Problem solving by search

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Notes

Outline

- ▶ Search problem.
- ▶ State space graphs.
- ▶ Search trees.
- ▶ Strategies: which tree branches to choose?
- ▶ Strategy/Algorithm properties.
- ▶ Programming infrastructure.

Notes

Example: Traveling in Romania
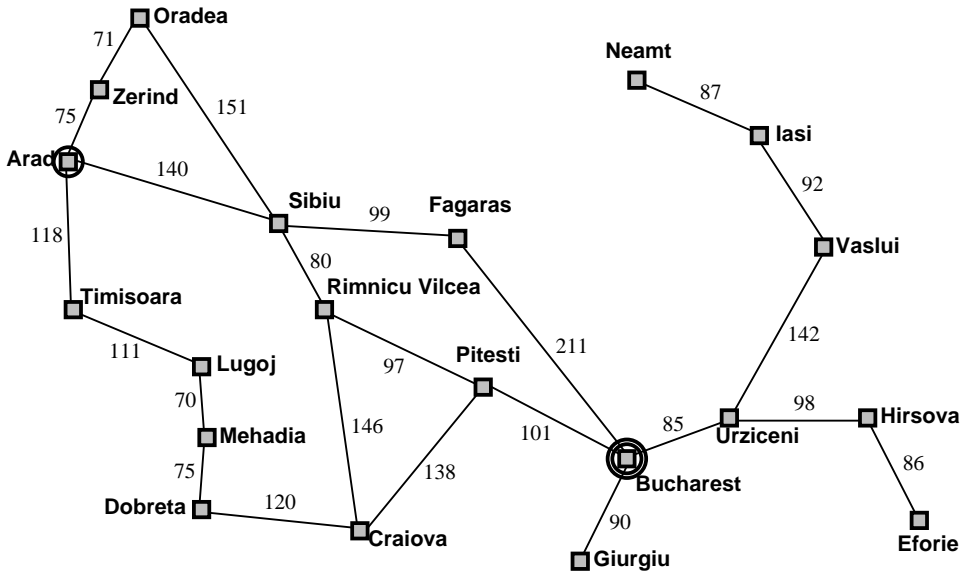
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Ok, start with a simple one, almost everybody knows about the navigation - path planning problem. Waze, Garmin, . . .

Can you think about more problems?

For example:

- Touring problems. Special case: Traveling salesperson problem – each city must be visited exactly once.
- Planning robot movements – mobile robot or manipulator.
- VLSI (chip) layout.
- ...

Example: Map of Romania

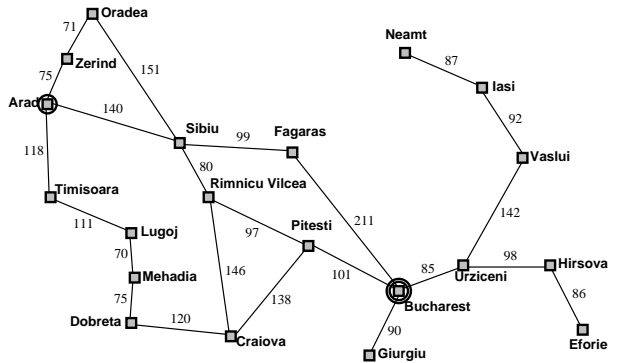### Goal:

be in Bucharest

Problem formulation:

states: position in a city (cities)

actions: drive between cities

Solution:

Sequence of cities (path)

(action sequence [2])

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Classical problem from the Book [2], we use it, too.

states and actions will be frequently discussed in several lectures and algorithms. It is important to fully understand them.

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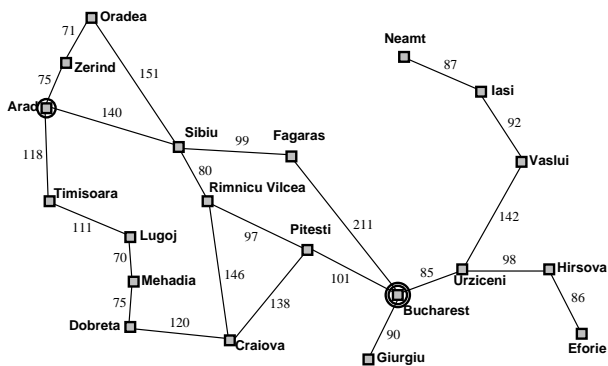
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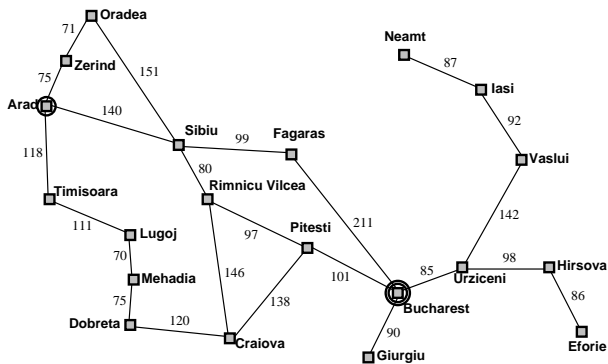   be in Bucharest

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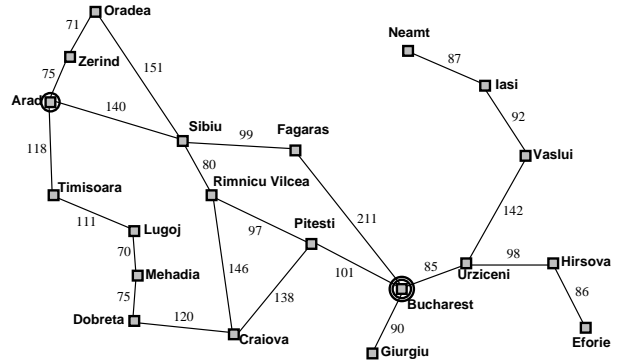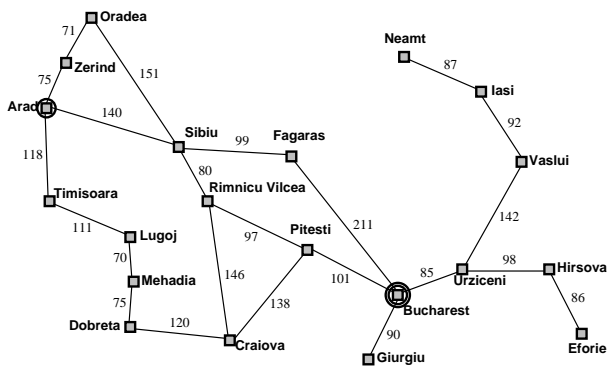
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Example: The 8-puzzle

7	2	4
5		6
8	3	1

Start State

1	2	3
4	5	6
7	8	

**Goal State**

states?
actions?
solution?
cost?

Notes

Also known as $n-1$ puzzle.

- States: Location of each of the 8 tiles and the blank.
- Number of states: 9!
- Initial state: any state. (Note that any given goal state can be reached from exactly half of the initial states.)
- Actions: Movements of the blank space: Left, Right, Up, Down (or a subset of these)
- Solution / goal test: Check whether state matches the goal configuration.
- Path cost: nr. steps in the path (each step costs 1)

Toy problem (3.2.1) from [2].

Example: Vacuum cleaner
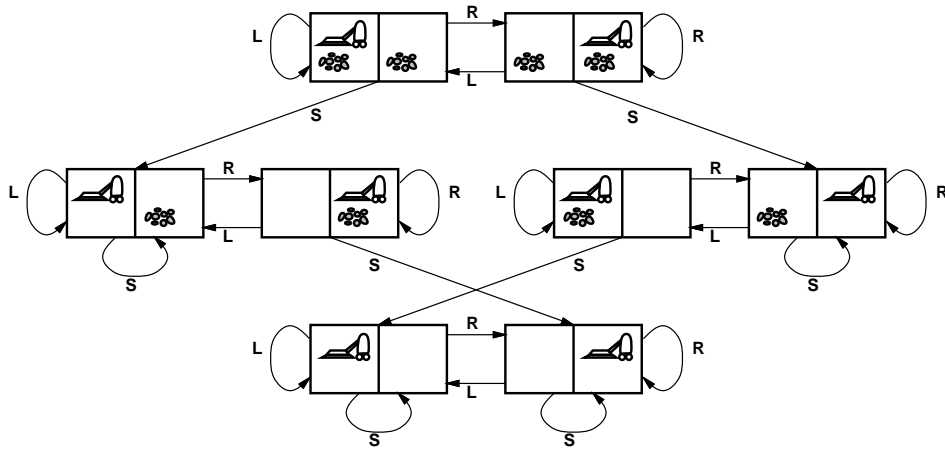
states?
actions?
solution?
cost?

─── **Notes** ───

- States: Determined by agent location and dirt location. The agent is in one of two locations, each of which may or may not contain dirt.

- Number of states: 2×2^2 (two possible choices for agent location; for every location, choice dirt vs. no dirt). For n locations: $n \times 2^n$

- Initial state: any state

- Actions: Left, Right, Suck (larger envs. can have also Up and Down)

- Solution / goal test: Are all squares clean?

- Path cost: nr. steps in the path (each step costs 1)

Toy problem (3.2.1) from [2].

A Search Problem

▶ State space (including Start/Initial state): position, board configuration,

▶ Actions : drive to, Up, Down, Left ...

▶ Transition model : Given state and action return state (and cost)

▶ Goal test : Are we done?

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We will use the terminology throught the next 5-6 lectures; also for Markov (Sequential) Decision Processes, Reinforcement Learning.

Make a mental test: You are a robot, going from home to school. What would be states, actions, transition model, goal test?

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State Space Graphs

State space graph: a representation of a search problem

- ▶ Graph Nodes – states – are abstracted world configurations
- ▶ Arcs represent action results
- ▶ Goal test – a set of goal nodes

Each state occurs only *once* in a state (search) space.
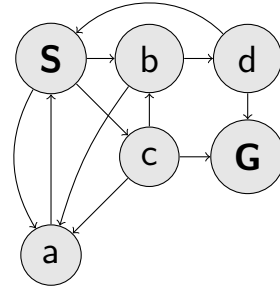
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Formalizing a real world problem – (creating) a state space graph – could be a problem in itself. I put creating into brackets as it may be also infinite.

Search Trees
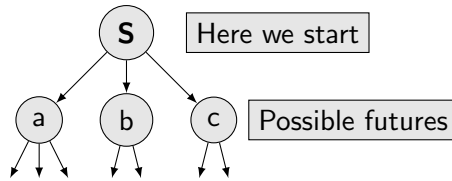
- ▶ A "what if" tree of plans and their outcomes
- ▶ Start node is the root
- ▶ Children are successors
- ▶ Nodes show/contains states, but correspond to *plans* that achieve those states

Notes

- What if decision about an action, repeats . . .
- Nodes in the search tree are not the same as the nodes in the state space graph.

State Space Graphs vs. Search Trees

How big is the search tree?

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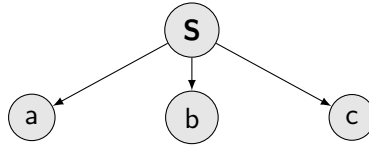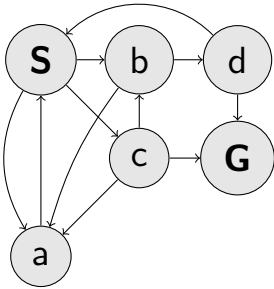
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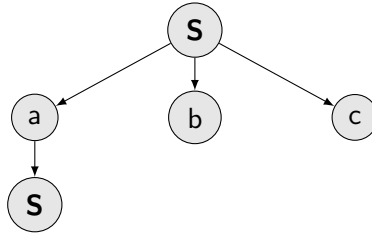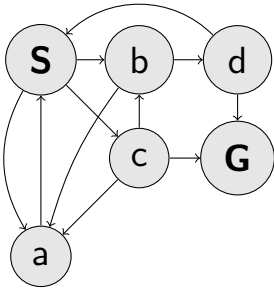
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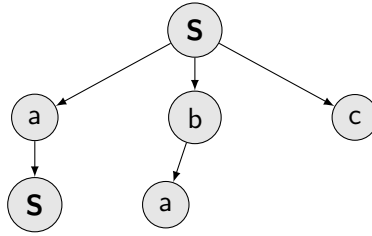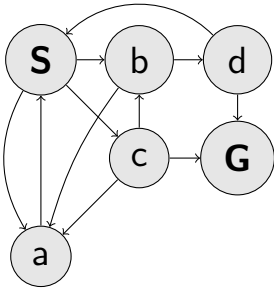
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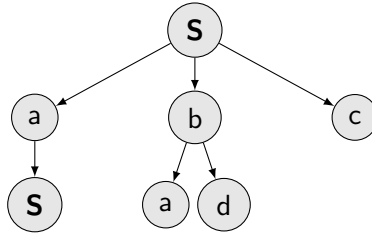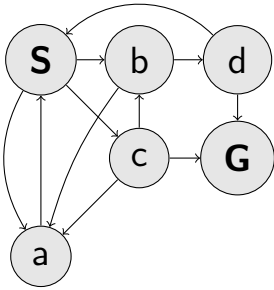
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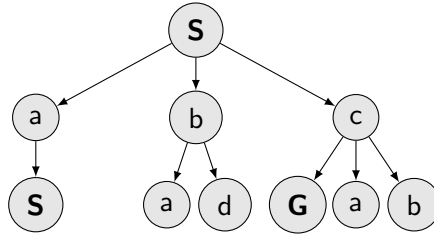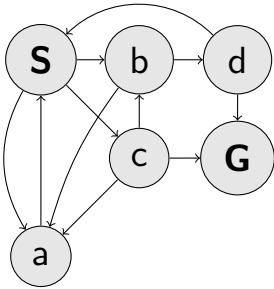
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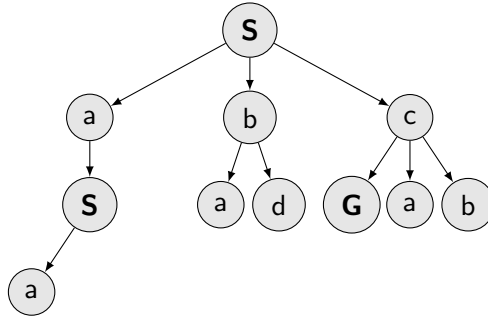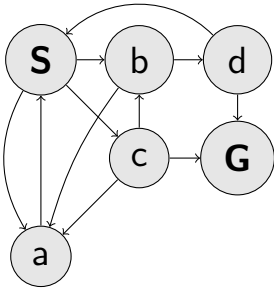
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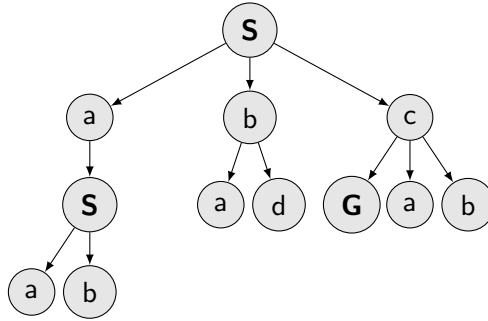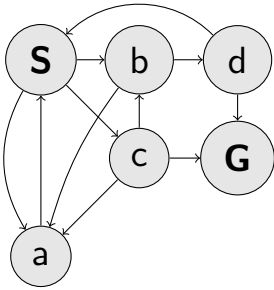
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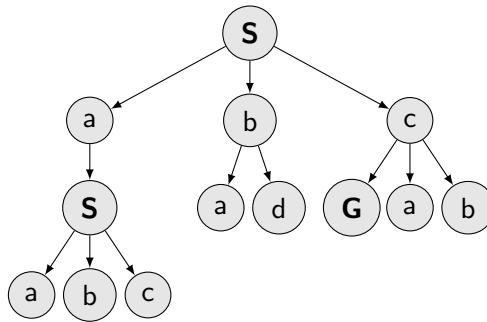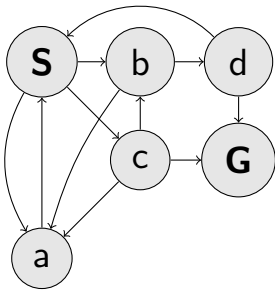
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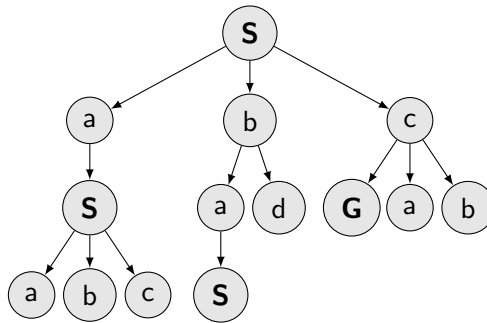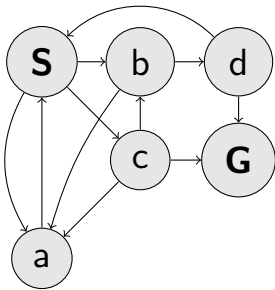
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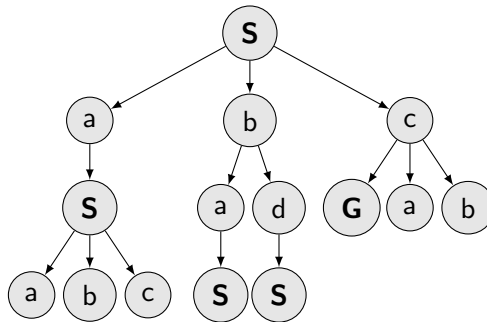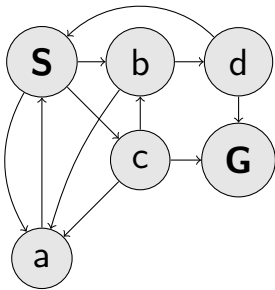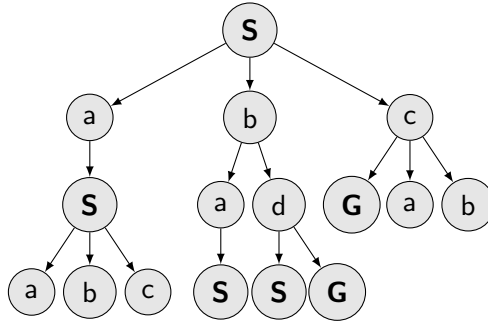
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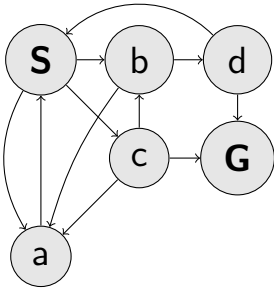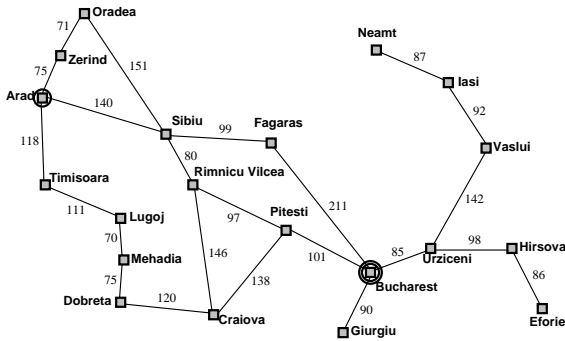
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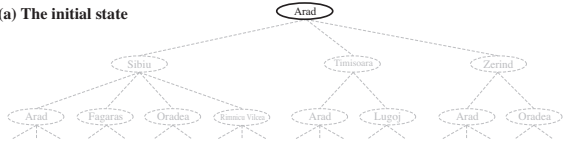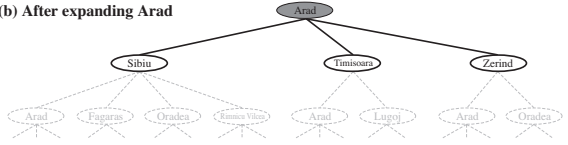# State Space Graphs vs. Search Trees

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Problem/transition graph is revealed incrementally.

The revealing strategy can be visualized as a search tree.

Notes

Images from [2].

Search elements - unvisited, dead, alive states

(c) After expanding Sibiu
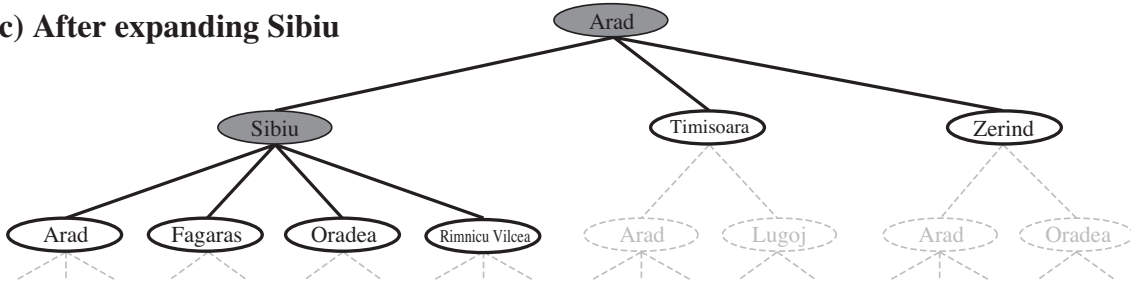
- ▶ Expand plans - possible ways (tree nodes).
- ▶ Manage/Maintain fringe (or frontier) of plans under consideration.
- ▶ Expand new nodes *wisely(?).*

Notes

Tree search algorithm

function TREE_SEARCH(problem) **return** a solution or failure
  initialize by using the initial state of the problem
  loop
    if no candidates for expansion then return failure
    else choose a leaf node for expansion
    end if
    if the node contains a goal state then return the solution
    end if
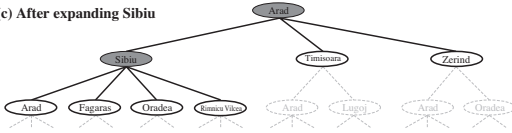   Expand the node and add the resulting nodes to the tree
  end loop
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A *general* tree search algorithm. Individual search algorithms vary primarily in how they choose which state to expand next – the "search strategy".

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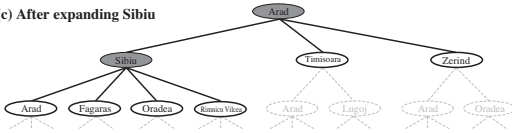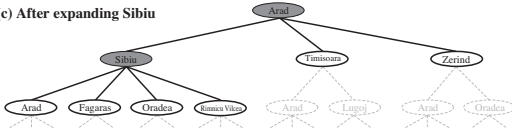   Expand the node and add the resulting nodes to the tree
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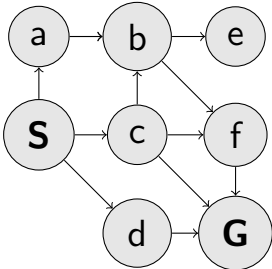   Expand the node and add the resulting nodes to the tree
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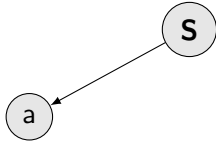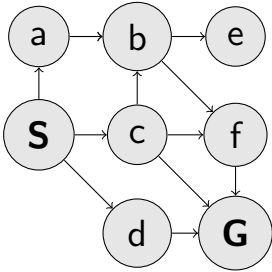
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Example of a tree search

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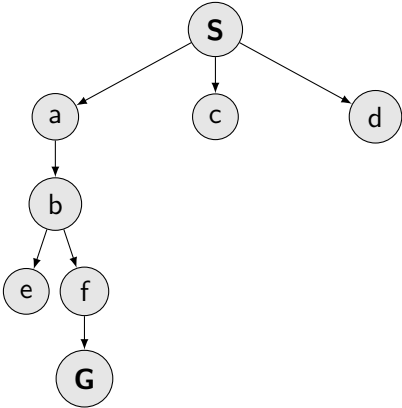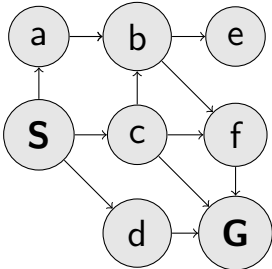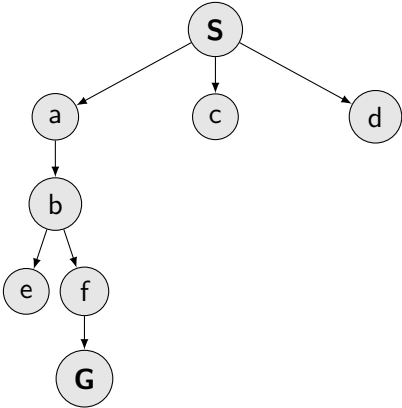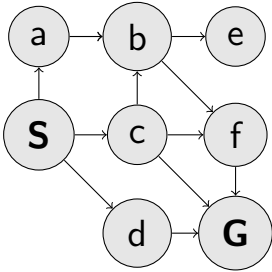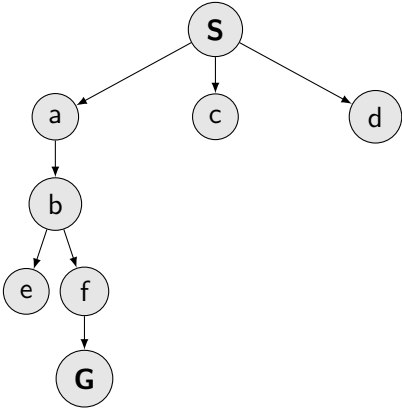
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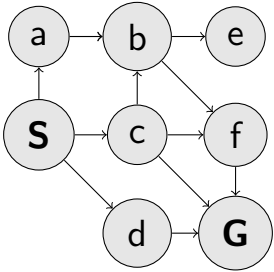
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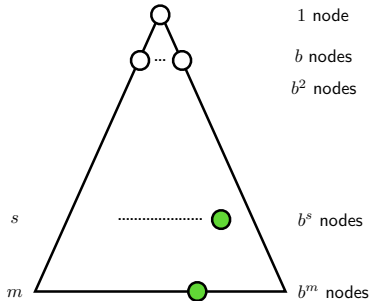
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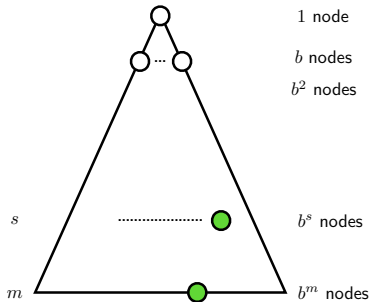
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How many nodes in a (search) tree? What are tree parameters?

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Draw a (symbolic–think about a triangle) sketch of a (search) tree. It may grow upwards or downwards. How would you characterize/parametrize *size* of a tree.

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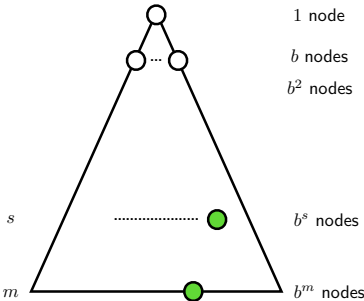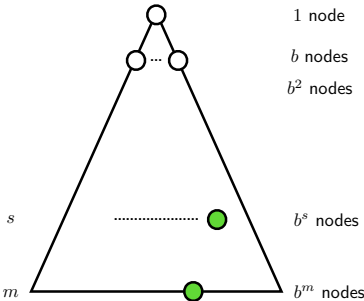
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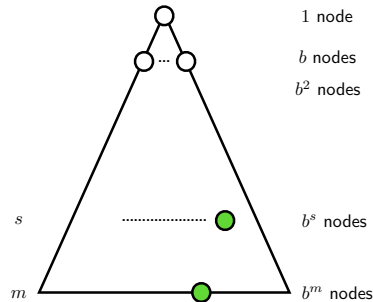
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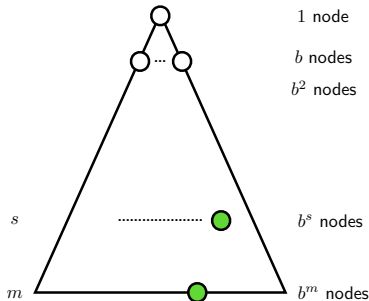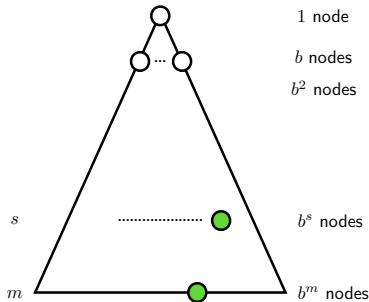
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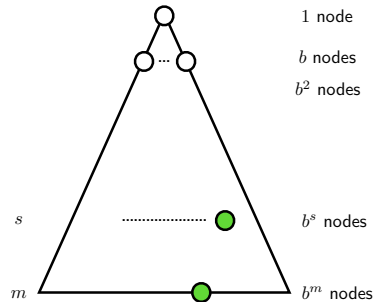
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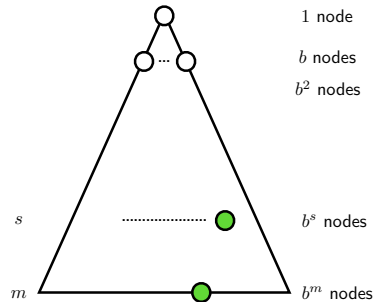
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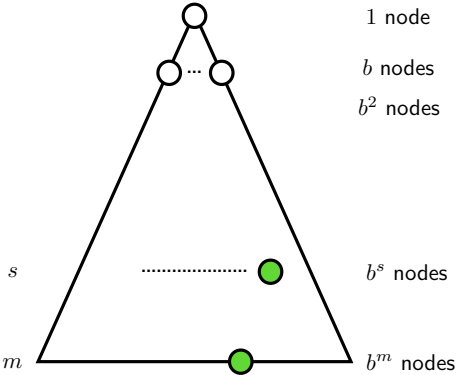
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Strategies

How to traverse/build a search tree?

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It is perhaps worth to remember that the search tree is built as the algorithm goes. Or better said, the tree is a human friendly representation of the machine run.

Depth-First Search (DFS)

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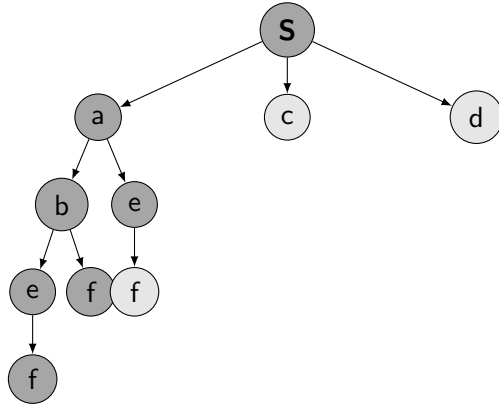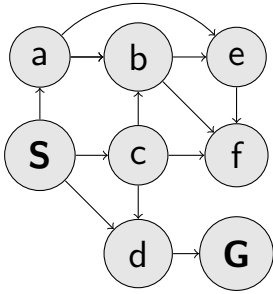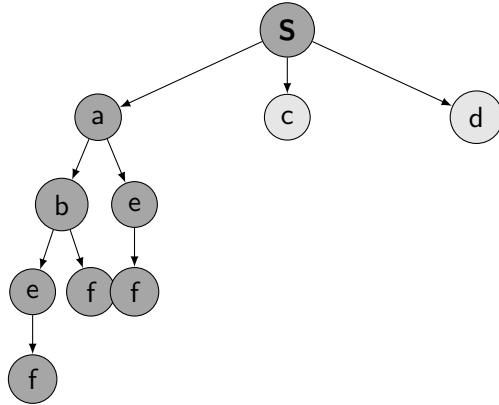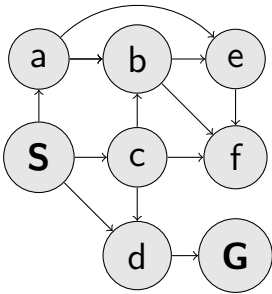
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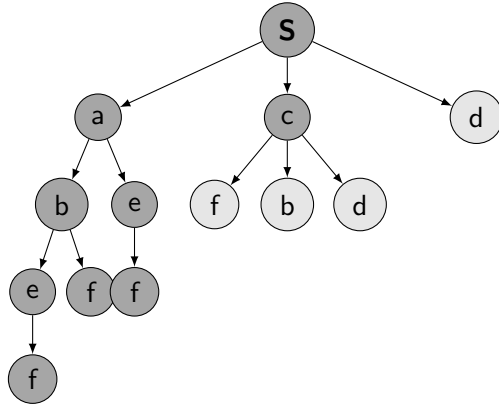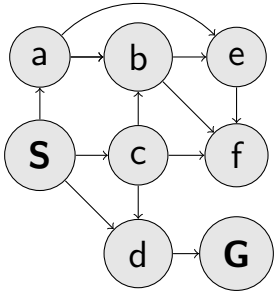
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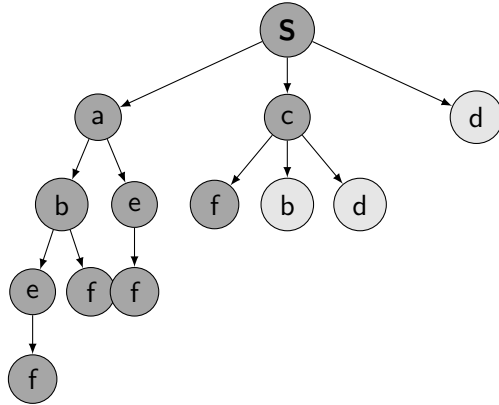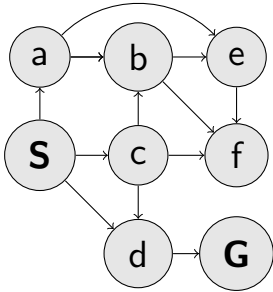
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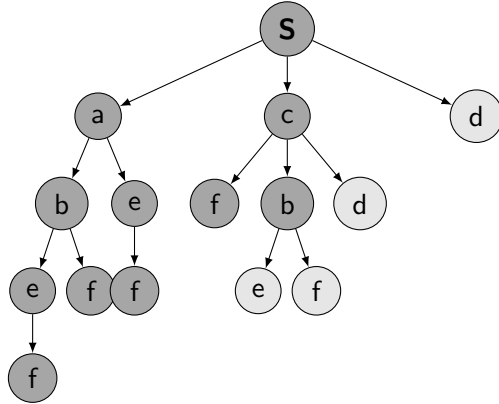
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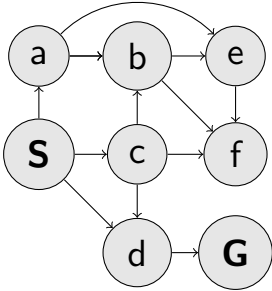
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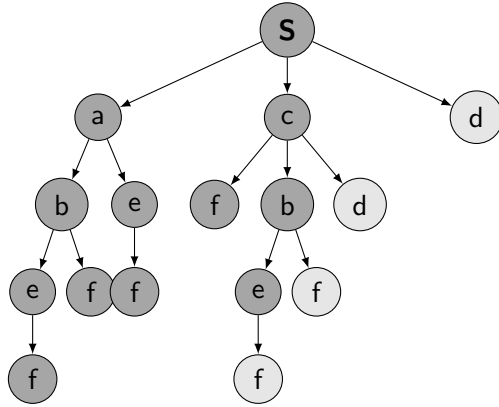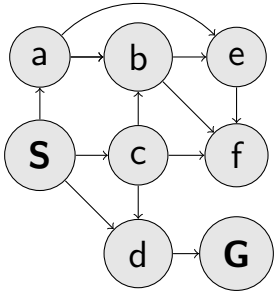
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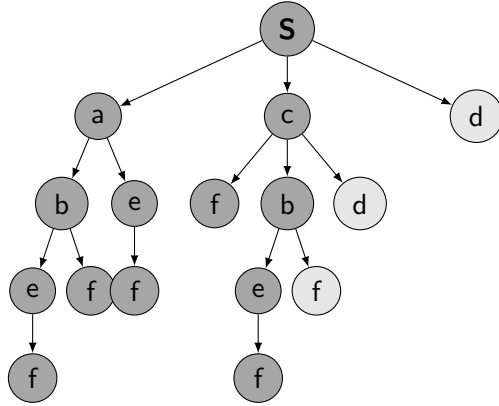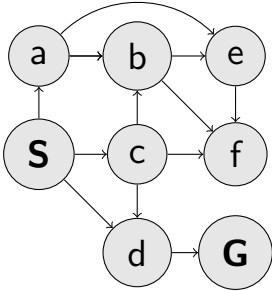
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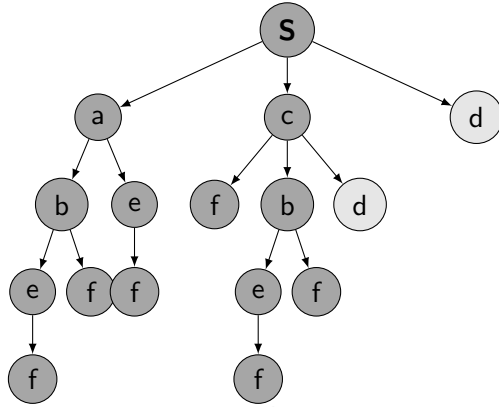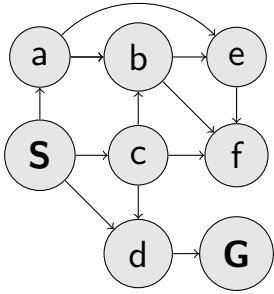
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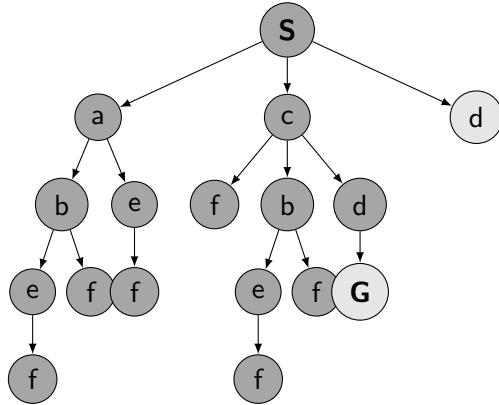- Thinking about optimality, what is the best solution we seek?
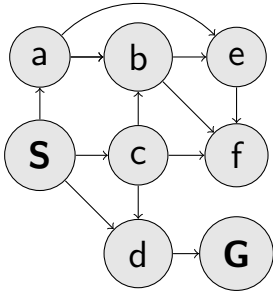
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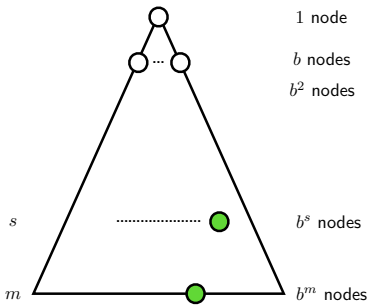- Thinking about optimality, what is the best solution we seek?

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DFS properties

▶ Time complexity?
▶ Space complexity?
▶ Complete?
▶ Optimal?

b, m, s, Time complexity?

A $\mathcal{O}(bm)$
B $\mathcal{O}(b^m)$
C $\mathcal{O}(m^b)$
D ∞
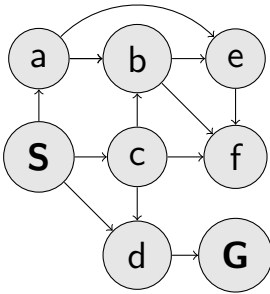
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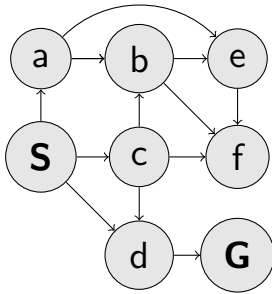- Optimality: No! It just takes the first solution found.

DFS properties

► Time complexity?

► Space complexity?

► Complete?

► Optimal?

b, m, s, Space complexity?

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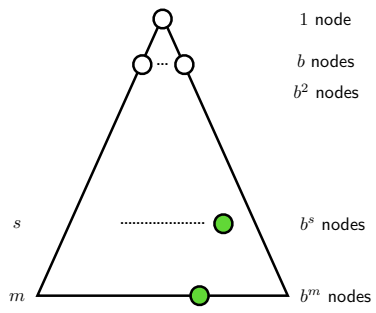
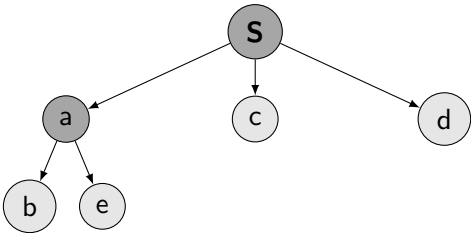# Breadth-First Search (BFS)
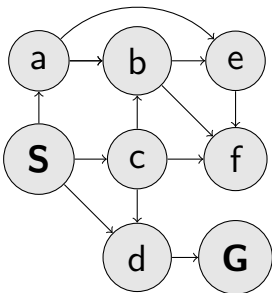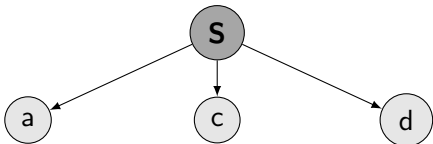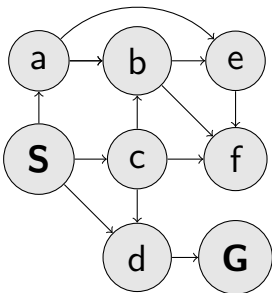
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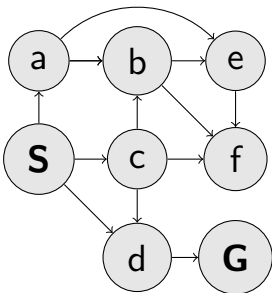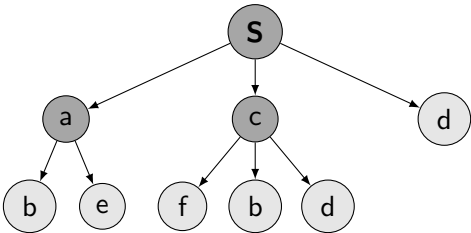# Breadth-First Search (BFS)

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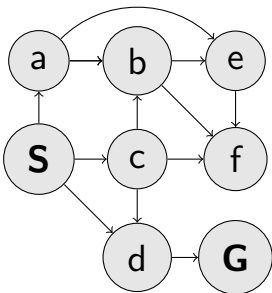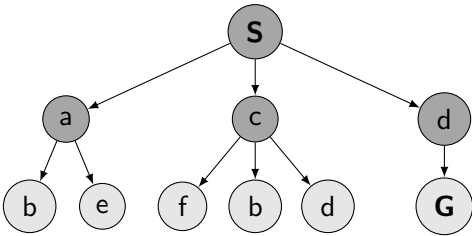
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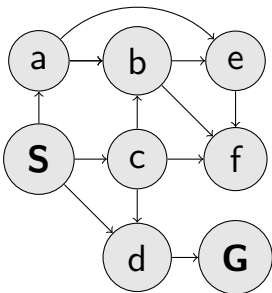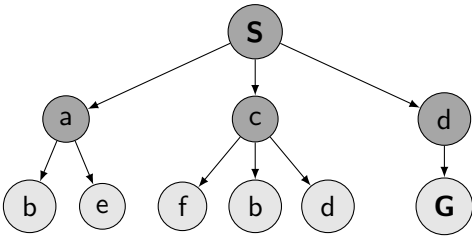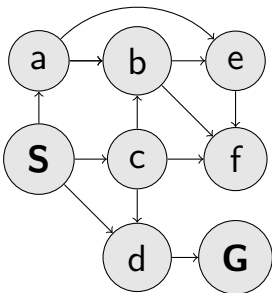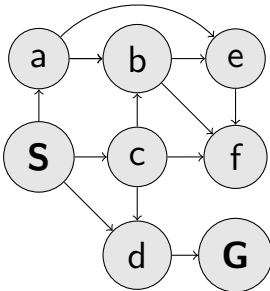
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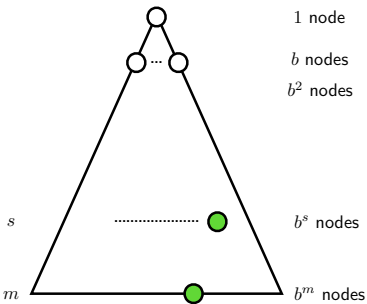**Notes**

BFS properties

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$b, m, s,$ Time complexity?

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- Space, all the frontier: b^s

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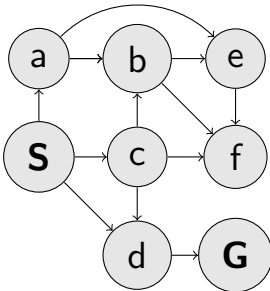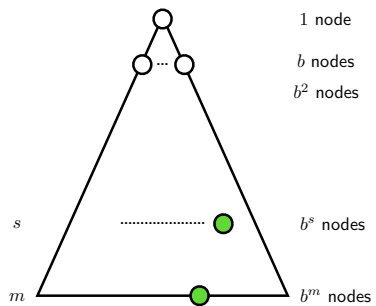- Optimality, it does not miss the shallowest solution, hence if all the transition costs are 1: Yes!

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What are (dis)advantages of the individual strategies?

(S)   (S)

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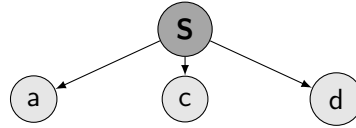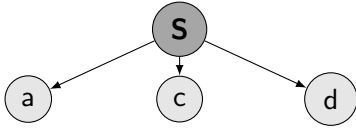What is the impression from the animation? BFS seems better.

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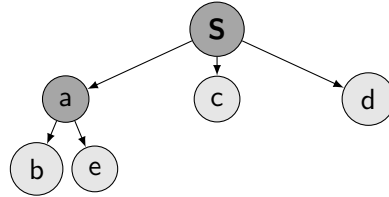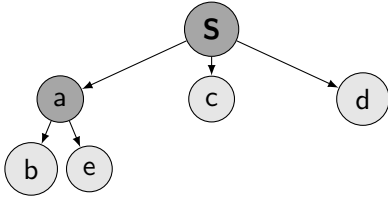What is the impression from the animation? BFS seems better.

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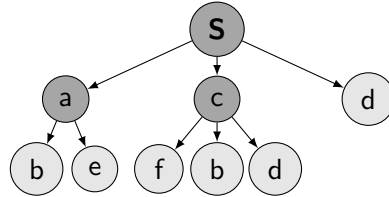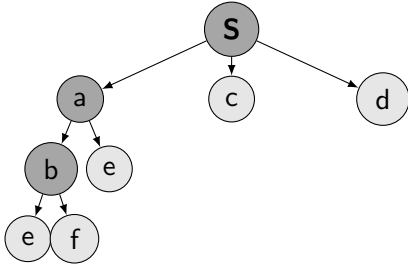What is the impression from the animation? BFS seems better.

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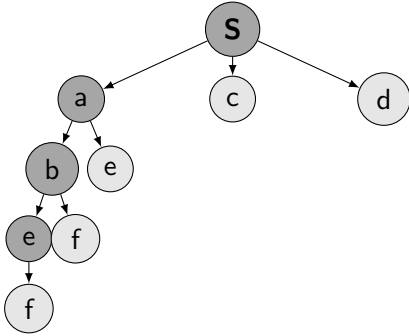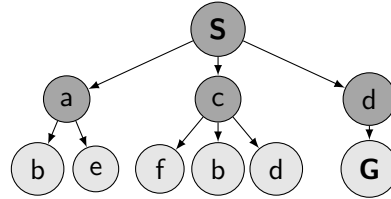What is the impression from the animation? BFS seems better.

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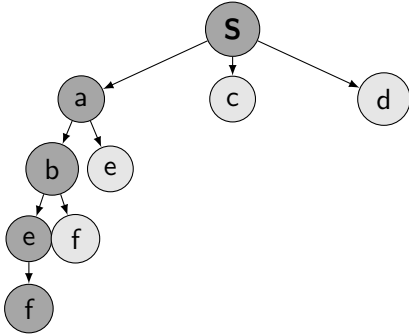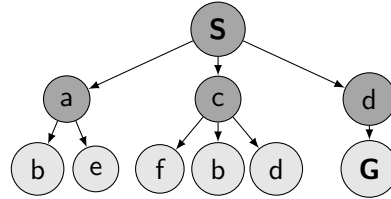What is the impression from the animation? BFS seems better.

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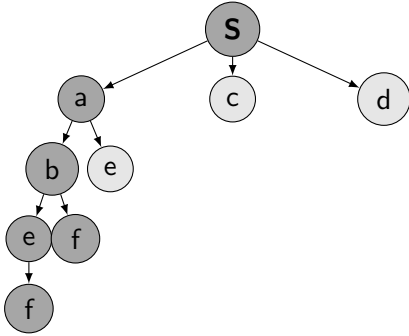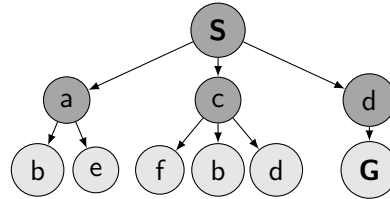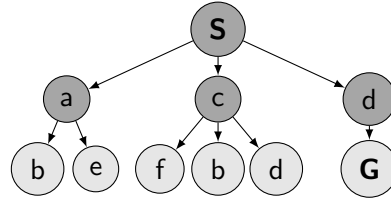What is the impression from the animation? BFS seems better.

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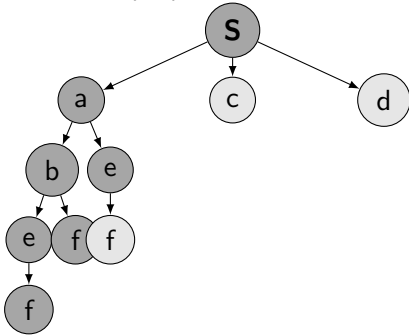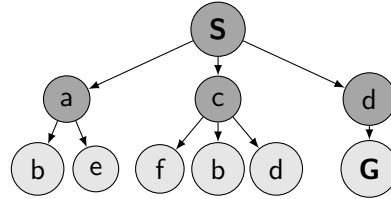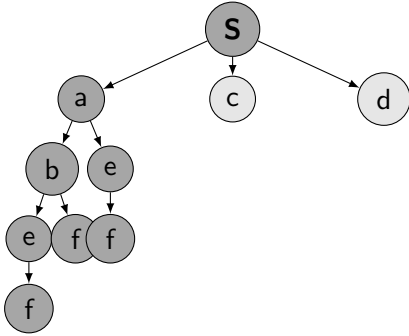What is the impression from the animation? BFS seems better.

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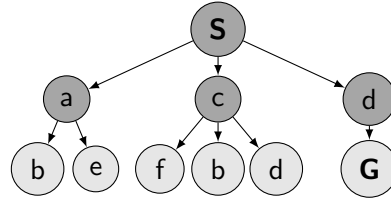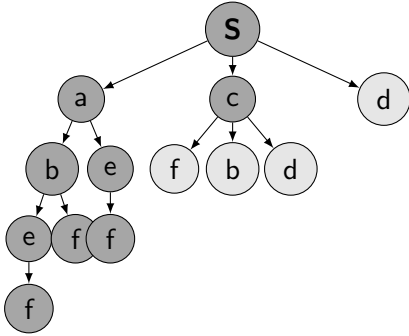What is the impression from the animation? BFS seems better.

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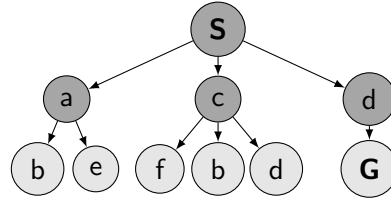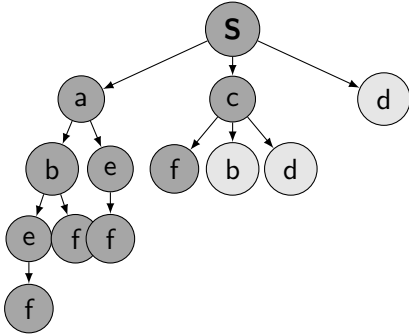What is the impression from the animation? BFS seems better.

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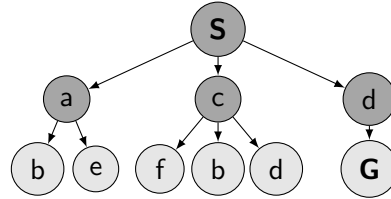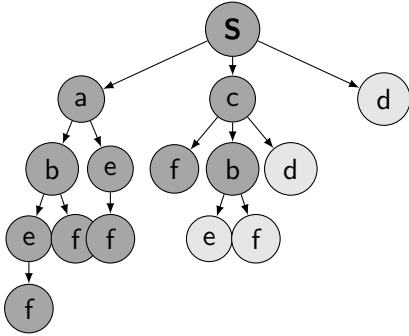What is the impression from the animation? BFS seems better.

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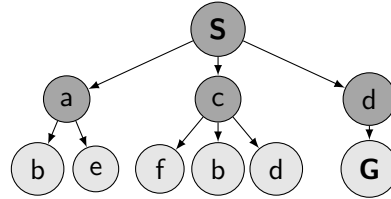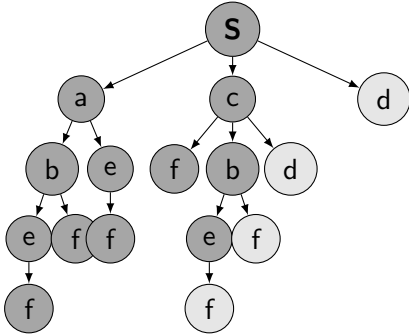What is the impression from the animation? BFS seems better.

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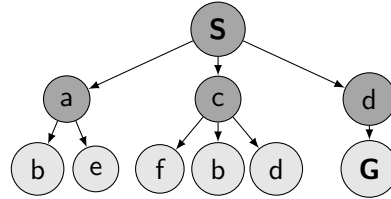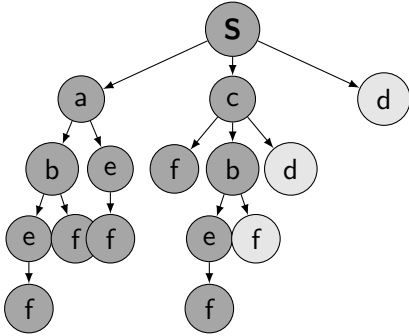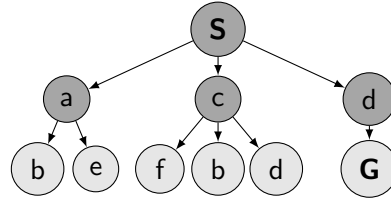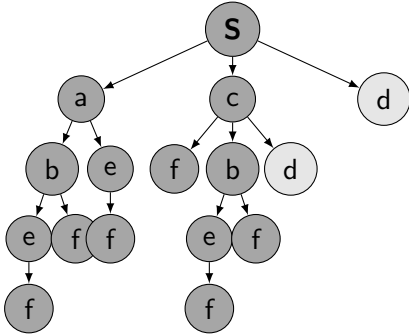What is the impression from the animation? BFS seems better.

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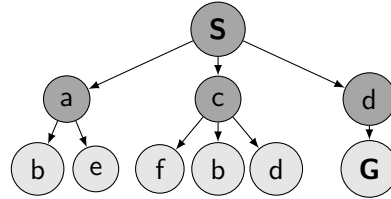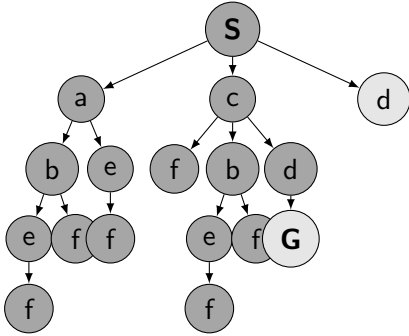What is the impression from the animation? BFS seems better.

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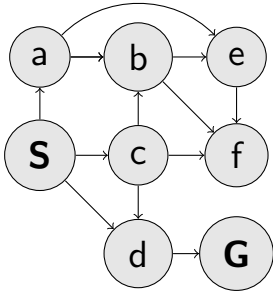What is the impression from the animation? BFS seems better.

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DFS with limited depth, `maxdepth=2`

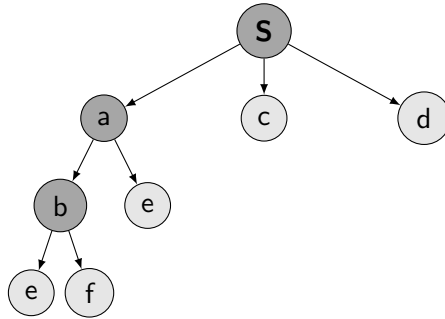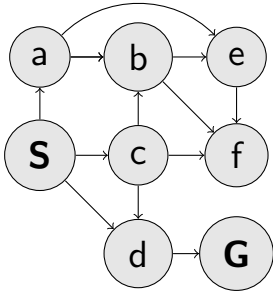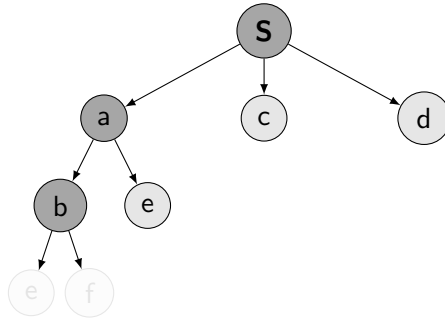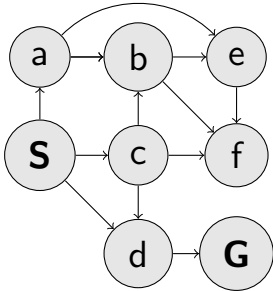Do not follow nodes with `depth > maxdepth`

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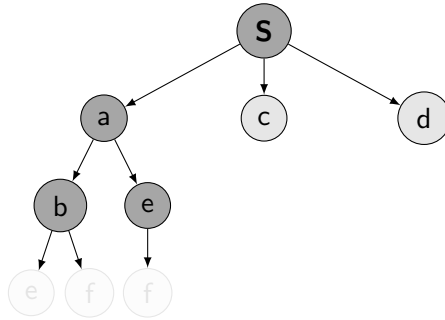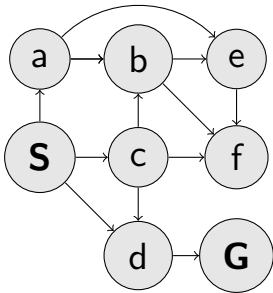Do not follow nodes with `depth > maxdepth`

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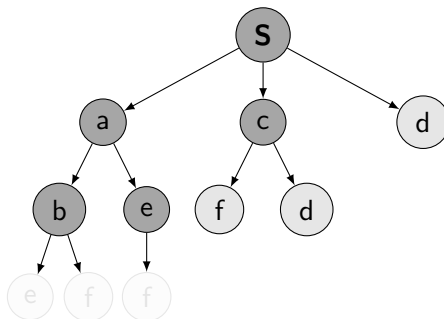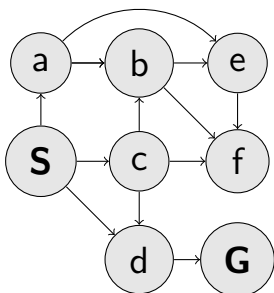Do not follow nodes with depth $>$ maxdepth

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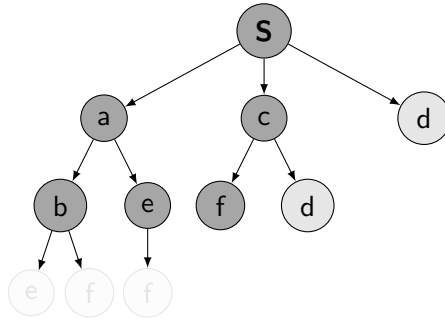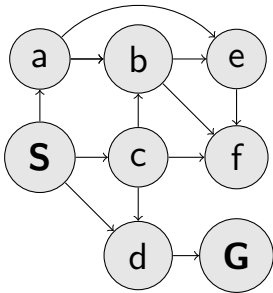Do not follow nodes with `depth > maxdepth`

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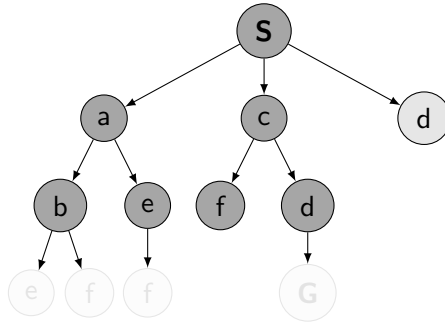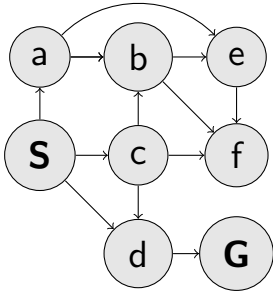Do not follow nodes with `depth` > `maxdepth`

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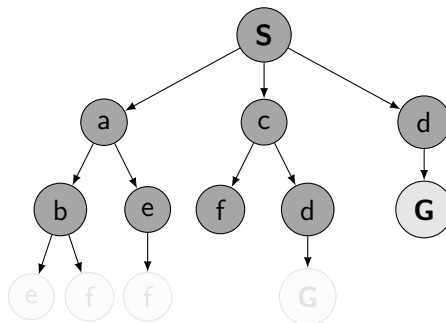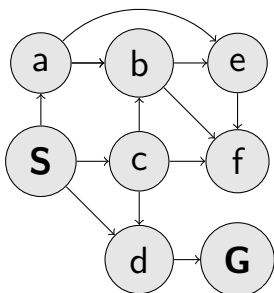Do not follow nodes with `depth` > `maxdepth`

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Iterative deepening DFS (ID-DFS)

▶ Start with `maxdepth = 1`

▶ Perform DFS with limited depth. Report success or failure.

▶ If failure, forget everything, increase `maxdepth` and repeat DFS

Is it not a terrible waste to forget everything between steps?

─── **Notes** ───

Really, how much do we repeat/waste? The "upper levels", close to the root, are repeated many times. However, in a tree, most nodes are the bottom levels and nr. nodes traversed is what counts. More specifically, for a solution at depth s, the nodes on the bottm level are generated only once, those on the next-to-bottom level 2x ... children of the root are generated $s\times$. Compare the number of nodes generated ID-DFS vs. BFS:
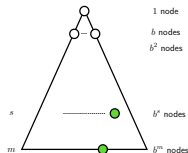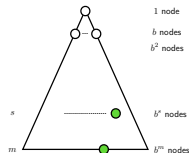
$$N(\text{ID-DFS}) = (s)b + (s-1)b^2 + (s-2)b^3 + \cdots + (1)b^s$$

$$N(\text{BFS}) = b + b^2 + b^3 + \cdots + b^s$$

Try some calculations for various s and b. For $b = 10$ and $d = 5$:

$$N(\text{ID-DFS}) = 50 + 400 + 3000 + 20000 + 100000 = 123450$$

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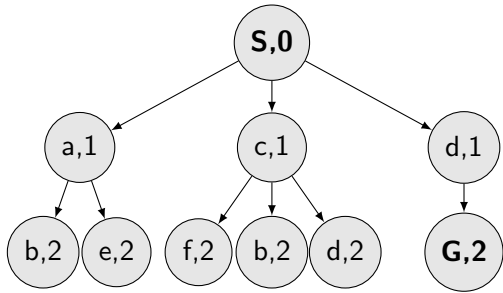

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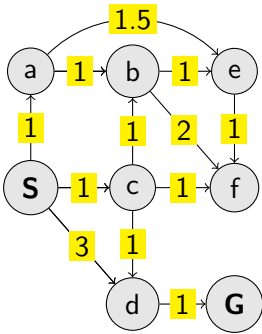$$N(\text{BFS}) = 10 + 100 + 1000 + 10000 + 100000 = 111110$$

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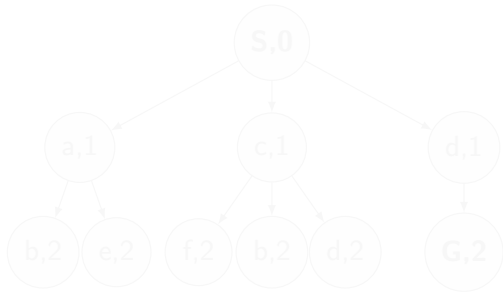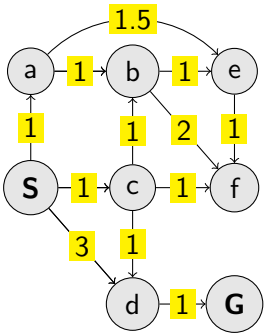
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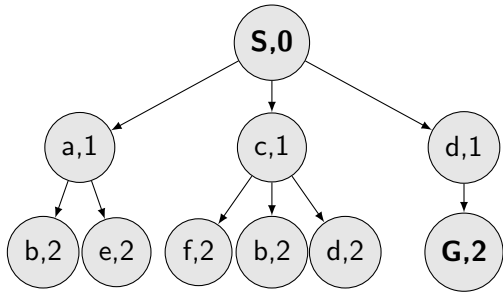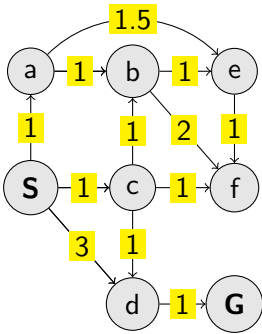
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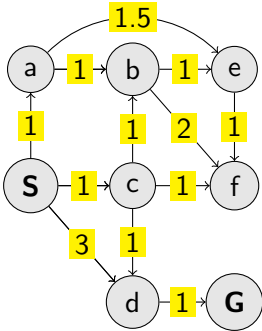- How to evaluate nodes with path cost?

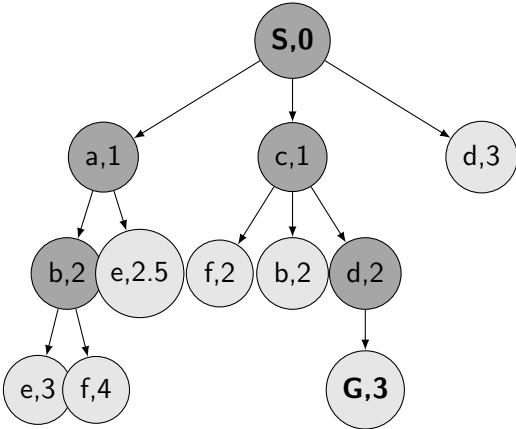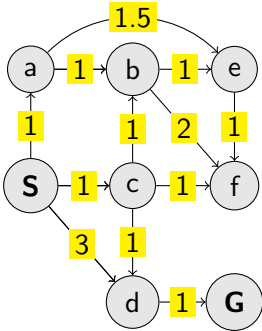**Notes**

Uniform Cost Search (UCS)

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Simple extension of BFS. Instead of expanding shallowest node, the node with smallest path cost so far is expanded.
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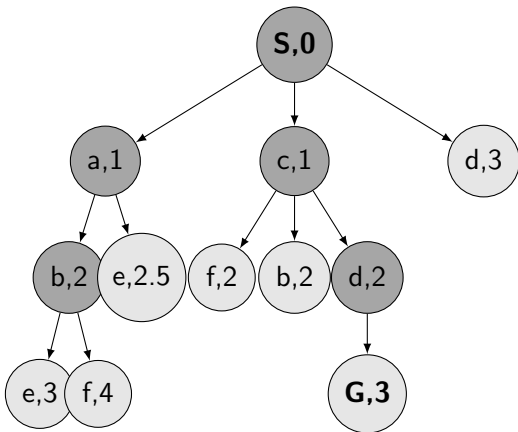
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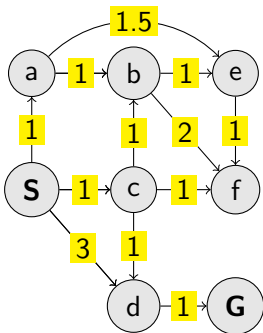When to check the goal (and stop) the search? When visiting or expanding the node?
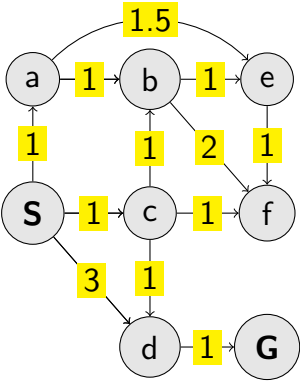
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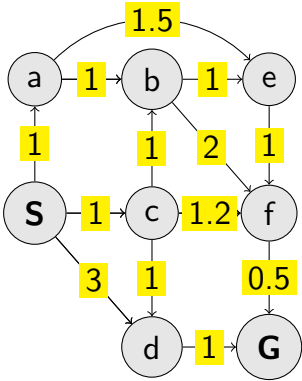- Test is added in case a better path is found to a node currently on the frontier.

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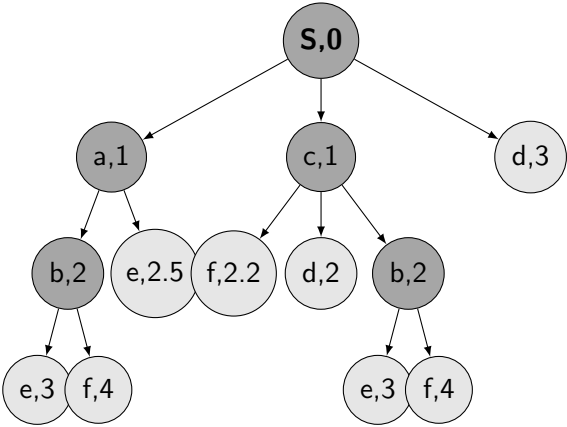- Test is added in case a better path is found to a node currently on the frontier.
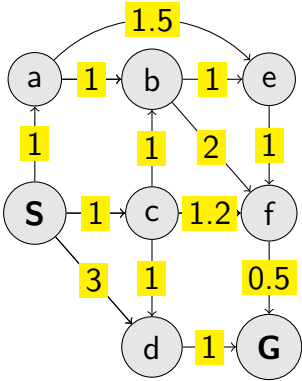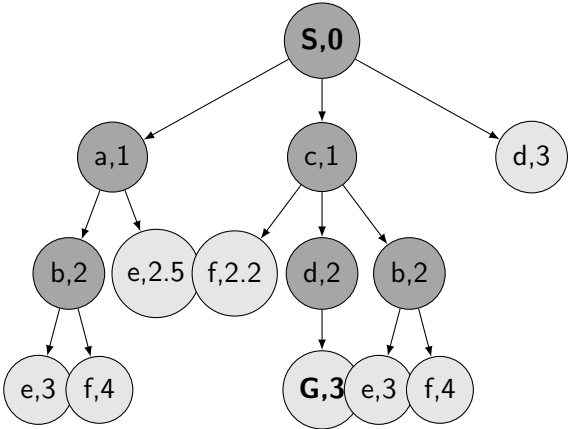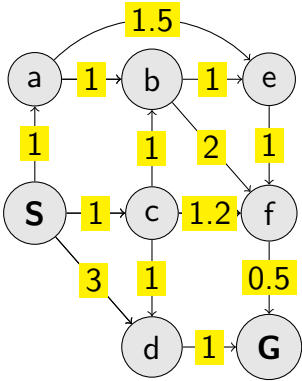
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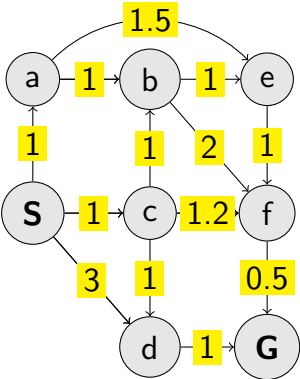
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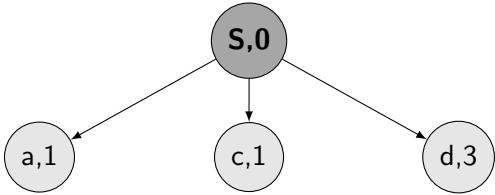
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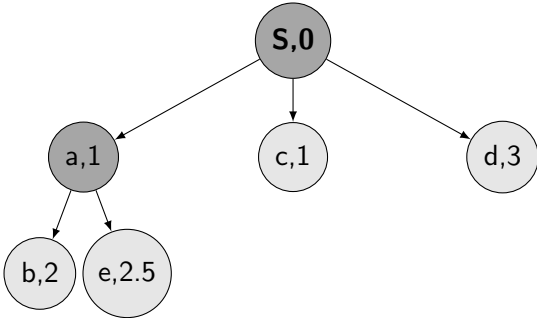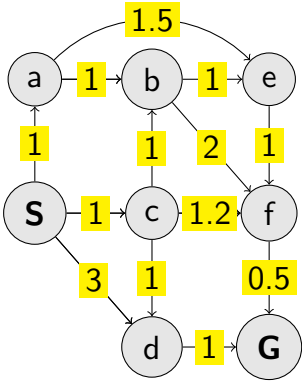
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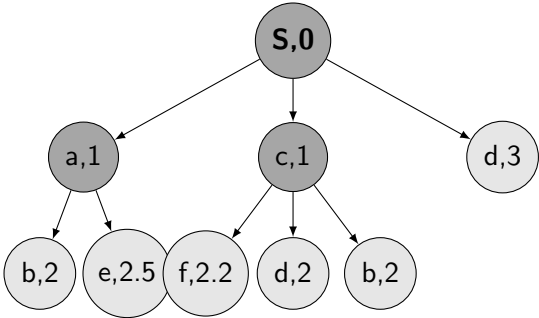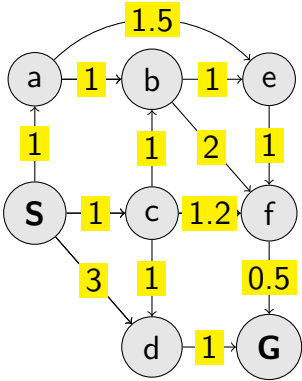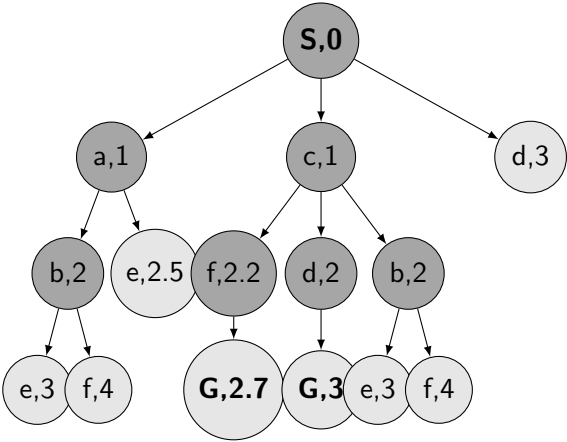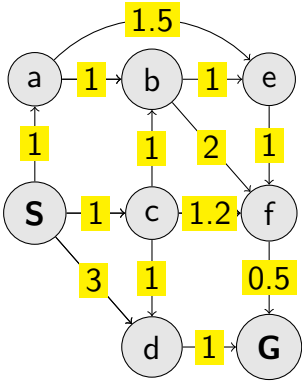
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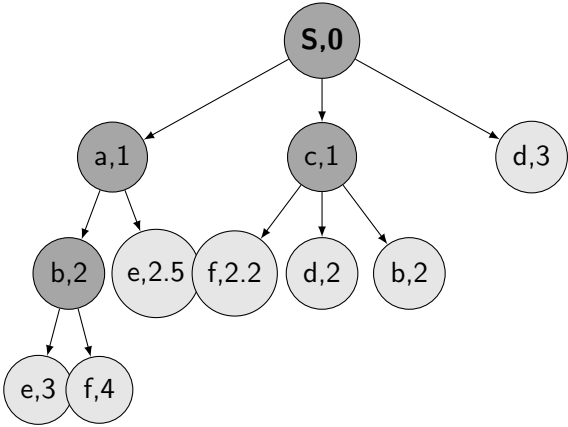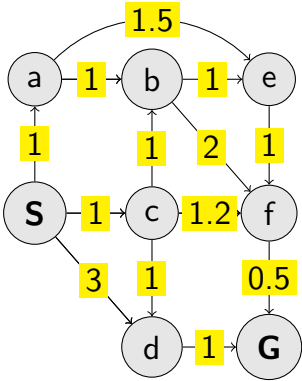
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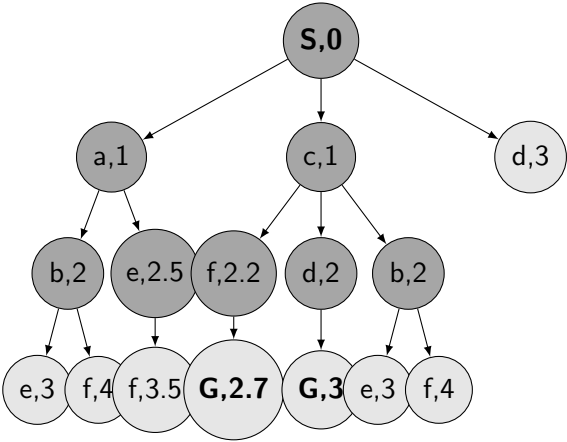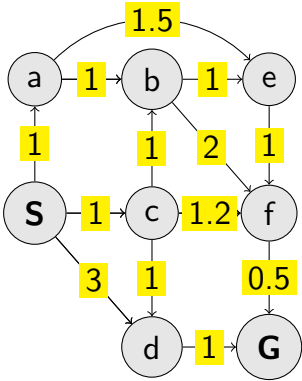
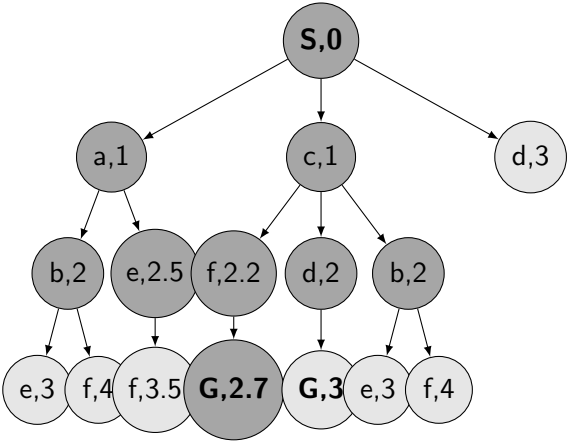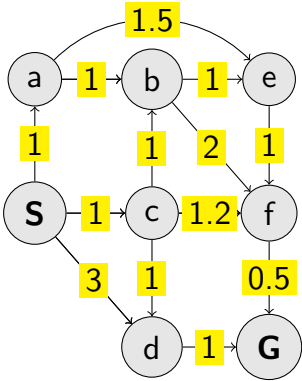# When to stop, when visiting or expanding?

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UCS properties

- ▶ Time complexity?
- ▶ Space complexity?
- ▶ Complete?
- ▶ Optimal?

Notes

Solution cost C^*, transition cost at least ϵ. Effective depth, roughly C^*/ϵ.

- Time: $b^{C^*/\epsilon}$
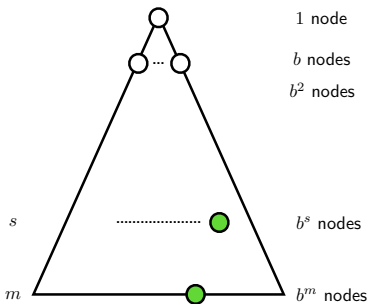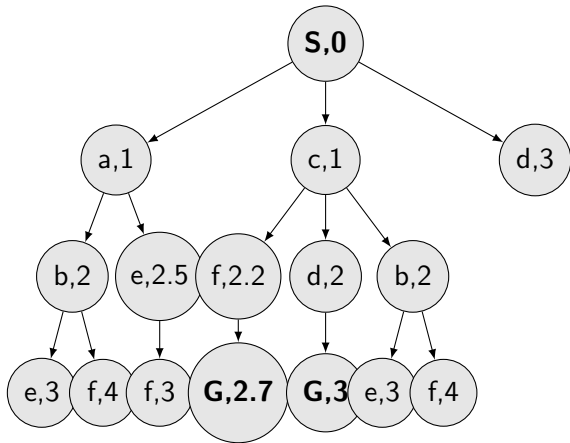- Space: $b^{C^*/\epsilon}$
- Completness: Yes!
- Optimality: Yes! Why?

UCS properties

- ▶ Time complexity?
- ▶ Space complexity?
- ▶ Complete?
- ▶ Optimal?

Notes

Solution cost C^*, transition cost at least ϵ. Effective depth, roughly C^*/ϵ.

- Time: $b^{C^*/\epsilon}$
- Space: $b^{C^*/\epsilon}$
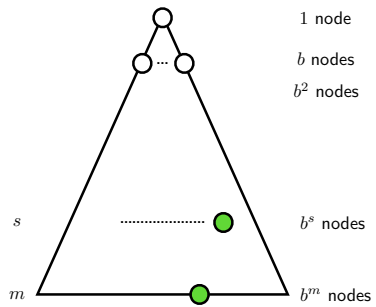- Completness: Yes!
- Optimality: Yes! Why?

Example: Graph with costs
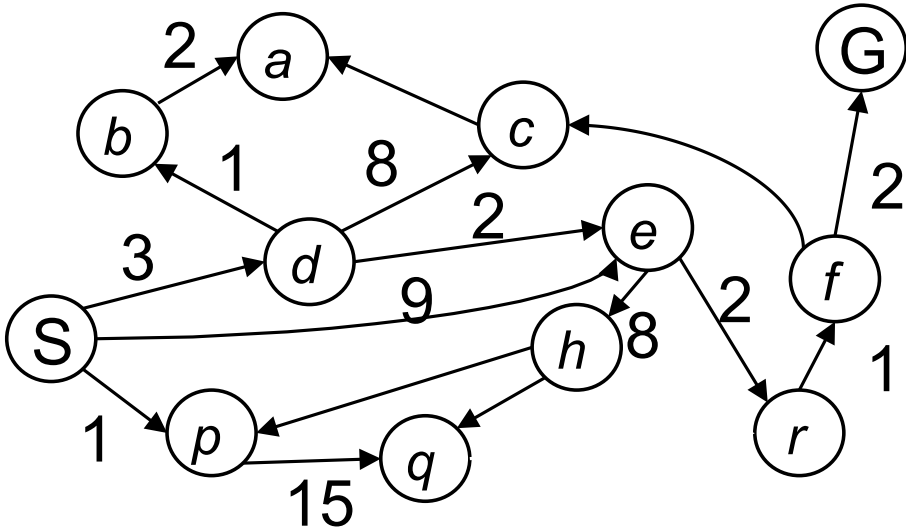
Notes

Try it on paper, mark which nodes are in frontier, mark lines of equal cost.

Infrastructure for (tree) search algorithms

What should a tree node n know?

- ▶ n.state
- ▶ n.parent
- ▶ n.pathcost

Perhaps we may add something later, if needed . . .

Notes

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Notes

How to organize nodes?

The Python examples are just suggestions, ...

- ▶ A dynamically linked structure (`list()`).
- ▶ Add a node (`list.insert(node)`).
- ▶ Take a node and remove from the structure (`node=list.pop()`).
- ▶ Check the Python modules `heapq`[1] and `queue`[2] for inspiration.

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[1] https://docs.python.org/3.5/library/heapq.html
[2] https://docs.python.org/3.5/library/queue.html

Notes

Very likely, you discussed `heapq` and `queue` in some programming, algorithms or data structures related courses.

What is the solution?

- ▶ We stop when Goal is reached.
- ▶ How do we contruct the path?

Notes

References, further reading

Some figures if from [2]. Chapter 2 in [1] provides a compact/dense intro into search algorithms.

[1] Steven M. LaValle.
   Planning Algorithms.
   Cambridge, 1st edition, 2006.
   Online version available at: http://planning.cs.uiuc.edu.

[2] Stuart Russell and Peter Norvig.
   Artificial Intelligence: A Modern Approach.
   Prentice Hall, 3rd edition, 2010.
   http://aima.cs.berkeley.edu/.

Notes