3D Computer Vision

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Open Informatics Master's Course

Module II

Perspective Camera

2.1 Basic Entities: Points, Lines

2.2 Homography: Mapping Acting on Points and Lines

2.3 Canonical Perspective Camera

2.4 Changing the Outer and Inner Reference Frames

2.5 Projection Matrix Decomposition

2.6 Anatomy of Linear Perspective Camera

2.7 Vanishing Points and Lines

covered by

[H&Z] Secs: 2.1, 2.2, 3.1, 6.1, 6.2, 8.6, 2.5, Example: 2.19

▶Basic Geometric Entities, their Representation, and Notation

- entities have names and representations
- names and their components:

entity	in 2-space	in 3-space
point	$m = (u, v)$	$X = (x, y, z)$
line	n	O
plane		π, φ

- associated vector representations

$$\mathbf{m} = \begin{bmatrix} u \\ v \end{bmatrix} = [u, v]^\top, \quad \mathbf{X} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad \mathbf{n}$$

will also be written in an 'in-line' form as $\mathbf{m} = (u, v)$, $\mathbf{X} = (x, y, z)$, etc.

- vectors are always meant to be columns $\mathbf{x} \in \mathbb{R}^{n \times 1}$
- associated homogeneous representations

$$\underline{\mathbf{m}} = [m_1, m_2, m_3]^\top, \quad \underline{\mathbf{X}} = [x_1, x_2, x_3, x_4]^\top, \quad \underline{\mathbf{n}}$$

'in-line' forms: $\underline{\mathbf{m}} = (m_1, m_2, m_3)$, $\underline{\mathbf{X}} = (x_1, x_2, x_3, x_4)$, etc.

- matrices are $\mathbf{Q} \in \mathbb{R}^{m \times n}$, linear map of a $\mathbb{R}^{n \times 1}$ vector is $\mathbf{y} = \mathbf{Q}\mathbf{x}$
- j-th element of vector $\mathbf{m}_i$ is $(\mathbf{m}_i)_j$; element i, j of matrix $\mathbf{P}$ is $\mathbf{P}_{ij}$

▶Image Line (in 2D)

$(u, v, 1)$

a finite line in the 2D (u, v) plane $\qquad \lambda\,(a\,u + b\,v + c) = 0 \qquad \lambda \neq 0$

has a parameter (homogeneous) vector $\qquad \underline{\mathbf{n}} \simeq (a,\, b,\, c)\,, \quad \|\underline{\mathbf{n}}\| \neq 0$

and there is an equivalence class for $\lambda \in \mathbb{R}$, $\lambda \neq 0 \qquad (\lambda a,\, \lambda b,\, \lambda c) \simeq (a,\, b,\, c)$

'Finite' lines

- standard representative for finite $\underline{\mathbf{n}} = (n_1, n_2, n_3)$ is $\lambda\underline{\mathbf{n}}$, where $\lambda = \frac{\mathbf{1}}{\sqrt{n_1^2 + n_2^2}}$

assuming $n_1^2 + n_2^2 \neq 0$; $\mathbf{1}$ is the unit, usually $\mathbf{1} = 1$

'Infinite' line

- we augment the set of lines for a special entity called the **line at infinity** (ideal line)

$$\underline{\mathbf{n}}_\infty \simeq (0, 0, \mathbf{1}) \qquad \text{(standard representative)}$$

- the set of equivalence classes of vectors in $\mathbb{R}^3 \setminus (0, 0, 0)$ forms the projective space $\mathbb{P}^2$

a set of rays →21

- line at infinity is a proper member of $\mathbb{P}^2$
- I may sometimes wrongly use $=$ instead of $\simeq$, if you are in doubt, ask me

▶Image Point

if and only if

Finite point $\mathbf{m} = (u, v)$ is incident on a finite line $\underline{\mathbf{n}} = (a, b, c)$ iff   iff = works either way!

$$a\,u + b\,v + c = 0$$

can be rewritten as (with scalar product): $(u, v, \mathbf{1}) \cdot (a, b, c) = \boxed{\underline{\mathbf{m}}^\top \underline{\mathbf{n}} = 0}$

'Finite' points

- a finite point is also represented by a homogeneous vector $\underline{\mathbf{m}} \simeq (u, v, \mathbf{1})$, $\|\underline{\mathbf{m}}\| \neq 0$
- the equivalence class for $\lambda \in \mathbb{R}, \lambda \neq 0$ is $(m_1, m_2, m_3) = \lambda\,\underline{\mathbf{m}} \simeq \underline{\mathbf{m}}$
- the standard representative for finite point $\underline{\mathbf{m}}$ is $\lambda\,\underline{\mathbf{m}}$, where $\lambda = \frac{\mathbf{1}}{m_3}$ assuming $m_3 \neq 0$
- when $\mathbf{1} = 1$ then units are pixels and $\lambda\underline{\mathbf{m}} = (u, v, 1)$
- when $\mathbf{1} = f$ then all elements have a similar magnitude, $f \sim$ image diagonal

use $\mathbf{1} = 1$ unless you know what you are doing;
all entities participating in a formula must be expressed in the same units

'Infinite' points

- we augment for **points at infinity** (ideal points) $\underline{\mathbf{m}}_\infty \simeq (m_1, m_2, 0)$

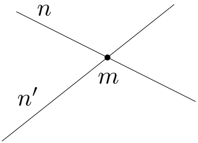
proper members of $\mathbb{P}^2$

- all such points lie on the line at infinity (ideal line) $\underline{\mathbf{n}}_\infty \simeq (0, 0, 1)$, i.e. $\underline{\mathbf{m}}_\infty^\top \underline{\mathbf{n}}_\infty = 0$

▶Line Intersection and Point Join

The point of **intersection** m of image lines n and n', $n \not\simeq n'$ is

$$\underline{\mathbf{m}} \simeq \underline{\mathbf{n}} \times \underline{\mathbf{n}}'$$

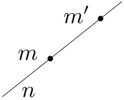


proof: If $\underline{\mathbf{m}} = \underline{\mathbf{n}} \times \underline{\mathbf{n}}'$ is the intersection point, it must be incident on both lines. Indeed, using known equivalences from vector algebra

$$\underline{\mathbf{n}}^\top \underbrace{(\underline{\mathbf{n}} \times \underline{\mathbf{n}}')}_{\underline{\mathbf{m}}} \equiv \underline{\mathbf{n}}'^\top \underbrace{(\underline{\mathbf{n}} \times \underline{\mathbf{n}}')}_{\underline{\mathbf{m}}} \equiv 0$$

The **join** n of two image points m and m', $m \not\simeq m'$ is

$$\underline{\mathbf{n}} \simeq \underline{\mathbf{m}} \times \underline{\mathbf{m}}'$$



Paralel lines intersect (somewhere) on the line at infinity $\underline{\mathbf{n}}_\infty \simeq (0, 0, 1)$:

$$a\,u + b\,v + c = 0,$$
$$a\,u + b\,v + d = 0, \qquad d \neq c$$
$$(a, b, c) \times (a, b, d) \simeq (b, -a, 0)$$

- all such intersections lie on $\underline{\mathbf{n}}_\infty$
- line at infinity therefore represents the set of (unoriented) directions in the plane
- Matlab: `m = cross(n, n_prime);`

Thank You