-(s,a)-(s')-(s',a')$  tree to discuss the next-action selection.

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## Technicalities for the Q-learning agent

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- ▶ How to represent the Q-function?
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## Technicalities for the Q-learning agent

- ▶ How to represent the Q-function?
- ▶ What is the value for terminal?  $Q(s, \text{Exit})$  or  $Q(s, \text{None})$
- ▶ How to drive? Where to drive next? Does it change over the course?

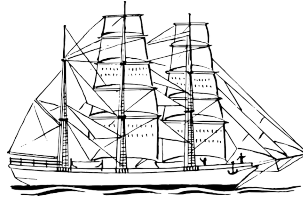
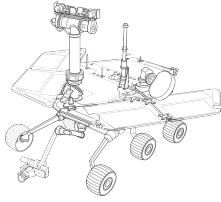
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### Notes

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# Exploration vs. Exploitation

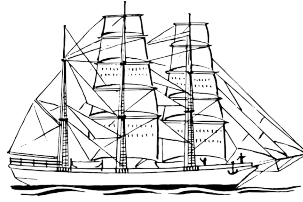
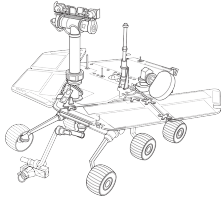
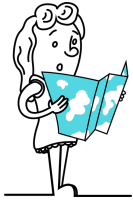


► Drive the known road or try a new one?

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Notes

# Exploration vs. Exploitation

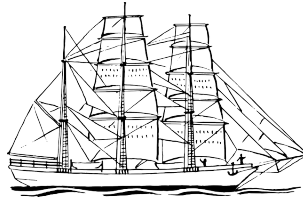
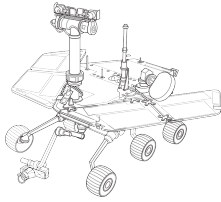


- ▶ Drive the known road or try a new one?
- ▶ Go to the university menza or try a nearby restaurant?

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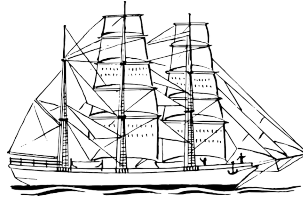
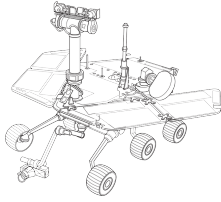
Notes

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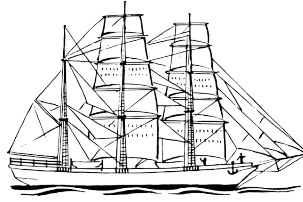
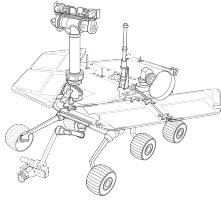
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- ▶ ...

# How to explore?

Random ( $\epsilon$ -greedy):

---

## Notes

- We can think about lowering  $\epsilon$  as the learning progresses.
- Favor unexplored states - be optimistic - exploration functions -  $f(u, n) = u + k/n$ , where  $u$  is the value estimated, and  $n$  is the visit count, and  $k$  is the training/simulation episode.

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## Problems with randomness?

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- ▶  $\epsilon$  same everywhere?

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# References

Further reading: Chapter 21 of [2]. More detailed discussion in [3], chapters 5 and 6.

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