

## Logical reasoning and programming, lab session 3

(October 4, 2021)

**3.1** Check the details of two watched literals, for example, these and these slides illustrate the data structure.

**3.2** Decide the satisfiability of

$$\{\{\bar{p}, r, s, t\}, \{\bar{r}, s, t\}, \{\bar{p}, r, \bar{s}\}, \{p, q\}, \{p, \bar{q}\}, \{\bar{p}, \bar{t}\}, \{\bar{r}, \bar{s}, t\}\}$$

by DPLL. Use the order of branching:  $p, q, r, \dots$

**3.3** Decide the satisfiability of

$$\begin{aligned} &\{\{p_1, p_{13}\}, \{\bar{p}_1, \bar{p}_2, p_{14}\}, \{p_3, p_{15}\}, \{p_4, p_{16}\}, \\ &\{\bar{p}_5, \bar{p}_3, p_6\}, \{\bar{p}_5, \bar{p}_7\}, \{\bar{p}_6, p_7, p_8\}, \{\bar{p}_4, \bar{p}_8, \bar{p}_9\}, \{\bar{p}_1, p_9, \bar{p}_{10}\}, \\ &\{p_9, p_{11}, \bar{p}_{14}\}, \{p_{10}, \bar{p}_{11}, p_{12}\}, \{\bar{p}_2, \bar{p}_{11}, \bar{p}_{12}\}\} \end{aligned}$$

by CDCL using the first UIP. Use the order of branching:  $p_1, p_2, p_3, \dots$

**3.4** Given the formula  $\{\{p_1, p_2\}, \{\bar{p}_1, p_3\}, \{p_2, \bar{p}_3\}, \{\bar{p}_2, \bar{p}_4\}, \{\bar{p}_3, p_4\}\}$ , what clause will a CDCL solver learn first if it begins by deciding that  $p_1$  is true?

**3.5** Assume that our version of CDCL fails to produce a satisfiable valuation for  $\varphi$  and hence  $\varphi$  is unsatisfiable. How would you produce a resolution proof of this fact from the run of CDCL?

**3.6** How many symmetries does your formulation of  $\text{PHP}_n^{n+1}$  have?

**3.7** We can define the lexicographic order on two bit vectors  $x_1, \dots, x_n$  and  $y_1, \dots, y_n$ , denoted  $x_1 \dots x_n \leq_{lex} y_1 \dots y_n$ , as follows

$$\bigwedge_{i=1}^n ((\bar{x}_i \vee y_i \vee \bar{a}_{i-1}) \wedge (\bar{x}_i \vee a_i \vee \bar{a}_{i-1}) \wedge (y_i \vee a_i \vee \bar{a}_{i-1})),$$

where  $\bar{a}_0$  is always false, using new auxiliary variables  $a_0, a_1, \dots, a_{n-1}, a_n$ .

(a) What is the purpose of auxiliary variables?

*Hint:* When is it necessary to satisfy  $x_i \leq y_i$ ?

(b) Why is  $\bar{a}_0$  always false and hence useless?

(c) Why can we replace  $(\bar{x}_n \vee y_n \vee \bar{a}_{n-1}) \wedge (\bar{x}_n \vee a_n \vee \bar{a}_{n-1}) \wedge (y_n \vee a_n \vee \bar{a}_{n-1})$  just by  $(\bar{x}_n \vee y_n \vee \bar{a}_{n-1})$ ? Hence we need only  $3n - 2$  clauses and  $n - 1$  auxiliary variables ( $a_n$  is also useless).

(d) How does the meaning of the formula change if you replace  $(\bar{x}_n \vee y_n \vee \bar{a}_{n-1})$  by  $(\bar{x}_n \vee \bar{a}_{n-1}) \wedge (y_n \vee \bar{a}_{n-1})$ ?

**3.8** How can we exploit the lexicographic order to decrease the number of symmetries in  $\text{PHP}_n^{n+1}$ ?

*Hint:* Order hole-occupancy or pigeon-occupancy vectors.

- 3.9** A very nice symmetry breaker for  $\text{PHP}_n^{n+1}$  is based on columnwise symmetry, namely we can add the following clauses

$$p_{i(i+1)} \vee \overline{p_{ij}}$$

for  $1 \leq i < j \leq n$ , where  $p_{kl}$  means that pigeon  $k$  is in hole  $l$ , for  $1 \leq k \leq n+1$  and  $1 \leq l \leq n$ . Why?

- 3.10** Try PicoSAT/pycosat and PySAT on  $\text{PHP}_n^{n+1}$  with various symmetry breakers.
- 3.11** Symmetry breaking and  $\text{PHP}_n^{n+1}$  (cont'd). For details see Knuth's TAOCP on satisfiability or slides Symmetry in SAT: an overview.
- 3.12** Try BreakID.