Uncertainty, Chance, and Utilities

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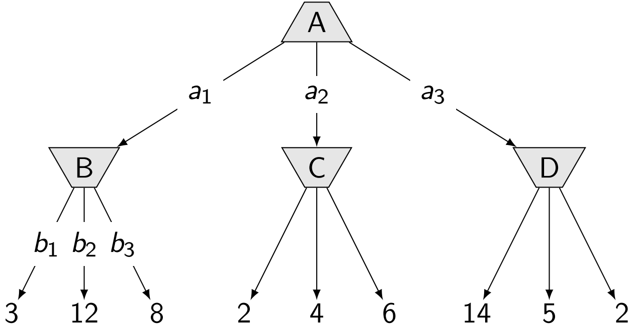
March 10, 2022

Deterministic opponent → stochastic environment



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CHANCE nodes are "virtual", b_1, b_2, b_3 are not actions!

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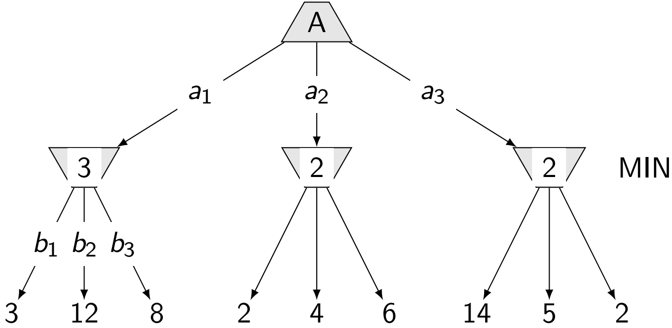

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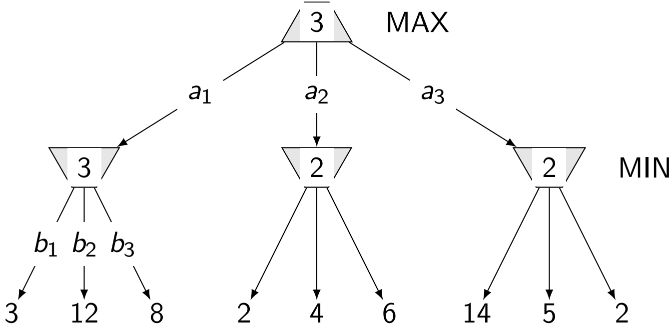

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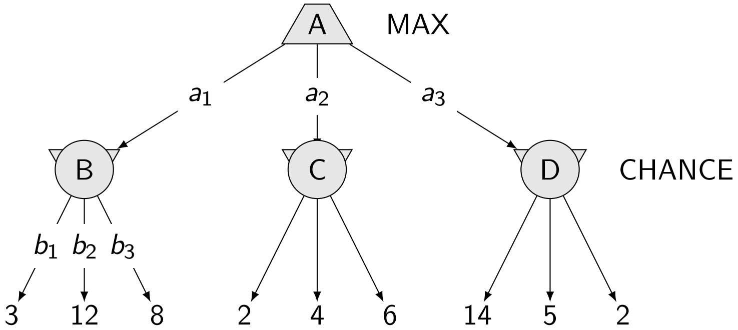

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Why? Actions may fail, . . .

Video: Slipping robot. Vision for Robotics and Autonomous Systems, http://cyber.felk.cvut.cz/vras, https://youtu.be/kvEEHNyCHMs

Why? Action costs not deterministic, . . . , getting to work

A At home

tram *bike* *car*

Random variable: Situation on rails R

r_1 free rails

r_2 accident

r_3 congestion

MAX/MIN depends on what the $r_?$ options and terminal numbers mean. The goal may be to get to work as fast as possible.

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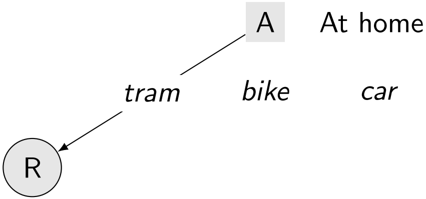

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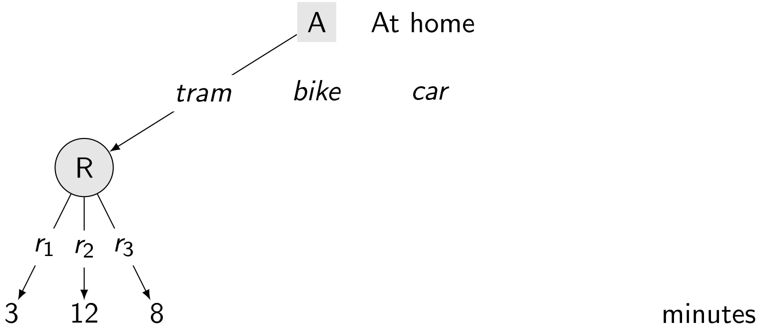

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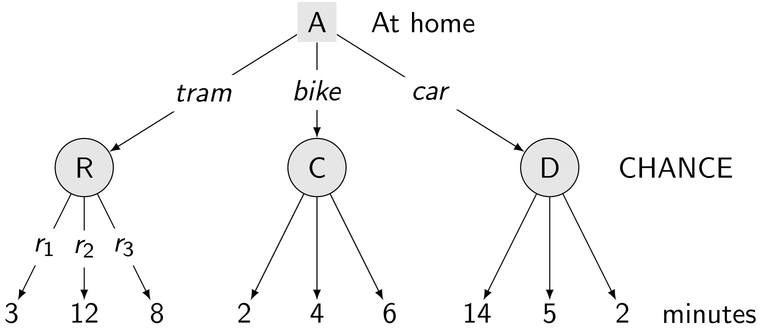

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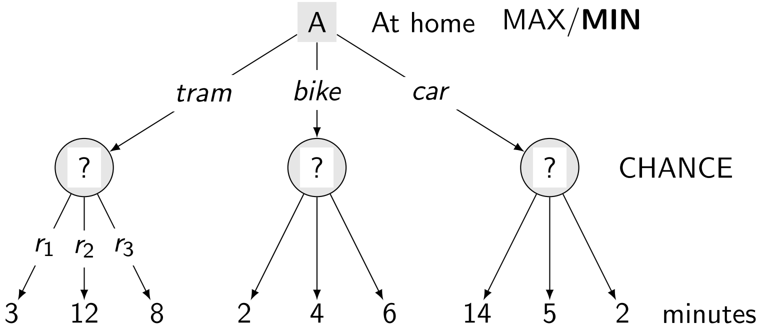

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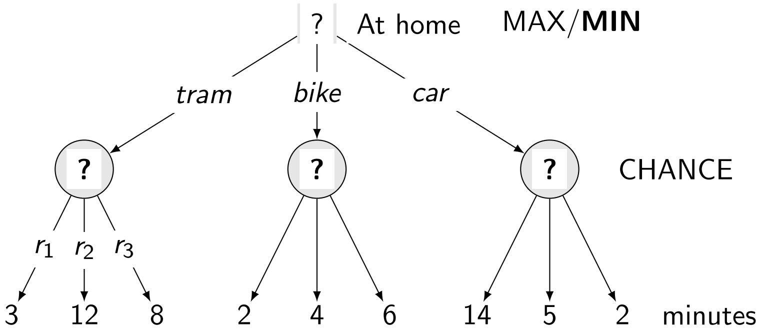

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- Calculate expected utilities . . .
- i.e. take weighted average (expectation) of successors

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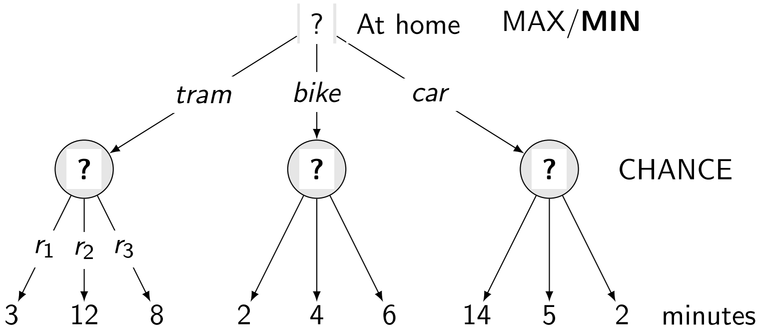

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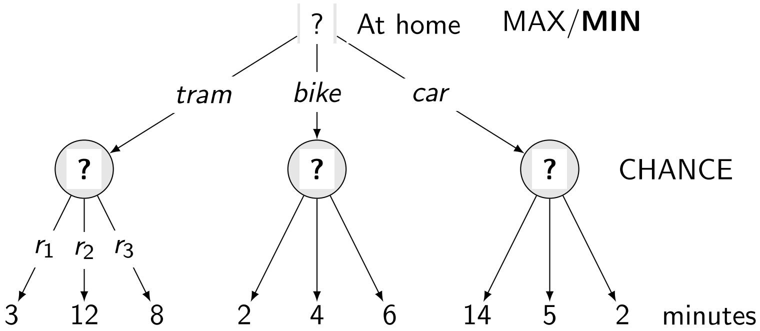

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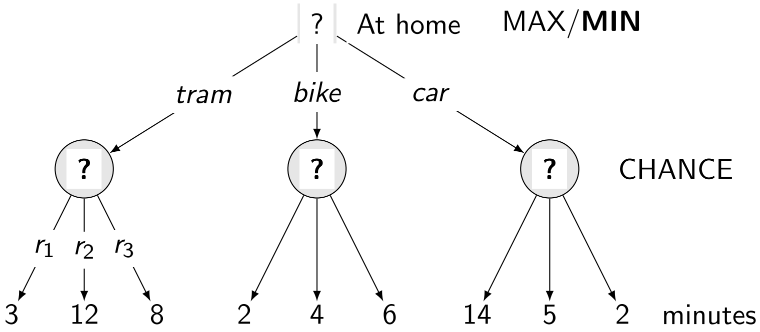

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Expectimax

function EXPECTIMAX(state) **return** a value
  if IS-TERMINAL(state): **return** UTILITY(state)
  if state (next agent) is **MAX**: **return** MAX-VALUE(state)
  if state (next agent) is **CHANCE**: **return** EXP-VALUE(state)
end function

---

function MAX-VALUE(state) **return** value v
  $v \leftarrow -\infty$
  for a **in** ACTIONS(state) **do**
    $v \leftarrow$ **max**(v, EXPECTIMAX(RESULT(state,a)))
  end for
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function EXP-VALUE(state) **return** value v
  $v \leftarrow 0$
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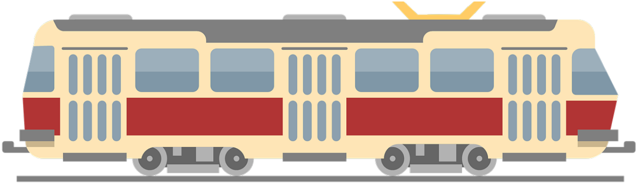
Random variables, probability distribution, . . .

- Random variable - an event with unknown outcome
- Probability distribution - assignment of weights to the outcomes



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Few reminders from laws of probability. Probabilities:

- always non-negative,
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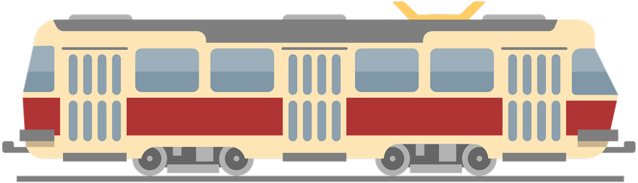

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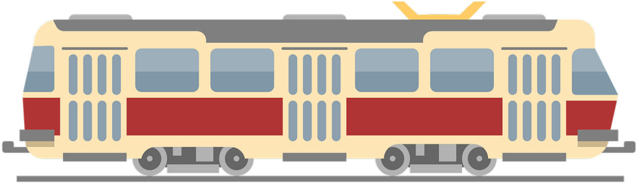

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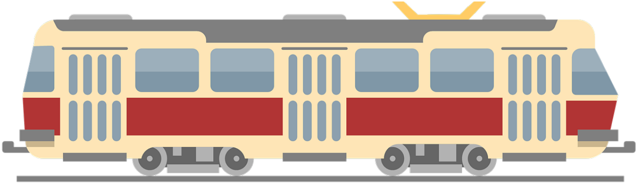

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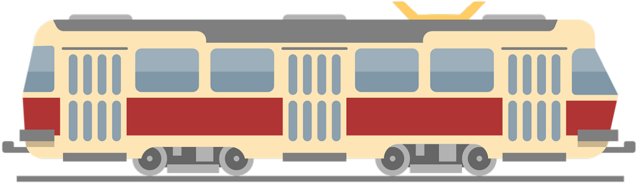

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Expectations, . . .

How long does it take to go to work by tram?

- Depends on the random variable R - situation on rails with possible events r_1, r_2, r_3.
- What is the expectation of the time?

$$t = P(r_1)t_1 + P(r_2)t_2 + P(r_3)t_3$$

Weighted average.

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- Is there any space for randomness?
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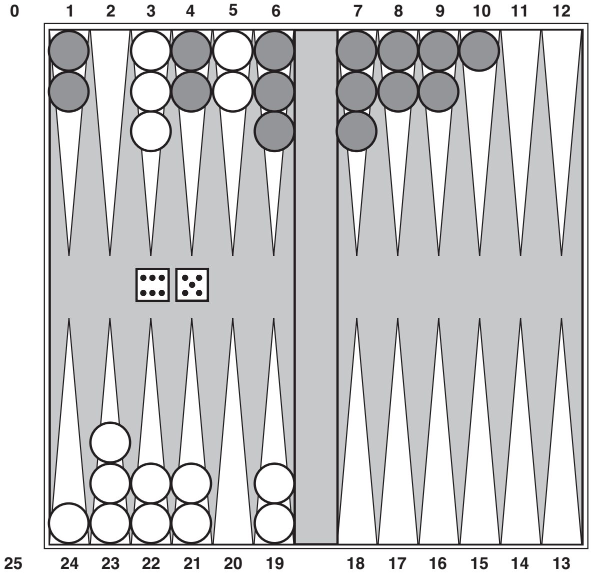
Games with chance **and** strategy



Random variable: Throwing two dice

Do we care which die comes first?

What is the probability of , ?[1]

A 1/24

B 1/36

C 1/18

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[1]Source of dice images: https://flyclipart.com/dice-clipart-tool-rolling-dice-clipart-248574

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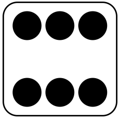
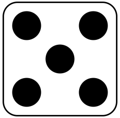
What is the probability of ,  ?[1]

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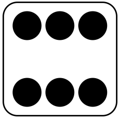
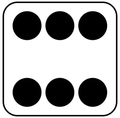
What is the probability of ,  ?[1]

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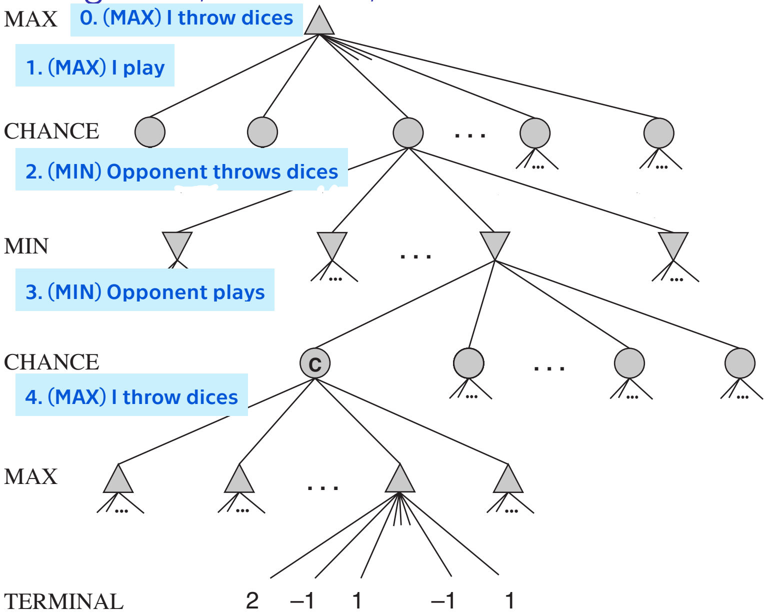
Mixing MAX, CHANCE, and MIN nodes



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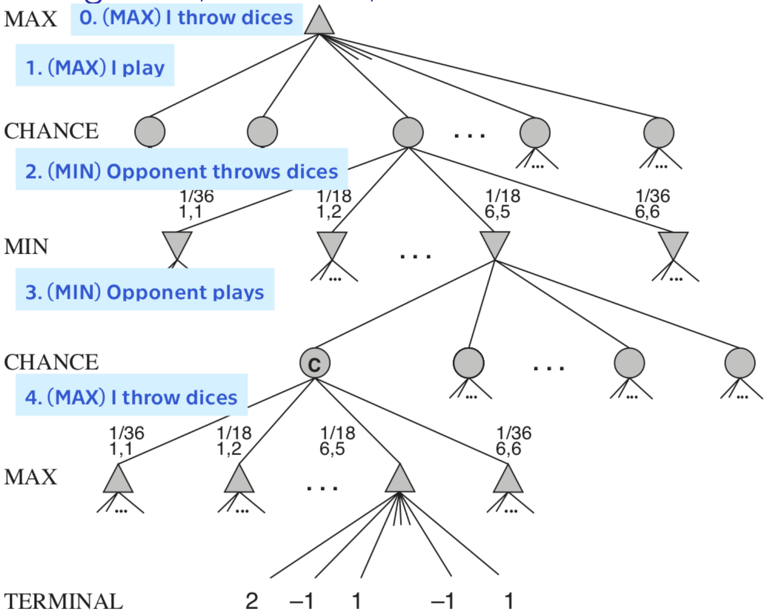

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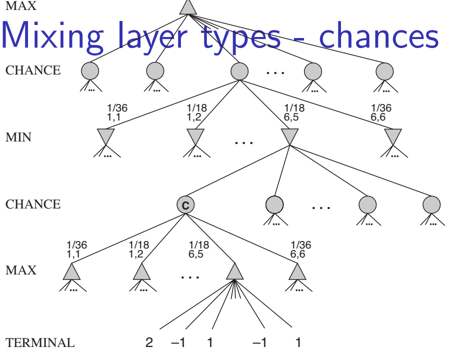

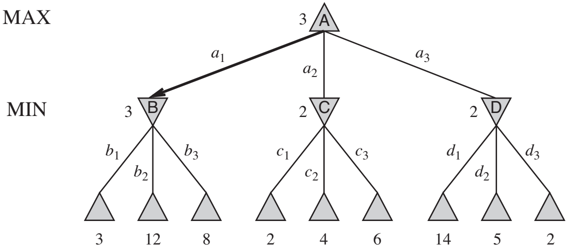

Extra random agent that moves after each MAX and MIN agent

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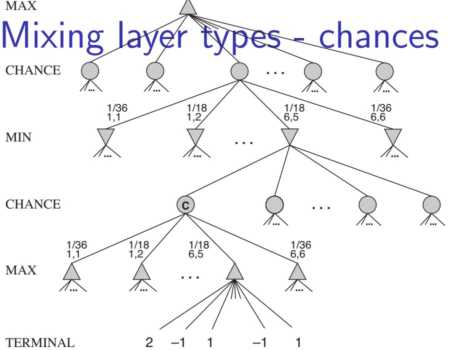

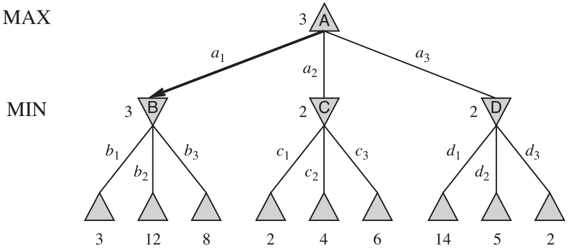

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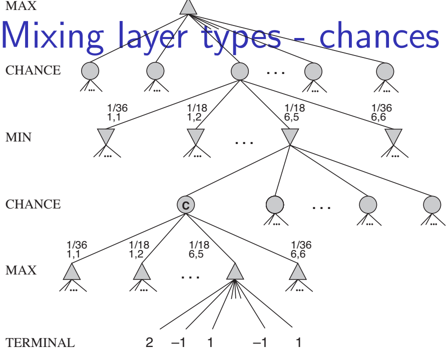

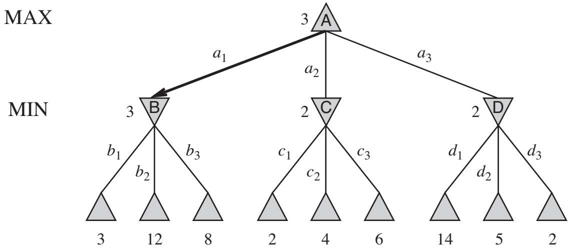

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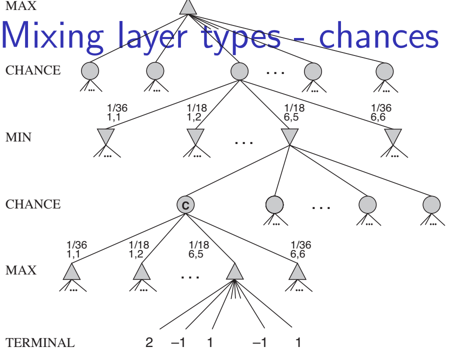

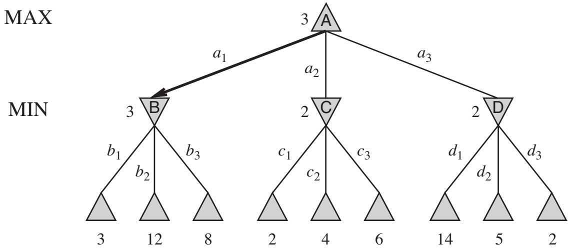

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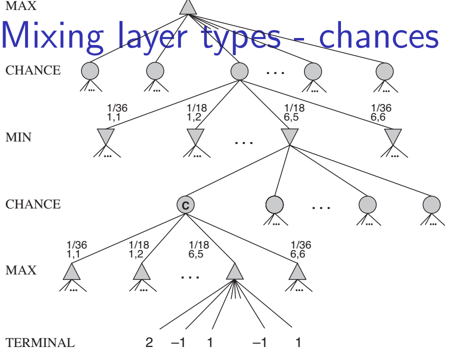

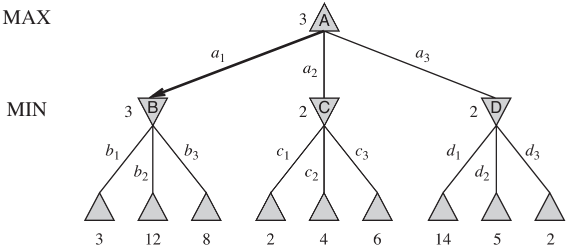

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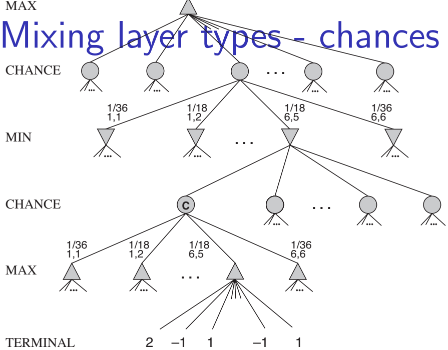

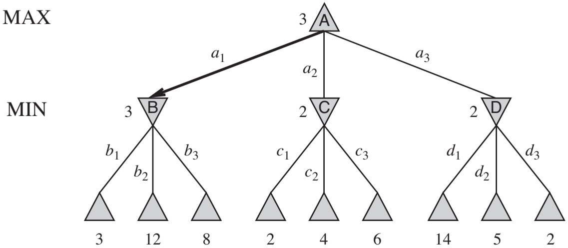

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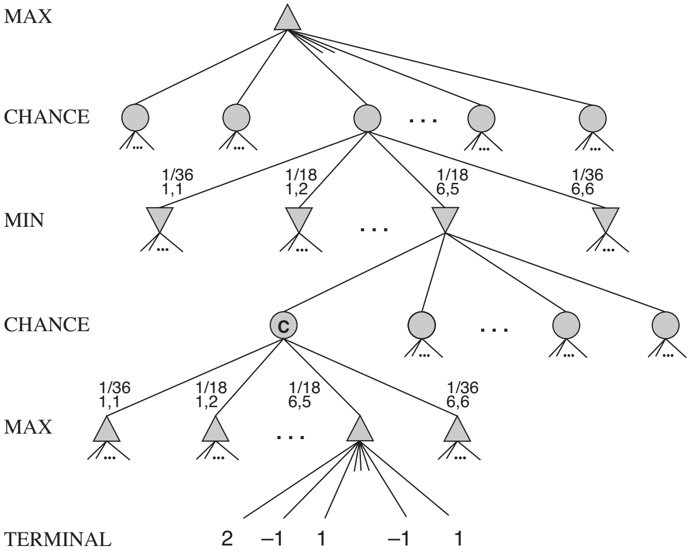
Mixing chance into min/max tree. How big is the tree going to be?



- b branching factor
- m maximum depth
- n number of distinct rolls

What is the time complexity of EXPECTIMINIMAX?

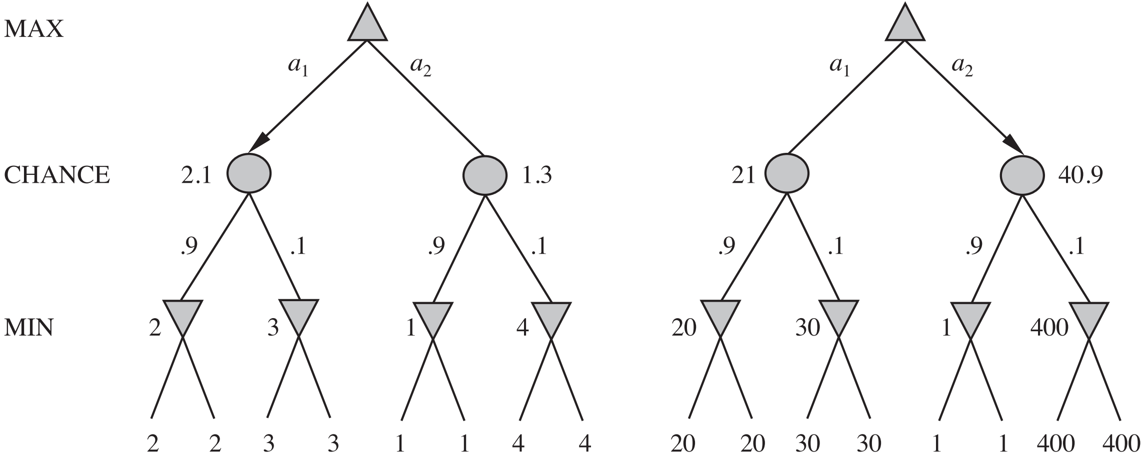
A $O(b^{mn})$

B $O(b^m n)$

C $O(b^m n^b)$

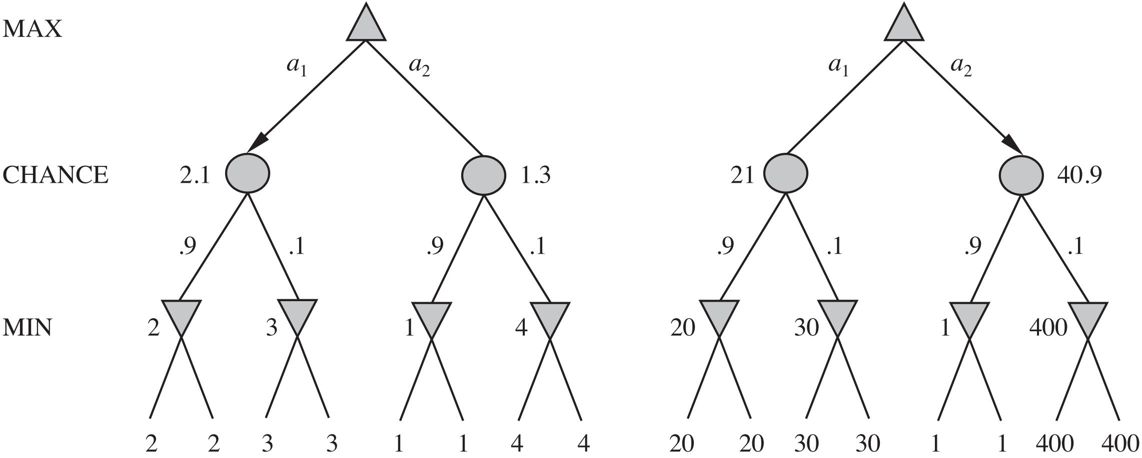
D $O(b^m n^m)$

Evaluation function



- Left: a_1 is the best. Right: a_2 is the best. Ordering of the (terminal) leaves is the same.
- Scale matters! Not only ordering.
- Can we prune the tree? (α, β like?)

Evaluation function



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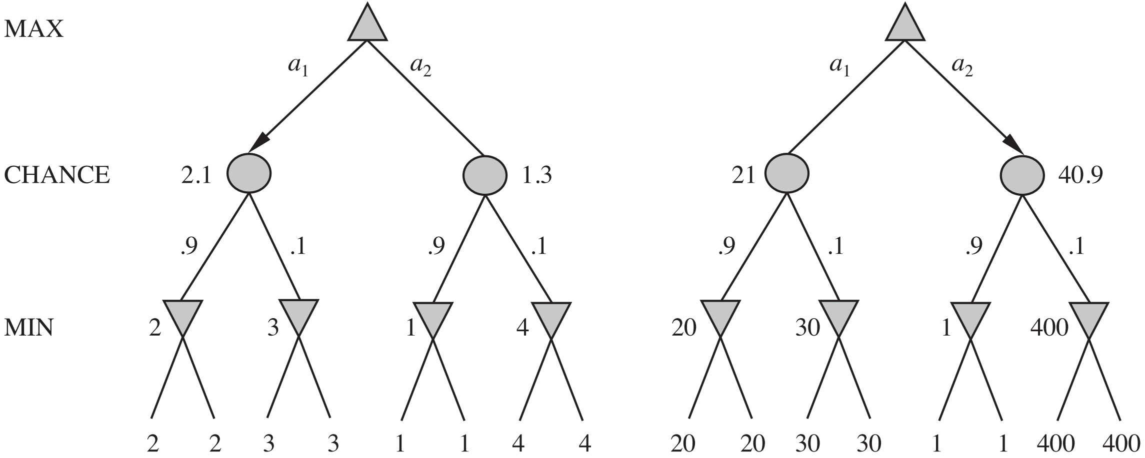

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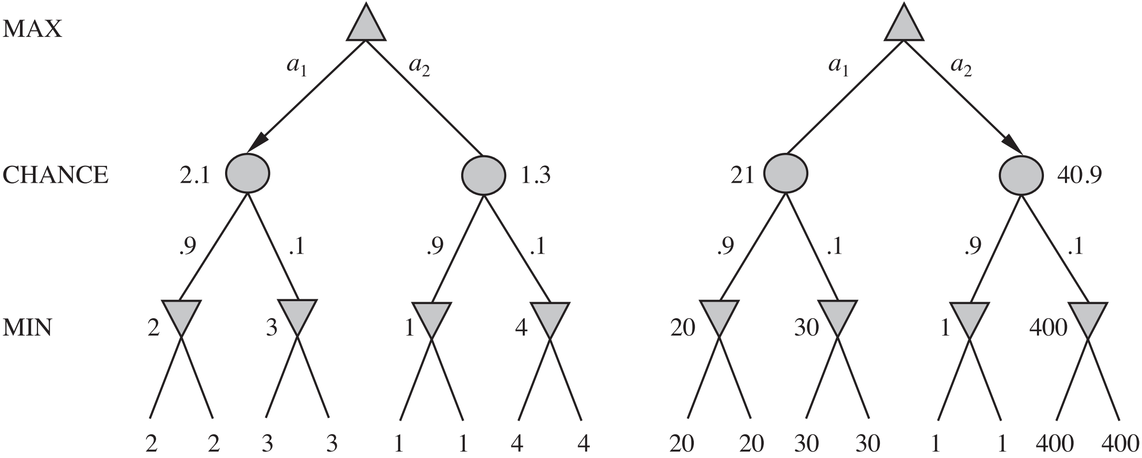

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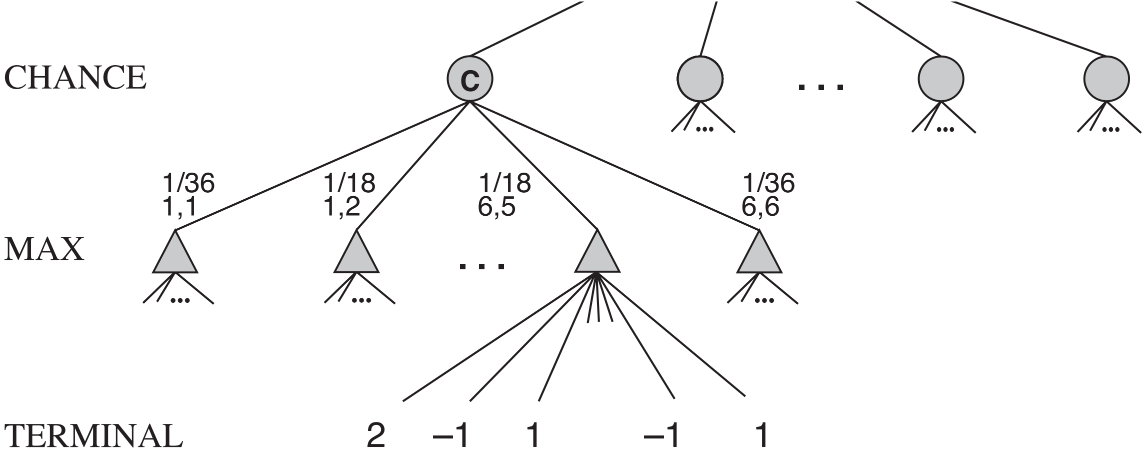
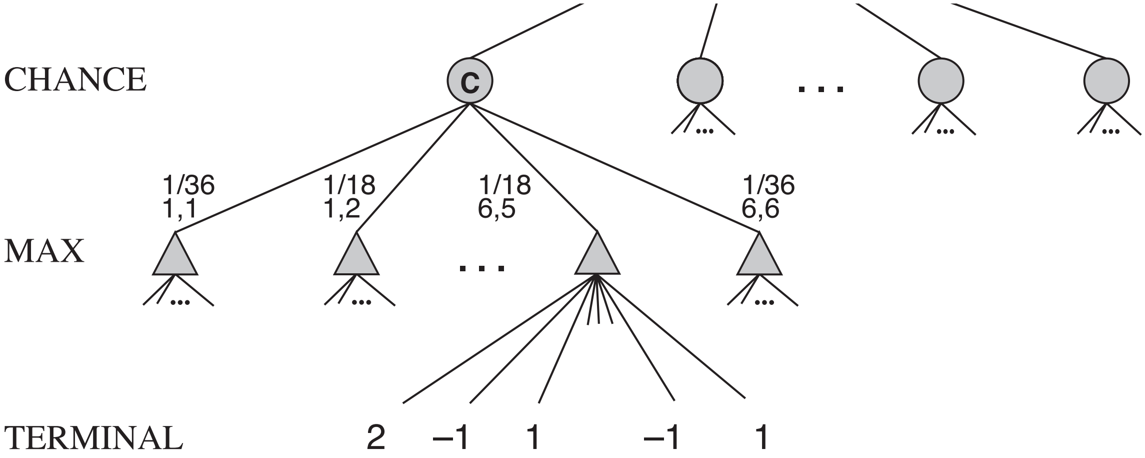
Pruning expectiminimax tree



- Bounds on terminal utilities needed. Terminal values from −2 to 2.
- Monte Carlo simulation for evaluation of a position (state).

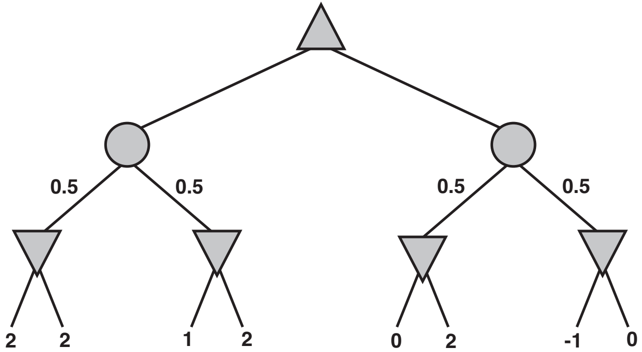
Pruning expectiminimax tree

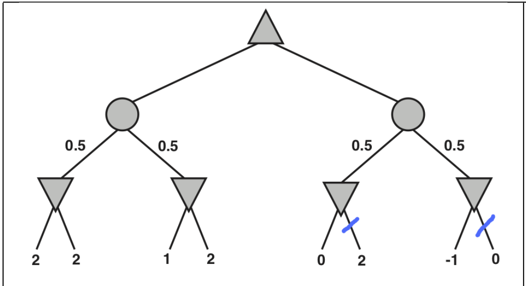


- Bounds on terminal utilities needed. Terminal values from −2 to 2.
- Monte Carlo simulation for evaluation of a position (state).

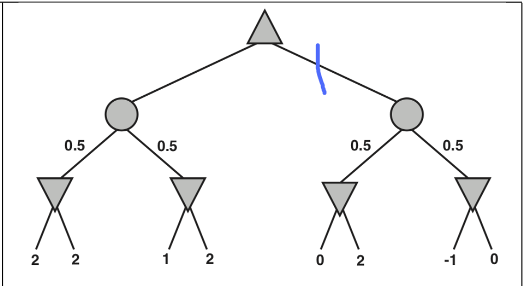
Where to prune the Expectimax tree

- Assume terminal nodes bounded to -2 to 2, inclusive
- Going from left to right.
- Which branches can be pruned out?

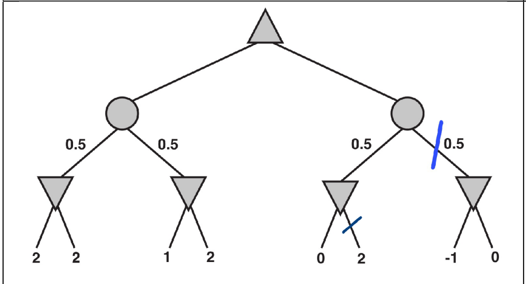




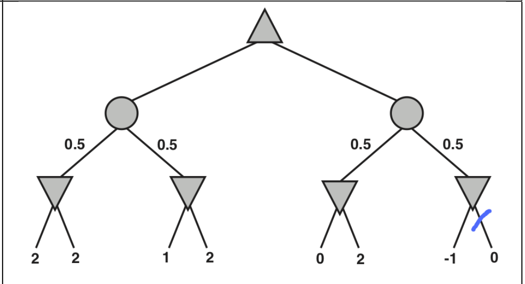
A



B



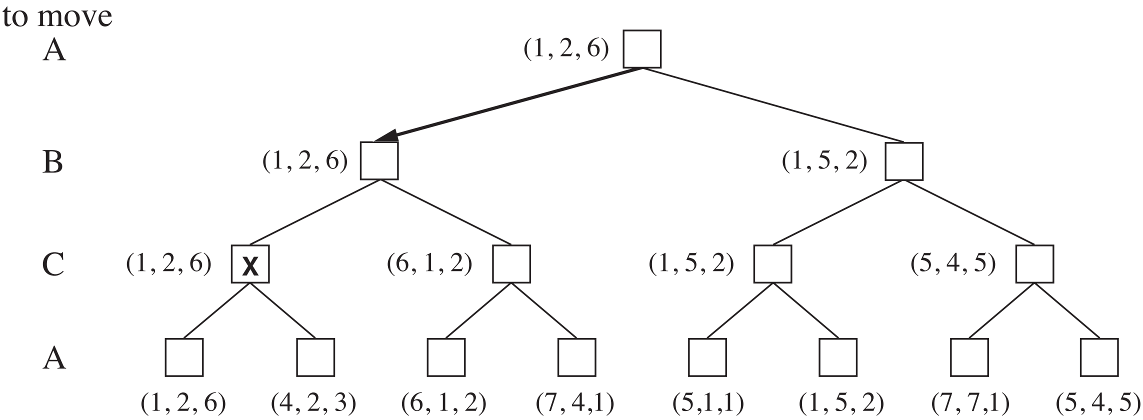
C



D

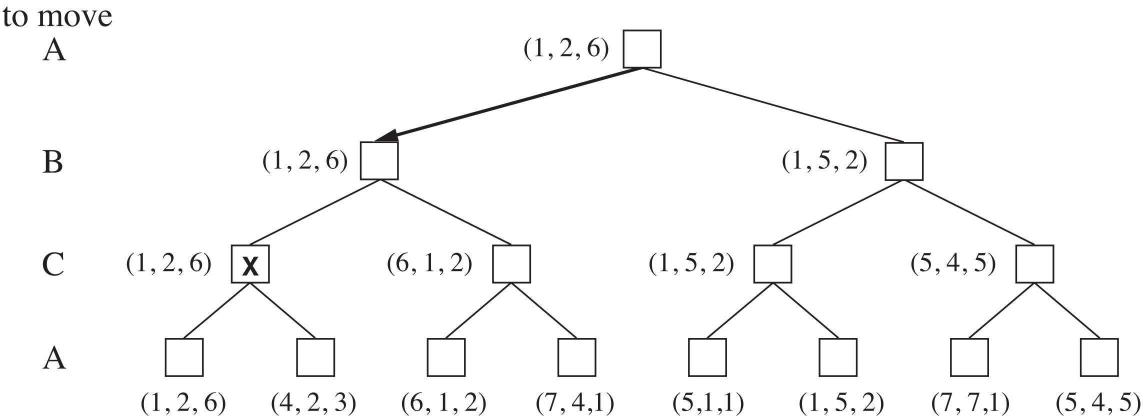
Assume terminal nodes bounded to -2 to 2, inclusive. Going from left to right.

Multi-player games



- Utility tuples
- Each player maximizes its own
- Coalitions, cooperations, competitions may be dynamic

Multi-player games



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Uncertainty recap (enough games, back to the robots/agents)

- Uncertain outcome of an action.
- Robot/Agent may not know the current state!

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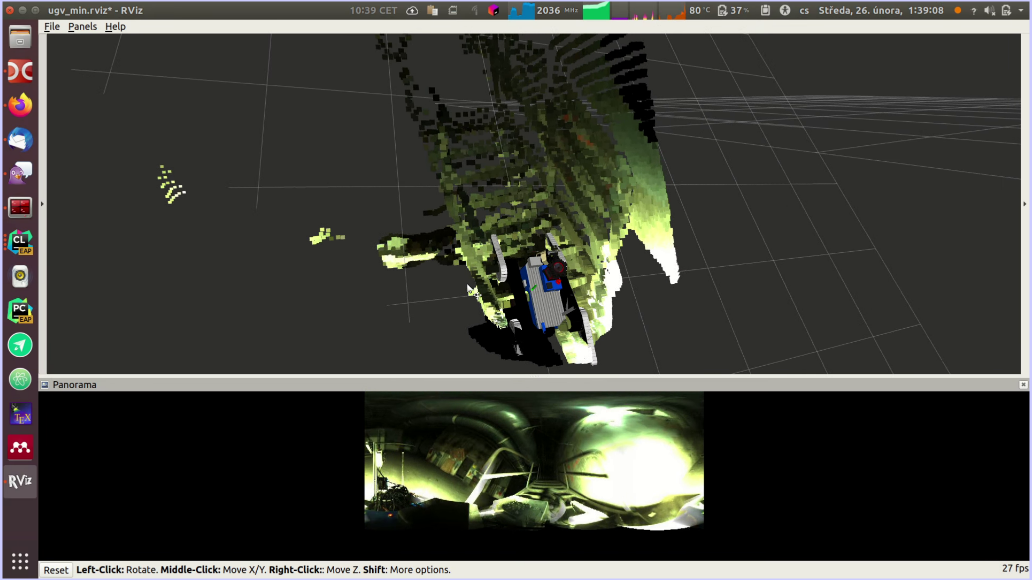


Video: Climbing stairs failure, From: http://robotics.fel.cvut.cz/cras/darpa-subt/

Uncertain, partially observable environment

- Current state s may be unknown, observations $\mathbf{e}$
- Uncertain outcome, random variable $\textsc{Result}(a)$
- Probability of outcome s' given $\mathbf{e}$ is

$$P(\textsc{Result}(a) = s' | a, \mathbf{e})$$

- Utility function $U(s)$ corresponds to agent preferences.
- Expected utility of an action a given $\mathbf{e}$:

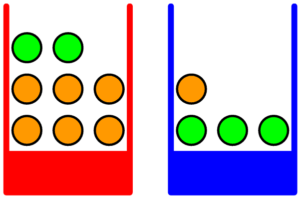
$$EU(a|\mathbf{e}) = \sum_{s'} P(\textsc{Result}(a) = s' | a, \mathbf{e}) U(s')$$

Amatrice, Italy, 2016.

About (conditional) probabilities

Two kind of boxes in a dark warehouse. State – box type color – is not directly observable.

- red box: 2 apples, 6 oranges
- blue box: 3 apples, 1 orange

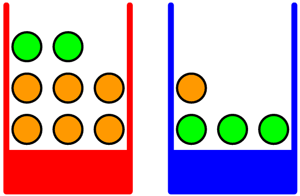


- Scenario: Pick a box at random. *Then* pick a fruit at random.
- (Frequent) questions:
  - What is the overall probability that the selection procedure will pick an apple?
  - Given that we have chosen an orange, what is the probability that it was from the blue box?

Example from Chapter 1.2 [1]

Picking fruits. What is the probability that . . . ?

- red box: 2 apples, 6 oranges
- blue box: 3 apples, 1 orange



Procedure: Pick a box (say red box in 40% cases), then pick a fruit at random.

Quiz 1: What is the probability that the selection procedure will pick an apple?

A: 11/20

B: 6/8

C: 1/2

D: Different value.

Picking fruits. What is the probability that . . . ?

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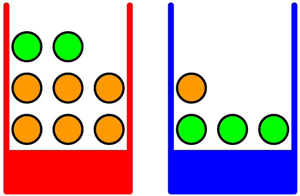

Procedure: Pick a box (say red box in 40% cases), then pick a fruit at random.

Quiz 2: Given that we have chosen an orange, what is the probability that it was from the blue box?

A: 1/4

B: 3/5

C: 1/3

D: Different value.

Rational agent

Agent's expected utility of an action a given $\mathbf{e}$:

$$EU(a|\mathbf{e}) = \sum_{s'} P(\text{RESULT}(a) = s'|a, \mathbf{e})U(s')$$

What should a rational agent do?

Is it then all solved? Do we know all what we need?

- $P(\text{RESULT}(a) = s'|a, \mathbf{e})$
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Utilities

- Where do utilities come from?
- Does averaging make sense?
- Do they exist?
- What if our preferences can't be described by utilities?

Agent/Robot Preferences

- Prizes A, B
- Lottery: uncertain prizes $L = [p, A; (1-p), B]$

Preference, indifference, ...

- Robot prefers A over B: $A \succ B$
- Robot has no preferences: $A \sim B$
- in between: $A \succsim B$

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Rational preferences

- Transitivity: $(A \succ B) \wedge (B \succ C) \Rightarrow (A \succ C)$
- Completeness: $(A \succ B) \vee (B \succ A) \vee (A \sim B)$
- Continuity: $(A \succ B \succ C) \Rightarrow \exists p \, [p, A; 1-p, C] \sim B$
- Substituability: $A \sim B \Rightarrow [p, A; 1-p, C] \sim [p, B; 1-pC]$. The same for $\succ$ and $\sim$.
- Monotonocity: $A \succ B \Rightarrow (p > q) \Leftrightarrow [p, A; 1-p, B] \succ [q, A; 1-q, B]$. Agent must prefer a lottery with higher chance to win.
- Decomposability, compressing compound lotteries into one: $[p, A; 1-p, [q, B; 1-q, C]] \sim [p, A; (1-p)q, B; (1-p)(1-q), C]$

Axioms of utility theory.

Motivation: if agent/robot violates an axiom $\Rightarrow$ irrational agent/robot.

Rational preferences

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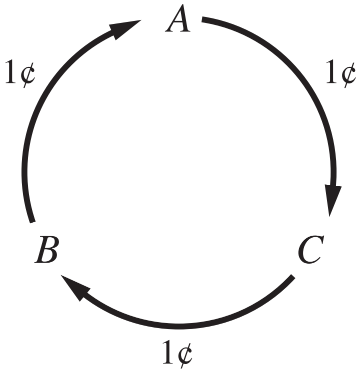
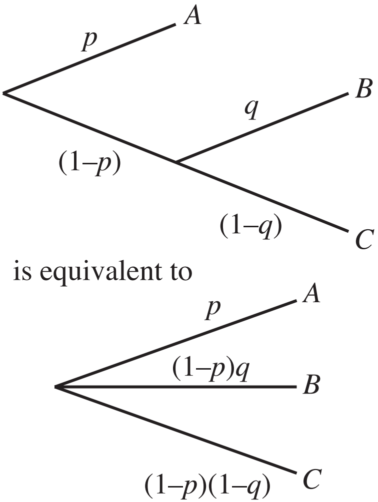
Axioms of utility theory.
Motivation: if agent/robot violates an axiom $\Rightarrow$ irrational agent/robot.

Transitivity and decomposability

Goods A, B, C and (nontransitive) preferences of an (irrational) agent $A \succ B \succ C \succ A$.



(a)



(b)

Maximum expected utility principle

Given the rational preferences (constraints), there exists a real valued function u such that:

$$u(A) > u(B) \quad \Leftrightarrow \quad A \succ B$$
$$u(A) = u(B) \quad \Leftrightarrow \quad A \sim B$$

Expected utility of a Lotery L (outcomes s_i with probabilities p_i):

$$L([p_1, S_1; \cdots ; p_n, S_n]) = \sum_i p_i u(S_i)$$

Proof in [5].
Is a utility u function unique?

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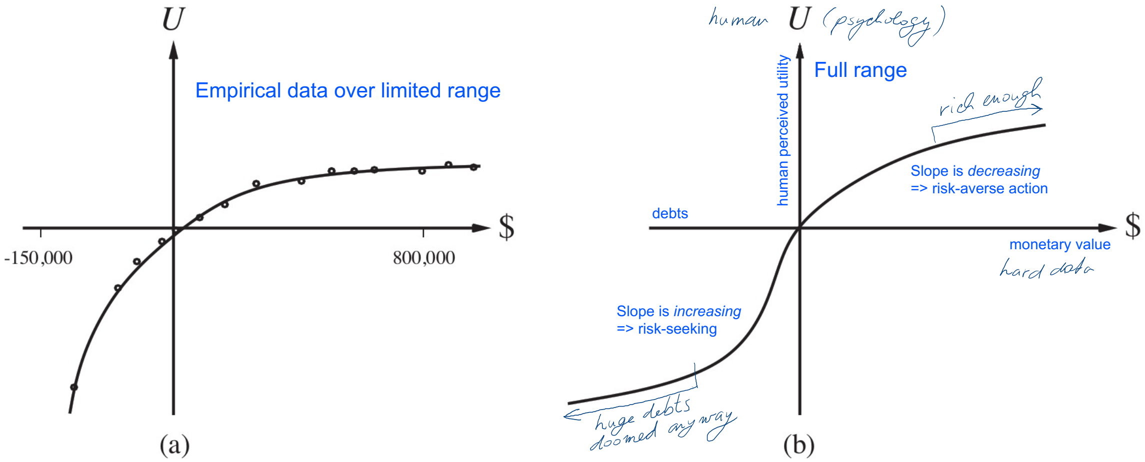
Human utilities

Utility of money

You triumphed in a TV show!

a) Take $\$1,000,000$ ... or
b) Flip a coin and loose all or win $\$2,500,000$

Utility of money: human psychology vs. hard data



References I

Some figures from [3], Chapters 5, 16. Human utilities are discussed in [2]. This lecture has been also greatly inspired by the 7th lecture of CS 188 at http://ai.berkeley.edu as it convenietly bridges the world of deterministic search and sequential decisions in uncertain worlds.

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