

## Logical reasoning and programming, lab session 1

(September 20, 2021)

**I.1** Decide which of the following formulae are tautologies:

- (a)  $((p \rightarrow q) \rightarrow q) \rightarrow q$ ,
- (b)  $((p \rightarrow q) \rightarrow p) \rightarrow p$ ,
- (c)  $(p \rightarrow q) \vee (q \rightarrow p)$ ,
- (d)  $((p \rightarrow q) \wedge q) \rightarrow p$ ,
- (e)  $\neg p \rightarrow \neg(p \vee (p \wedge q))$ .

Although formulae contain only two variables, try to find a better method than truth tables if possible.

**I.2** Decide which of the following claims are true:

- (a)  $\neg(\varphi \leftrightarrow \psi) \models \varphi \wedge \psi$ ,
- (b)  $\neg(\varphi \leftrightarrow \psi) \models \varphi \vee \psi$ ,
- (c)  $\neg(\varphi \leftrightarrow \psi) \models \varphi \rightarrow \psi$ .

**I.3** Are the following two C programs equivalent?

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if (!a && !b) h();           if (a) f();
else if (!a) g();           else if (b) g();
else f();                   else h();
```

Prove their equivalence formally, or provide a counterexample. Please, check your solution against these slides.

**I.4** If  $\varphi \rightarrow \psi \in \text{TAUT}$  and  $\psi \rightarrow \chi \in \text{TAUT}$ , then  $\varphi \rightarrow \chi \in \text{TAUT}$ . Why? Does the claim still hold if we replace TAUT with SAT and why?

**I.5** Let  $\text{Cl}(\Gamma) = \{\varphi : \Gamma \models \varphi\}$ . What is  $\text{Cl}(\emptyset)$  equivalent to? Let  $\Gamma$  and  $\Delta$  be sets of formulae. Check whether:

- (a)  $\Gamma \subseteq \text{Cl}(\Gamma)$ ,
- (b)  $\text{Cl}(\text{Cl}(\Gamma)) = \text{Cl}(\Gamma)$ ,
- (c)  $\text{Cl}(\Gamma \cup \Delta) = \text{Cl}(\Gamma) \cup \text{Cl}(\Delta)$ .

If the equality does not hold in (b) or (c), does at least one of the inclusions hold?

**I.6** Recall that a Boolean function of  $n$ -variables is a function  $f: \{0, 1\}^n \rightarrow \{0, 1\}$ . Describe all functions of one variable. How many distinct Boolean functions of  $n$  variables exist? Try  $n = 2$  first. Do you know, why are functions NAND ( $\uparrow$ ) and NOR ( $\downarrow$ ) interesting?<sup>1</sup>

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<sup>1</sup>These connectives are also called Sheffer stroke and Peirce arrow, respectively. They are defined as  $x \uparrow y := \neg(x \wedge y)$  and  $x \downarrow y := \neg(x \vee y)$ .

**I.7** If we use standard rewriting rules for producing a CNF, then we usually conclude by some simplifications—remove duplicate clauses and literals. Why can we do that? Is it correct that there is no need for a variable to occur more than once in a clause?

**I.8** Produce a formula in CNF which is equivalent to

$$\varphi = (a \rightarrow (c \wedge d)) \vee (b \rightarrow (c \wedge e)).$$

Then use the Tseytin transformation to produce a formula in CNF which is equisatisfiable to  $\varphi$ .

**I.9** Try solving SAT problems by hand in The SAT Game.