

# Multi-goal Planning

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Lecture 05

**B4M36UIR – Artificial Intelligence in Robotics**



# Overview of the Lecture

- Part 1 – Multi-goal Planning
  - Inspection Planning
  - Multi-goal Planning
- Part 2 – Unsupervised Learning for Multi-goal Planning
  - Unsupervised Learning for Multi-goal Planning
  - TSPN in Multi-goal Planning with Localization Uncertainty



# Part I

## Part 1 – Multi-goal Planning



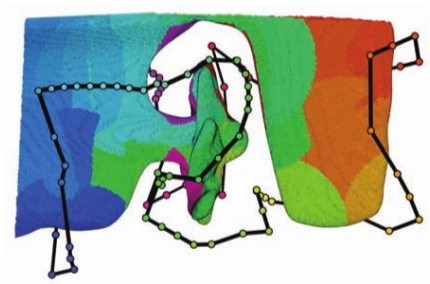
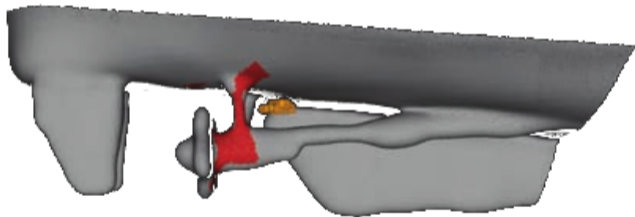
# Outline

- Inspection Planning
- Multi-goal Planning



## Robotic Information Gathering in Inspection of Vessel's Propeller

- The planning problem is to determine a shortest inspection path for an Autonomous Underwater Vehicle (AUV) to inspect the vessel's propeller.



[https://www.youtube.com/watch?v=8azP\\_9VnMtM](https://www.youtube.com/watch?v=8azP_9VnMtM)

Englot, B., Hover, F.S.: *Three-dimensional coverage planning for an underwater inspection robot*, International Journal of Robotics Research, 32(9–10):1048–1073, 2013.



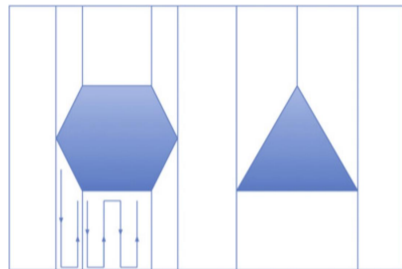
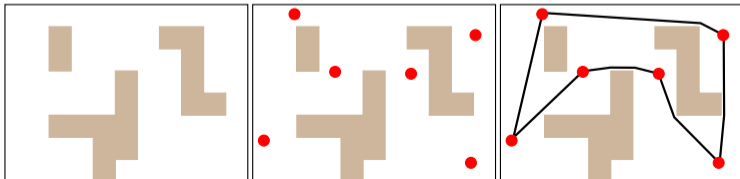


# Inspection Planning

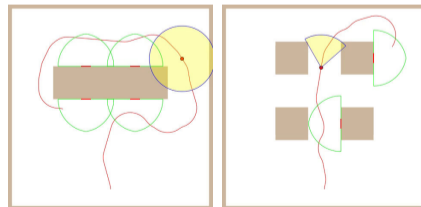
- **Inspection/coverage planning** stands to determine a plan (path) to inspect/cover the given areas or point of interest.
- We can directly find inspection/coverage plan using
  - predefined covering patterns such as *ox-plow* motion;
  - a “general” path satisfying coverage constraints.

Galceran, E., Carreras, M.: *A survey on coverage path planning for robotics*, Robotics and Autonomous Systems, 61(12):1258–1276, 2013.

- **Decoupled** approach where locations to be visited are determined before path planning as the **sensor placement** problem.



Trapezoidal decomposition method

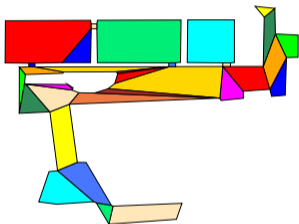


Kafka, Faigl, Váňa: ICRA 2016

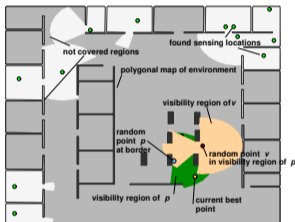
## Inspection Planning – Decoupled Approach

1. Determine sensing locations such that the whole environment would be inspected (seen) by visiting them (**Sampling design problem**).

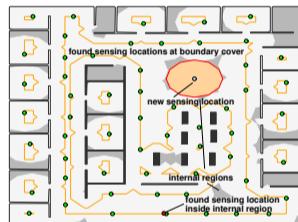
*In the geometrical-based approach, a solution of the **Art Gallery Problem**.*



Convex Partitioning  
(Kazazakis and Argyros, 2002)



Randomized Dual Sampling  
(González-Baños et al., 1998)



Boundary Placement  
(Faigl et al., 2006)

*The problem is related to the **sensor placement** and **sampling design**.*

2. Create a roadmap connecting the sensing location.
3. Find the inspection path visiting all the sensing locations as a solution of the multi-goal path planning (a solution of the robotic TSP).

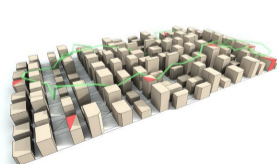
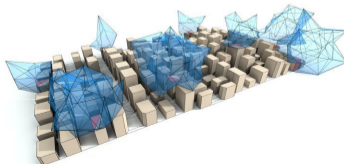
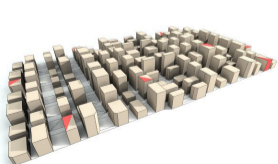
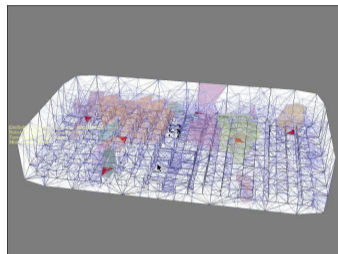
*E.g., using visibility graph or randomized sampling based approaches.*





## Planning to Capture Areas of Interest using UAV

- Determine a cost-efficient path from which a given set of target regions is covered.
- For each target region a subspace  $S \subset \mathbb{R}^3$  from which the target can be covered is determined. *S represents the neighborhood.*
- **We search for the best sequence of visits to the regions.**  
*Combinatorial optimization*
- The PRM is utilized to construct the planning roadmap (a graph).  
PRM – Probabilistic Roadmap Method – sampling-based motion planner, see lecture 8.
- The problem can be formulated as **the Traveling Salesman Problem with Neighborhoods**, as it is not necessary to visit exactly a single location to capture the area of interest.



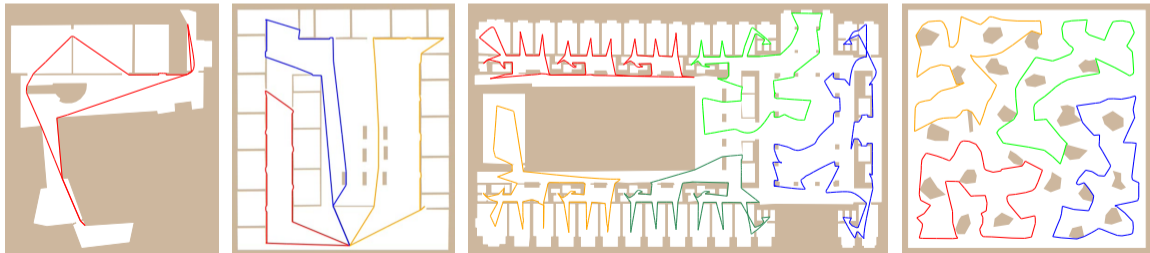
Janoušek and Faigl, ICRA 2013



## Inspection Planning – “Continuous Sensing”

- If we do not prescribe a discrete set of sensing locations, we can formulate the problem as the **Watchman route problem**.

Given a map of the environment  $\mathcal{W}$  determine the shortest, closed, and collision-free path, from which the whole environment is covered by an omnidirectional sensor with the radius  $\rho$ .



Faigl, J.: *Approximate Solution of the Multiple Watchman Routes Problem with Restricted Visibility Range*, IEEE Transactions on Neural Networks, 21(10):1668-1679, 2010.



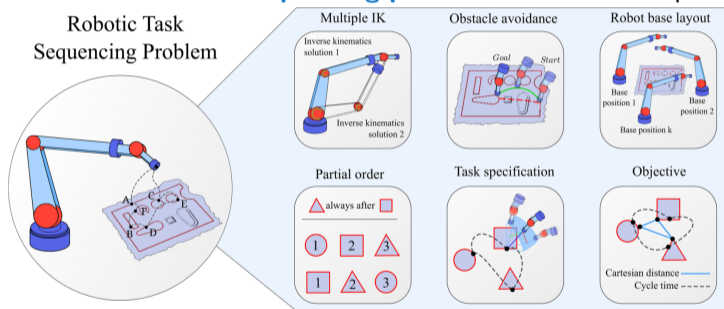
# Outline

- Inspection Planning
- Multi-goal Planning



# Multi-Goal Planning

- Having a **set of locations** to be visited, determine the cost-efficient path to visit them.
  - Locations where a robotic arm or mobile robot performs some task. *The operation can be repeated—closed path.*
- The problem is called **robotic task sequencing problem** for robotic manipulators.



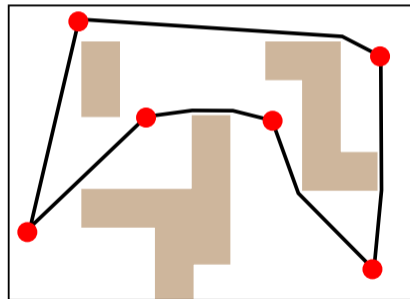
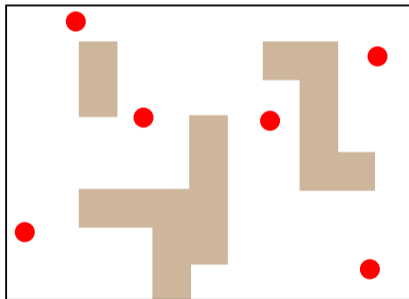
- The problem is also called **Multi-goal Path Planning** (MTP) problem or **Multi-goal Planning** (**MGP**).

*Also studied in its Multi-goal Motion Planning (MGMP) variant.*



## Multi-Goal Path Planning (MTP)

- Multi-goal planning problem is a problem how to visit the given set of locations.
- It consists of **point-to-point path planning** on how to reach one location from another.
- The *challenge* is to determine the optimal sequence of the visits to the locations w.r.t. cost-efficient path to visit all the given locations.



- Determination the sequence of visits is a **combinatorial optimization problem** that can be formulated as the **Traveling Salesman Problem (TSP)**.



## Traveling Salesman Problem (TSP)

Given a set of cities and the distances between each pair of cities, what is the shortest possible route that visits each city exactly once and returns to the origin city.

- The TSP can be formulated for a graph  $\mathbf{G}(V, E)$ , where  $V$  denotes a set of locations (cities) and  $E$  represents edges connecting two cities with the associated travel cost  $c$  (distance), i.e., for each  $v_i, v_j \in V$  there is an edge  $e_{ij} \in E$ ,  $e_{ij} = (v_i, v_j)$  with the cost  $c_{ij}$ .
- If the associated cost of the edge  $(v_i, v_j)$  is the Euclidean distance  $c_{ij} = |(v_i, v_j)|$ , the problem is called the **Euclidean TSP** (ETSP).
- It is known, the TSP is NP-hard (its decision variant) and several algorithms can be found in literature.

*William J. Cook (2012) – In Pursuit of the Traveling Salesman: Mathematics at the Limits of Computation.*



## Traveling Salesman Problem (TSP)

- Let  $S$  be a set of  $n$  sensor locations  $S = \{\mathbf{s}_1, \dots, \mathbf{s}_n\}$ ,  $\mathbf{s}_i \in \mathbb{R}^2$  and  $c(\mathbf{s}_i, \mathbf{s}_j)$  is a cost of travel from  $\mathbf{s}_i$  to  $\mathbf{s}_j$
- Traveling Salesman Problem (TSP)** is a problem to determine a closed tour visiting each  $\mathbf{s} \in S$  such that the total tour length is minimal.
  - We are searching for the optimal **sequence of visits**  $\Sigma = (\sigma_1, \dots, \sigma_n)$  such that

$$\begin{aligned} \text{minimize } \Sigma \quad & L = \left( \sum_{i=1}^{n-1} c(\mathbf{s}_{\sigma_i}, \mathbf{s}_{\sigma_{i+1}}) \right) + c(\mathbf{s}_{\sigma_n}, \mathbf{s}_{\sigma_1}) \\ \text{subject to} \quad & \Sigma = (\sigma_1, \dots, \sigma_n), 1 \leq \sigma_i \leq n, \sigma_i \neq \sigma_j \text{ for } i \neq j. \end{aligned} \quad (1)$$

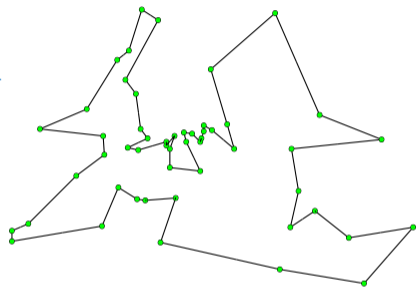
- The TSP can be considered on a graph  $G(V, E)$  where the set of vertices  $V$  represents sensor locations  $S$  and  $E$  are edges connecting the nodes with the cost  $c(\mathbf{s}_i, \mathbf{s}_j)$ .
- For simplicity we can consider  $c(\mathbf{s}_i, \mathbf{s}_j)$  to be Euclidean distance; otherwise, we also need to address the path/motion planning problem. **Euclidean TSP**
- If  $c(\mathbf{s}_i, \mathbf{s}_j) \neq c(\mathbf{s}_j, \mathbf{s}_i)$  it is the **Asymmetric TSP**.
- The TSP is known to be NP-hard unless  $P=NP$ .

Traveling vs Travelling – <http://www.math.uwaterloo.ca/tsp/history/travelling.html>



## Existing Approaches to the TSP

- Exact solutions
  - Branch&Bound, Branch&Cut, and Integer Linear Programming (ILP).  
 Concorde-<http://www.math.uwaterloo.ca/tsp/concorde.html>
- Approximation algorithms
  - Minimum Spanning Tree (MST) heuristic with  $L \leq 2L_{opt}$ .
  - Christofides's algorithm with  $L \leq \frac{3/2}{L} L_{opt}$ .
- Heuristic algorithms
  - Constructive heuristic – Nearest Neighborhood (NN) algorithm
  - 2-Opt – local search algorithm proposed by Croes 1958
  - LKH – K. Helsgaun efficient implementation of the Lin-Kernighan heuristic (1998). <http://www.akira.ruc.dk/~keld/research/LKH/>
- Combinatorial meta-heuristics
  - Variable Neighborhood Search (VNS)
  - Greedy Randomized Adaptive Search Procedure (GRASP)
- Soft-computing techniques, evolutionary methods, and unsupervised learning



*Problem Berlin52 from the TSPLIB*

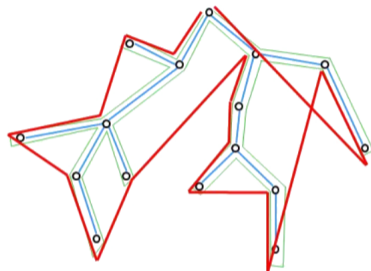




## MST-based Approximation Algorithm to the TSP

- Minimum Spanning Tree heuristic

1. Compute the MST (denoted  $T$ ) of the input graph  $G$ .
2. Construct a graph  $H$  by doubling every edge of  $T$ .
3. Shortcut repeated occurrences of a vertex in the tour.



- For the triangle inequality, the length of such a tour  $L$  is

$$L \leq 2L_{optimal},$$

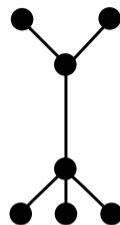
where  $L_{optimal}$  is the cost of the optimal solution of the TSP.



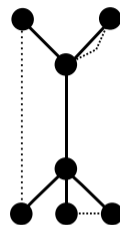
## Christofides's Algorithm to the TSP

### ■ Christofides's algorithm

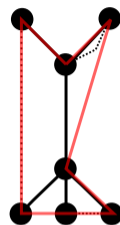
1. Compute the MST of the input graph  $G$ .
2. Compute the minimal matching on the odd-degree vertices.
3. Shortcut a traversal of the resulting Eulerian graph.



MST



Matching



Final tour

- For the triangle inequality, the length of such a tour  $L$  is

$$L \leq \frac{3}{2} L_{optimal},$$

where  $L_{optimal}$  is the cost of the optimal solution of the TSP.

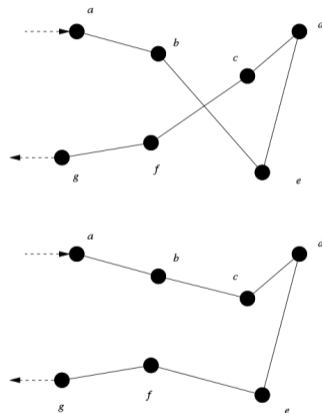
Length of the MST is  $\leq L_{optimal}$

Sum of lengths of the edges in the matching  $\leq \frac{1}{2} L_{optimal}$



## 2-Opt Heuristic

1. Use a construction heuristic to create an initial route
  - NN algorithm, cheapest insertion, farther insertion
2. Repeat until no improvement is made
  - 2.1 Determine swapping that can shorten the tour  $(i, j)$  for  $1 \leq i \leq n$  and  $i + 1 \leq j \leq n$ 
    - route[0] to route[i-1];
    - route[i] to route[j] in reverse order;
    - route[j] to route[end];
    - Determine length of the route;
    - Update the current route if the length is shorter than the existing solution.



Croes, G.A.: *A method for solving traveling salesman problems*, Operations Research 6:791–812, 1958.



# Overview of the Variable Neighborhood Search (VNS) for the TSP

- The **Variable Neighborhood Search (VNS)** is a metaheuristic for solving combinatorial optimization and global optimization problems by searching distant neighborhoods of the current **incumbent solution** using **shake** and **local search** procedures.

*Mladenović and Hansen, 1997*

- Shake** explores the neighborhood of the current solution to escape from a local minima using operators

- Insert** – moves one element;
- Exchange** – exchanges two elements.

- Local search** improves the solution by

- Path insert** – moves a subsequence;
- Path exchange** – exchanges two subsequences.

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## Algorithm 1: VNS-based Solver to the TSP

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**Input:**  $S$  – Set of the target locations to be visited.

**Output:**  $\Sigma$  – Found sequence of visits to locations  $S$ .

$\Sigma^* \leftarrow$  Initial sequence found by cheapest insertion

**while** *terminal condition is not met* **do**

$\Sigma' \leftarrow$  shake( $\Sigma^*$ )

**for**  $n^2$ -times **do**

$\Sigma'' \leftarrow$  localSearch( $\Sigma'$ )

**if**  $\Sigma''$  is "better" than  $\Sigma'$  **then**

$\Sigma' \leftarrow \Sigma''$  // Select  $\Sigma''$  instead of  $\Sigma'$

**if**  $\Sigma'$  is "better" than  $\Sigma^*$  **then**

$\Sigma^* \leftarrow \Sigma'$  // Replace the incumbent sequence.

**return**  $\Sigma^*$

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### Insert



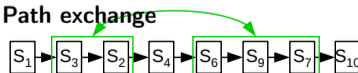
### Exchange



### Path insert



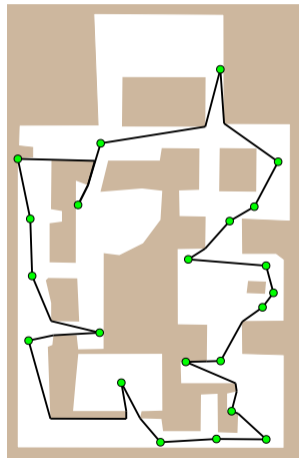
### Path exchange



## Multi-Goal Path Planning (MTP) Problem

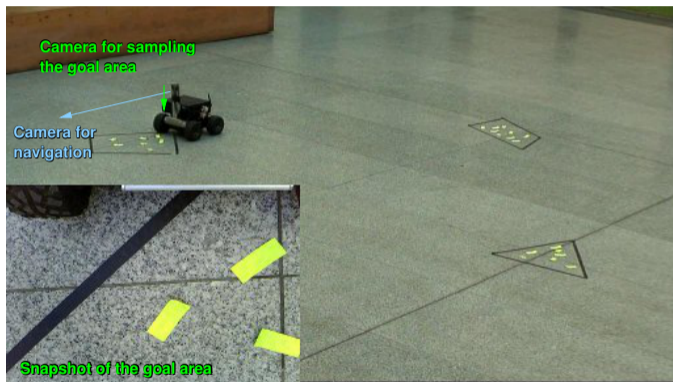
- **MTP problem** is a **robotic variant of the TSP** with the edge costs as the length of the *shortest* path connecting the locations.
- Variants of the **robotic TSP** includes additional constraints arising from limitations of real robotic systems such as
  - obstacles, curvature-constraints, sensing range, location precision.
- For  $n$  locations, we need to compute up to  $n^2$  shortest paths.
- Having a **roadmap** (graph) representing  $\mathcal{C}_{free}$ , the paths can be found in the graph (roadmap), from which the  $G(V, E)$  for the TSP can be constructed.  
*Visibility graph as a roadmap for a point robot provides a straight forward solution, but such a shortest path may not be necessarily feasible for more complex robots.*
- We can determine the roadmap using randomized sampling-based motion planning techniques.

See lecture 8.



## Multi-goal Path Planning with Goal Regions

- It may be sufficient to visit a goal region instead of the particular point location.



Not only a **sequence** of goals visit has to be determined, but also an **appropriate location** at each region has to be found.

The problem with goal regions can be considered as a variant of the **Traveling Salesman Problem with Neighborhoods (TSPN)**.



## Traveling Salesman Problem with Neighborhoods

Given a set of  $n$  regions (neighbourhoods), what is the shortest closed path that visits each region.

- The problem is NP-hard and APX-hard, it cannot be approximated to within factor  $2 - \epsilon$ , where  $\epsilon > 0$ .  
*Safra and Schwartz (2006) – Computational Complexity*
- Approximate algorithms exist for particular problem variants such as disjoint unit disk neighborhoods.
- **TSPN provides a suitable problem formulation for planning various inspection and data collection missions.**
- It enables to exploit non-zero sensing range, and thus find shortest (more cost-efficient) **data collection plans.**

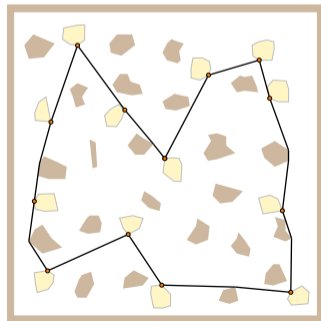


## Traveling Salesman Problem with Neighborhoods (TSPN)

- Instead visiting a particular location  $\mathbf{s} \in S$ ,  $\mathbf{s} \in \mathbb{R}^2$  as in the TSP, we request to visit a set of regions  $R = \{r_1, \dots, r_n\}$ ,  $r_i \subset \mathbb{R}^2$  to save travel cost.
- The TSP becomes the **TSP with Neighborhoods (TSPN)** where, in addition to the determination of the **sequence**  $\Sigma$ , we determine a suitable locations of visits  $P = \{\mathbf{p}_1, \dots, \mathbf{p}_n\}$ ,  $\mathbf{p}_i \in r_i$ .
- The problem is a combination of combinatorial optimization to determine  $\Sigma$  with **continuous optimization** to determine  $P$ .

$$\text{minimize}_{\Sigma, P} \quad L = \left( \sum_{i=1}^{n-1} c(\mathbf{p}_{\sigma_i}, \mathbf{p}_{\sigma_{i+1}}) \right) + c(\mathbf{p}_{\sigma_n}, \mathbf{p}_{\sigma_1})$$

$$\begin{aligned} \text{subject to} \quad & R = \{r_1, \dots, r_n\}, r_i \subset \mathbb{R}^2 \\ & P = \{\mathbf{p}_1, \dots, \mathbf{p}_n\}, \mathbf{p}_i \in r_i \\ & \Sigma = (\sigma_1, \dots, \sigma_n), 1 \leq \sigma_i \leq n, \\ & \sigma_i \neq \sigma_j \text{ for } i \neq j \\ & \text{Foreach } r_i \in R \text{ there is } \mathbf{p}_i \in r_i. \end{aligned}$$





## Approaches to the TSPN

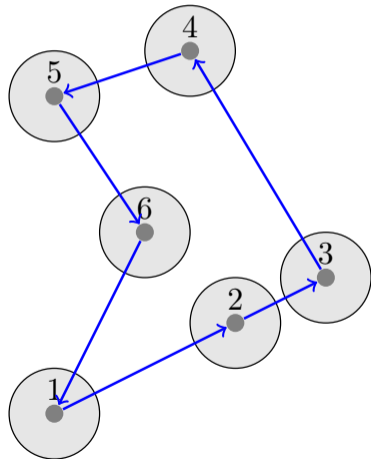
- A direct solution of the TSPN – approximation algorithms and heuristics
  - E.g., using evolutionary techniques or [unsupervised learning](#)
- Euclidean TSPN with, disk-shaped  $\delta$  neighborhoods is called **Closed Enough TSP (CETSP)**.
  - Simplified variant with regions as disks with radius  $\delta$  – remote sensing with the  $\delta$  communication range.
- **Decoupled approach**
  1. Determine sequence of visits  $\Sigma$  independently on the locations  $P$ , e.g., as a solution of the TSP using centroids of the (convex) regions  $R$ .
  2. For the sequence  $\Sigma$  determine the locations  $P$  to minimize the total tour length using
    - Touring polygon problem (TPP);
    - Sampling possible locations and use a forward search for finding the best locations;
    - Continuous optimization such as hill-climbing.

E.g., [Local Iterative Optimization](#) (LIO), Váňa & Faigl (IROS 2015)
- **Sampling-based** approaches
  - For each region, sample possible locations of visits into a discrete set of locations for each region.
  - The problem can be then formulated as the **Generalized Traveling Salesman Problem (GTSP)**.

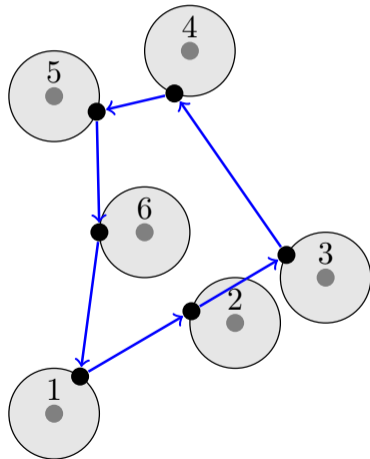


# Close Enough Traveling Salesman Problem (CETSP)

- **Close Enough TSP (CETSP)** is a variant of the TSPN with disk shaped  $\delta$ -neighborhoods.



A solution of the TSP for the centers of the disks



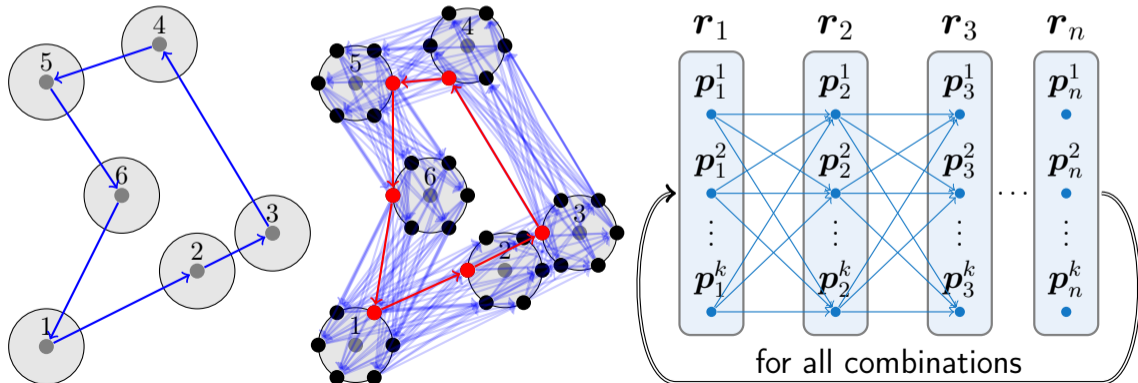
A solution of the CETSP



## Decoupled Sampling-based Solution of the TSPN / CETSP

- **Decoupled** – Determine sequence of visits as a solution of the Euclidean TSP for the representatives of the regions  $R$ , e.g., using centroids.
- Sample each region (neighborhood) with  $k$  samples, e.g.,  $k = 6$ .
- Construct graph and find the shortest tour in by graph search in  $\mathcal{O}(nk^3)$  for  $n$  regions and  $nk^2$  edges in the sequence.

For the closed path, we need to examine all  $k$  possible starting locations.



# Iterative Refinement in the Multi-goal Planning Problem with Regions

- Let the sequence of  $n$  polygon regions be  $R = (r_1, \dots, r_n)$ .

Li, F., Klette, R.: Approximate algorithms for touring a sequence of polygons. 2008

1. Sampling regions into a discrete set of points and determine all shortest paths between each sampled points in the sequence of visits to the regions.

*E.g., using visibility graph*

2. *Initialization:* Construct an initial touring polygons path using a sampled point of each region. Let the path be defined by  $P = (\mathbf{p}_1, \mathbf{p}_2, \dots, \mathbf{p}_n)$ , where  $\mathbf{p}_i \in r_i$  and  $L(P)$  be the length of the shortest path induced by  $P$ .

3. *Refinement:* **For**  $i = 1, 2, \dots, n$ :

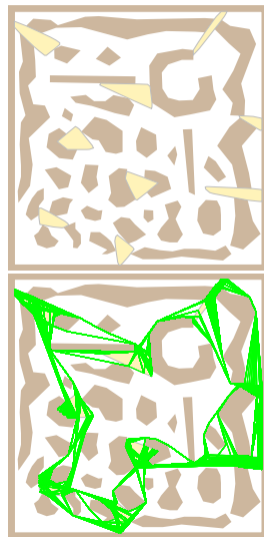
- Find  $\mathbf{p}_i^* \in r_i$  minimizing the length of the path  $d(\mathbf{p}_{i-1}, \mathbf{p}_i^*) + d(\mathbf{p}_i^*, \mathbf{p}_{i+1})$ , where  $d(\mathbf{p}_k, \mathbf{p}_l)$  is the path length from  $\mathbf{p}_k$  to  $\mathbf{p}_l$ ,  $\mathbf{p}_0 = \mathbf{p}_n$ , and  $\mathbf{p}_{n+1} = \mathbf{p}_1$ .
- If the total length of the current path over point  $\mathbf{p}_i^*$  is shorter than over  $\mathbf{p}_i$ , replace the point  $\mathbf{p}_i$  by  $\mathbf{p}_i^*$ .

4. Compute the path length  $L_{new}$  using the refined points.

5. *Termination condition:* If  $L_{new} - L < \epsilon$  Stop the refinement. Otherwise  $L \leftarrow L_{new}$  and go to Step 3.

6. *Final path construction:* Use the last points and construct the path using the shortest paths among obstacles between two consecutive points.

On-line sampling during the iterations – [Local Iterative Optimization \(LIO\)](#),  
Vána & Faigl (IROS 2015).



## Part II

# Part 2 – Unsupervised Learning for Multi-goal Planning



# Outline

- Unsupervised Learning for Multi-goal Planning
- TSPN in Multi-goal Planning with Localization Uncertainty



## Unsupervised Learning based Solution of the TSP

- Iterative learning procedure where neurons (nodes) adapt to the target locations.
- Based on self-organizing map by T. Kohonen.

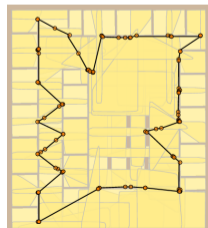
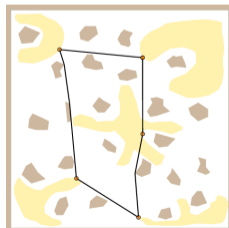
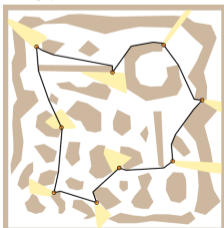
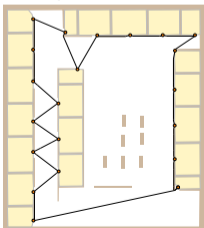
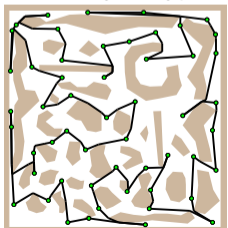
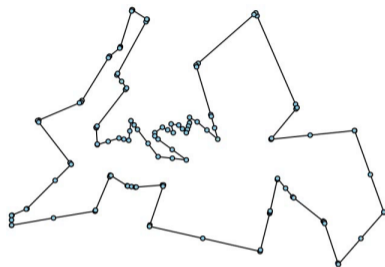
Somhom, S., Modares, A., Enkawa, T. (1999)

- Deployed in robotic problems such as inspection and search-and-rescue planning.

Faigl, J. et al. (2011)

- Generalized to polygonal domain with (overlapping) regions.
- Evolved to **Growing Self-Organizing Array (GSOA)**.

A general heuristic for various routing problems with neighborhoods; including routing problems with profit aka the orienteering problem.



# Unsupervised Learning based Solution of the TSP

Kohonen's type of **unsupervised** two-layered neural network (**Self-Organizing Map**)

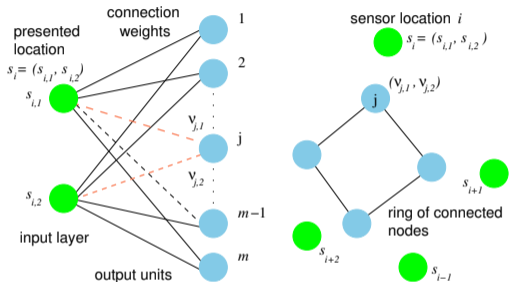
- Neurons' **weights** represent **nodes**  $\mathcal{N} = \{\nu_1, \dots, \nu_m\}$  in a **plane** (input space  $\mathbb{R}^2$ ).
- Nodes are organized into a **ring** that evolved in the output space  $\mathbb{R}^2$ .
- Target locations  $\mathbf{S} = \{s_1, \dots, s_n\}$  are presented to the network in a **random** order.
- Nodes **compete** to be winner according to their distance to the presented goal  $\mathbf{s}$

$$\nu^* = \operatorname{argmin}_{\nu \in \mathcal{N}} |\mathcal{D}(\{\nu, \mathbf{s}\})|.$$

- The **winner** and its **neighbouring** nodes are adapted (**moved**) towards the target according to the neighbouring function  $\nu' \leftarrow \mu f(\sigma, d)(\nu - \mathbf{s})$

$$f(\sigma, d) = \begin{cases} e^{-\frac{d^2}{\sigma^2}} & \text{for } d < m/n_f, \\ 0 & \text{otherwise,} \end{cases}$$

- Best matching unit  $\nu$  to the presented prototype  $\mathbf{s}$  is determined according to the distance function  $|\mathcal{D}(\nu, \mathbf{s})|$ .



- For the Euclidean TSP,  $\mathcal{D}$  is the Euclidean distance
- However, for problems with obstacles, the multi-goal path planning,  $\mathcal{D}$  should correspond to the length of the shortest, collision-free path.

Fort, J.C. (1988), Angéniol, B. et al. (1988), Somhom, S. et al. (1997), and further improvements.



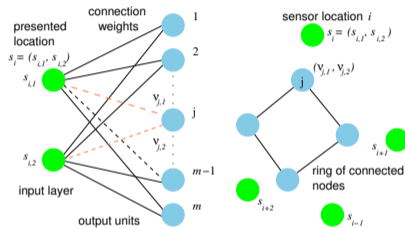


# Unsupervised Learning based Solution of the TSP - Detail

- Target (sensor) locations  $S = \{\mathbf{s}_1, \dots, \mathbf{s}_n\}$ ,  $\mathbf{s}_i \in \mathbb{R}^2$ ; Neurons  $\mathcal{N} = (\nu_1, \dots, \nu_m)$ ,  $\nu_i \in \mathbb{R}^2$ ,  $m = 2.5n$ .
  - Learning gain  $\sigma$ ; epoch counter  $i$ ; gain decreasing rate  $\alpha = 0.1$ ; learning rate  $\mu = 0.6$ .
1.  $\mathcal{N} \leftarrow$  init ring of neurons as a small ring around some  $\mathbf{s}_i \in S$ , e.g., a circle with radius 0.5.
  2.  $i \leftarrow 0$ ;  $\sigma \leftarrow 12.41n + 0.06$ ;
  3.  $I \leftarrow \emptyset$  // clear inhibited neurons
  4. **foreach**  $\mathbf{s} \in \Pi(S)$  (a permutation of  $S$ )
    - 4.1  $\nu^* \leftarrow \operatorname{argmin}_{\nu \in \mathcal{N} \setminus I} \|(\nu, \mathbf{s})\|$
    - 4.2 **foreach**  $\nu$  in  $d$  neighborhood of  $\nu^*$ 

$$\nu \leftarrow \nu + \mu f(\sigma, d)(\mathbf{s} - \nu)$$

$$f(\sigma, d) = \begin{cases} e^{-\frac{d^2}{\sigma^2}} & \text{for } d < 0.2m, \\ 0 & \text{otherwise,} \end{cases}$$
    - 4.3  $I \leftarrow I \cup \{\nu^*\}$  // inhibit the winner
  5.  $\sigma \leftarrow (1 - \alpha)\sigma$ ;  $i \leftarrow i + 1$ ;
  6. If (**termination condition** is not satisfied) Goto Step 3; Otherwise retrieve solution.



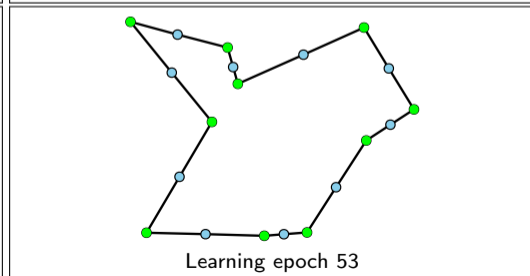
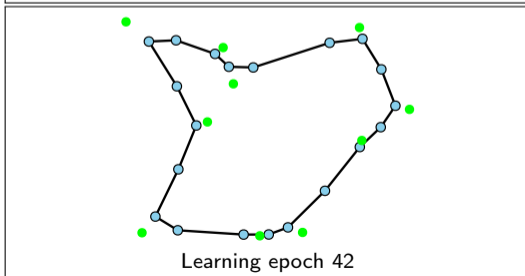
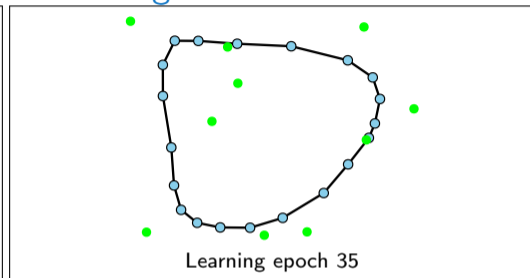
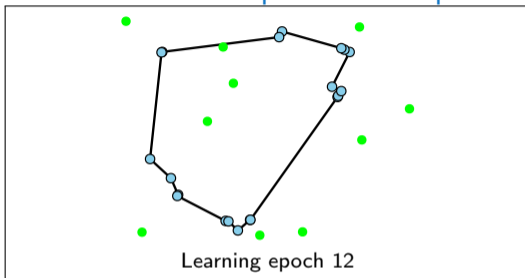
**Termination condition** can be

- Maximal number of learning epochs  $i \leq i_{max}$ , e.g.,  $i_{max} = 120$ .
- Winner neurons are negligibly close to sensor locations, e.g., less than 0.001.

Somhom, S., Modares, A., Enkawa, T. (1999): [Competition-based neural network for the multiple travelling salesmen problem with minmax objective](#). Computers & Operations Research.  
 Faigl, J. et al. (2011): [An application of the self-organizing map in the non-Euclidean Traveling Salesman Problem](#). Neurocomputing.



## Example of Unsupervised Learning for the TSP



## Unsupervised Learning for the Multi-Goal Path Planning

- Unsupervised learning procedure for the Multi-goal Path Planning (**MTP**) problem a robotic variant of the Traveling Salesman Problem (TSP).

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### Algorithm 2: SOM-based MTP solver

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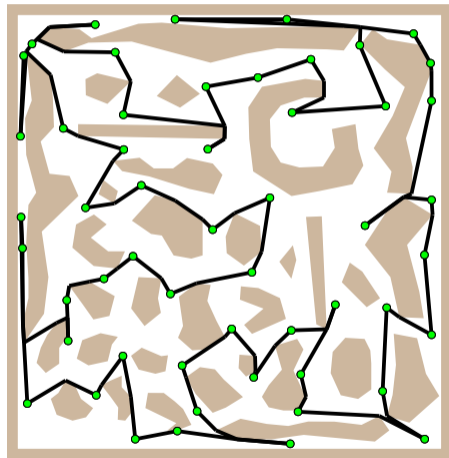
```

 $\mathcal{N} \leftarrow \text{initialization}(\nu_1, \dots, \nu_m);$ 
repeat
   $error \leftarrow 0;$ 
  foreach  $g \in \Pi(\mathcal{S})$  do
     $\nu^* \leftarrow$ 
    selectWinner  $\text{argmin}_{\nu \in \mathcal{N}} |S(g, \nu)|;$ 
    adapt  $(S(g, \nu), \mu f(\sigma, l) |S(g, \nu)|);$ 
     $error \leftarrow \max\{error, |S(g, \nu^*)|\};$ 
   $\sigma \leftarrow (1 - \alpha)\sigma;$ 
until  $error \leq \delta;$ 

```

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- For multi-goal path planning – the **selectWinner** and **adapt** procedures are based on the solution of the **path planning problem**.

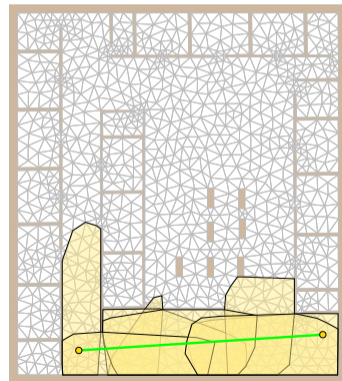
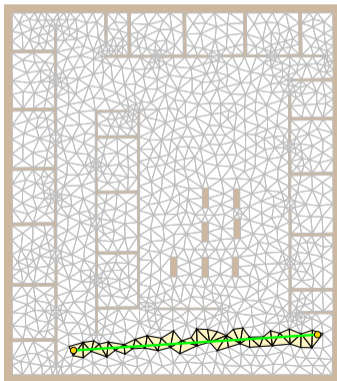
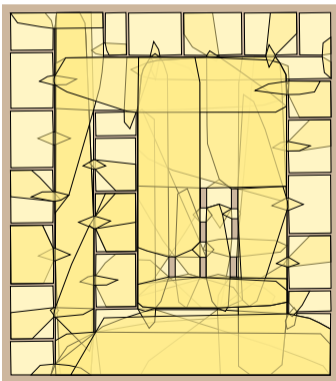


Faigl, J., Kulich, M., Vonásek, V., Přebil, L.: *An Application of Self-Organizing Map in the non-Euclidean Traveling Salesman Problem*, *Neurocomputing*, 74(5):671-679, 2011.

## SOM for the TSP in the Watchman Route Problem – *Inspection Planning*

During the unsupervised learning, we can compute *coverage* of  $\mathcal{W}$  from the current *ring* (solution represented by the neurons) and *adapt* the network *towards uncovered parts* of  $\mathcal{W}$ .

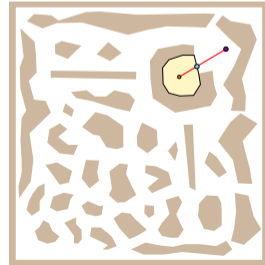
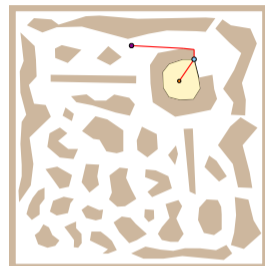
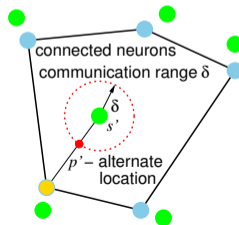
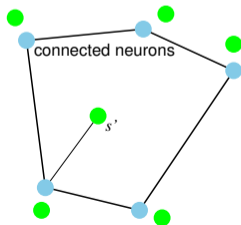
- Convex cover set of  $\mathcal{W}$  created on top of a triangular mesh.
- Incident convex polygons with a straight line segment are found by walking in a triangular mesh.



Faigl, J.: *Approximate solution of the multiple watchman routes problem with restricted visibility range*, IEEE Transactions on Neural Networks, 21(10):1668-1679, 2010.

## Unsupervised Learning for the TSPN

- A suitable location of the region can be sampled during the winner selection.
- We can use the centroid of the region for the shortest path computation from  $\nu$  to the region  $r$  presented to the network.
- Then, an intersection point of the path with the region can be used as an alternate location.
  - Faigl, J. et al. (2013): [Visiting convex regions in a polygonal map](#). Robotics and Autonomous Systems.
- For the Euclidean TSPN with disk-shaped  $\delta$  neighborhoods, we can compute the alternate location directly from the Euclidean distance.

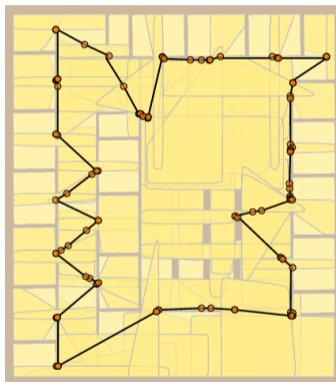


# SOM for the Traveling Salesman Problem with Neighborhoods (TSPN)

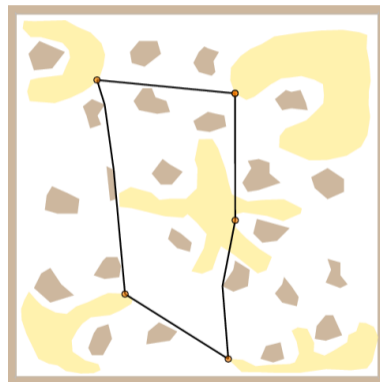
- Unsupervised learning of the SOM for the TSP allows to generalize the adaptation procedure to the TSPN.
- It also provides solutions for non-convex regions, overlapping regions, and coverage problems.



Polygonal Goals  
 $n=9$ ,  $T=0.32$  s



Convex Cover Set  
 $n=106$ ,  $T=5.1$  s



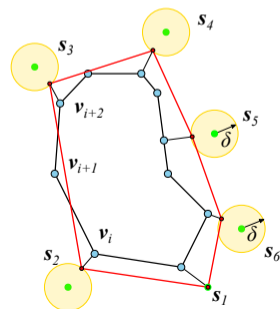
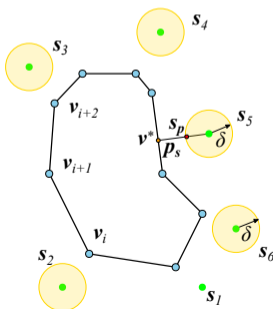
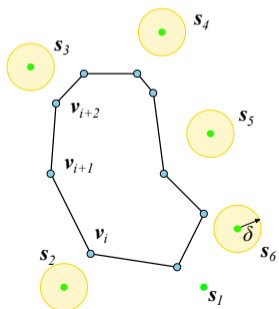
Non-Convex Goals  
 $n=5$ ,  $T=0.1$  s

Faigl, J., Vonásek, V., Přeučil, L.: *Visiting Convex Regions in a Polygonal Map*, Robotics and Autonomous Systems, 61(10):1070–1083, 2013.



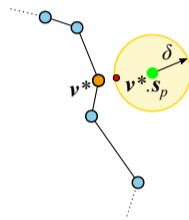
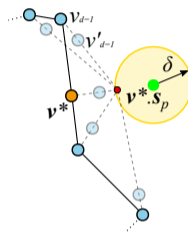
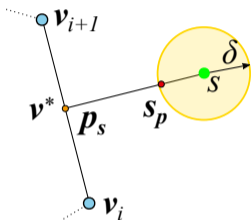
## Growing Self-Organizing Array (GSOA)

- **Growing Self-Organizing Array (GSOA)** is generalization of the unsupervised learning to routing problems motivated by data collection planning, i.e., routing with neighborhoods such as the **Close Enough TSP**.
- The GSOA is an array of nodes  $\mathcal{N} = \{\nu_1, \dots, \nu_M\}$  that evolves in the problem space using unsupervised learning.
- The array adapts to each  $s \in S$  (in a random order) and for each  $s$  a **new winner node**  $\nu^*$  is determined together with the corresponding  $s_p$ , such that  $\|(s_p, s)\| \leq \delta(s)$ . It **adaptively adjusts** the number of nodes.
- The winner and its neighborhoods are adapted (moved) towards  $s_p$ .
- After the adaptation to all  $s \in S$ , each  $s$  has its  $\nu$  and  $s_p$ , and the array defines the sequence  $\Sigma$  and the requested waypoints  $P$ .



## GSOA – Winner Selection and Its Adaptation

- Selecting winner node  $\nu^*$  for  $s$  and its waypoint  $s_p$
- Winner adaptation



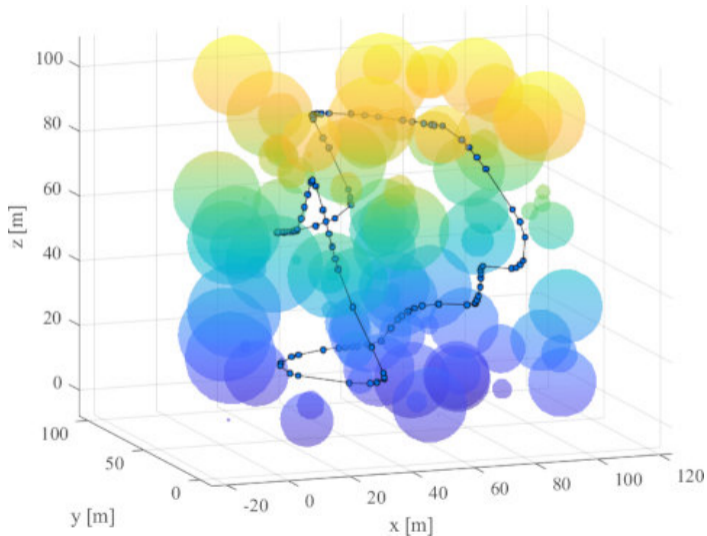
- For each  $s \in S$ , we create new node  $\nu^*$ , and therefore, all not winning nodes are removed after processing all locations in  $S$  (one learning epoch) to balance the number of nodes in the GSOA.
- After **each learning epoch**, the **GSOA encodes a feasible solution of the CETSP**.
- The power of adaptation is decreasing using a cooling schedule after each learning epoch.
- The GSOA converges to a stable solution in tens of epochs. Number of epochs can be set.

Faigl, J. (2018): **GSOA: Growing Self-Organizing Array - Unsupervised learning for the Close-Enough Traveling Salesman Problem and other routing problems**. Neurocomputing 312: 120-134 (2018).





# GSOA Evolution in solving the 3D CETSP



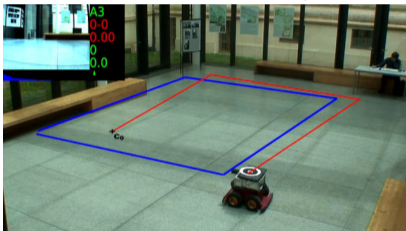
# Outline

- Unsupervised Learning for Multi-goal Planning
- TSPN in Multi-goal Planning with Localization Uncertainty



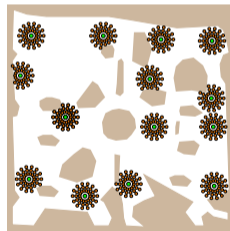
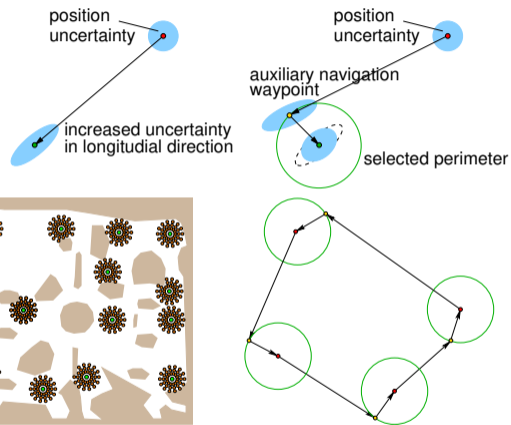
## Example – TSPN for Planning with Localization Uncertainty

- Teach-and-repeat autonomous navigation using vision-based bearing corrections that are more precise than estimation of the traveled distance based on odometry measurements.



Krajník, T., Faigl, J., Vonásek, V., Košnar, K., Kulich, M., and Přeučil, L.: *Simple yet stable bearing-only navigation*, *Journal of Field Robotics*, 27(5):511-533, 2010.

- The localization uncertainty can be decreased by visiting auxiliary navigation waypoints prior the target locations.
- It can be formulated as a variant of the TSPN with auxiliary navigation waypoints.

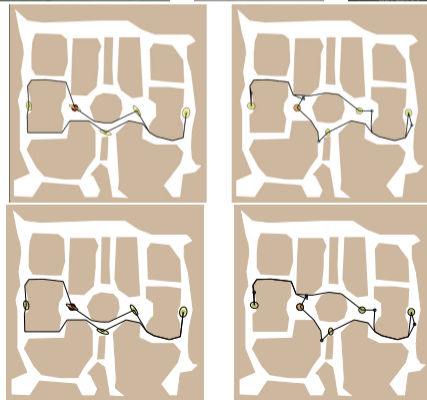
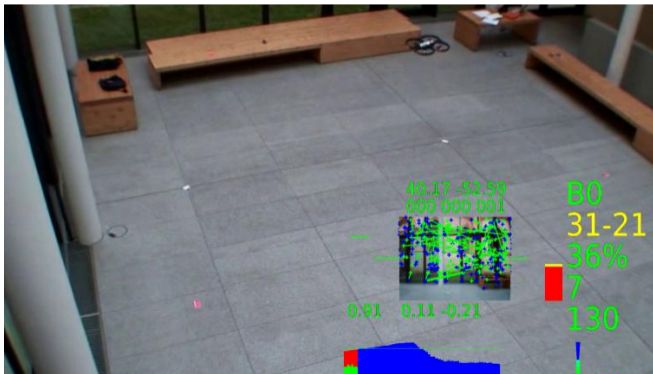


- The adaptation procedure is modified to select the auxiliary navigation waypoint to decrease the expected localization error at the target locations.

Faigl, J., Krajník, T., Vonásek, V., and Přeučil, L.: *On localization uncertainty in an autonomous inspection*, *IEEE International Conference on Robotics and Automation (ICRA)*, 2012, pp. 1119-1124.

# Example – Results on the TSPN for Planning with Localization Uncertainty

- Deployment of the method in indoor and outdoor environment with ground mobile robots and aerial vehicle in indoor environment.
- In the indoor with the small MMP5 robot, the error decreased from 16.6 cm  $\rightarrow$  12.8 cm.
- In the outdoor with the P3AT robot, the real overall error at the goals decreased from 0.89 m  $\rightarrow$  0.58 m (about 35%).
- Deployment with a small aerial vehicle the Parrot AR.Drone, the success of the locations' visits improved from 83% to 95%.



TSP:  $L=184$  m,  $E_{avg}=0.57$  m TSPN:  $L=202$  m,  $E_{avg}=0.35$  m

# Summary of the Lecture



# Topics Discussed

- Robotic information gathering in inspection missions
- Inspection planning and multi-goal path planning - coverage planning
- **Multi-goal path planning (MTP)**
  - Robotic Traveling Salesman Problem (TSP)
  - Traveling Salesman Problem with Neighborhoods (TSPN) and Close Enough Traveling Salesman Problem (CETSP)
    - **Decoupled** and **Sampling-based** approaches
    - TSP can be solved by efficient heuristics such as **LKH**
    - Optimal, approximation, and heuristics solutions
    - **Generalized TSP (GTSP)**
- **Next: Data collection planning**

